Evaluation of Target-Date Mutual Funds
Manulife problem

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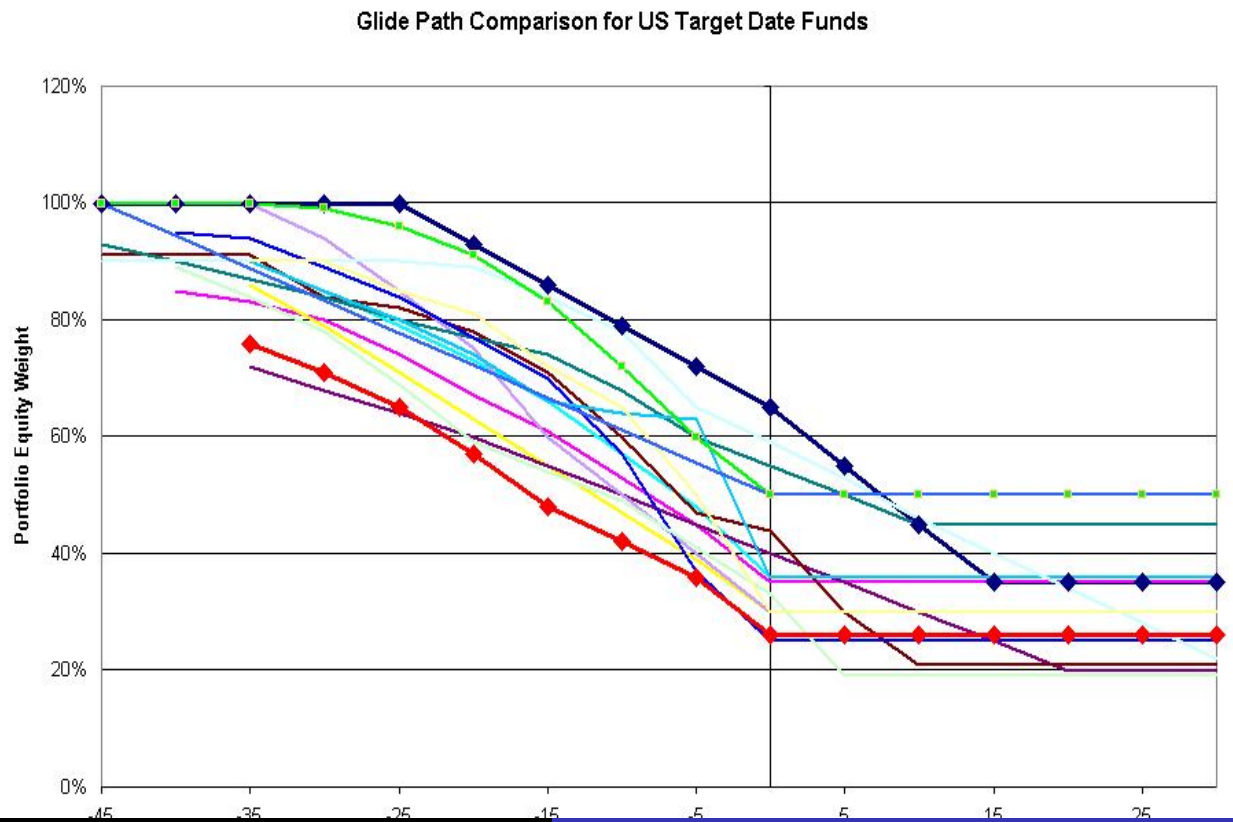
- The equity weight is continuously adjusted in time ($80\% - 90\%$ when you have 40 years before retirement to $40\% - 50\%$ at retirement).

Motivation

- There is no consensus in the industry on how to roll down the equity weight until retirement (e.i. how to select the optimal glide path)



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- How to rank the ability of FMs to pick better stocks?
- Are these FMs who actively manage the funds actively manage the funds actually adding value to the overall return? (i.e. adding value by tweaking the glide path around the target value)
- What is the optimal design of the glide path?

Questions

- The first two questions correspond more to the paradigm of portfolio management and there are many tools out there for answering these questions
- The last one can be more relevant in defining fund's characteristics and for promotion. It is related to the following situations:
  - How a given glide path matches the needs of a particular firm's employees
    - Use unique labor income distribution among employees
    - Use particular employees risk preferences
  - By this approach managers can run comparisons and present them to pension directors for promotion of their instruments.

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- We want to find a useful model for comparison of strategies among different target-date funds and be able to come up with a ranking system

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- Formulate the problem as a stochastic control problem: to find the "optimal strategy" that maximizes the expected utility of terminal wealth at retirement
- We considered two different utility functions (CRRA, HARA) to capture different preferences

The market and labor

Reference: Bodie and Treussard (2007)

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- For S_t and e_t, we consider the same source of randomness (perfectly correlated assets)

Wealth

- If ω_t is the fraction of wealth invested in the risky asset S_t, and labor earnings flows into the retirement account at a rate $e_t dt$ then, the wealth process evolves according to

$$dX_t = e_t dt + X_t r dt + X_t \omega_t (\mu - r) dt + X_t \omega_t \sigma dZ_t$$

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- We represent consumer's preferences by a constant relative risk aversion utility function (CRRA) of terminal wealth

$$U(X_T) = \frac{X_T^{1-\gamma}}{1-\gamma},$$

where γ is the parameter of relative risk aversion

Related stochastic control problem

- We want to find the optimal strategy ω_t^* that maximizes utility from terminal wealth

$$v(t,x,e) = \max_{\omega_t} \mathbb{E}\left[\frac{X_T^{1-\gamma}}{1-\gamma} | X_0 = x, e_o = e\right]$$

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- Exploiting the similarity solutions properties to reduce the order of the problem

$$Y_t = \frac{X_t}{e_t}$$

and

$$v(t,x,e) = e^{1-\gamma} u(t,y)$$

Solution to the first problem

- In this case, one gets an analytic solution

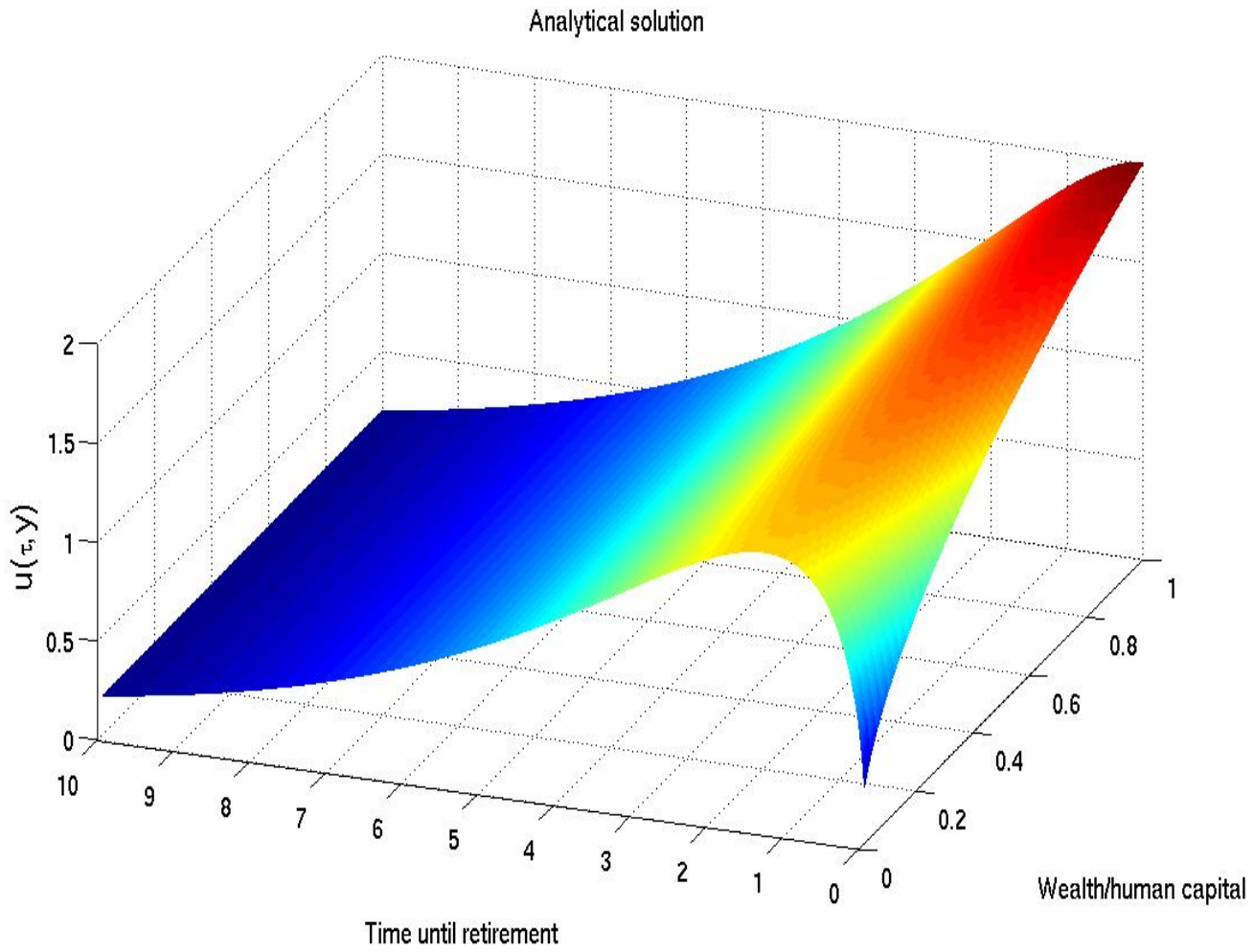
$$u(t,y) = \frac{(y+\alpha(t))^{1-\gamma}}{1-\gamma}\beta(t),$$

with

$$\begin{aligned}\alpha(t) &= \frac{1}{a_1}\left(1-\exp\{-a_1(T-t)\}\right),\\ \beta(t) &= \exp\left\{\left(a_o + a_1(1-\gamma) + a_2\left(\frac{1-\gamma}{\gamma}\right)\right)(T-t)\right\}\end{aligned}$$

where a_0, a_1 and a_2 are constants depending on the parameters.

Analytic solution



Optimal strategy

- Finally, the optimal strategy obtained is

$$\omega^*(t,y) = \frac{(\mu - r - \gamma\sigma\sigma_e)}{\gamma\sigma^2}\left(1 + \frac{1}{a_1 y}\left[1 - \exp\{-a_1(T-t)\}\right]\right) + \frac{\sigma_e}{\sigma}.$$

Results

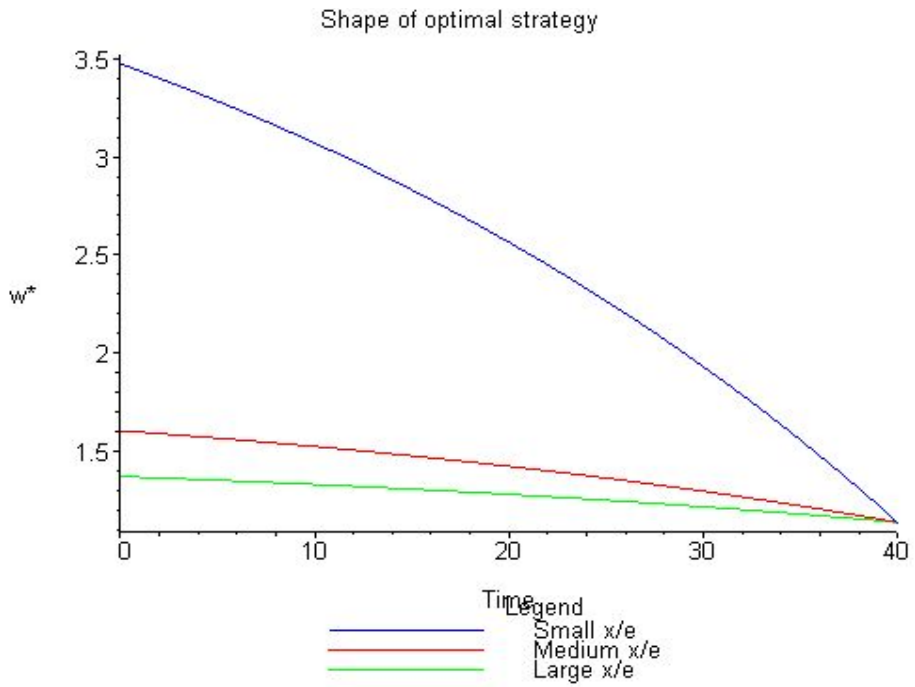
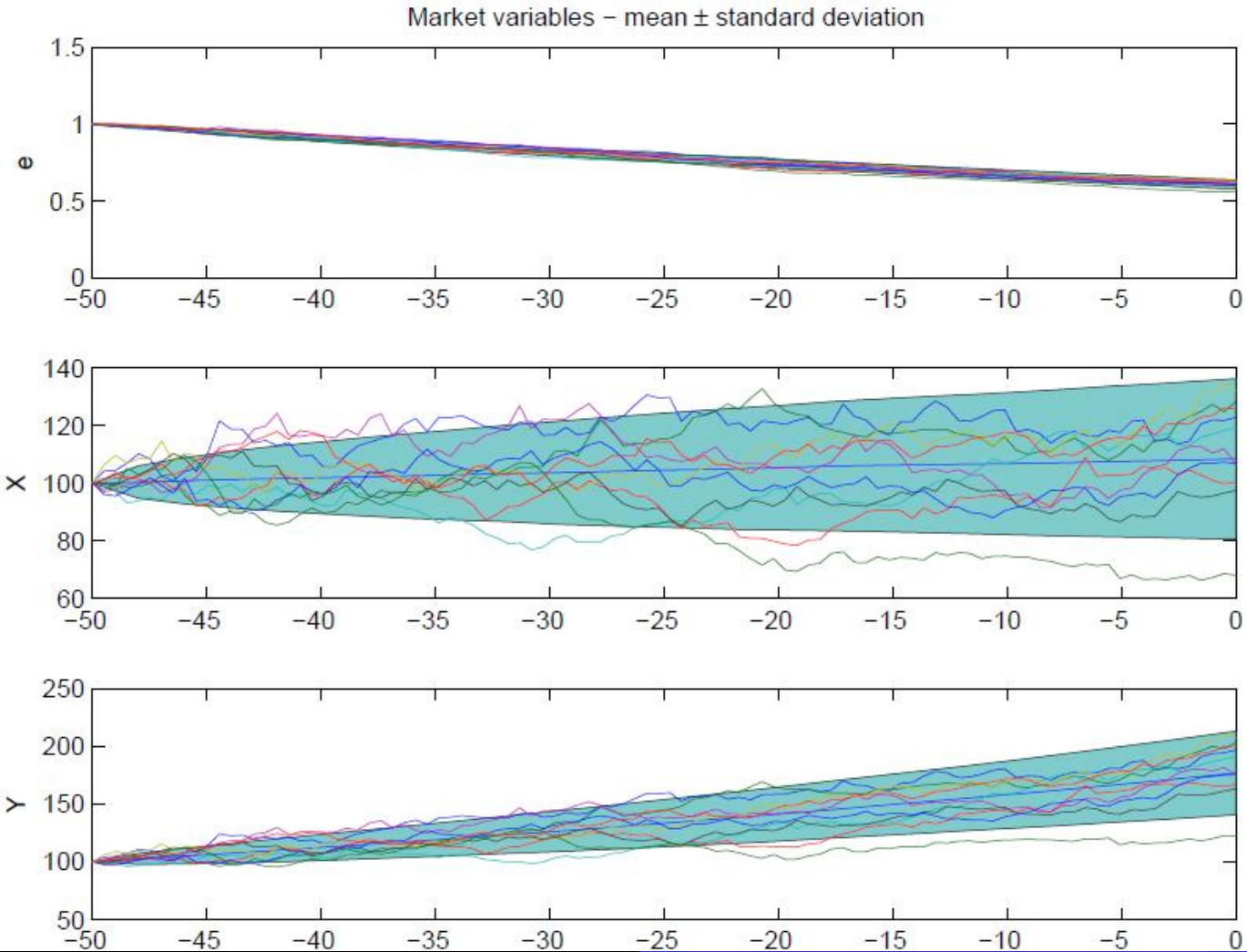


Figure: Optimal strategy as a function of time and for different ratios of initial wealth/initial labor income

$r = 0.02$, $\gamma = 1.1$, $\mu = 0.07$, $\mu_e = 0.01$, $\sigma = 0.2$, $\sigma_e = 0.05$.

Monte Carlo simulations



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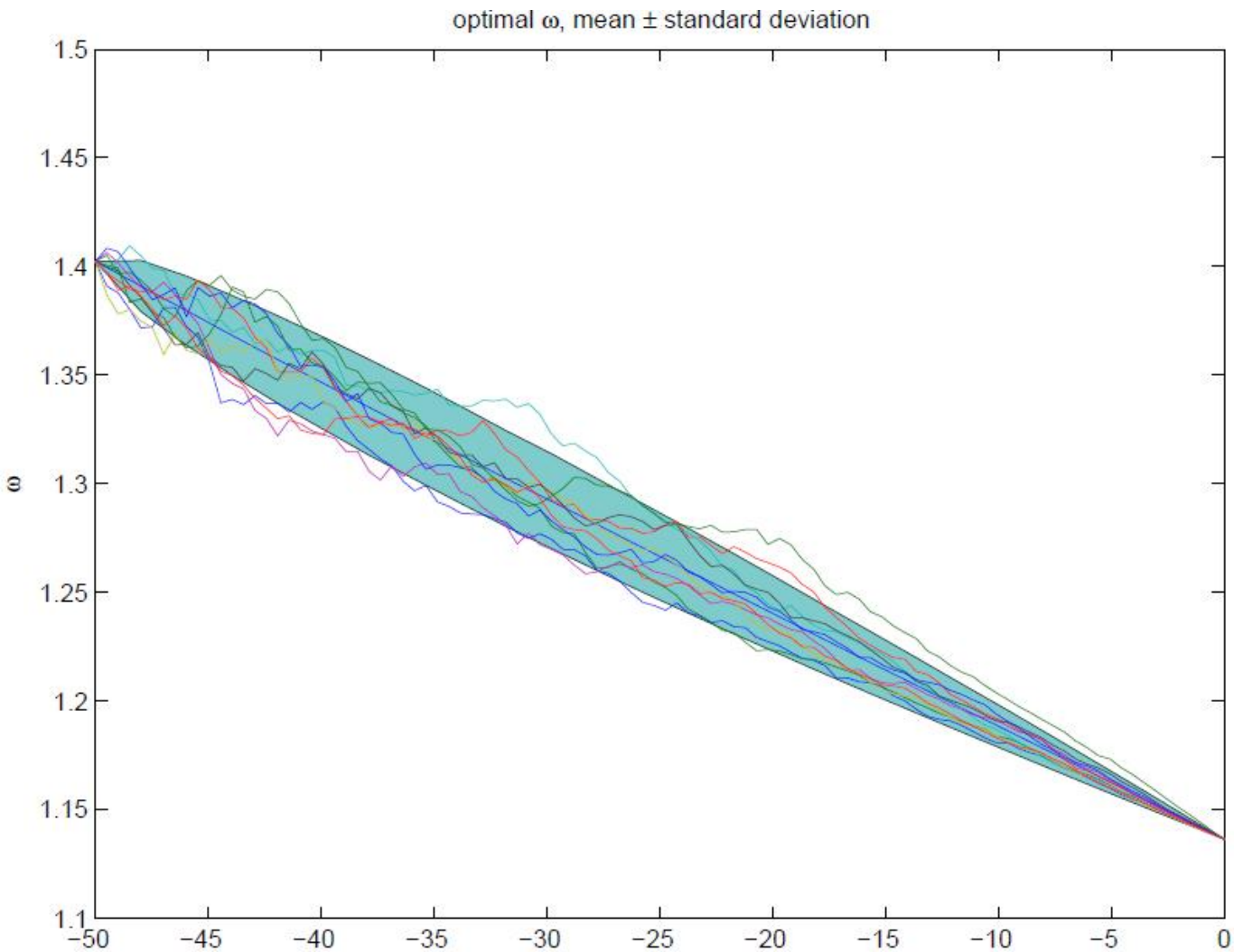

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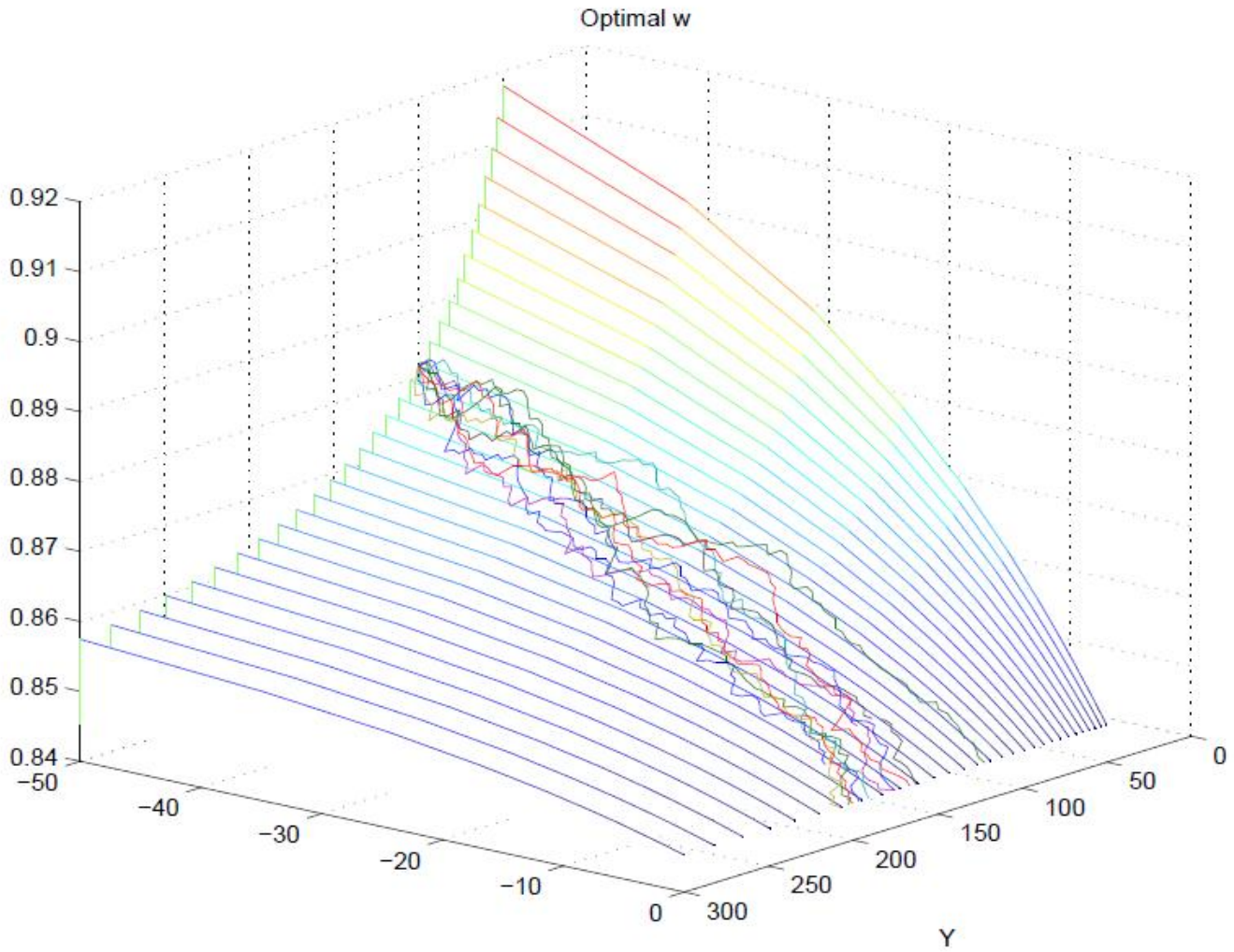
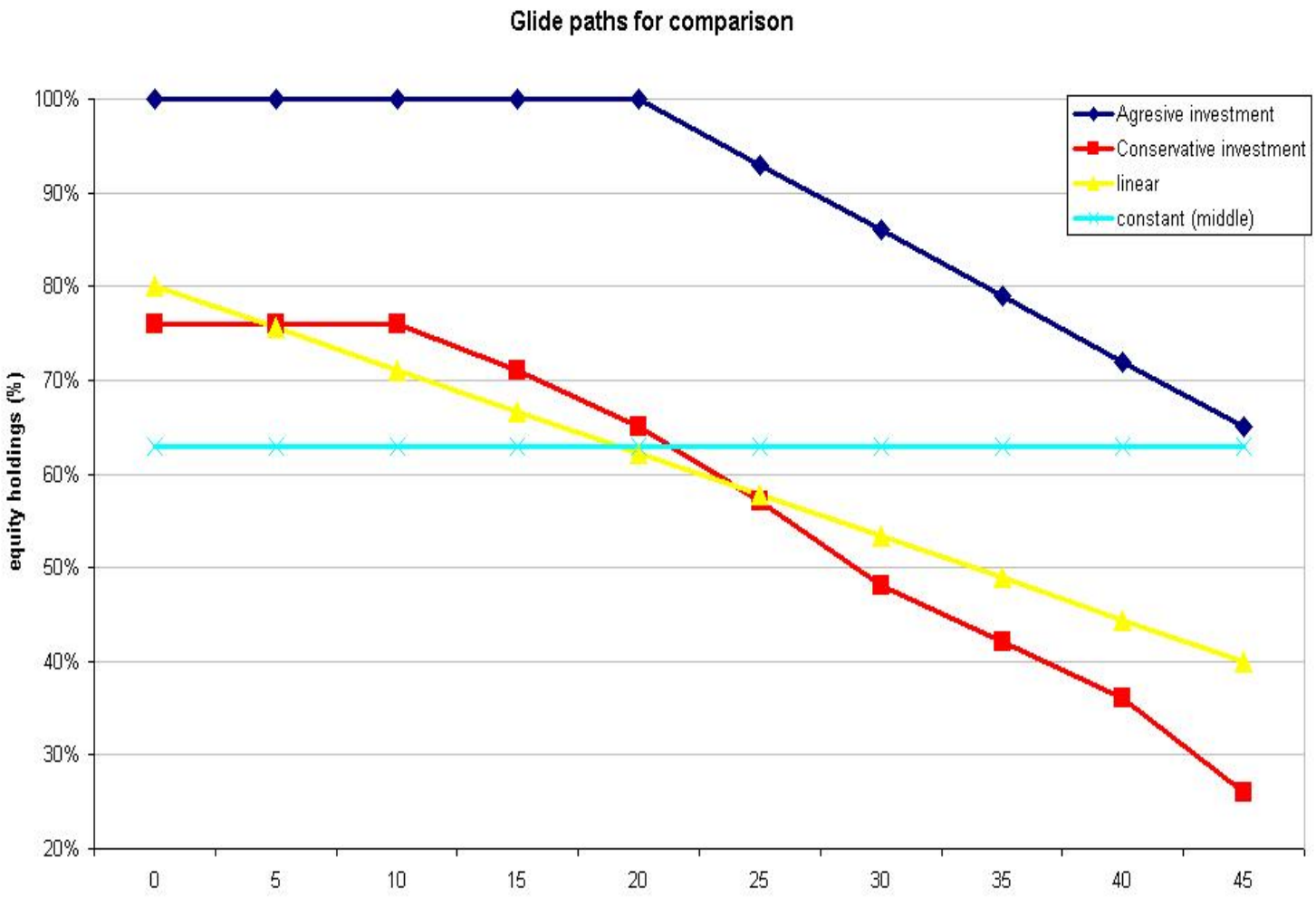
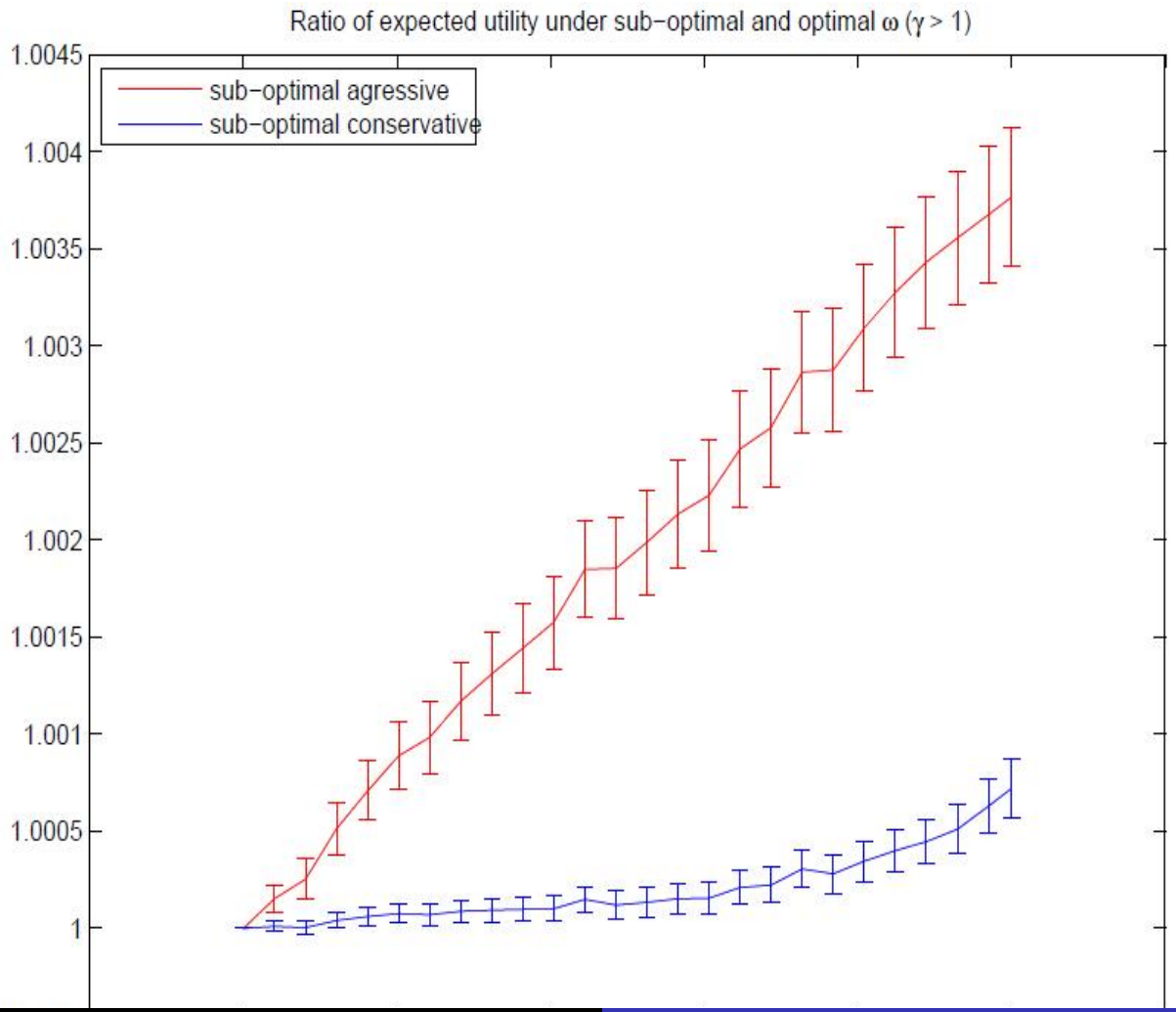


Manulife glide path examples



Comparison of glide paths vs the optimal strategy



The utility function

We also used a more realistic utility function (HARA). It sets a target $\hat{x}$ in the final wealth

- We find the optimal strategy ω_t^* that maximizes utility from terminal wealth, this is,

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- The MC simulation and the numerical implementation can be used for more general cases

Future work

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- Include other instruments for investment (derivatives, etc.)

The End

(a) The tension fund

(b) The pension fund fairy