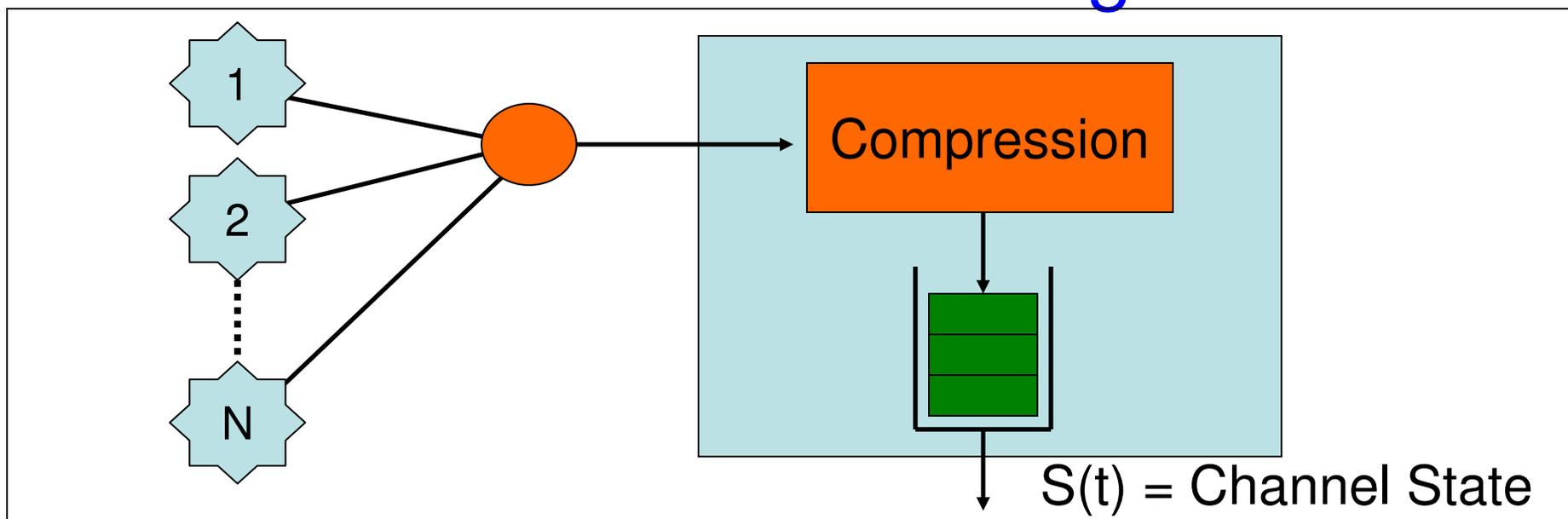




# Dynamic Data Compression for Wireless Transmission over a Fading Channel



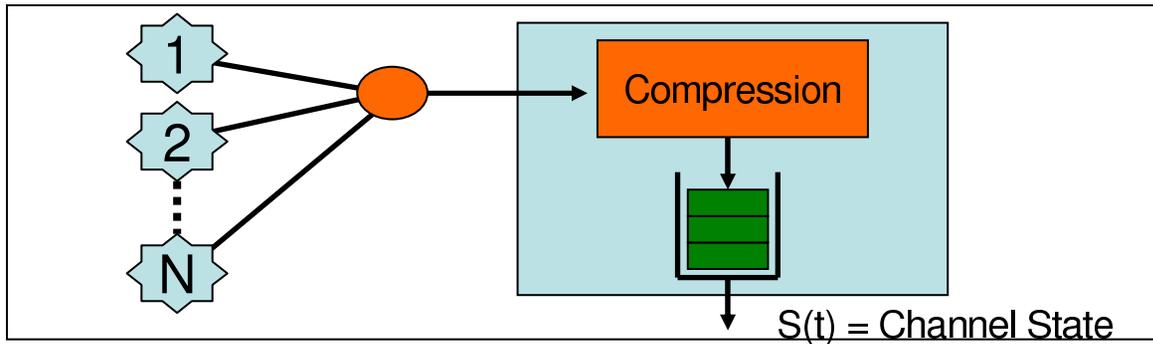
Toronto 2008

Michael J. Neely

University of Southern California

Conference paper in: CISS 2008 (<http://www-rcf.usc.edu/~mjneely>)

\*Sponsored in part by the DARPA IT-MANET Program, NSF OCE-0520324, NSF Career CCF-0747525



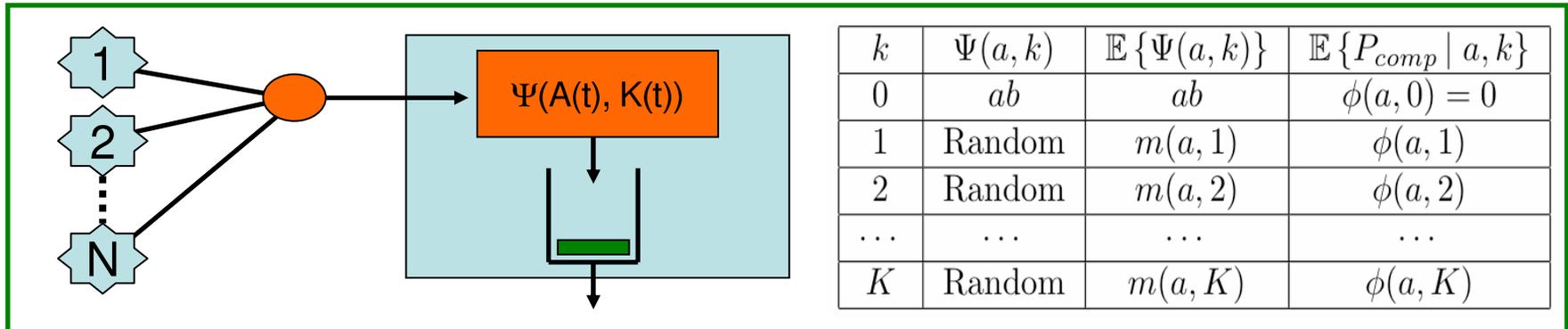
- Random Packet Arrivals  $A(t)$
- Stochastic Channel  $S(t)$
- Data must be Compressed, Stored, and Transmitted

Both **Compression** and **Transmission** expend power!  
 ↳ (signal processing consumes power)

Goal: Minimize Total Average Power Expenditure

$$\bar{P}_{\text{tot}} = \bar{P}_{\text{comp}} + \bar{P}_{\text{tran}}$$

## Compression Operation:



Timeslotted System:  $t = \{0, 1, 2, 3, \dots\}$

$A(t)$  = # packet arrivals at time  $t$

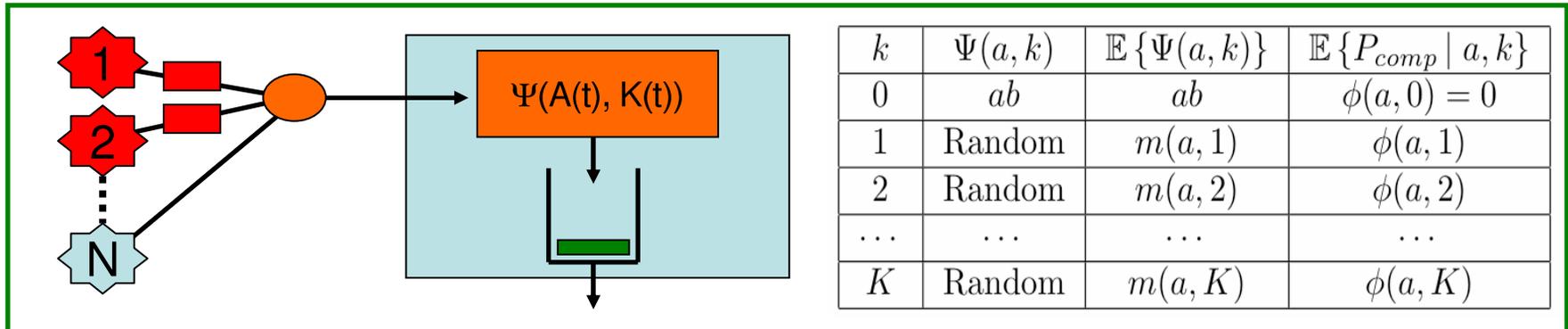
(fixed packets size  $B$  bits,  $A(t) \in \{0, 1, \dots, N\}$  )

$K(t)$  = Compression Decision Option at time  $t$

( $K(t) \in \{0, 1, \dots, K\}$ )

$\Psi(A(t), K(t))$  = Compression Function  
 = Random bit size output after compression

## Compression Operation:



Timeslotted System:  $t = \{0, 1, 2, 3, \dots\}$

$A(t)$  = # packet arrivals at time  $t$

(fixed packets size  $B$  bits,  $A(t) \in \{0, 1, \dots, N\}$ )

$K(t)$  = Compression Decision Option at time  $t$

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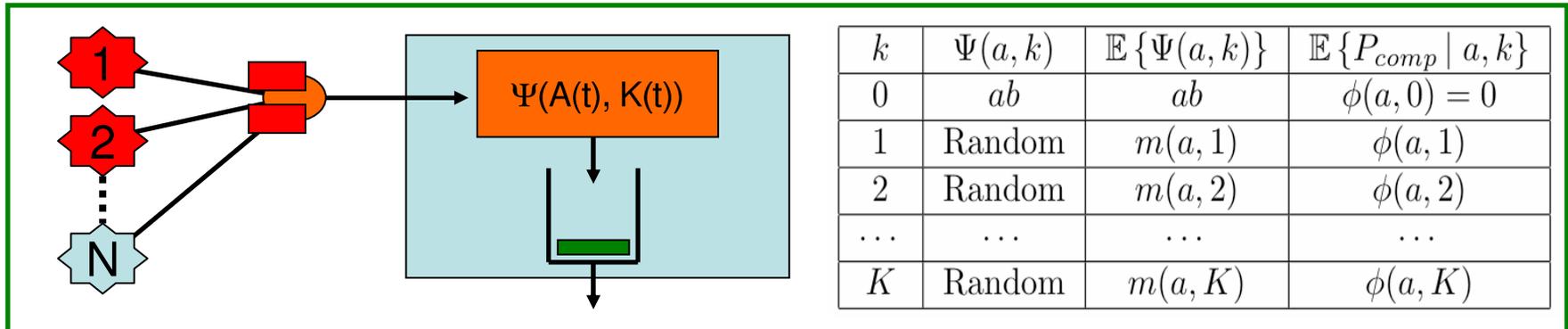
$\Psi(A(t), K(t))$  = Compression Function

Example:  $A(t) = 2, K(t) = k$

$\Psi(A(t), K(t)) = ??$  (random output)

$P_{comp}(t) = ??$  (random output)

# Compression Operation:



Timeslotted System:  $t = \{0, 1, 2, 3, \dots\}$

$A(t)$  = # packet arrivals at time  $t$

(fixed packets size  $B$  bits,  $A(t) \in \{0, 1, \dots, N\}$ )

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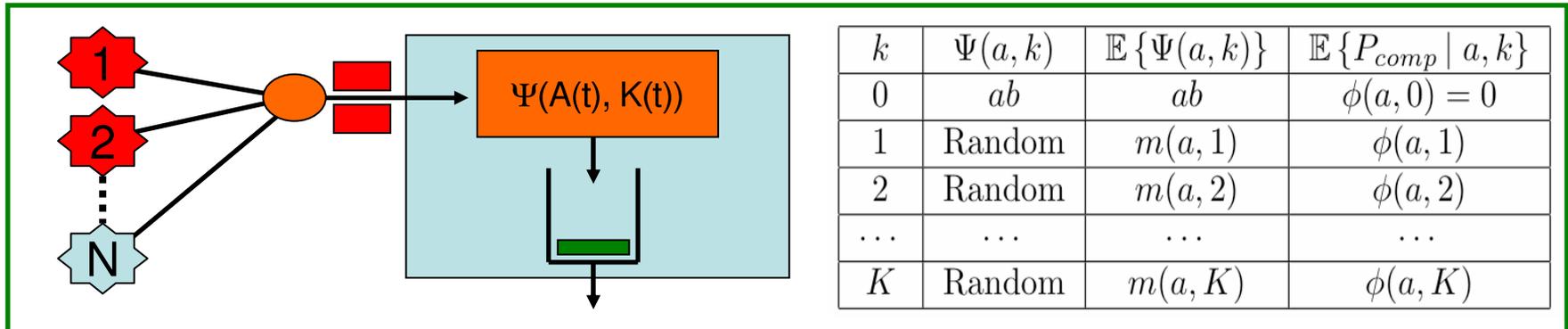
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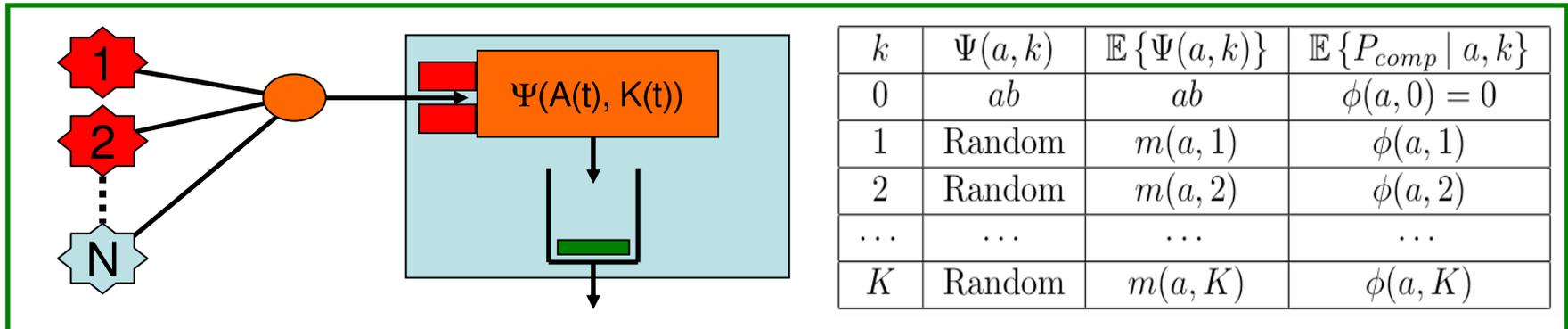
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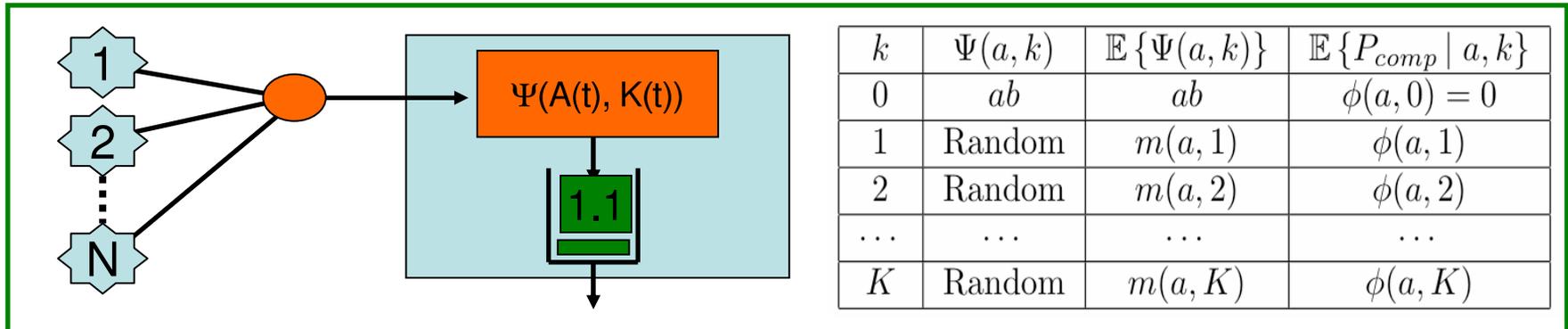
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## Compression Operation:



Timeslotted System:  $t = \{0, 1, 2, 3, \dots\}$

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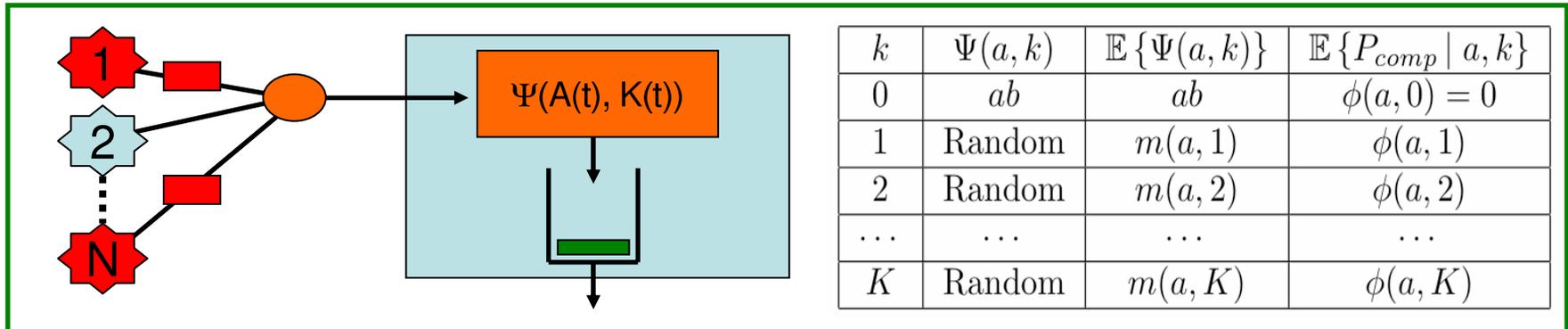
$\Psi(A(t), K(t)) =$  Compression Function

Example:  $A(t) = 2, K(t) = k$

$\Psi(A(t), K(t)) = (1.1)B$  bits (random output)

$P_{comp}(t) = .2$  mW (random output)

# Compression Operation:



Timeslotted System:  $t = \{0, 1, 2, 3, \dots\}$

$A(t)$  = # packet arrivals at time  $t$

(fixed packets size  $B$  bits,  $A(t) \in \{0, 1, \dots, N\}$  )

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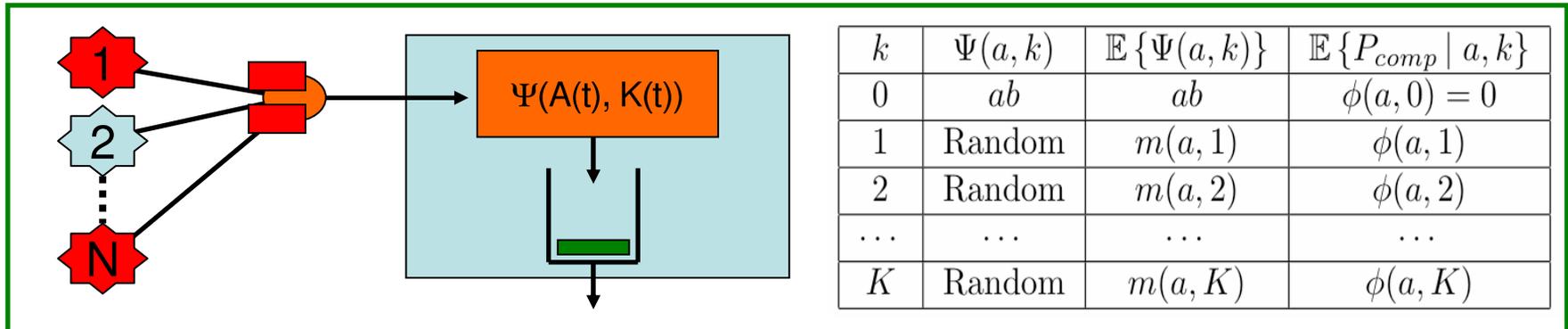
$\Psi(A(t), K(t))$  = Compression Function

Example 2:  $A(t) = 2, K(t) = k$

$\Psi(A(t), K(t)) = ??$  (random output)

$P_{comp}(t) = ??$  (random output)

# Compression Operation:



Timeslotted System:  $t = \{0, 1, 2, 3, \dots\}$

$A(t)$  = # packet arrivals at time  $t$

(fixed packets size  $B$  bits,  $A(t) \in \{0, 1, \dots, N\}$ )

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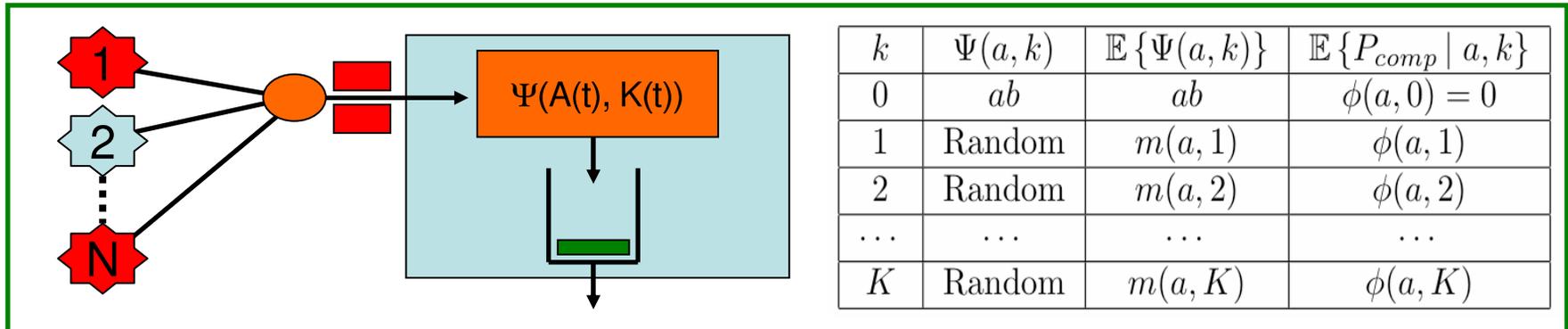
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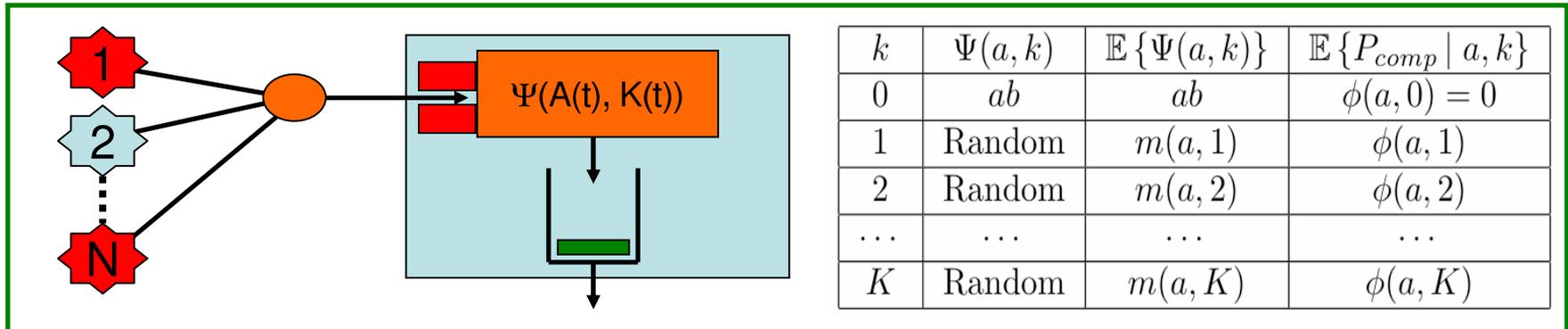
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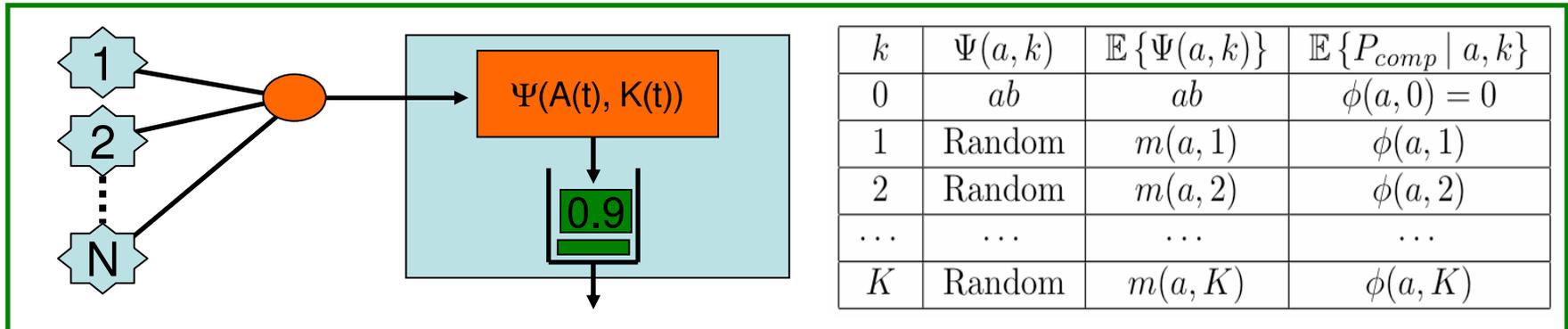
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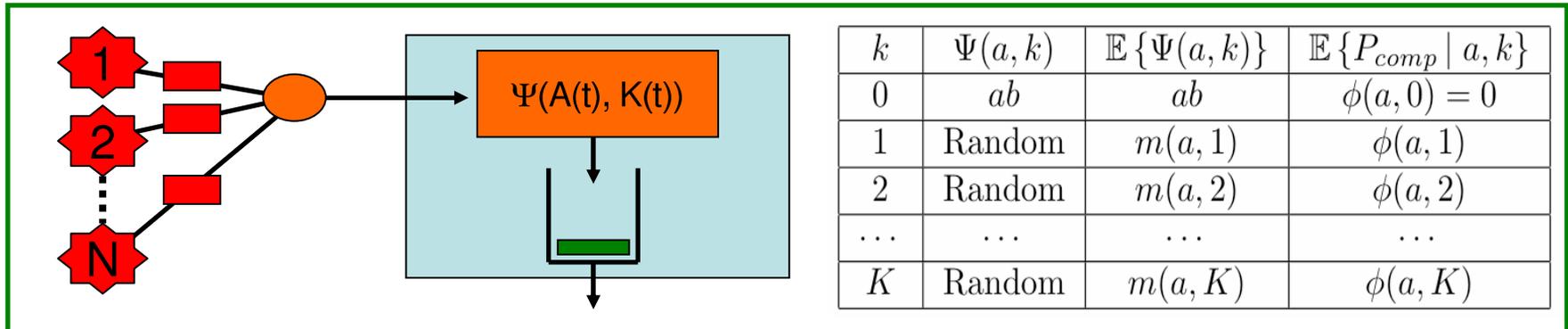
$\Psi(A(t), K(t))$  = Compression Function

Example 2:  $A(t) = 2, K(t) = k$

$\Psi(A(t), K(t)) = (0.9)B$  bits (random output)

$P_{comp}(t) = .3$  mW (random output)

# Compression Operation:



Timeslotted System:  $t = \{0, 1, 2, 3, \dots\}$

$A(t) = \#$  packet arrivals at time  $t$

(fixed packets size  $B$  bits,  $A(t) \in \{0, 1, \dots, N\}$ )

$K(t) =$  Compression Decision Option at time  $t$

( $K(t) \in \{0, 1, \dots, K\}$ )

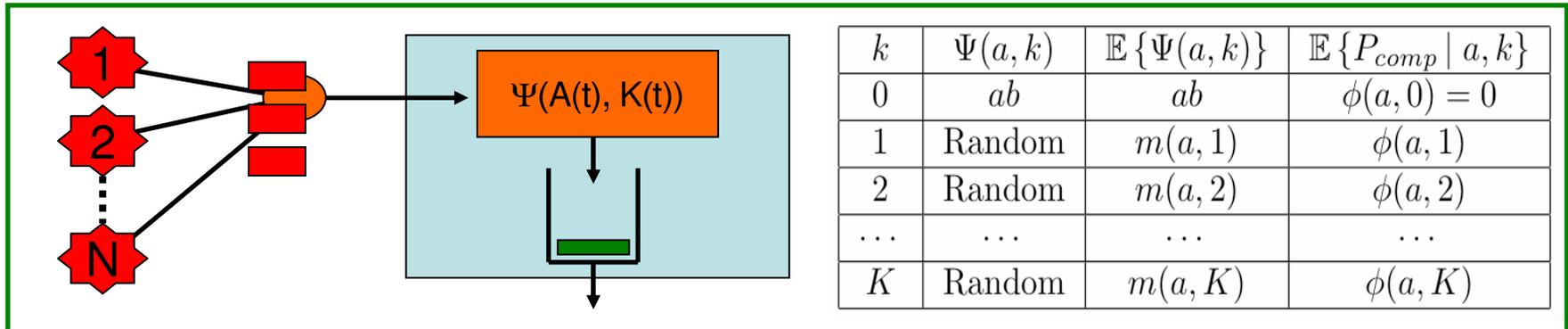
$\Psi(A(t), K(t)) =$  Compression Function

Example 3:  $A(t) = 3, K(t) = k$

$\Psi(A(t), K(t)) = ??$  (random output)

$P_{comp}(t) = ??$  (random output)

# Compression Operation:



Timeslotted System:  $t = \{0, 1, 2, 3, \dots\}$

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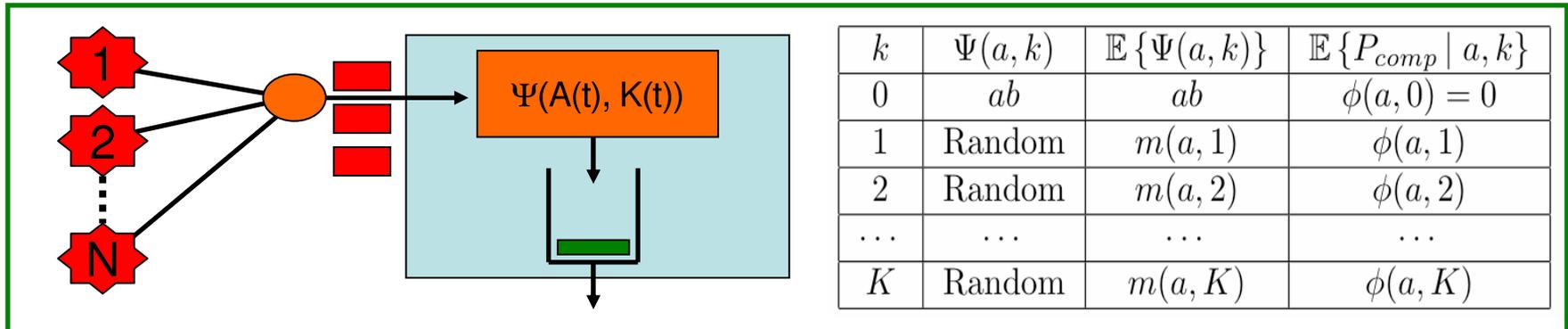
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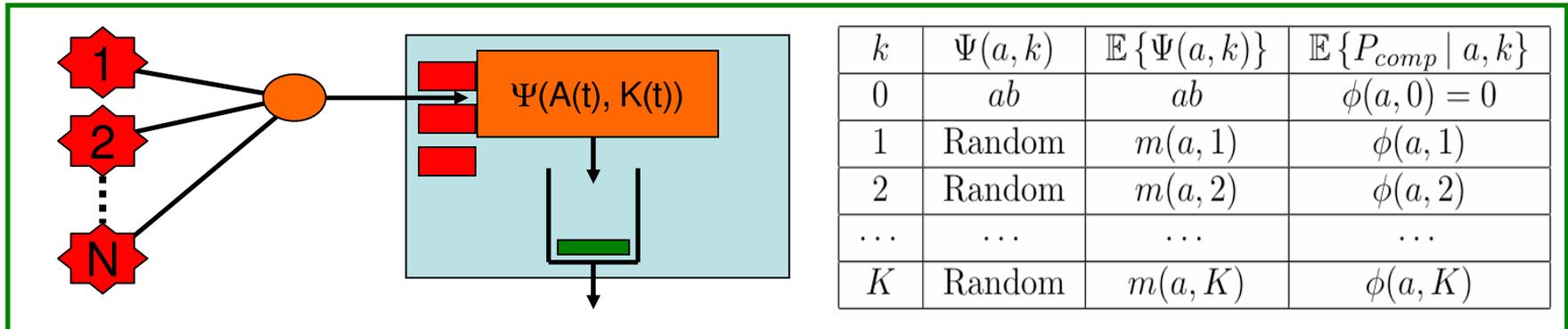
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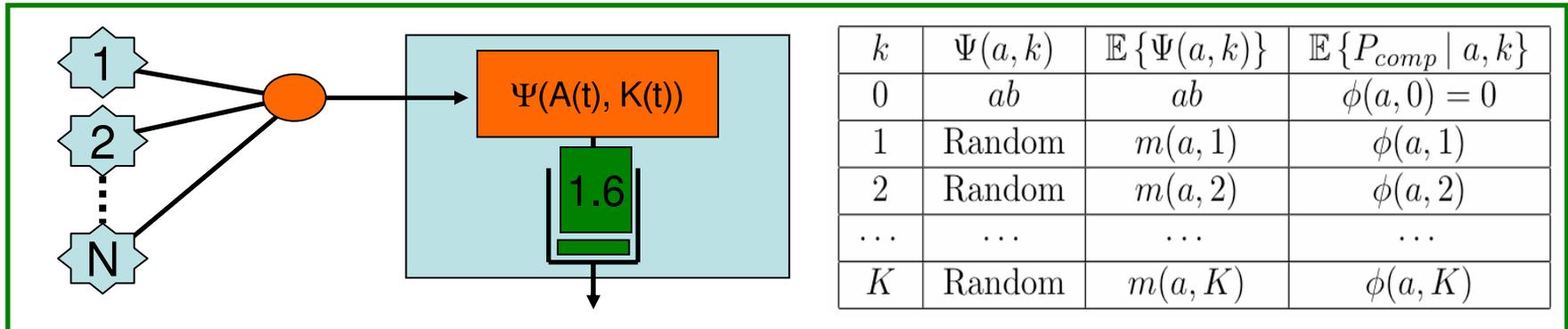
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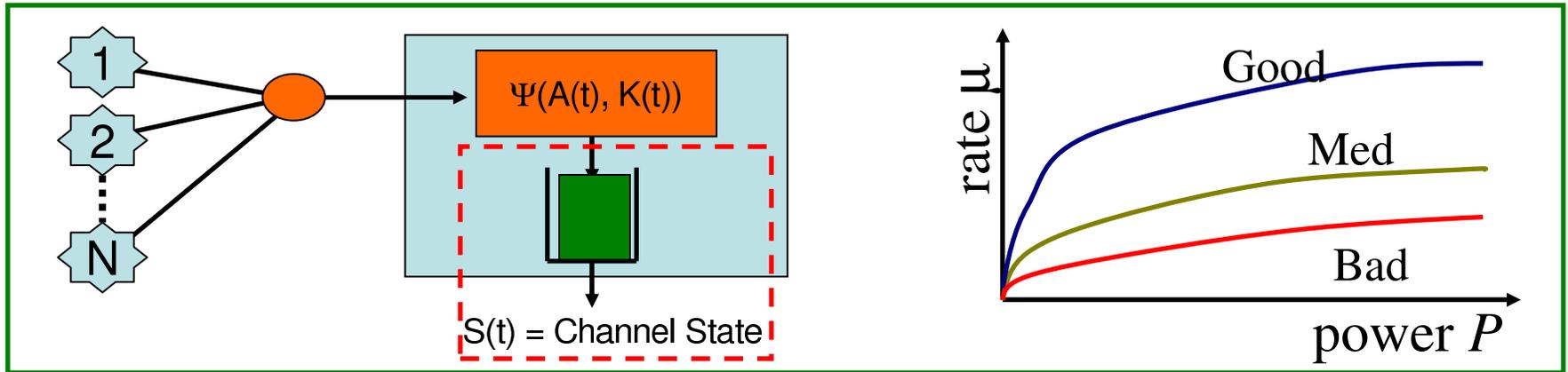
$\Psi(A(t), K(t))$  = Compression Function

Example 3:  $A(t) = 3, K(t) = k$

$\Psi(A(t), K(t)) = (1.6)B$  bits (random output)

$P_{comp}(t) = .4$  mW (random output)

## Transmission Operation:



$S(t)$  = Current Channel State on slot  $t$

**Example:**  $S(t) \in \{\text{"Good"}, \text{"Med"}, \text{"Bad"}\}$

$S(t) \in \{\text{attenuation}, 0 < S(t) < 1\}$

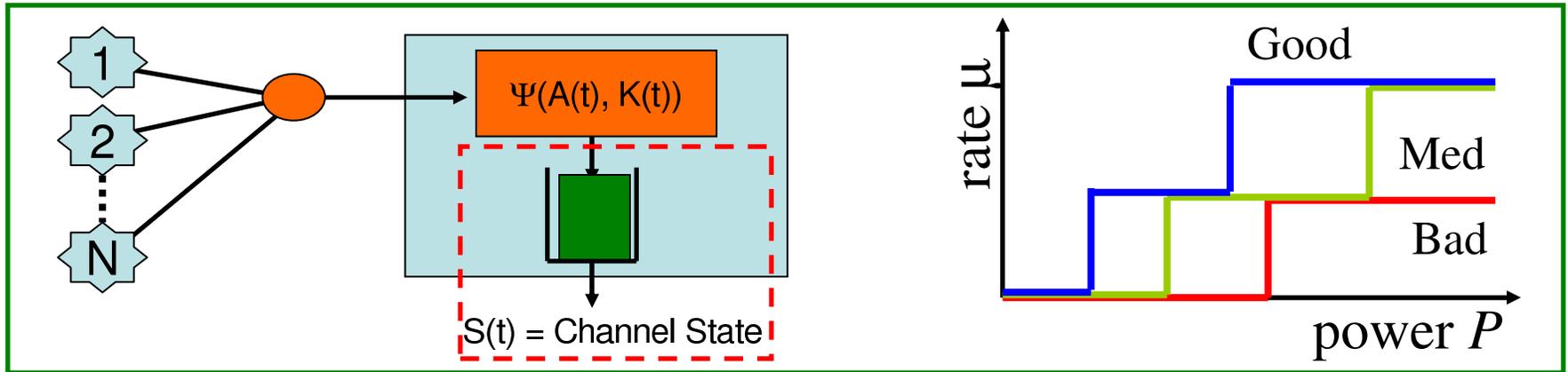
$P_{\text{tran}}(t)$  = Power Decision on slot  $t$  ( $\mathbf{P}$  = set of power options)

**Example:**  $\mathbf{P} \in \{0 < p < P_{\text{max}}\}$  or  $\mathbf{P} \in \{0, P_{\text{max}}/2, P_{\text{max}}\}$

$\mu(t) = C(P_{\text{tran}}(t), S(t))$  = transmission rate on slot  $t$

rate-power curve

## Transmission Operation:



$S(t)$  = Current Channel State on slot  $t$

**Example:**  $S(t) \in \{\text{“Good”, “Med”, “Bad”}\}$

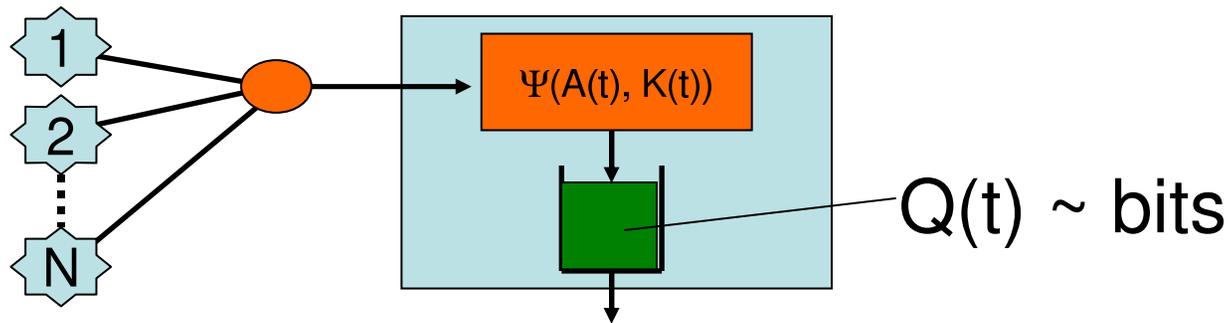
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$\mu(t) = C(P_{\text{tran}}(t), S(t))$  = transmission rate on slot  $t$

rate-power curve



## Queueing Dynamics:

$$Q(t+1) = \max[Q(t) - \mu(t), 0] + R(t)$$

$$\mu(t) = C(P_{\text{tran}}(t), S(t)) \quad , \quad R(t) = \Psi(A(t), K(t))$$

**Goal:** Design a joint strategy for choosing compression and transmission decisions  $\mathbf{K}(t)$ ,  $\mathbf{P}_{\text{tran}}(t)$  over time to ***support all traffic*** and ***minimize total average power***.

[Also want low delay!]  $\hookrightarrow \bar{P}_{\text{tot}} = \bar{P}_{\text{comp}} + \bar{P}_{\text{tran}}$

$k$	$\Psi(a, k)$	$\mathbb{E} \{ \Psi(a, k) \}$	$\mathbb{E} \{ P_{comp}   a, k \}$
0	$ab$	$ab$	$\phi(a, 0) = 0$
1	Random	$m(a, 1)$	$\phi(a, 1)$
2	Random	$m(a, 2)$	$\phi(a, 2)$
...	...	...	...
$K$	Random	$m(a, K)$	$\phi(a, K)$

**Intuition:** Consider a trivial case where all of the following hold...

- 1) Static Channel:  $S(t) = \text{Constant}$
- 2) Linear rate-power curve:  $C(P) = \alpha P$
- 3) Raw Data Rate small:  $E\{A(t)\}B < \mu_{\max} = \alpha P_{\max}$

If all 3 hold, easy to show  $\bar{P}_{\text{tran}}$  proportional to bits transmitted, and decision is trivial: choose  $K(t) = k \in \{0, \dots, K\}$  that minimizes:

$$\phi(a, k) + m(a, k)/\alpha$$

*If one or more of the above fail, decision is non-trivial!*

Prior Experimental Work in this case:

1. Barr, Asanovic “Energy Aware Lossless Data compression” [2003]
2. Sadler, Martonosi “Data Compression algorithms for energy-constrained devices” [SenSys2006]

Data Compression an important problem!

*[Richard Baraniuk's Plenary talk at CISS 08 was totally cool!]*

Intelligent Compression Can Significantly Improve Power Expenditure.

## Defining Optimality: The $h^*(r)$ and $g^*(r)$ functions

Assume:

- # arrivals  $A(t)$  iid over slots,  $p_A(a) = \Pr[A(t) = a]$
- $S(t)$  iid over slots,  $\pi_s = \Pr[S(t) = s]$

Distributions  $p_A(a)$  and  $\pi_s$  are potentially unknown to controller

Min bit rate out of compressor

$$r_{min} \triangleq \mathbb{E} \left\{ \min_{k \in \mathcal{K}} m(A(t), k) \right\}$$

$$r_{max} \triangleq \mathbb{E} \{ C(P_{max}, S(t)) \}$$

Max time avg. transmission rate

(assume  $r_{min} < r_{max}$ )

## Defining Optimality: The $h^*(r)$ and $g^*(r)$ functions

Assume:

- # arrivals  $A(t)$  iid over slots,  $p_A(a) = \Pr[A(t) = a]$
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Distributions  $p_A(a)$  and  $\pi_s$  are potentially unknown to controller

$$r_{min} \triangleq \mathbb{E} \left\{ \min_{k \in \mathcal{K}} m(A(t), k) \right\}$$

$$r_{max} \triangleq \mathbb{E} \{ C(P_{max}, S(t)) \}$$

Intuitively: There are two reasons to compress data:

- To stabilize queue, we may need to compress (if raw input rate  $> r_{max}$ )
- Power spent compressing can reduce trans. power.

# Defining Optimality: The $h^*(r)$ and $g^*(r)$ functions

$h^*(r) = \text{minimum } \bar{P}_{\text{comp}}$  to yield compressor output rate  $r$ .

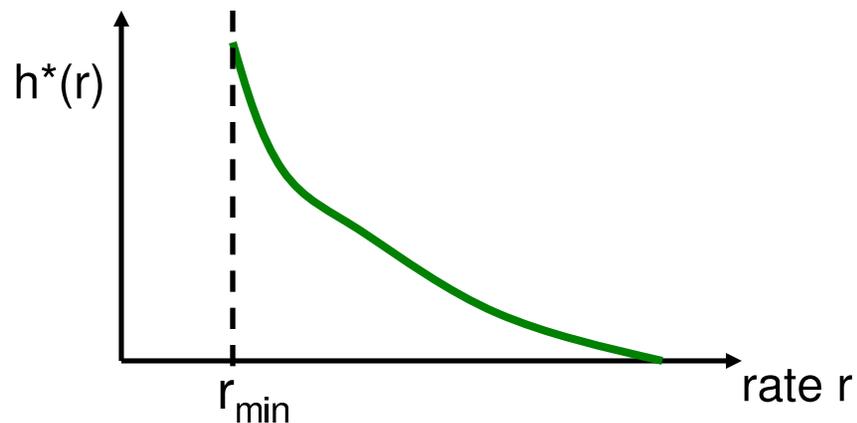
Min  $h$  such that:

$$\sum_{a=0}^N \sum_{k=1}^K p_A(a) \gamma_{a,k} \phi(a, k) = h$$

$$\sum_{a=0}^N \sum_{k=1}^K p_A(a) \gamma_{a,k} m(a, k) \leq r$$

$$\gamma_{a,k} \geq 0 \quad \text{for all } a, k$$

$$\sum_{k=1}^K \gamma_{a,k} = 1 \quad \text{for all } a$$



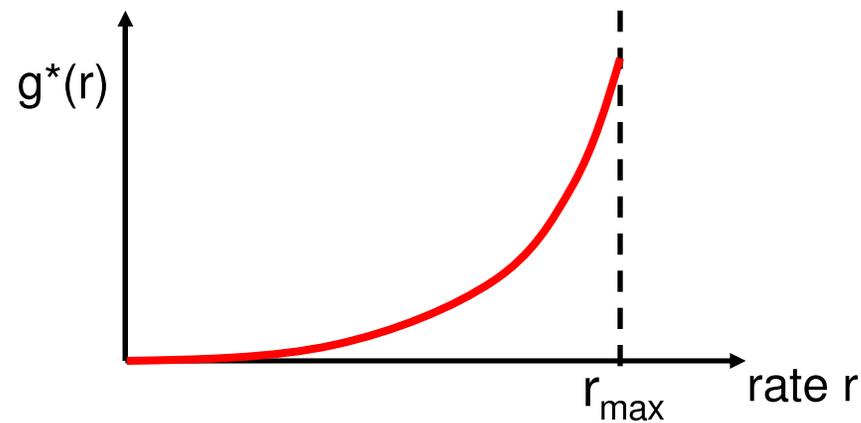
$g^*(r) = \text{minimum } \bar{P}_{\text{tran}}$  to yield avg. transmission rate  $r$ .

Min  $g$  such that:

$$\mathbb{E} \{P_{\text{tran}}(t)\} = g$$

$$\mathbb{E} \{C(P_{\text{tran}}(t), S(t))\} \geq r$$

(optimizing over all stationary randomized opportunistic policies)



## Theorem (characterizing minimum avg. power):

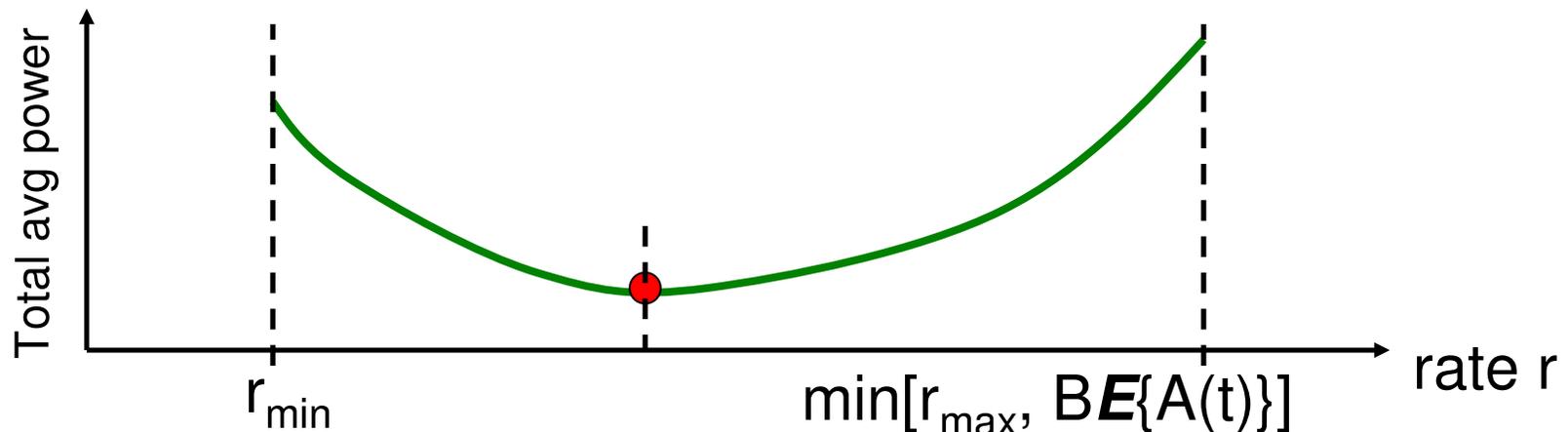
Any stabilizing policy yields avg. power such that:

$$\bar{P}_{\text{comp}} + \bar{P}_{\text{tran}} \geq P_{\text{av}}^*$$

Where  $P_{\text{av}}^*$  is the min avg. power and satisfies:

Minimize:  $h^*(r) + g^*(r)$

Subject to:  $r_{\min} \leq r \leq \min[r_{\max}, \mathbf{BE}\{A(t)\}]$



Want a dynamic control algorithm to minimize power...

Use **Joint Lyapunov Stability + Performance Optimization**

Technique for ***Stochastic Network Optimization***:

-Georgiadis, Neely, Tassiulas [NOW F&T in Networking, 2006]

-Neely [Phd thesis 2003, Infocom 2005, IT 2006]

-See also related technique in Stolyar [Queueing Systems 2005]

**Lyapunov Function:**  $L(Q) = (1/2) Q^2$

**Lyapunov Drift:**  $\Delta(Q(t)) = \mathbf{E}\{L(Q(t+1)) - L(Q(t)) \mid Q(t)\}$

**Technique:** Every slot, observe  $Q(t)$ , take control action to minimize (for a given constant  $V > 0$ ):

$$\Delta(Q(t)) + V \mathbf{E}\{\text{Cost}(t) \mid Q(t)\}$$

**Theorem:** 1)  $\mathbf{E}\{Q\} < O(V)$

2)  $\mathbf{E}\{\text{Cost}(\text{actual}) - \text{Cost}(\text{optimal})\} < O(1/V)$

# A brief history of **Lyapunov Drift** for Queueing Systems:

## **Lyapunov Stability:**

Tassiulas, Ephremides [91, 92, 93]

P. R. Kumar, S. Meyn [95]

McKeown, Anantharam, Walrand [96, 99]

Kahale, P. E. Wright [97]

Andrews, Kumaran, Ramanan, Stolyar, Whiting [2001]

Leonardi, Mellia, Neri, Marsan [2001]

Neely, Modiano, Rohrs [2002, 2003, 2005]

## **Lyapunov Stability with Stochastic Performance Optimization:**

Neely, Modiano [2003, 2005] (Fairness, Energy)

Georgiadis, Neely, Tassiulas [NOW Publishers, F&T, 2006]

## **Alternate Approaches to Stoch. Performance Optimization:**

Eryilmaz, Srikant [2005] (Fluid Model Transformations)

Stolyar [2005] (Fluid Model Transformations)

Lee, Mazumdar, Shroff [2005] (Stochastic Gradients)

# A brief history of **Lyapunov Drift** for Queueing Systems:

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## **Lyapunov Stability with Stochastic Performance Optimization:**

Neely, Modiano [2003, 2005] (Fairness, Energy)

Georgiadis, Neely, Tassiulas [NOW Publishers, F&T, 2006]

*Yields explicit performance/delay tradeoffs! [ $O(1/N)$ ,  $O(V)$ ]*

## **Alternate Approaches to Stoch. Performance Optimization:**

Eryilmaz, Srikant [2005] (Fluid Model Transformations)

Stolyar [2005] (Fluid Model Transformations)

Lee, Mazumdar, Shroff [2005] (Stochastic Gradients)

Want a dynamic control algorithm to minimize power...

Use **Joint Lyapunov Stability + Performance Optimization**

Technique for ***Stochastic Network Optimization***:

-Georgiadis, Neely, Tassiulas [NOW F&T in Networking, 2006]

-Neely [Phd thesis 2003, Infocom 2005, IT 2006]

-See also related technique in Stolyar [Queueing Systems 2005]

**Lyapunov Function:**  $L(Q) = (1/2) Q^2$

**Lyapunov Drift:**  $\Delta(Q(t)) = \mathbf{E}\{L(Q(t+1)) - L(Q(t)) \mid Q(t)\}$

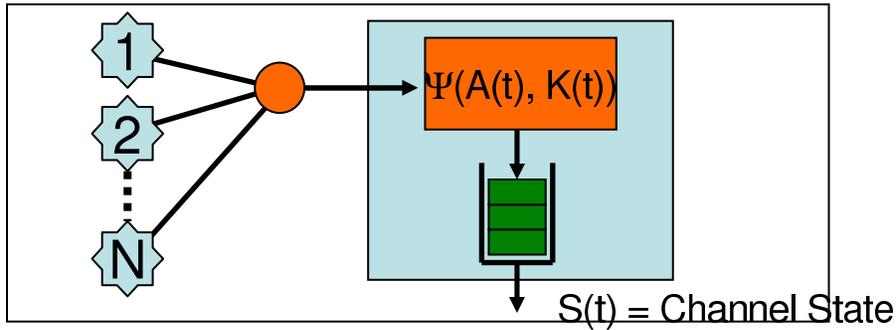
**Technique:** Every slot, observe  $Q(t)$ , take control action to minimize (for a given constant  $V > 0$ ):

$$\Delta(Q(t)) + V \mathbf{E}\{\text{Cost}(t) \mid Q(t)\}$$

\*(only need to minimize to within an additive constant)

**Theorem:** 1)  $\mathbf{E}\{Q\} < O(V)$

2)  $\mathbf{E}\{\text{Cost}(\text{actual}) - \text{Cost}(\text{optimal})\} < O(1/V)$



$k$	$\Psi(a, k)$	$\mathbb{E}\{\Psi(a, k)\}$	$\mathbb{E}\{P_{comp}   a, k\}$
0	$ab$	$ab$	$\phi(a, 0) = 0$
1	Random	$m(a, 1)$	$\phi(a, 1)$
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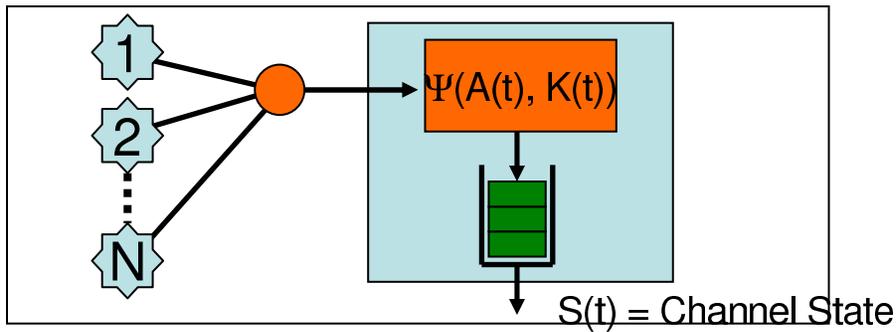
## The Dynamic Compression and Transmission Algorithm:

**Compression:** On slot  $t$ , observe  $A(t)$ . Choose  $K(t)$  such that:

$$k(t) = \arg \min_{k \in \mathcal{K}} [Q(t)m(A(t), k) + V\phi(A(t), k)]$$

**Transmission:** On slot  $t$ , observe  $S(t)$ . Choose  $P_{tran}(t)$  such that:

$$P_{tran}(t) = \arg \max_{P \in \mathcal{P}} [Q(t)C(P, S(t)) - VP]$$

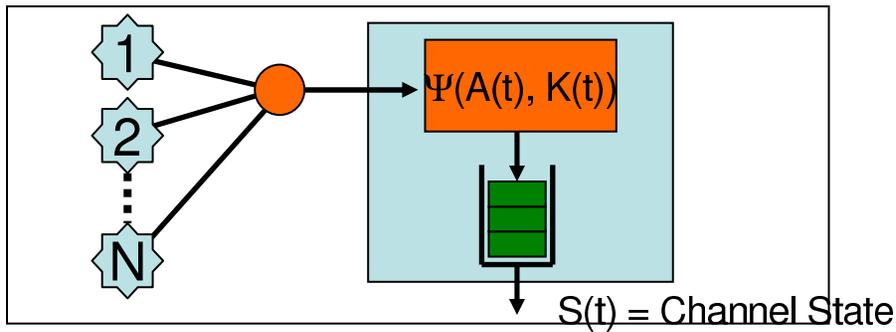


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**Concluding Theorem (Performance):** For any  $V > 0$  we have:

$$\begin{aligned} \overline{P}_{tot} &\leq P_{av}^* + B/V \\ \overline{Q} &\leq \frac{B + V(P_{max} + \phi_{max})}{(r_{max} - r_{min})} \end{aligned}$$

$$B = (1/2)[\mu_{max}^2 + B^2 E\{A^2\}]$$

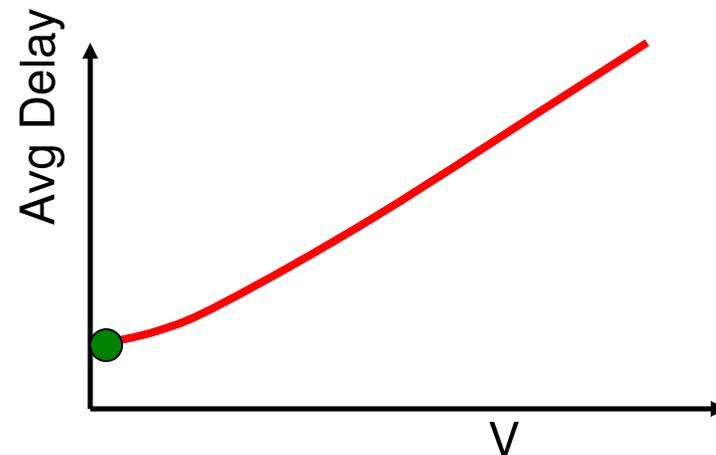
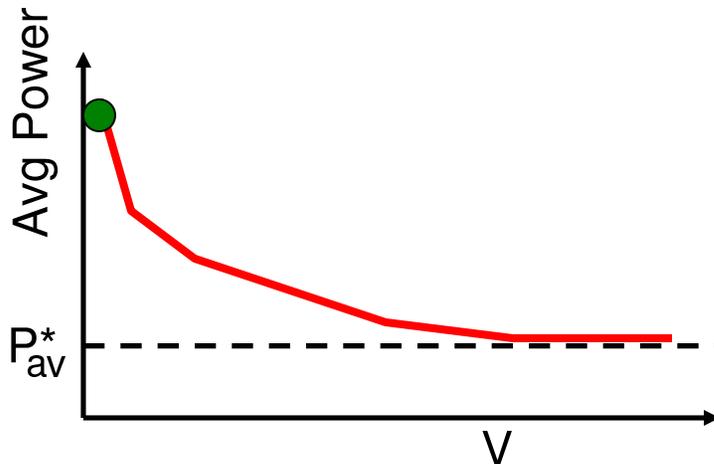


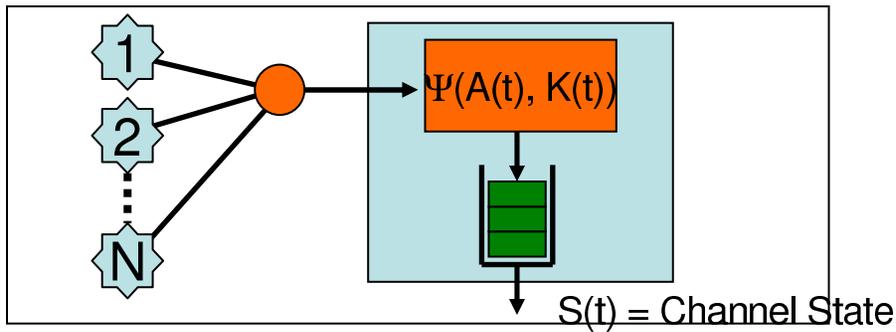
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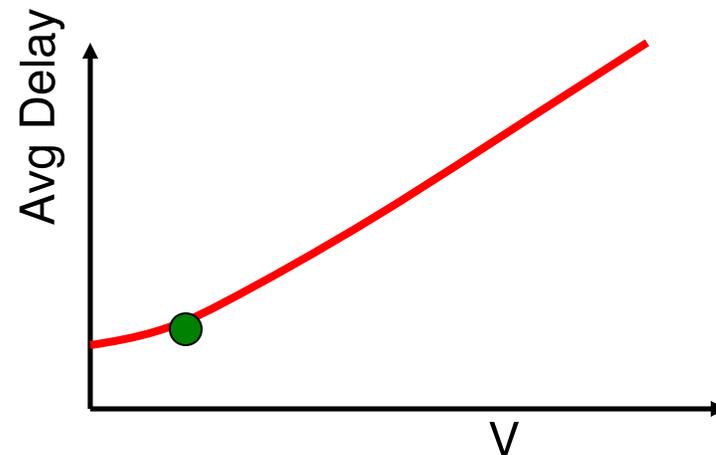
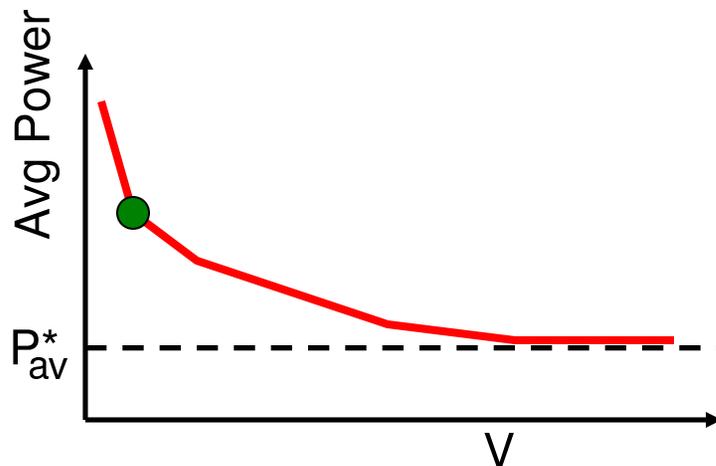


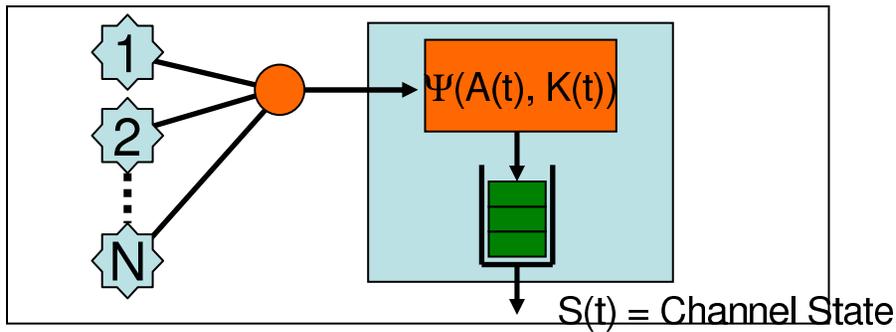
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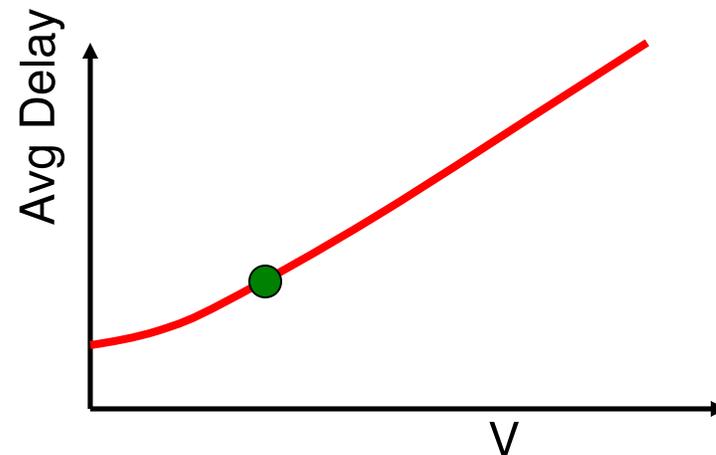
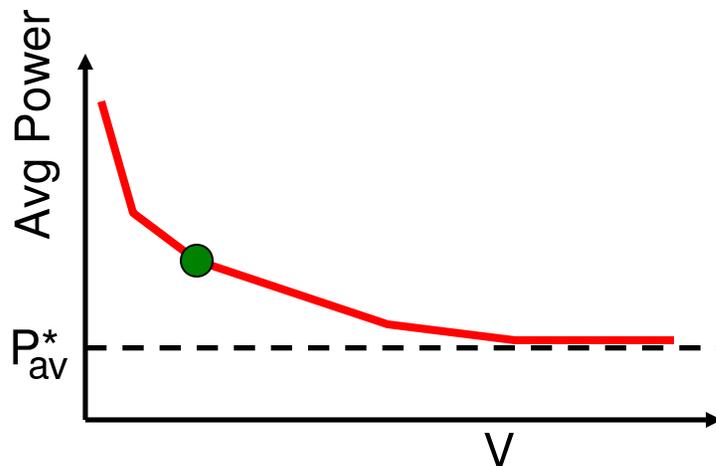


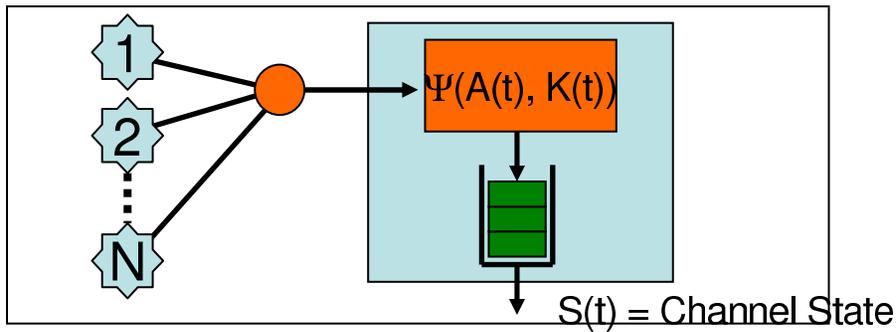
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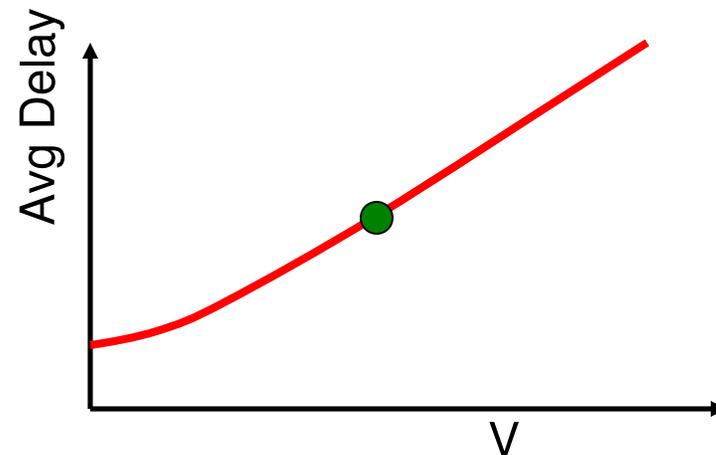
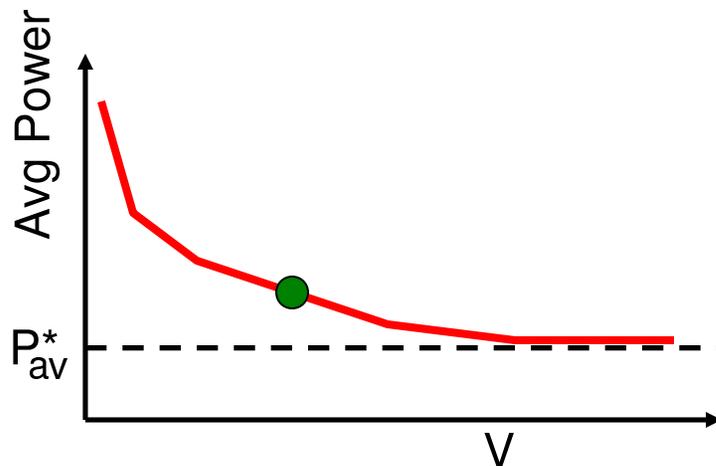


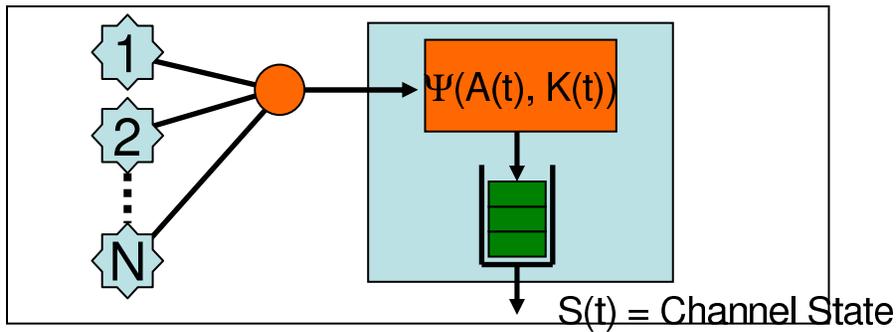
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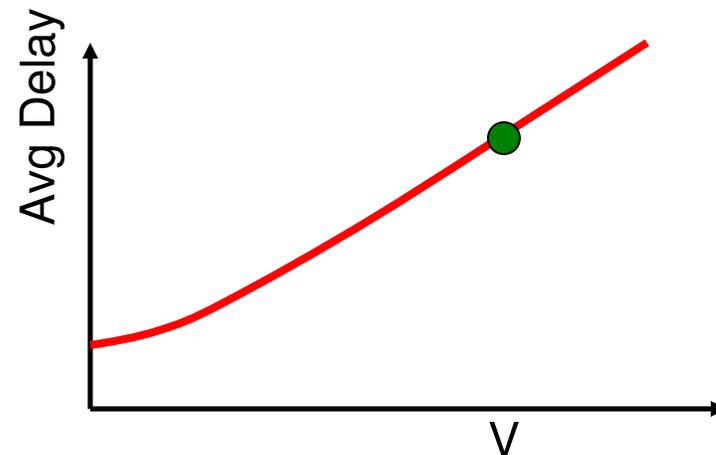
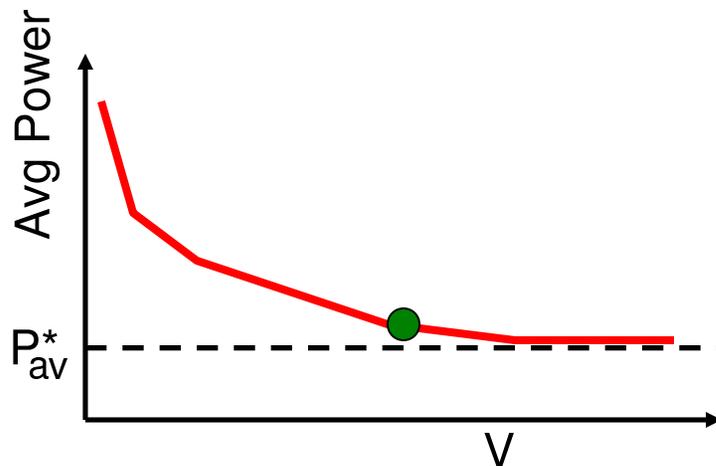


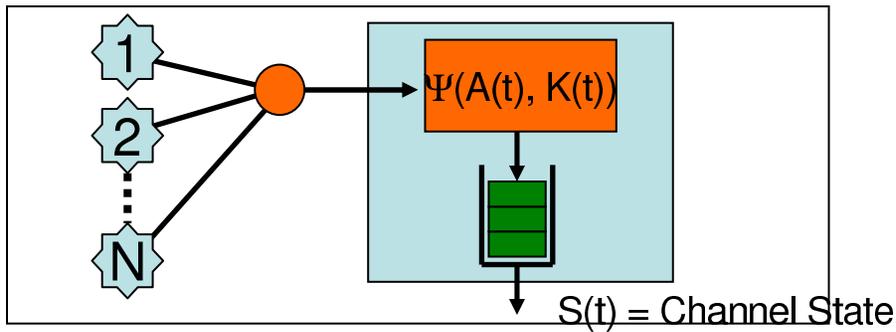
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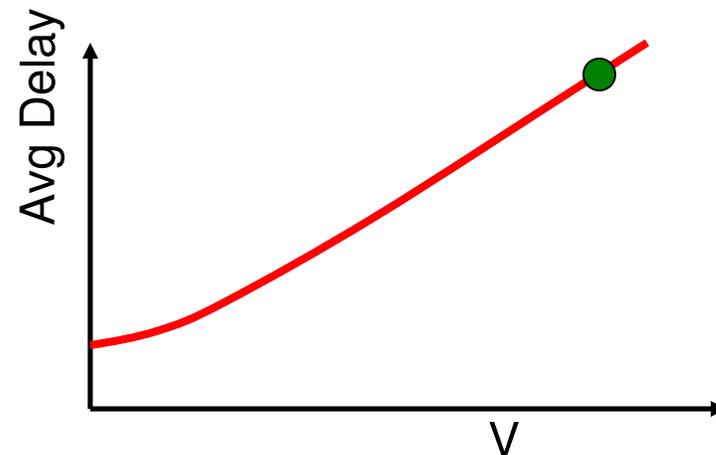
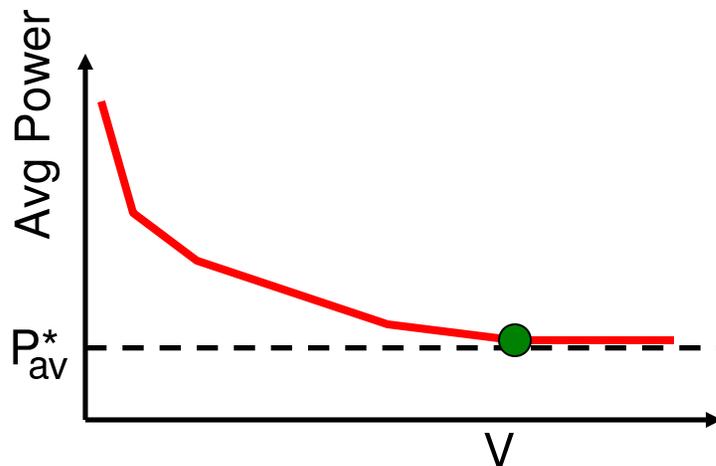


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A final note on energy-delay tradeoffs:

This algorithm achieves a  $[O(1/V), O(V)]$  energy-delay tradeoff!

See also:

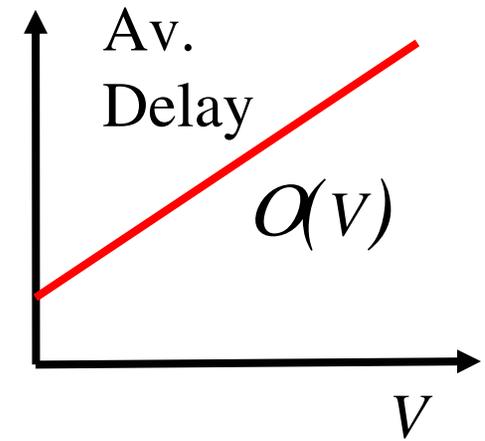
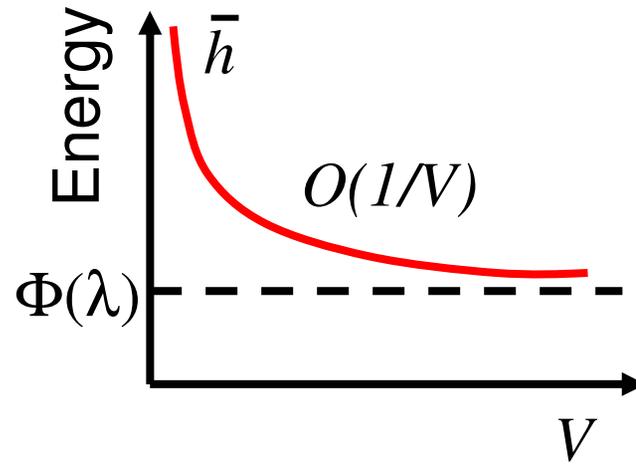
- $[O(1/V), O(V)]$  tradeoff for **general multi-hop nets** (without compression)  
[Neely Trans. Information Theory 2006]
- $[O(1/V), O(V)]$  tradeoff for **multi-receiver diversity** (“DIVBAR”)  
[Neely CISS 2006]
- $[O(1/V), O(V)]$  **fairness utility-delay tradeoffs for flow control**  
[Neely thesis 2003, INFOCOM 2005]

***Optimal Energy-Delay Tradeoffs:***

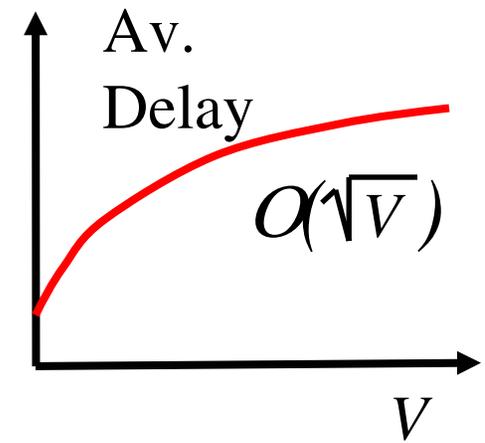
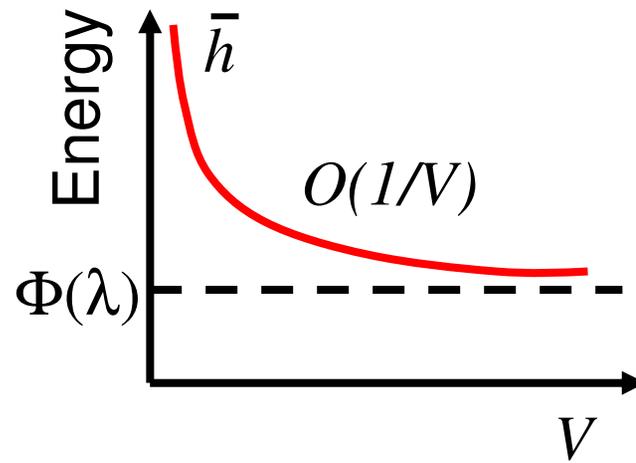
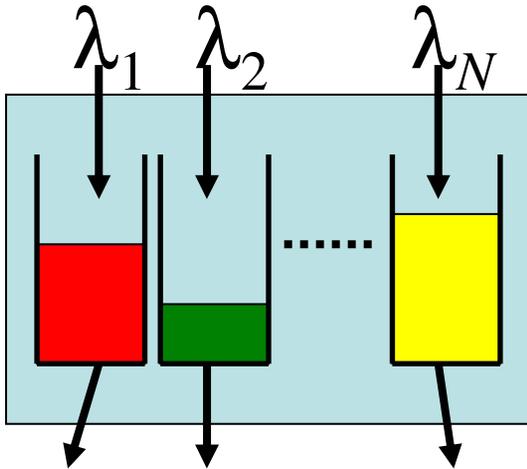
Berry-Gallager bound:  $[O(1/V), O(\text{Sqrt}(V))]$

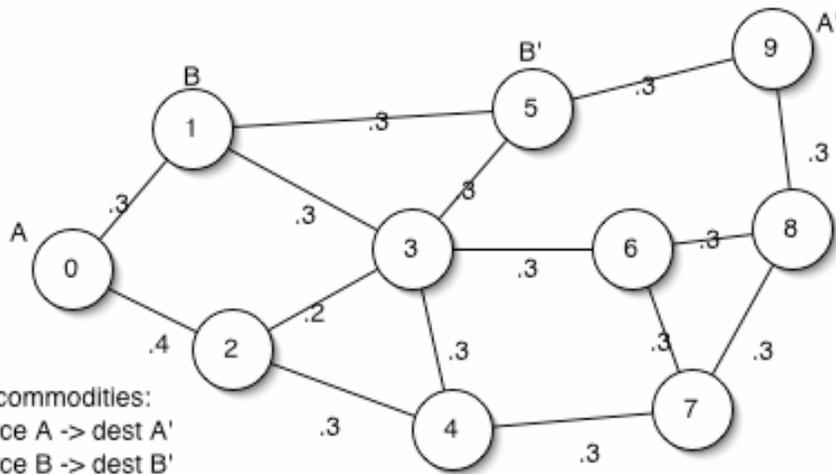
- offline, single user  $[O(1/V), O(\text{Sqrt}(V))]$  [Berry-Gallager IT 2002]
- online, multi-user  $[O(1/V), O(\text{Sqrt}(V))]$  [Neely IT 2007]
- online, flow control and/or piecewise linearity:  
 (“*beyond the bound Berry-Gallager Bound*”)  $[O(1/V), O(\log(V))]$   
 [Neely WiOpt 2006, IT 2007]

## Linear Tradeoff:

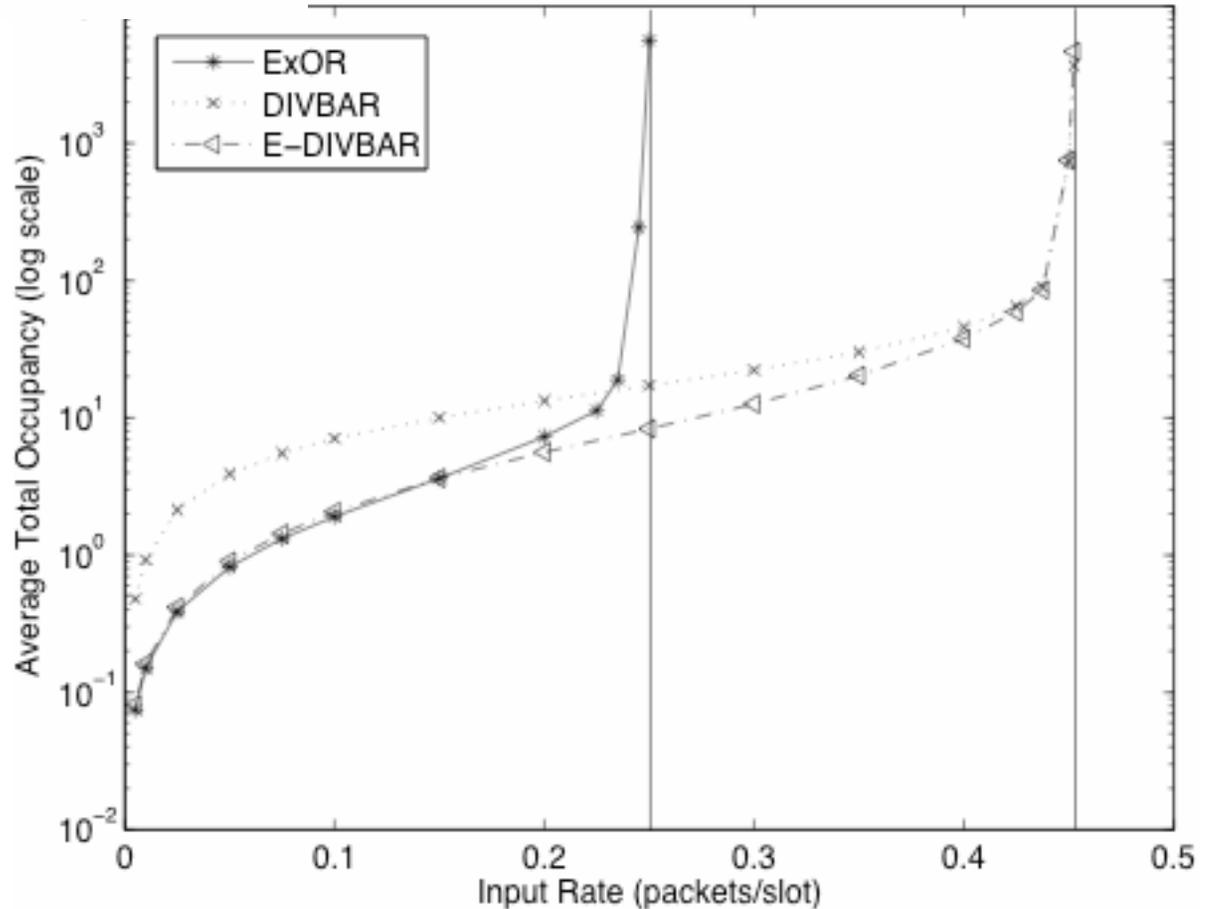


## Square Root Tradeoff:





Average Total Occupancy vs. Input Rate

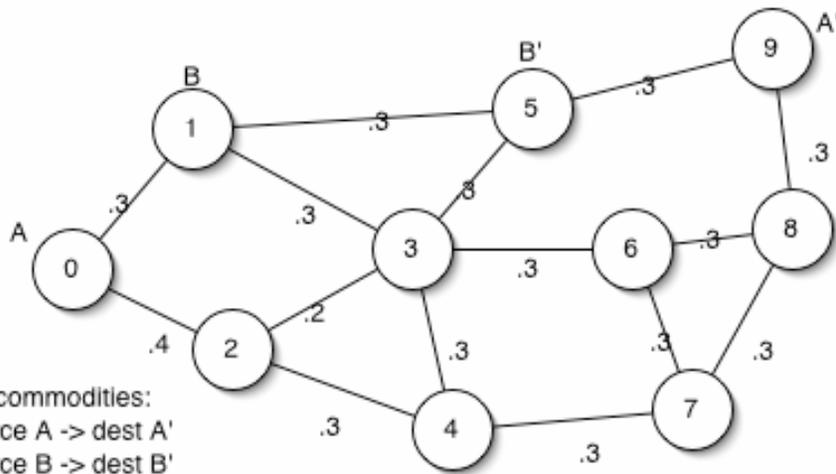


ExOR

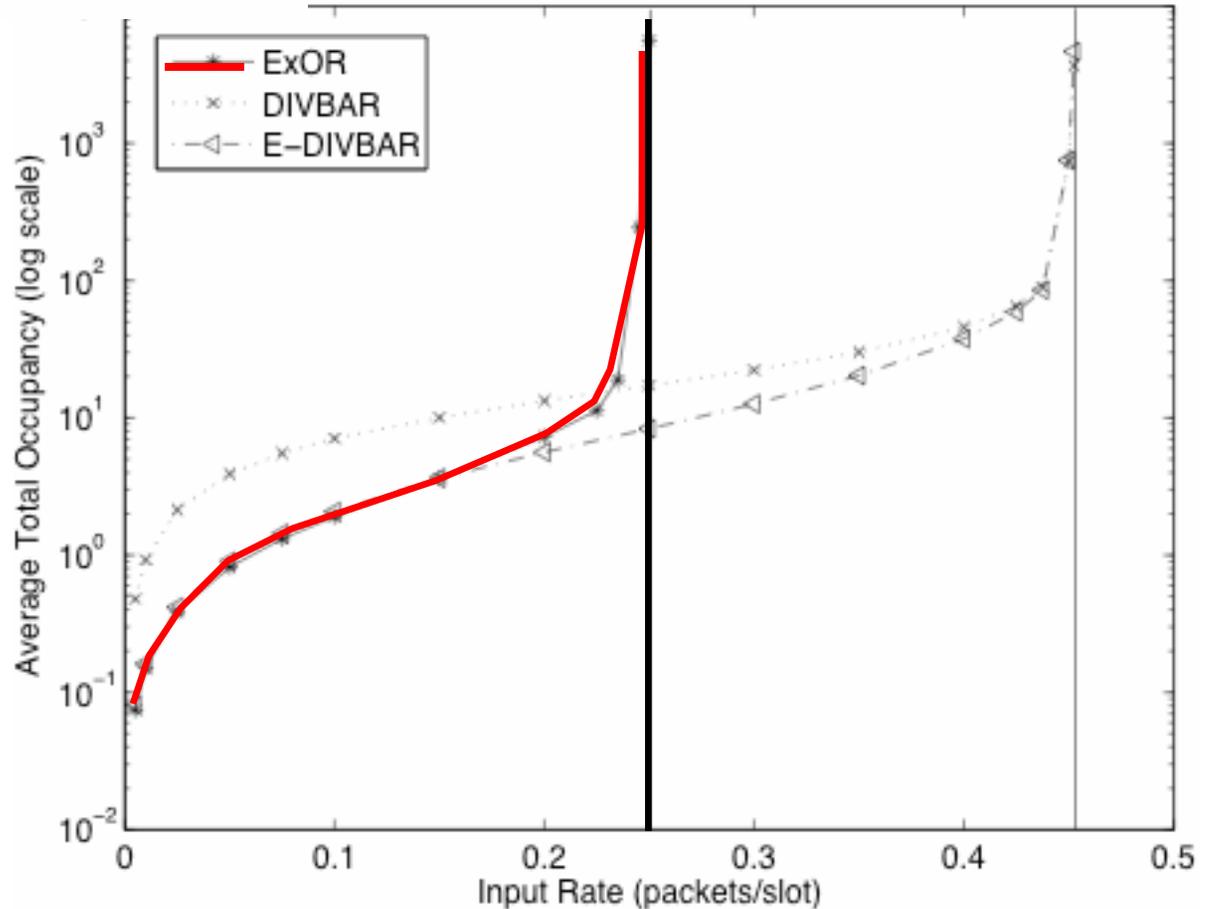
[Biswas, Morris 05]

DIVBAR, E-DIVBAR

[Neely, Urgaonkar  
 2006]



Average Total Occupancy vs. Input Rate

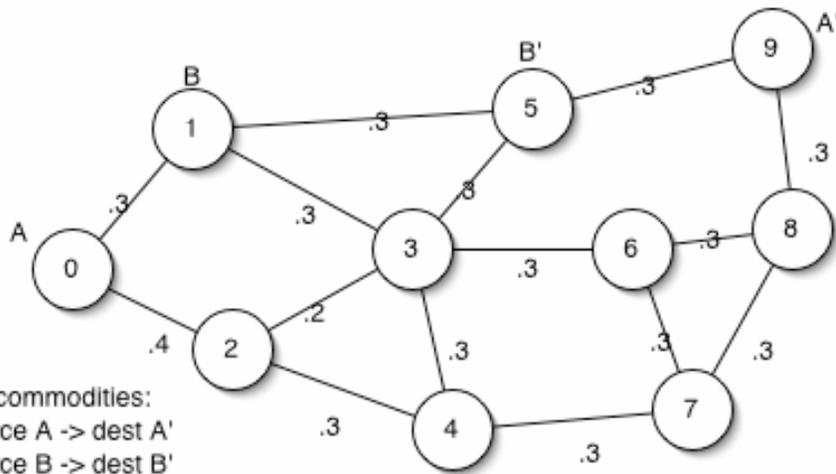


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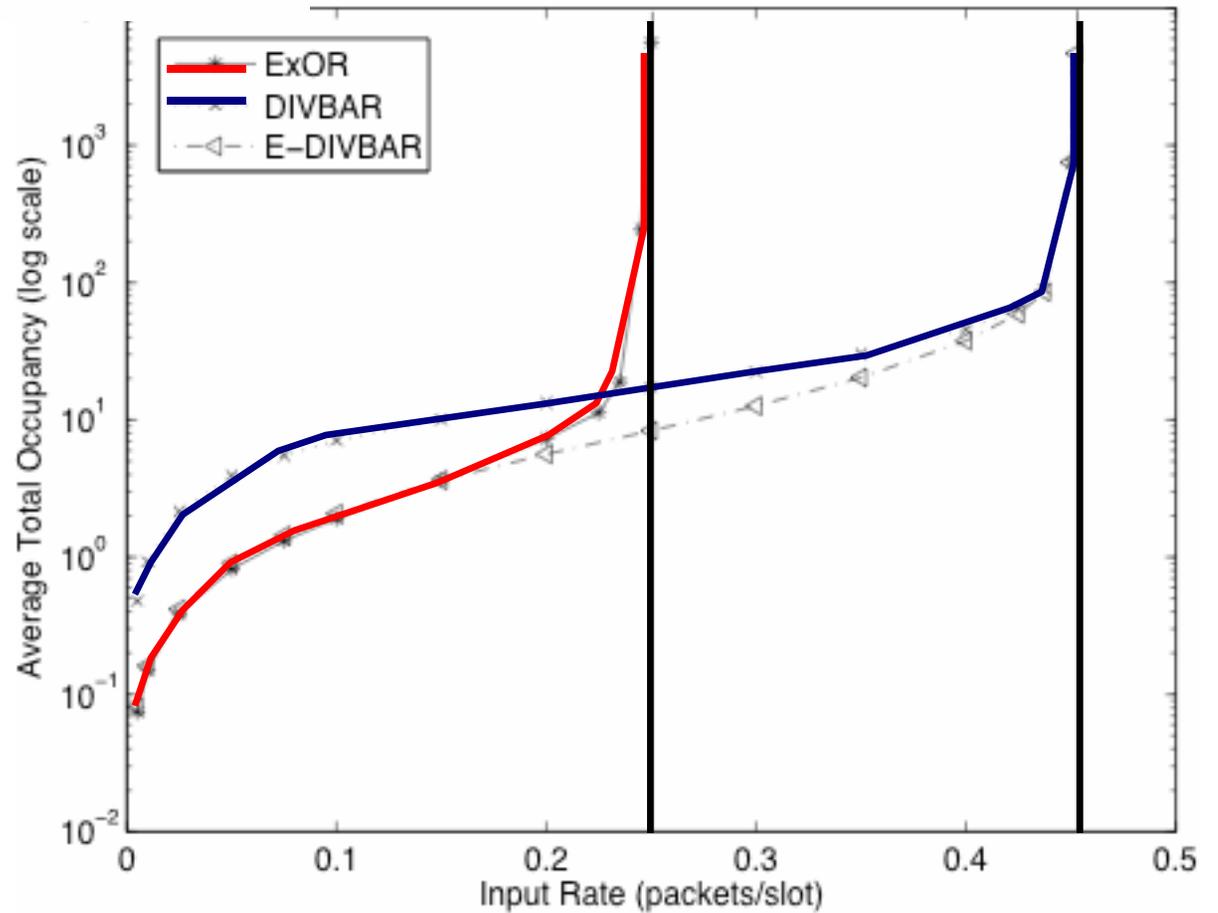
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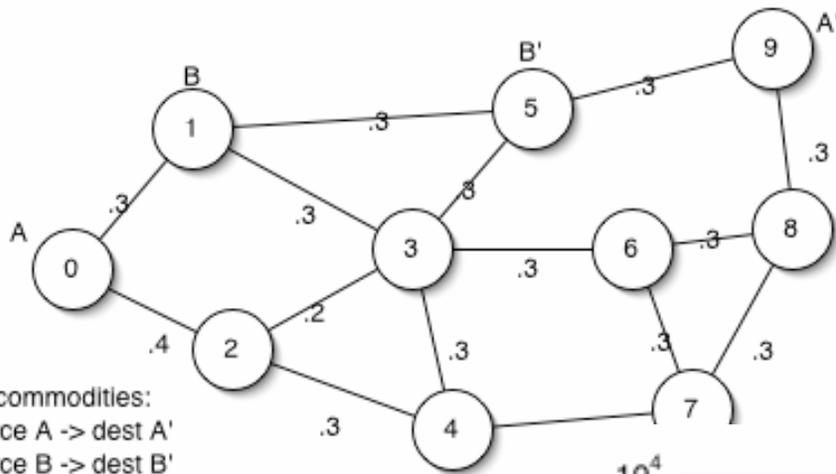


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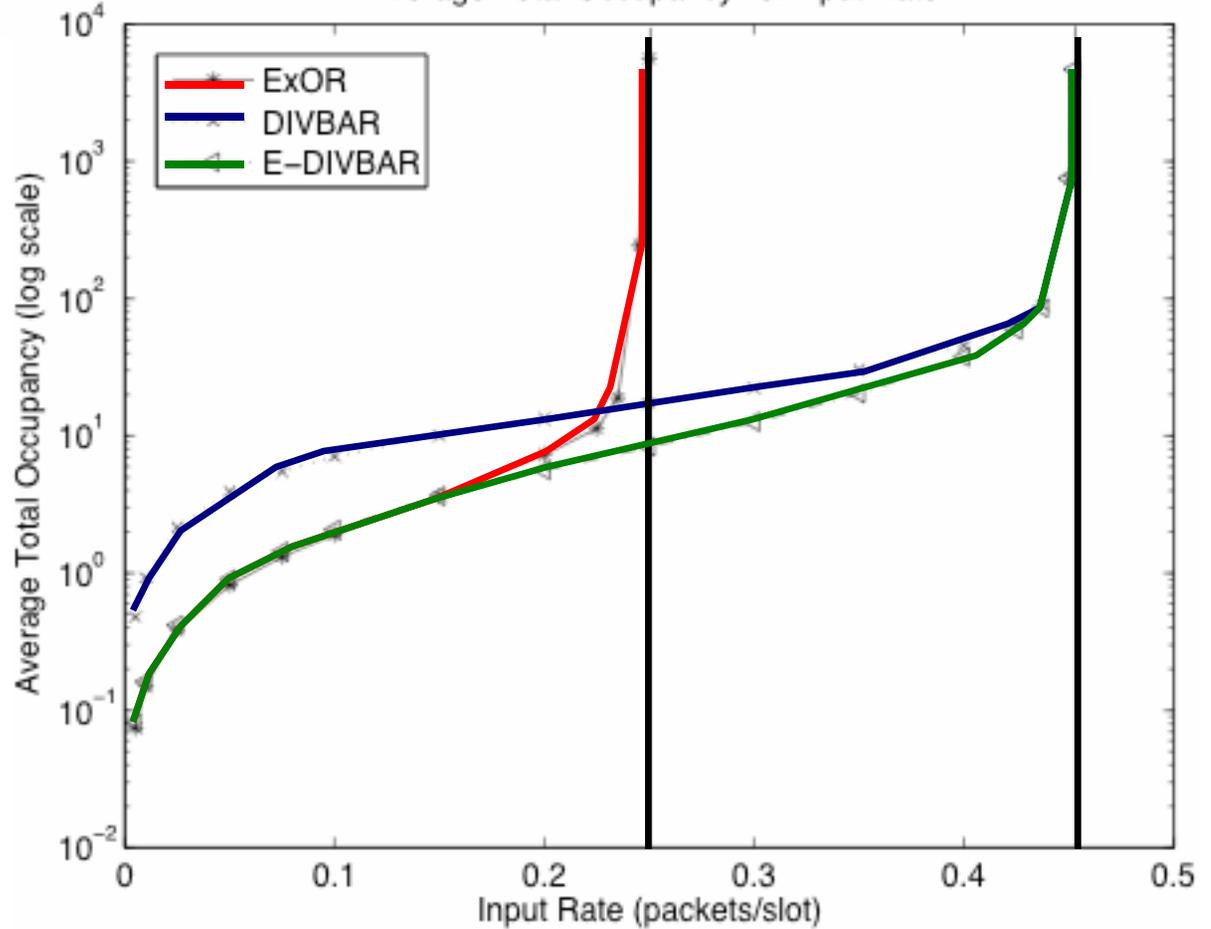
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Average Total Occupancy vs. Input Rate



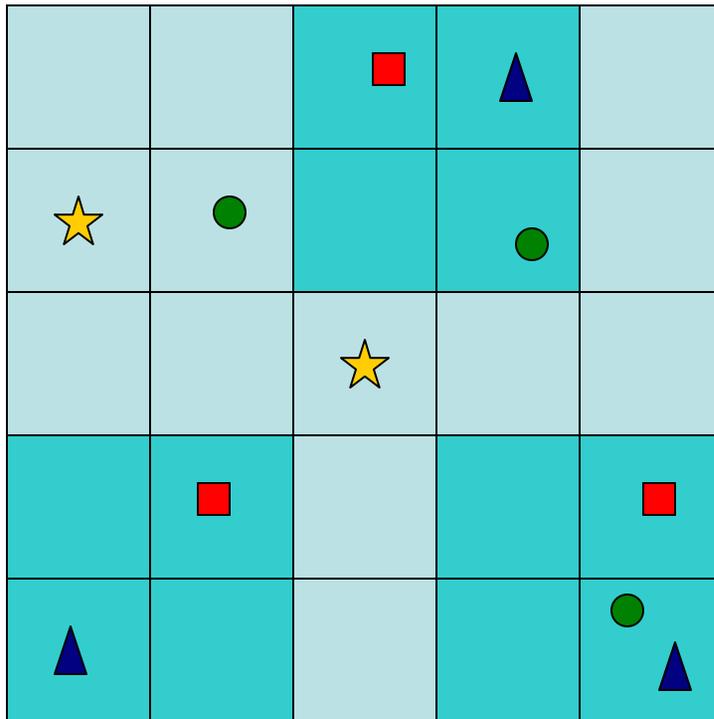
ExOR

[Biswas, Morris 05]

DIVBAR, E-DIVBAR

[Neely, Urgaonkar  
2006]

# DIVBAR for Mobile Networks



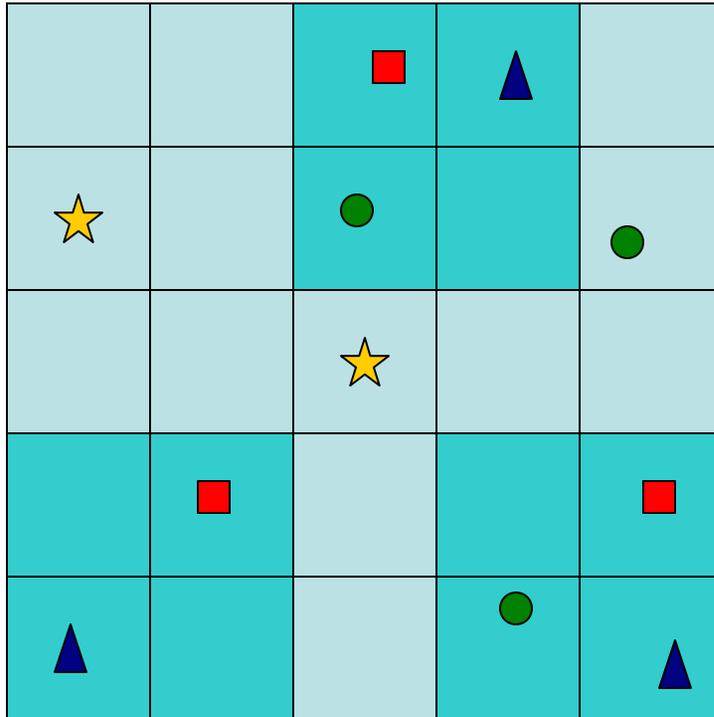
▲ = Stationary Node

■ = Locally Mobile Node

● = Fully Mobile Node

★ = Sink

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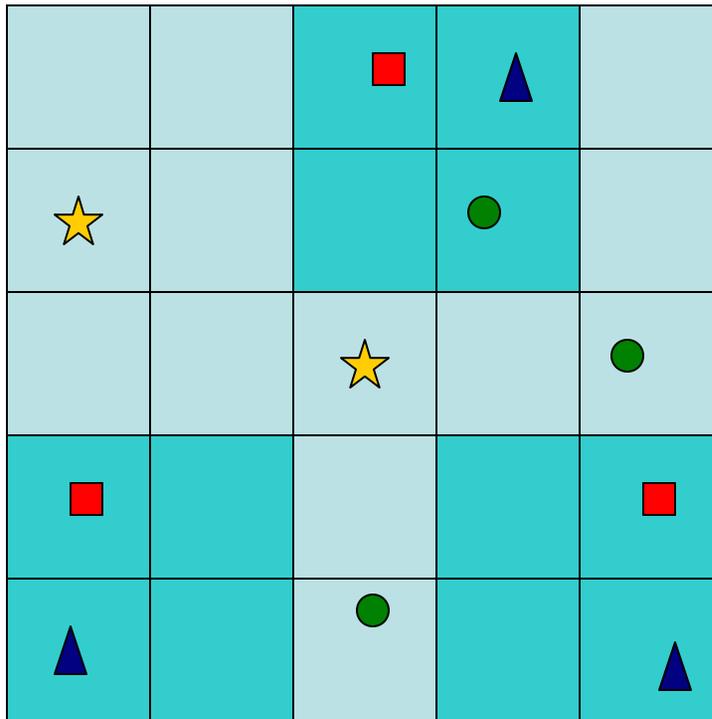
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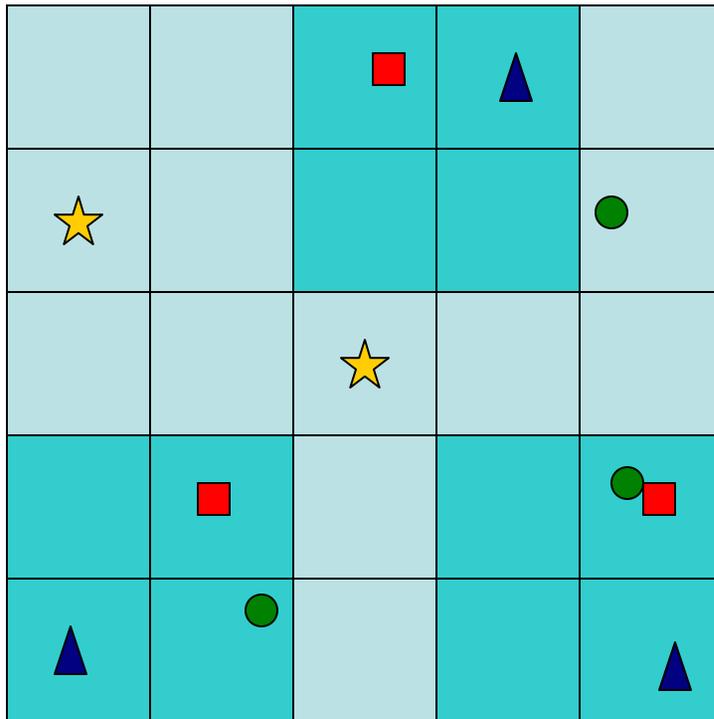
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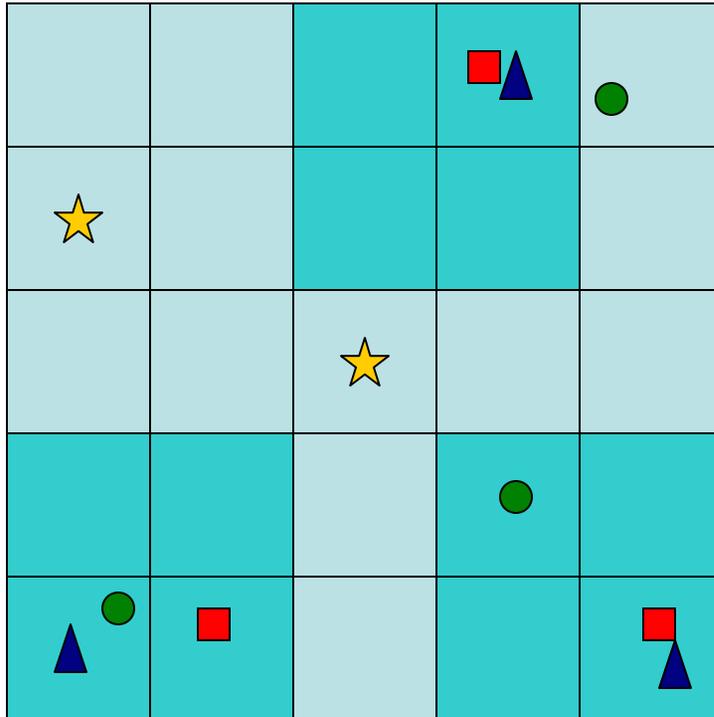
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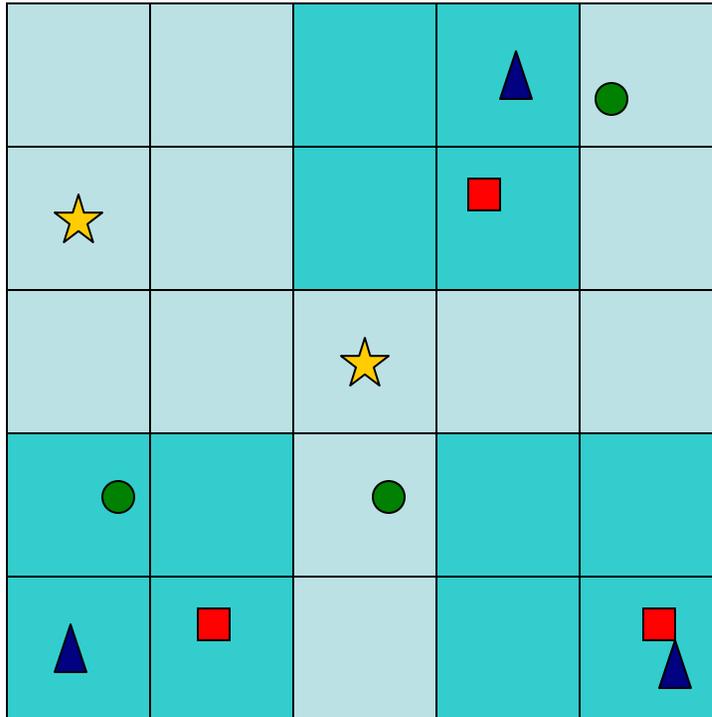
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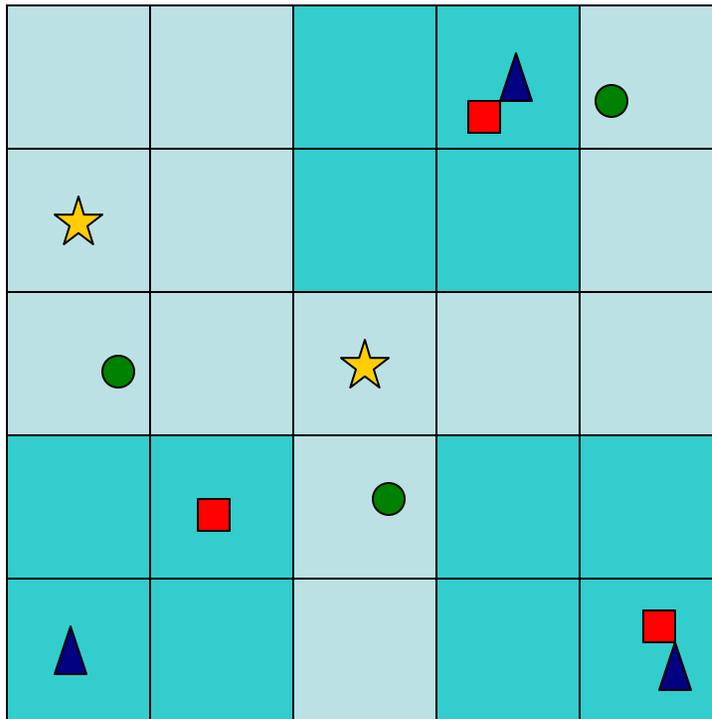
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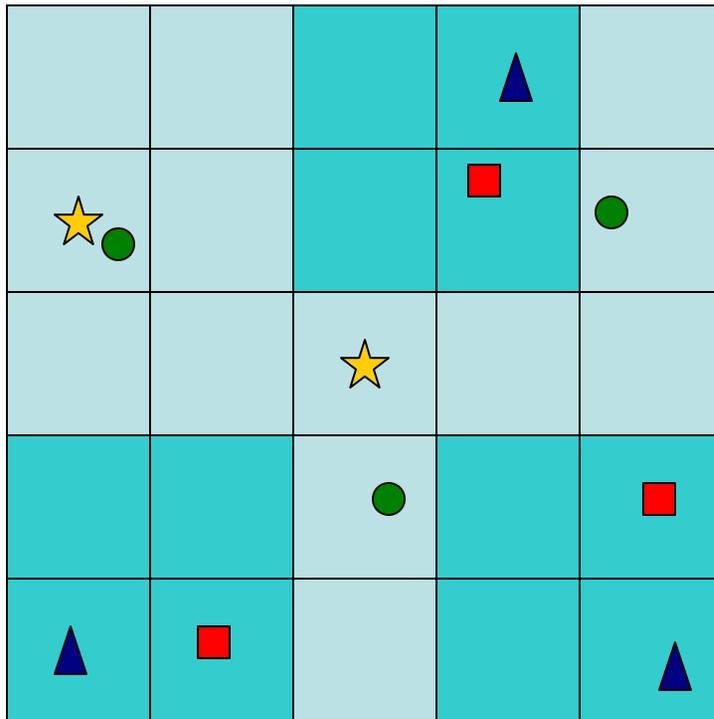
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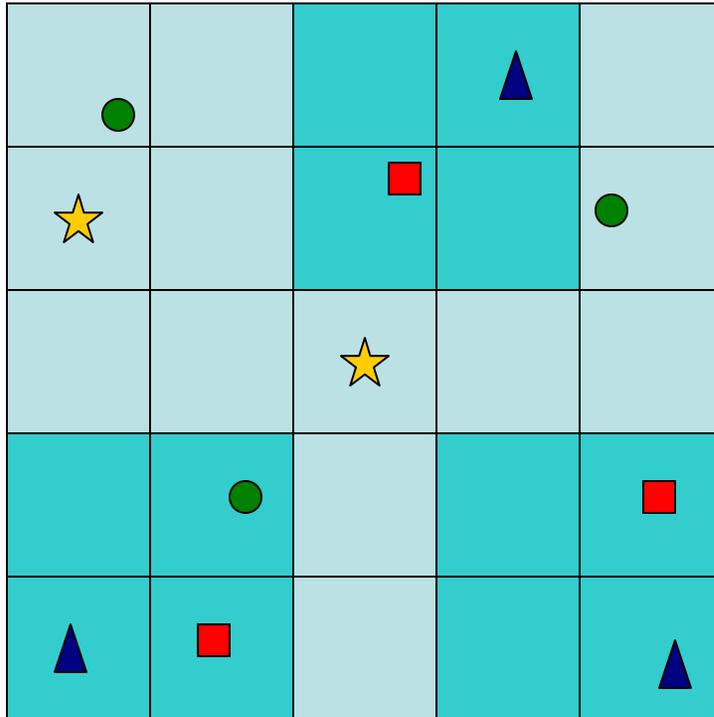
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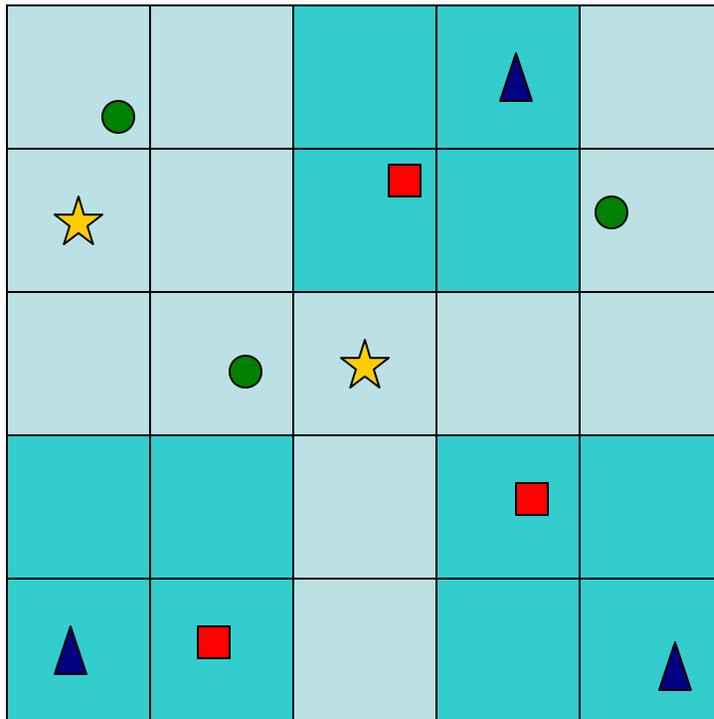
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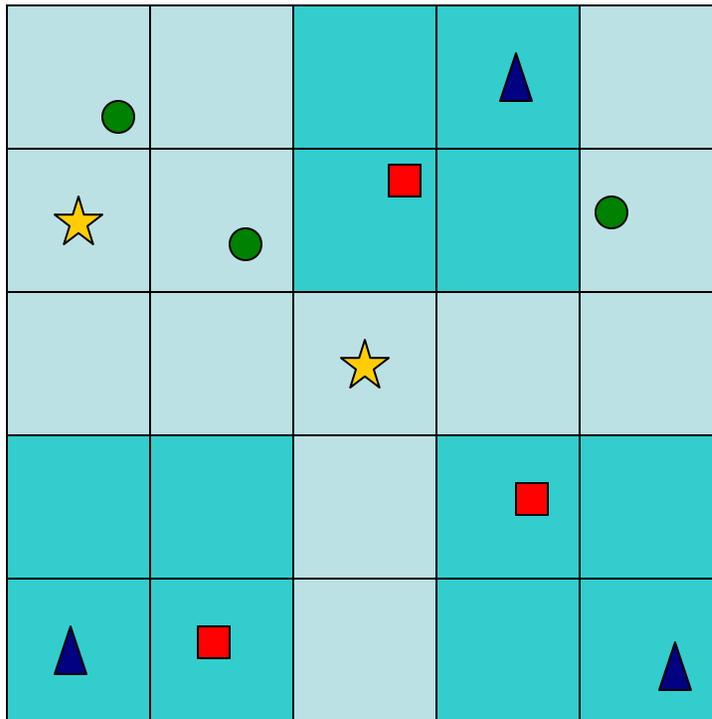
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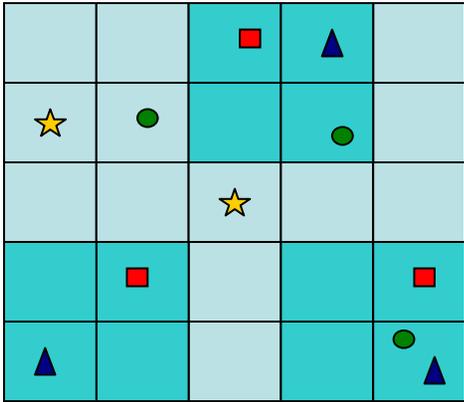
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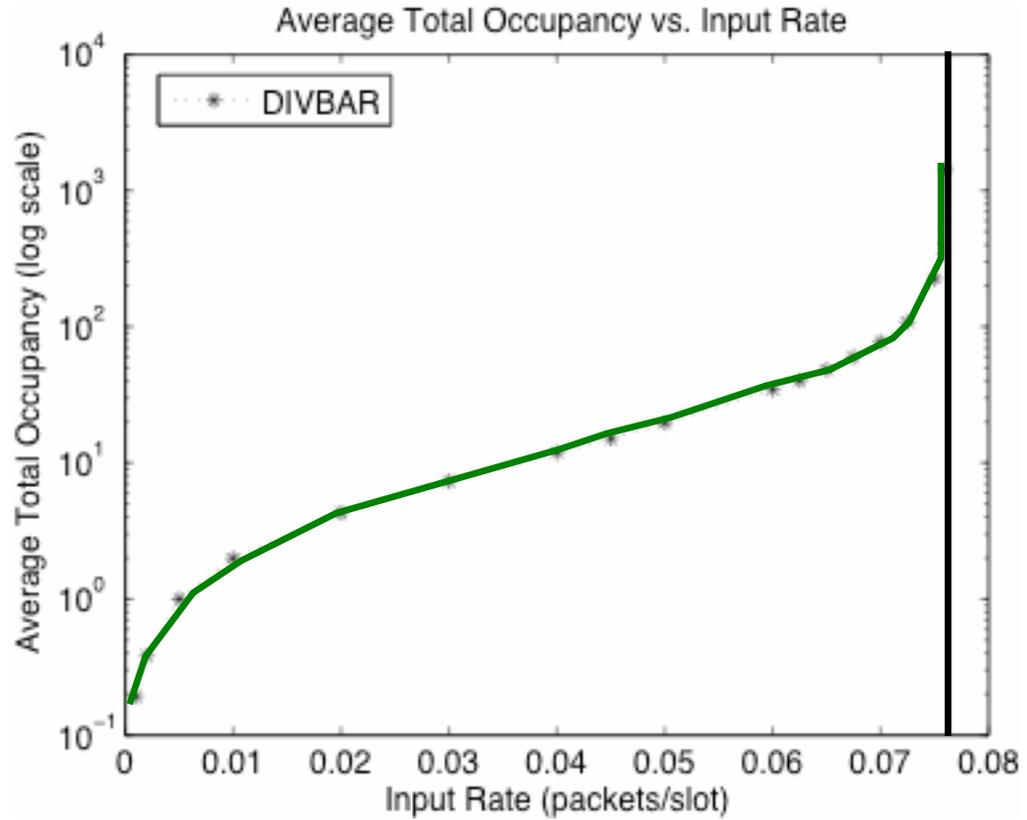
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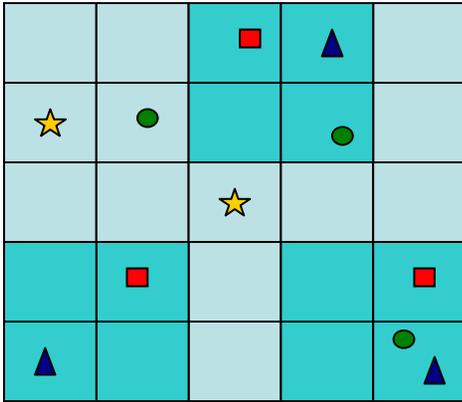
★ = Sink



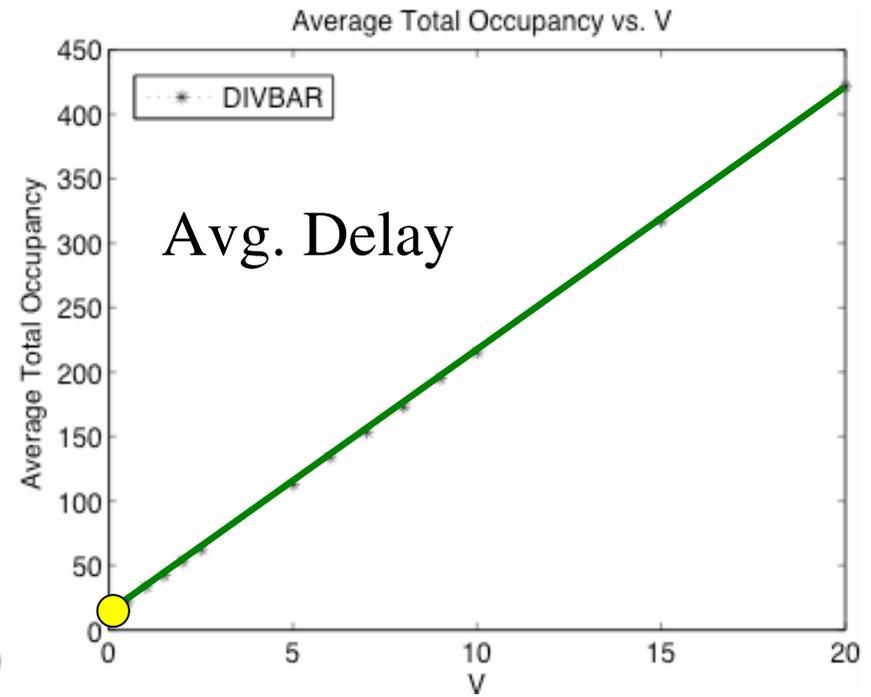
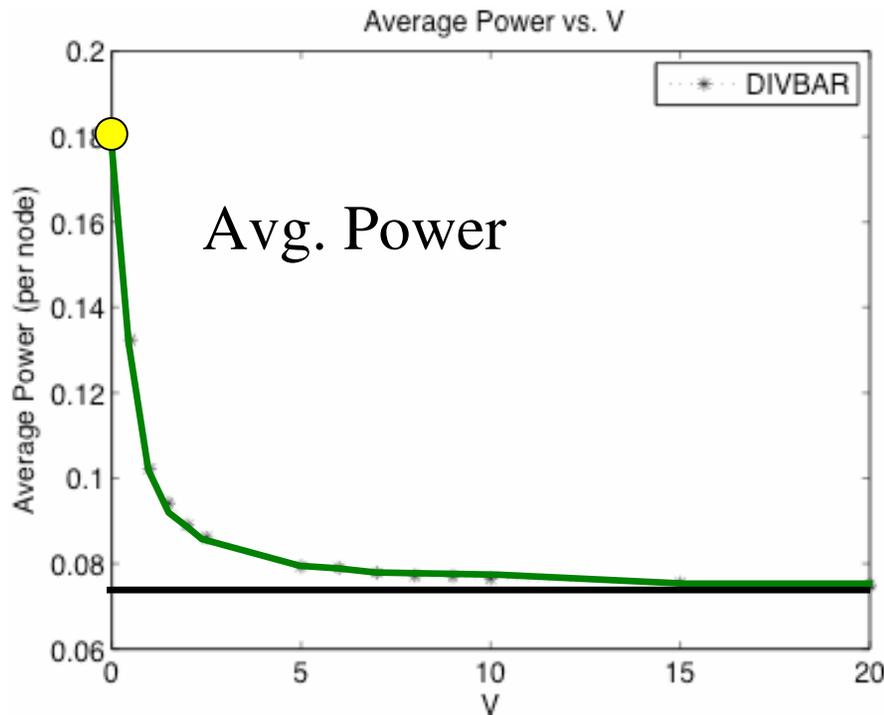


# Average Total Backlog Versus Input rate for Mobile DIVBAR

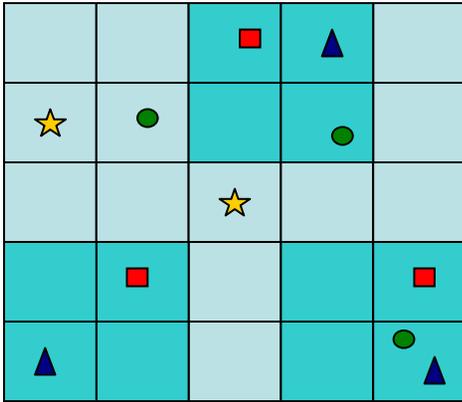




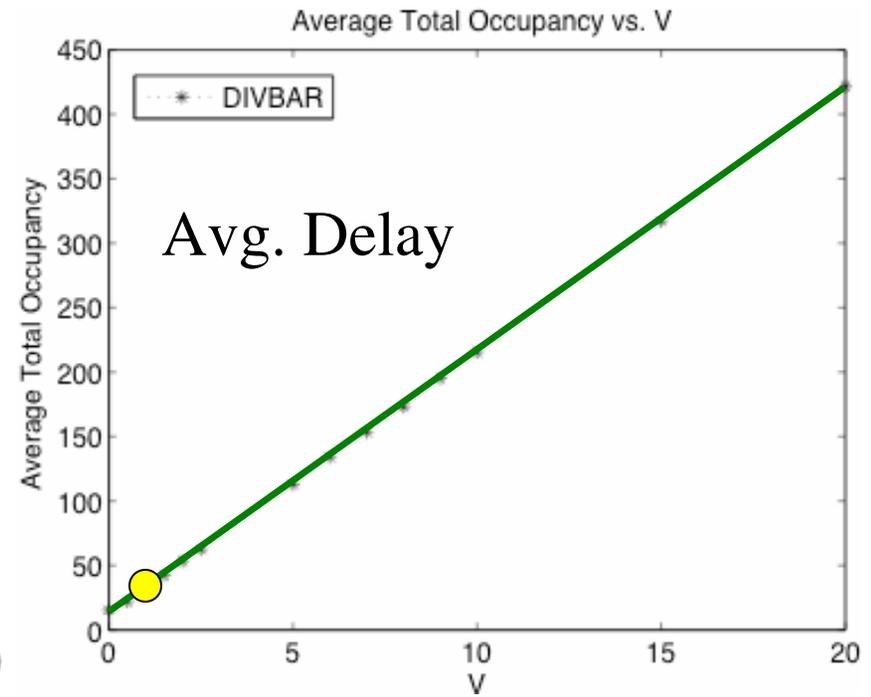
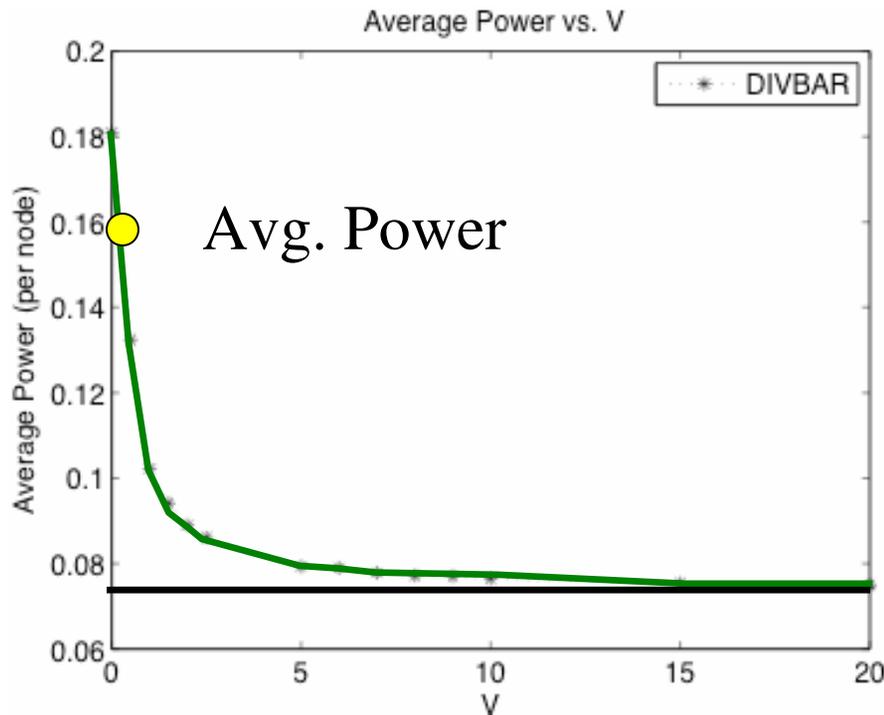
## Average Power Versus Delay (Fix a set of transmission rates for each node)



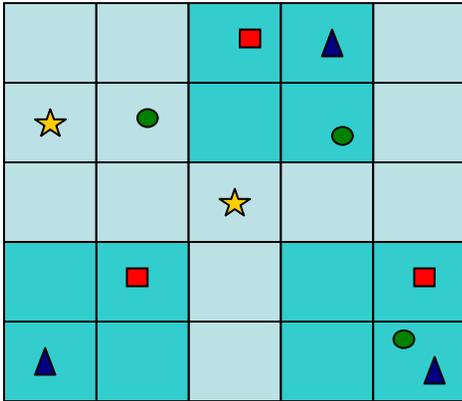
Performance-Delay Tradeoff:  $[O(1/V), O(V)]$



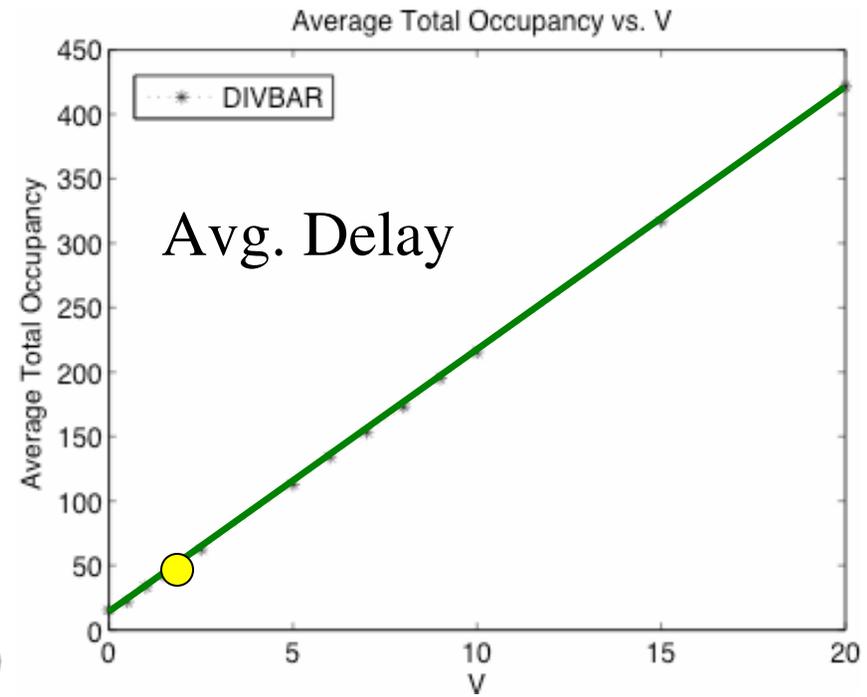
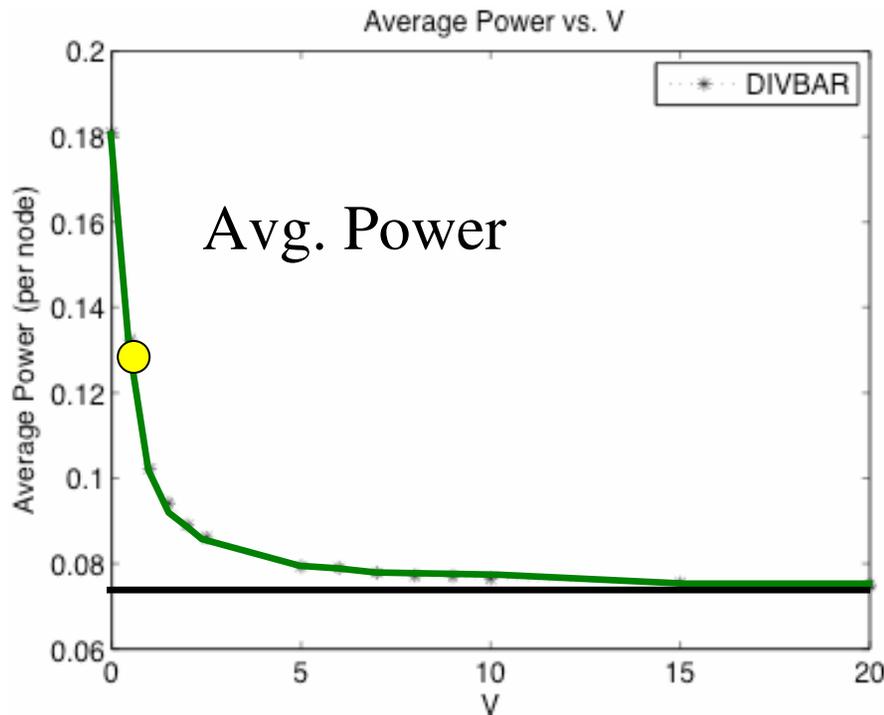
## Average Power Versus Delay (Fix a set of transmission rates for each node)



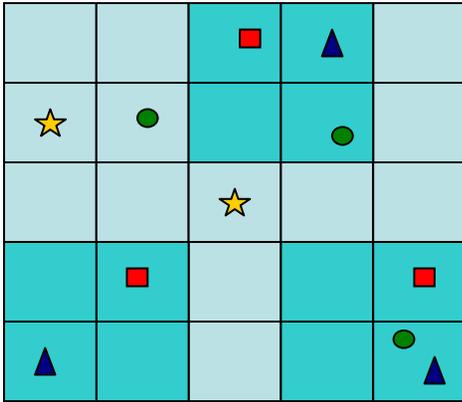
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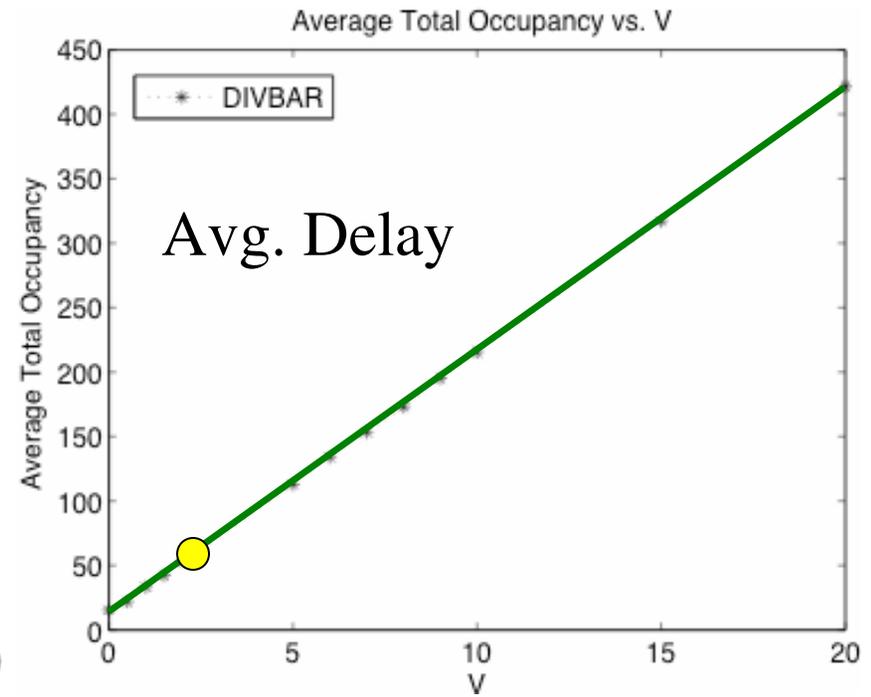
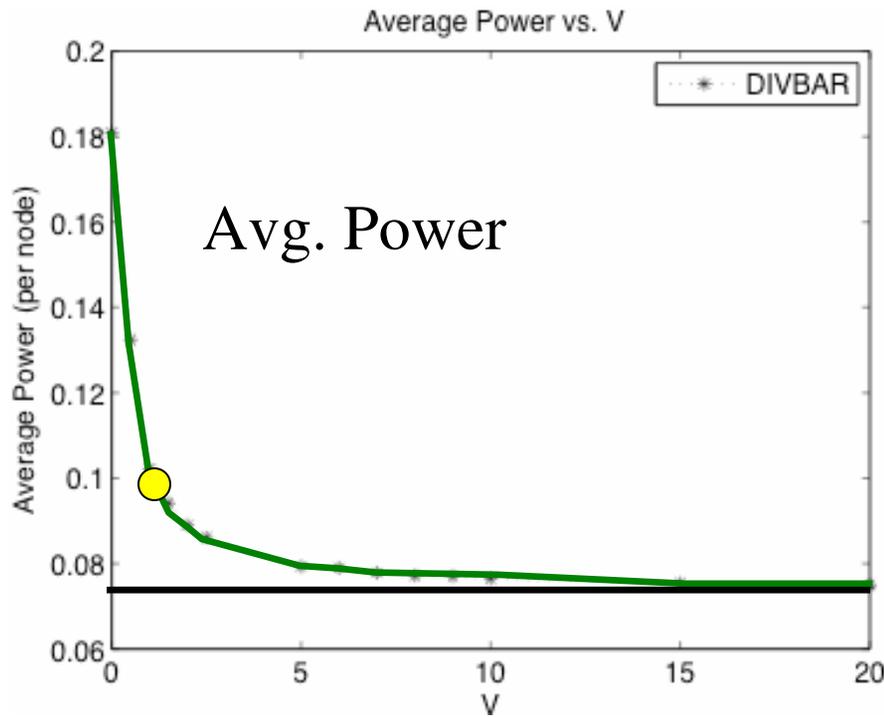
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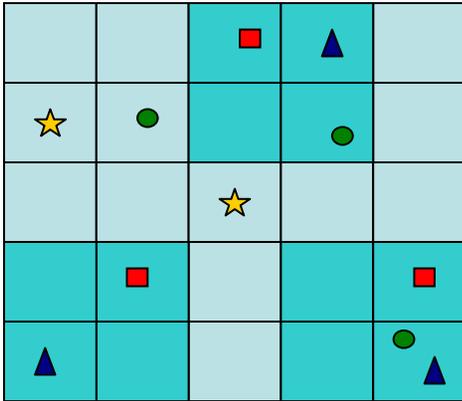
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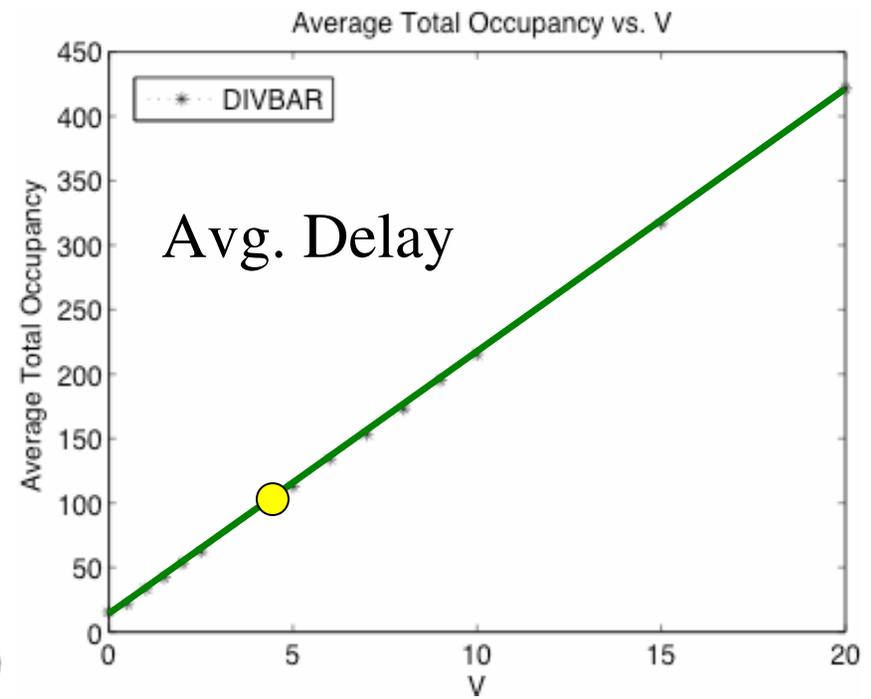
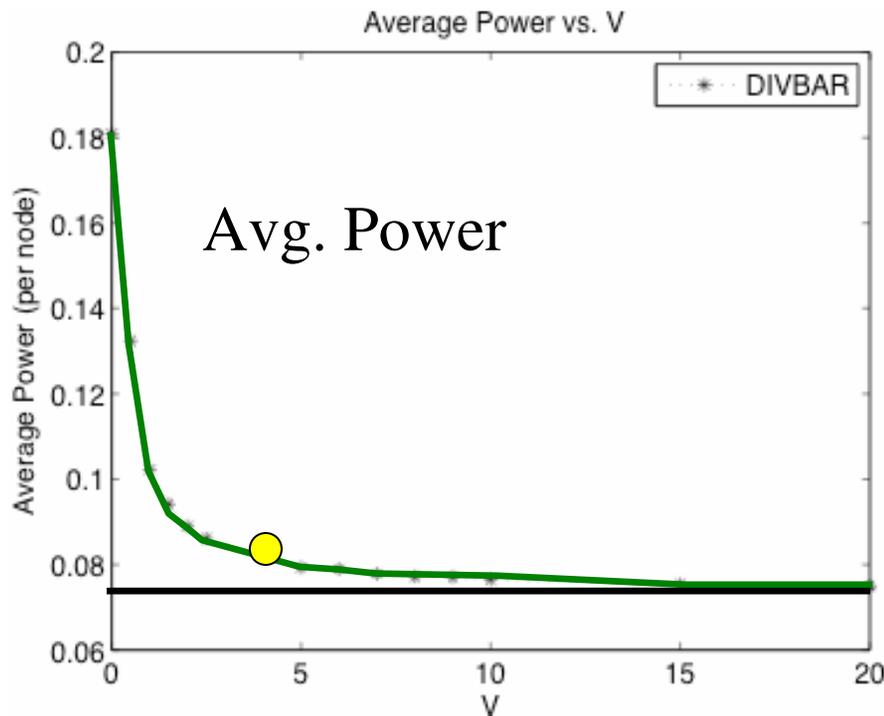
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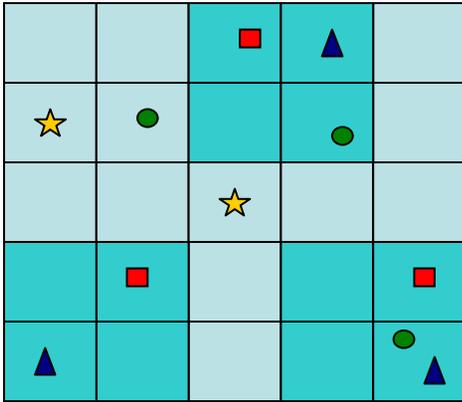
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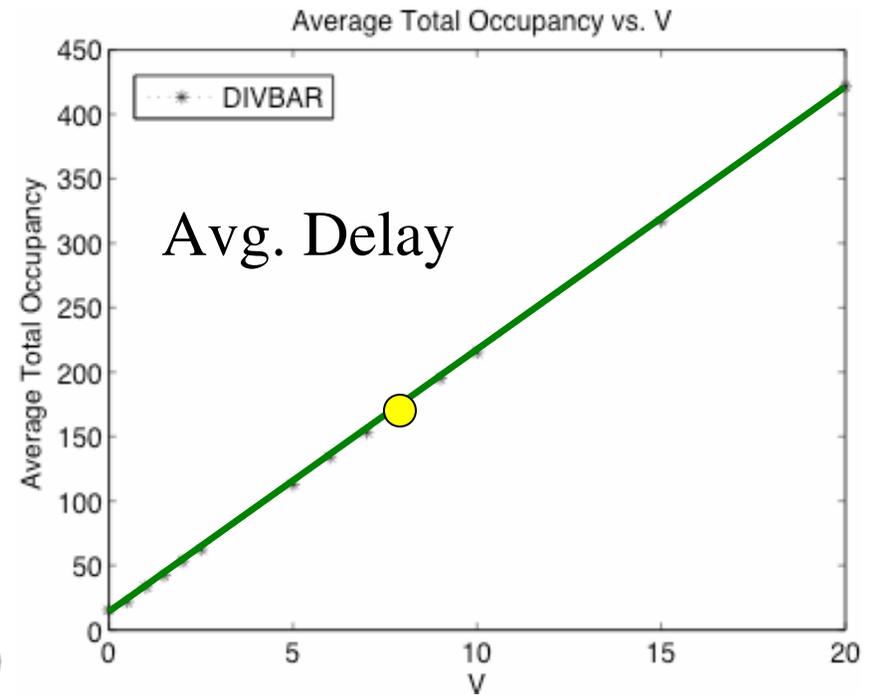
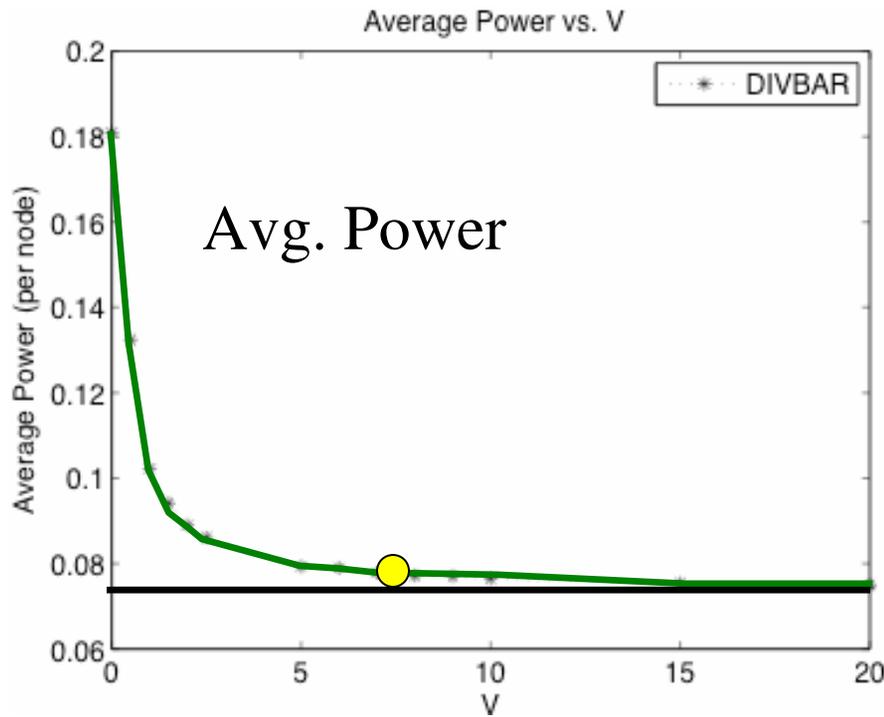
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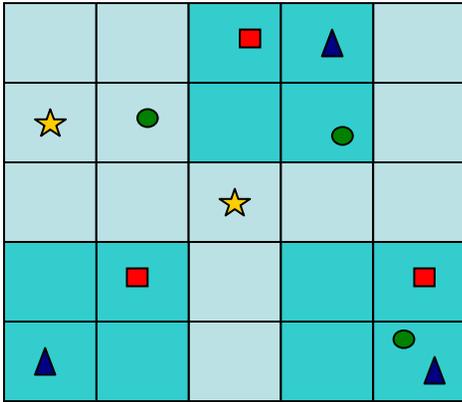
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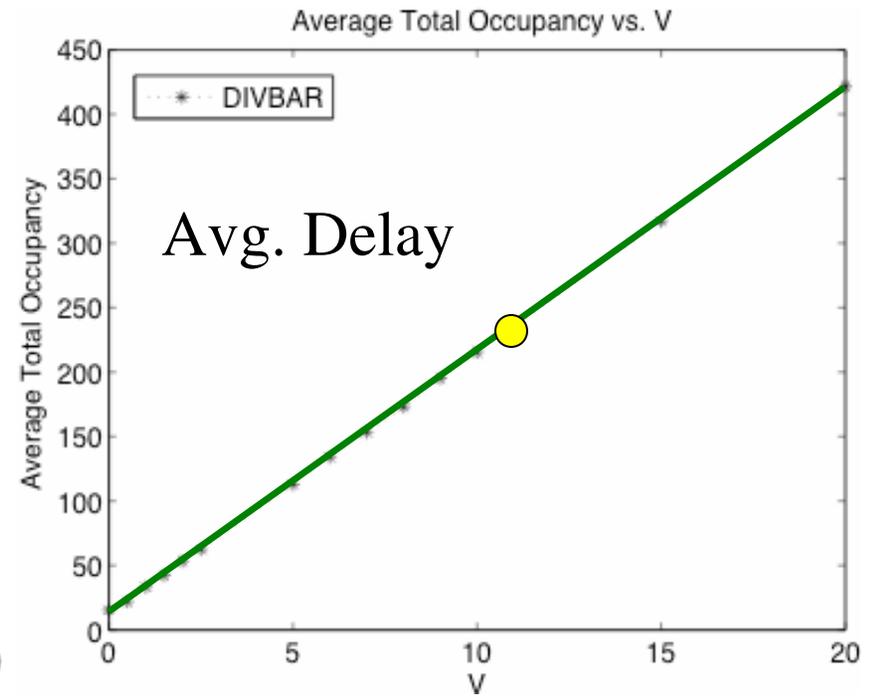
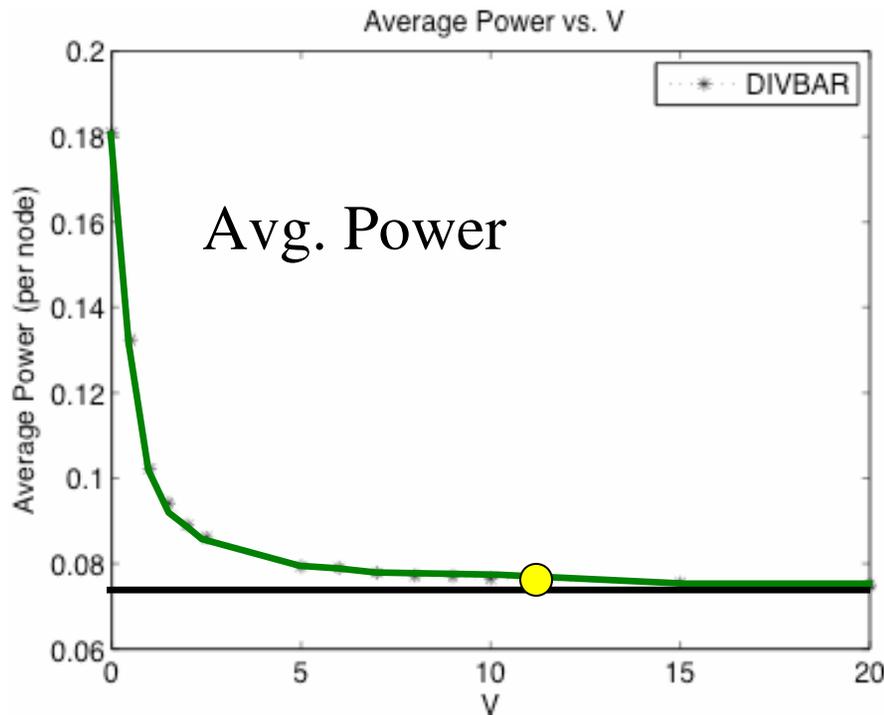
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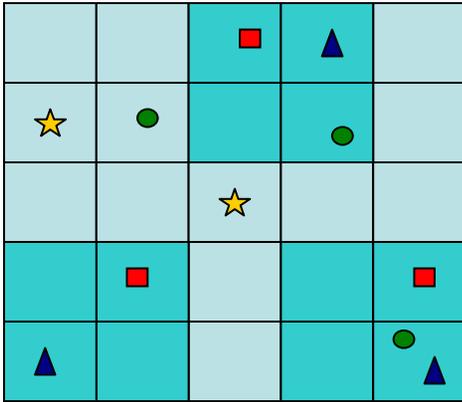
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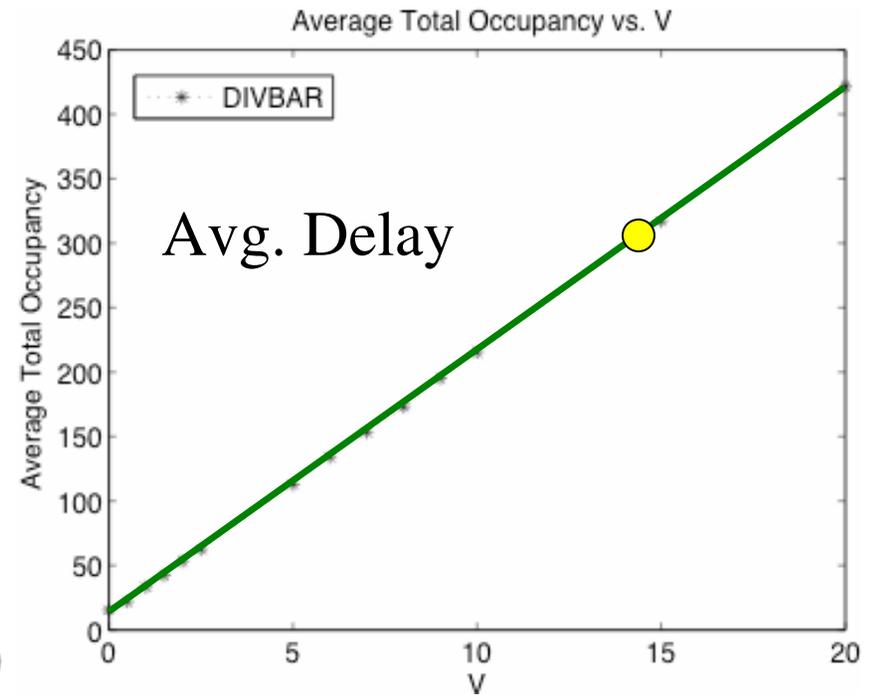
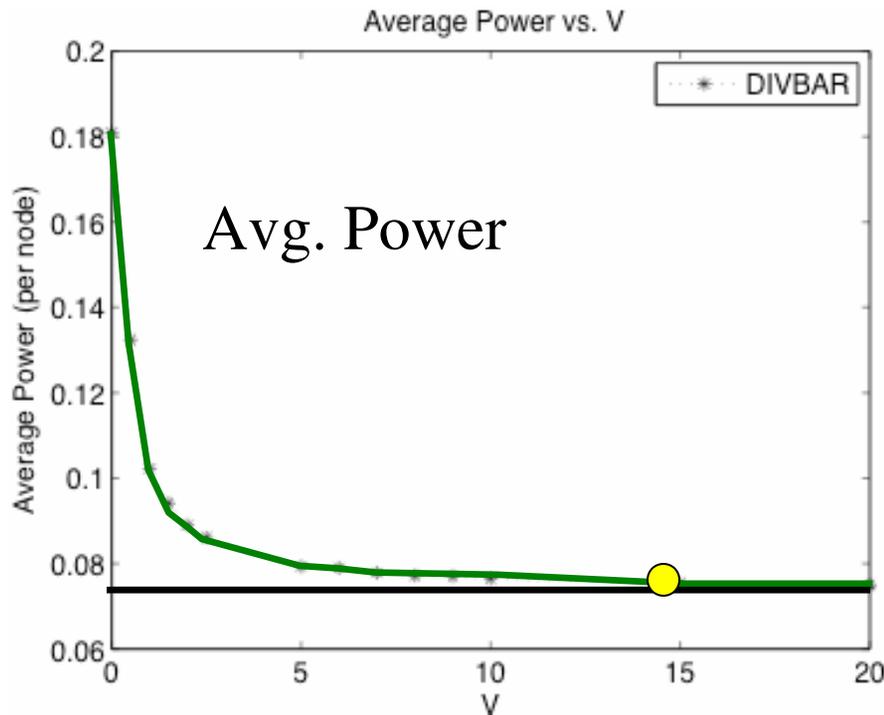
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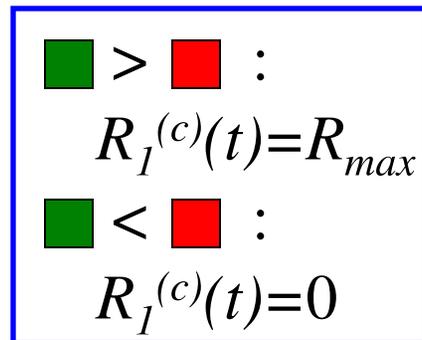
DIVBAR can easily be integrated with other cross-layer performance objectives using stochastic Lyapunov optimization, using techniques of *Virtual Power Queues*, *Auxiliary Variables*, *Flow State Queues* developed in:

Flow Control, Fairness, Energy:

[Neely, Modiano 2003, 2005] (fairness, stochastic utility opt.)

[Neely Infocom 2005] (energy optimal control)

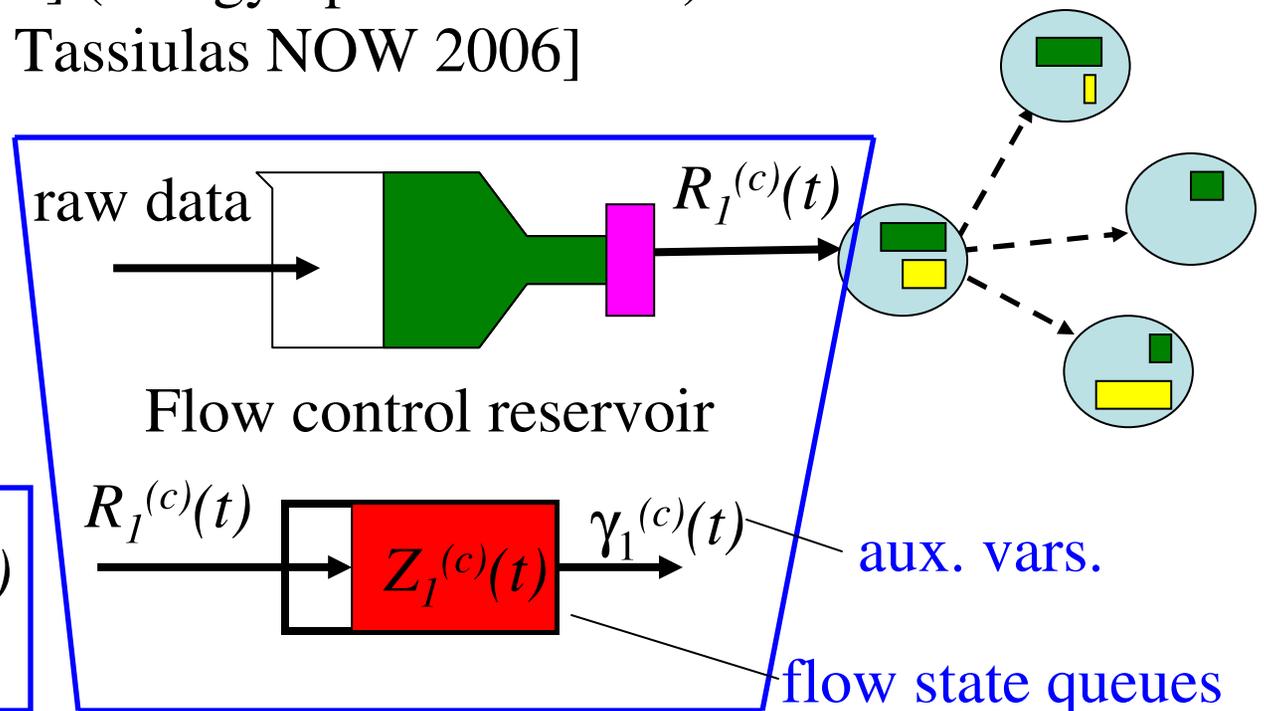
[Georgiadis, Neely, Tassiulas NOW 2006]



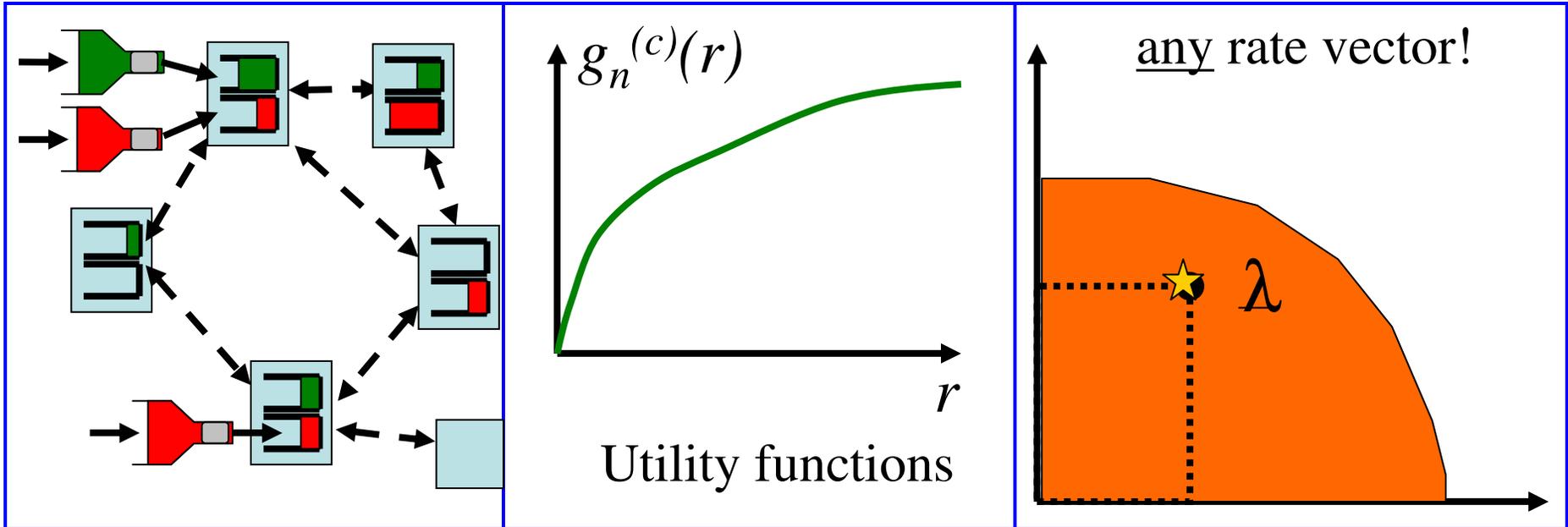
max:

$$Vg_1^{(c)}(\gamma_1^{(c)}) - \gamma_1^{(c)}Z_1^{(c)}(t)$$

$$0 \leq \gamma_1^{(c)} \leq R_{max}$$

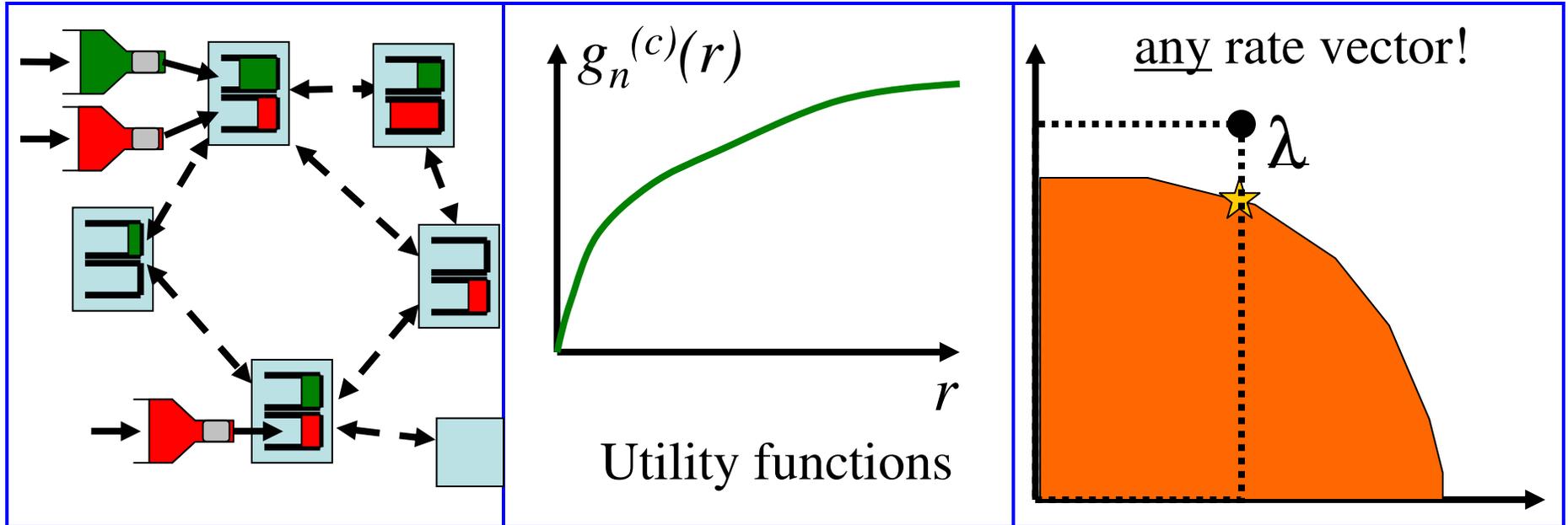


# Flow Control + DIVBAR (similar to Neely, Modiano 03, 05)



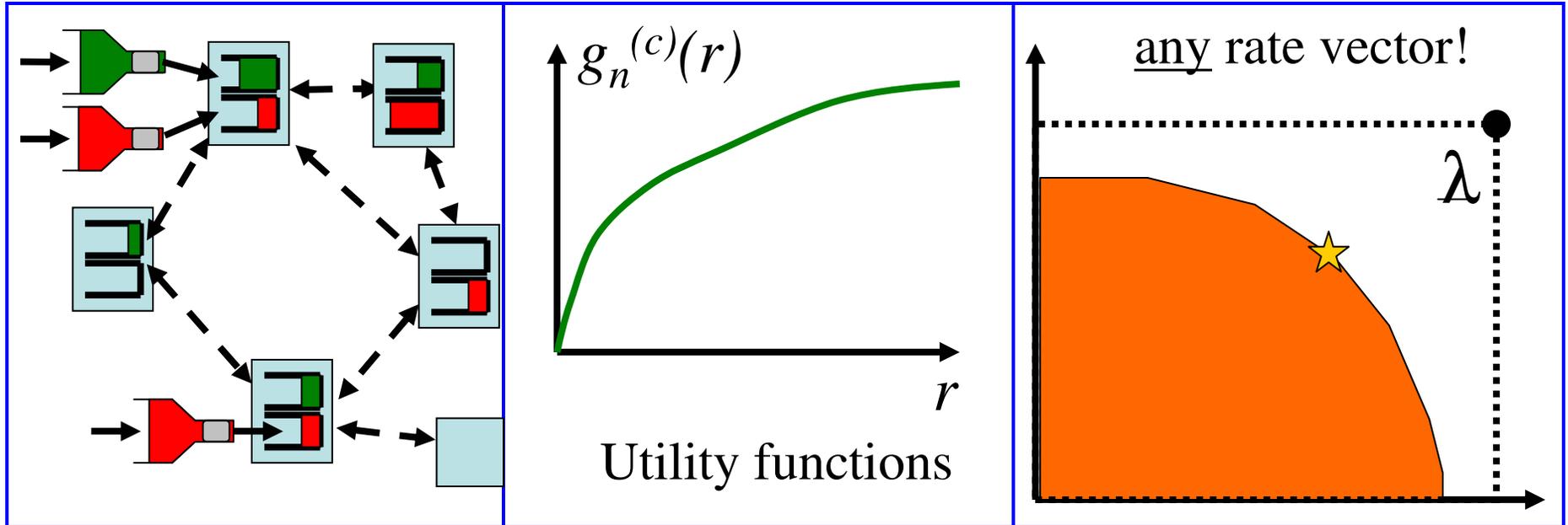
Achieves:  $[O(1/V), O(V)]$  utility-delay tradeoff!

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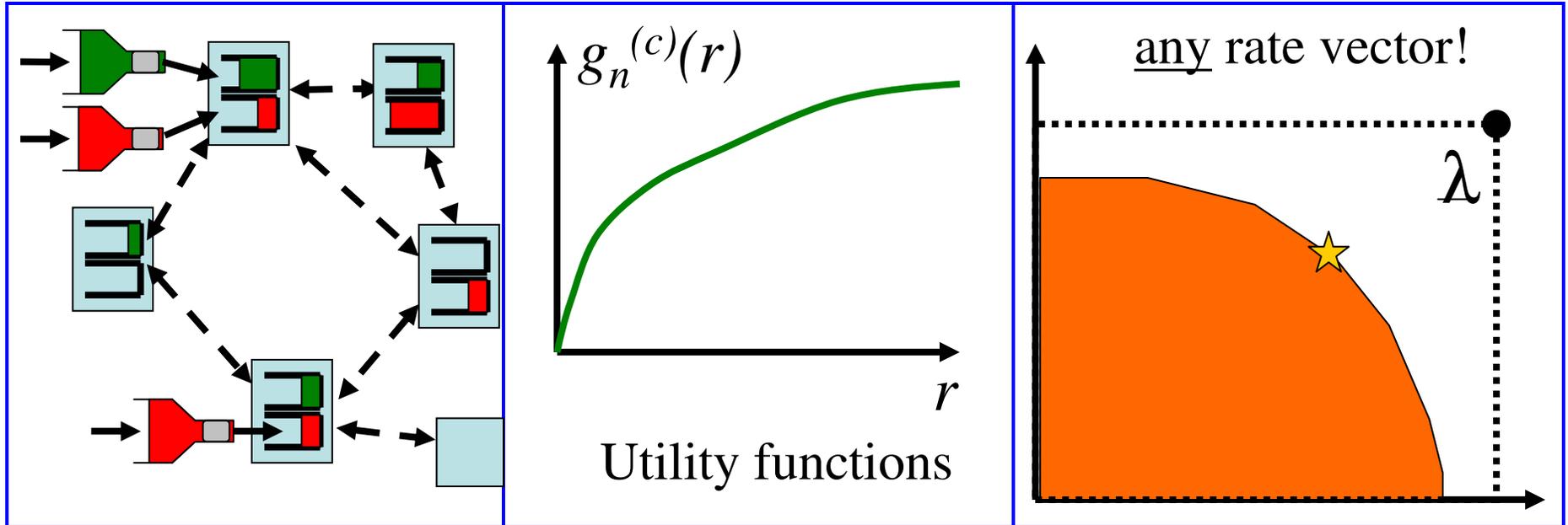
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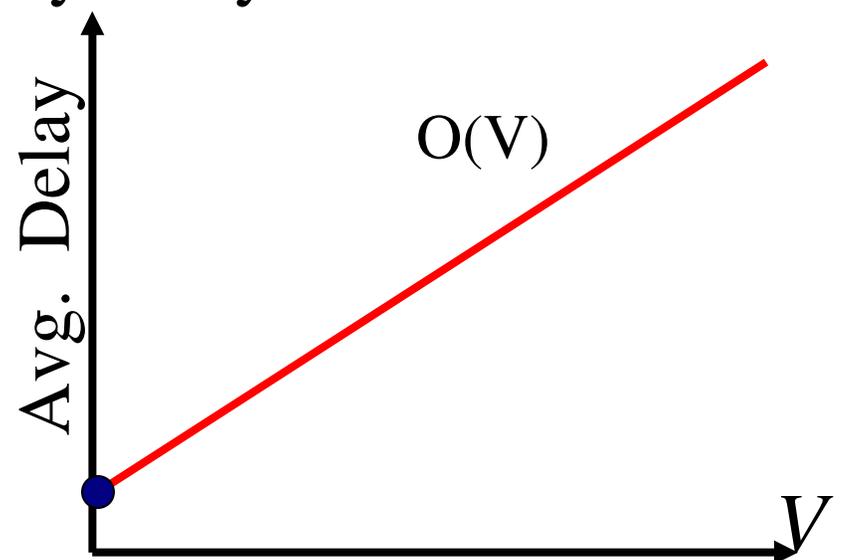
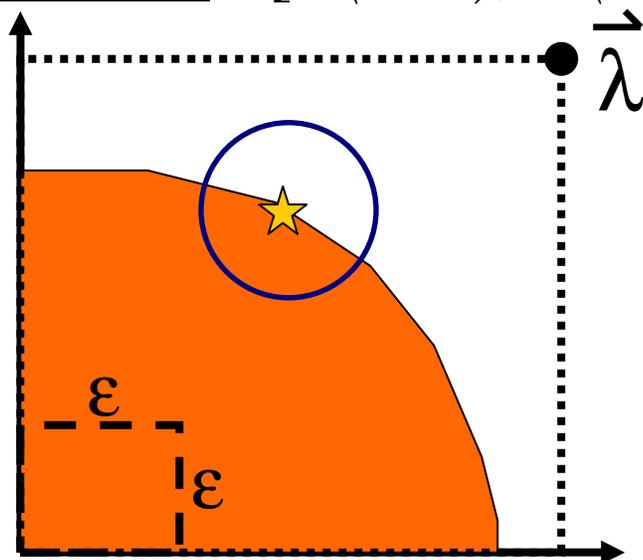


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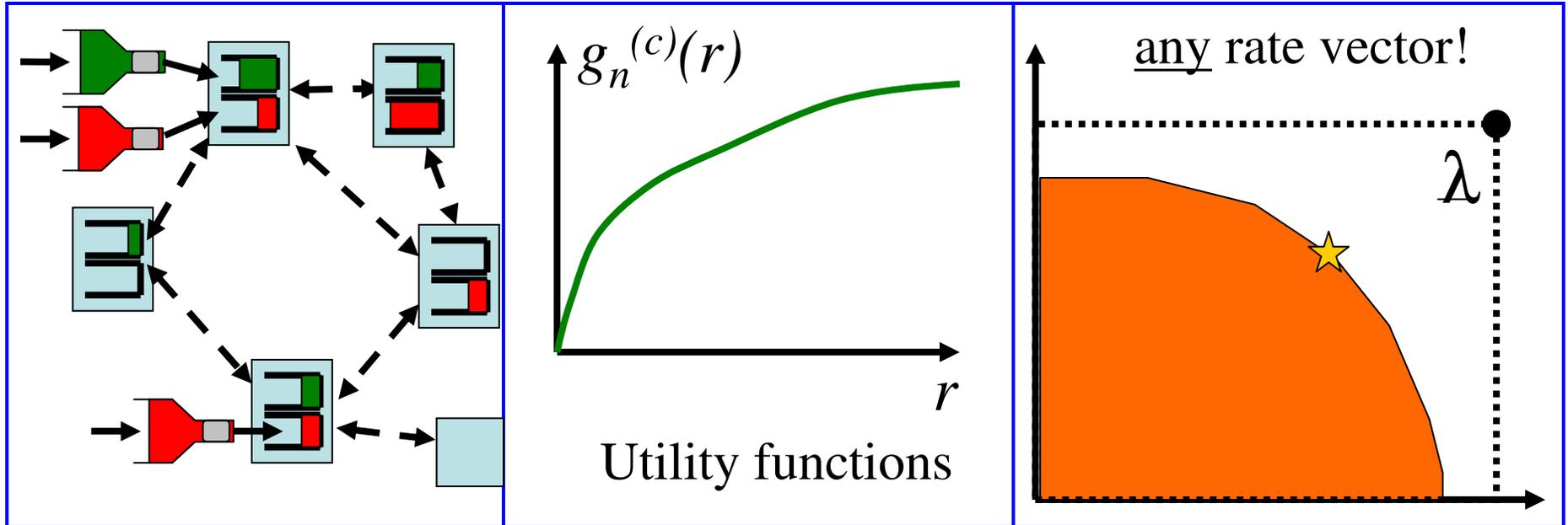
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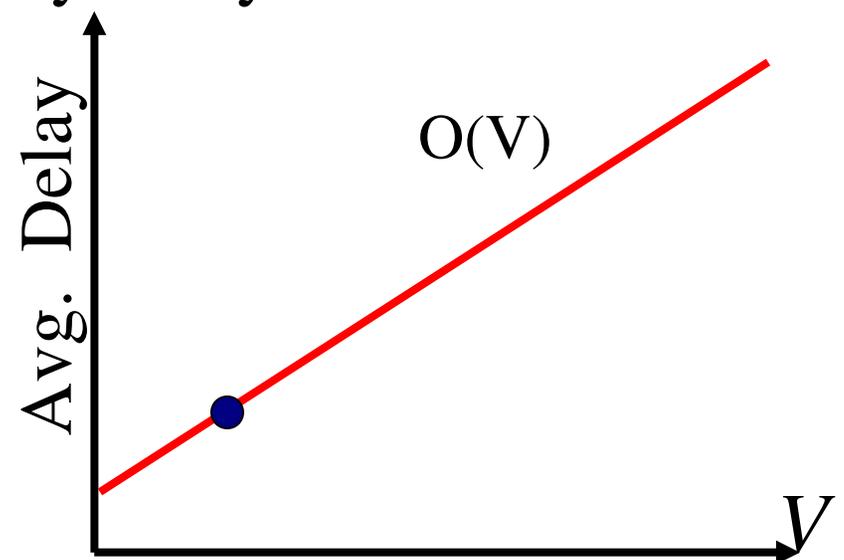
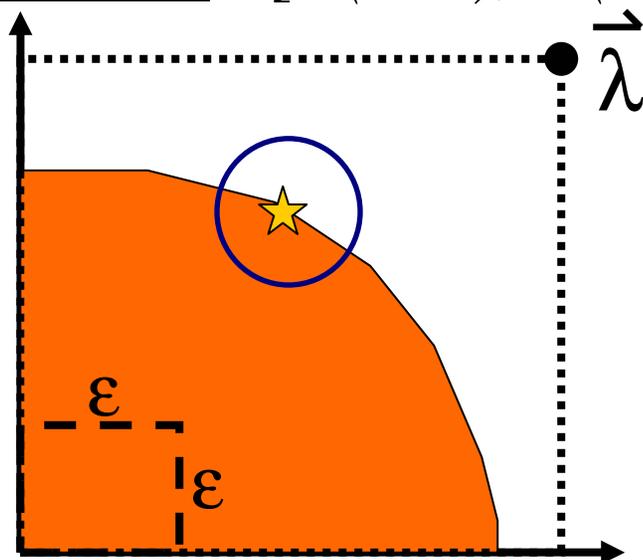
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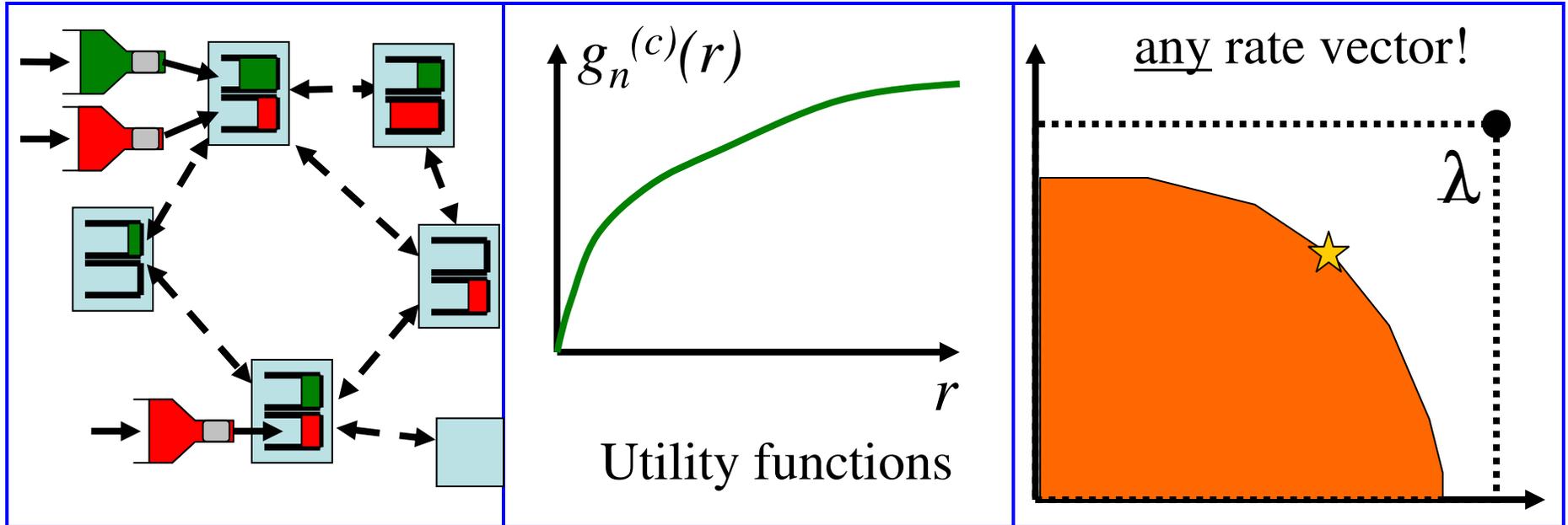
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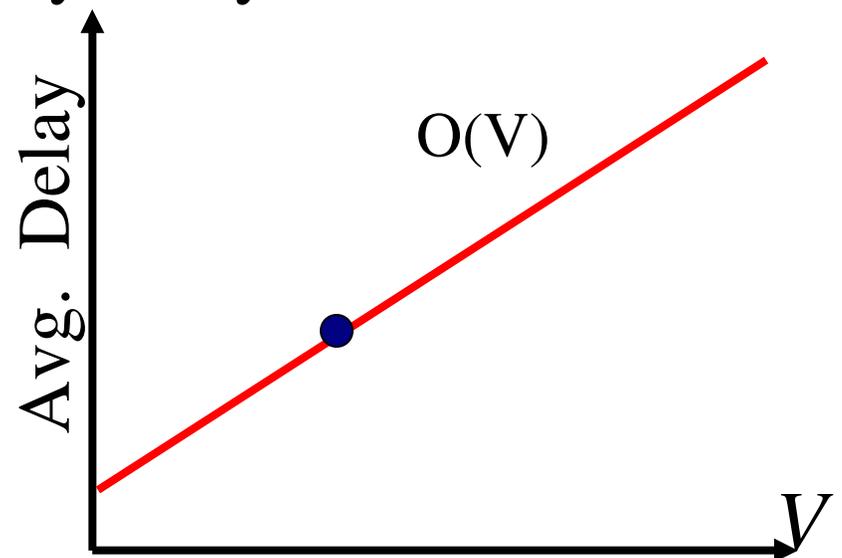
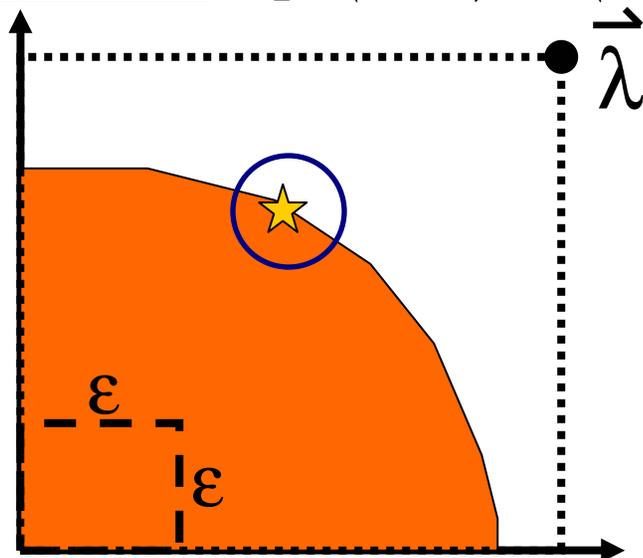
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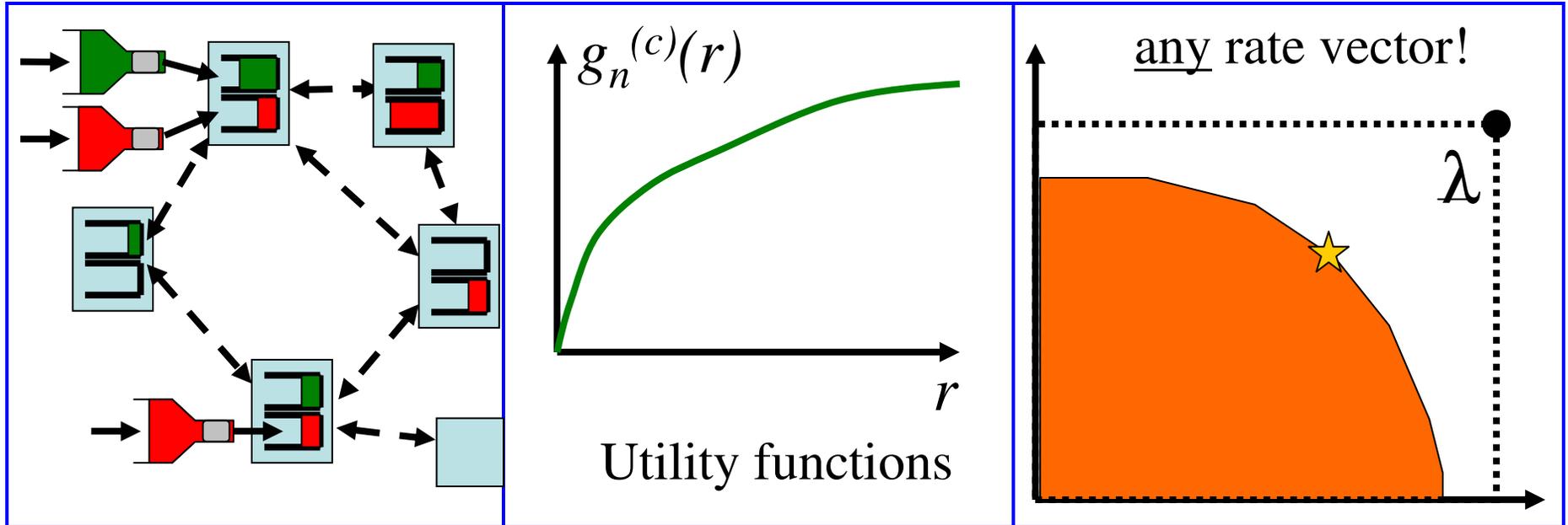
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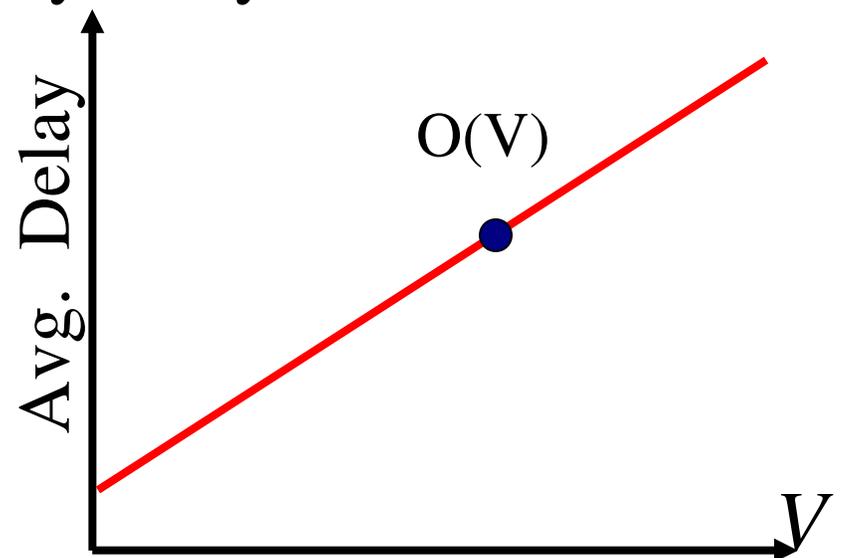
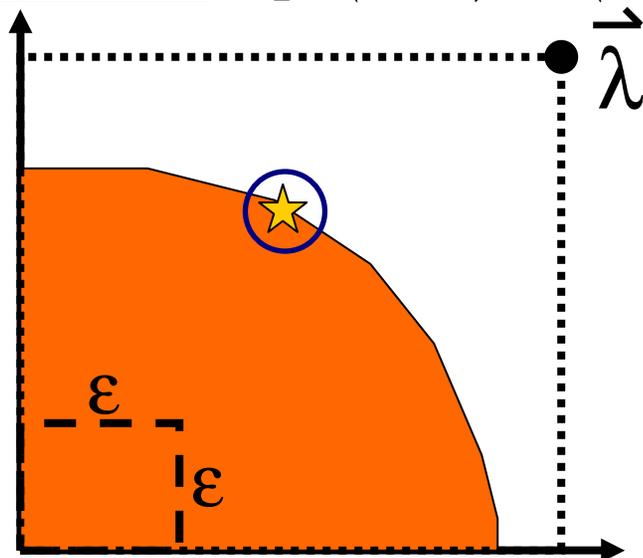
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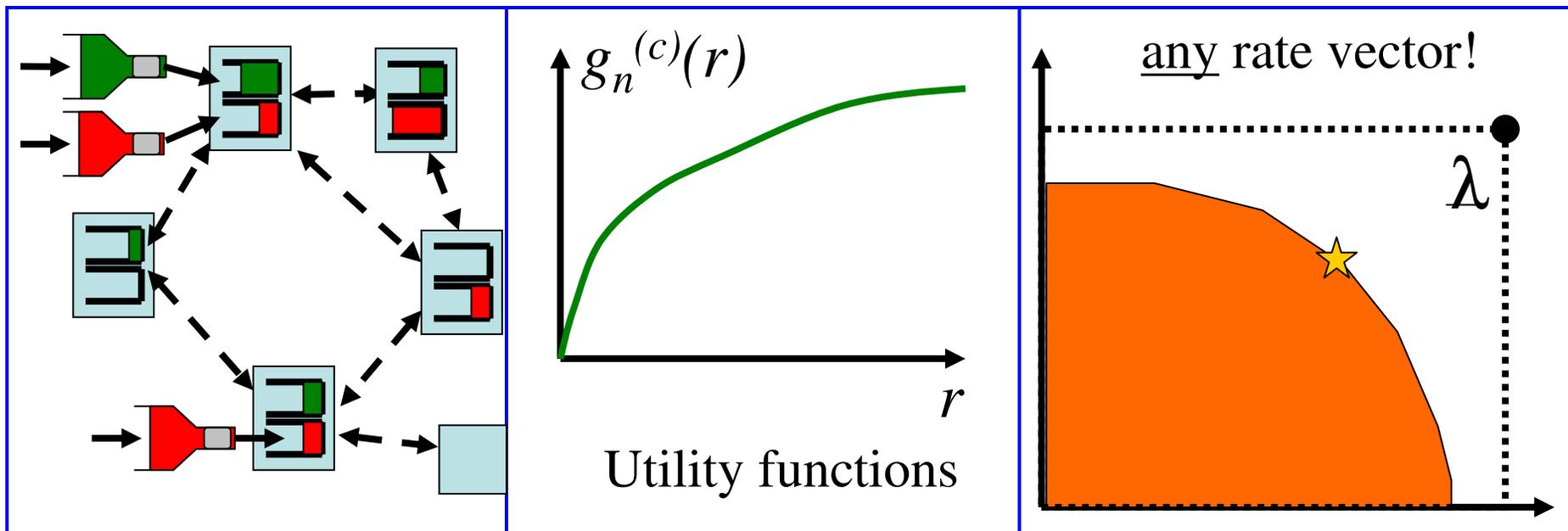
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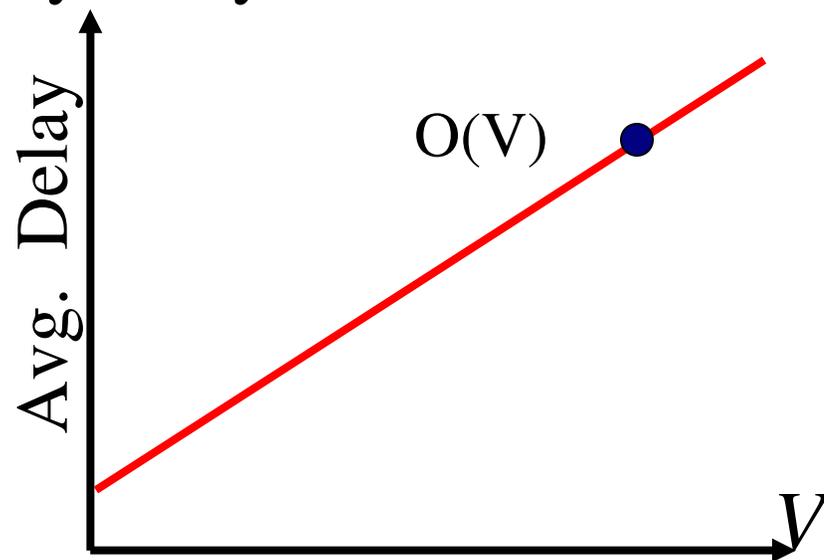
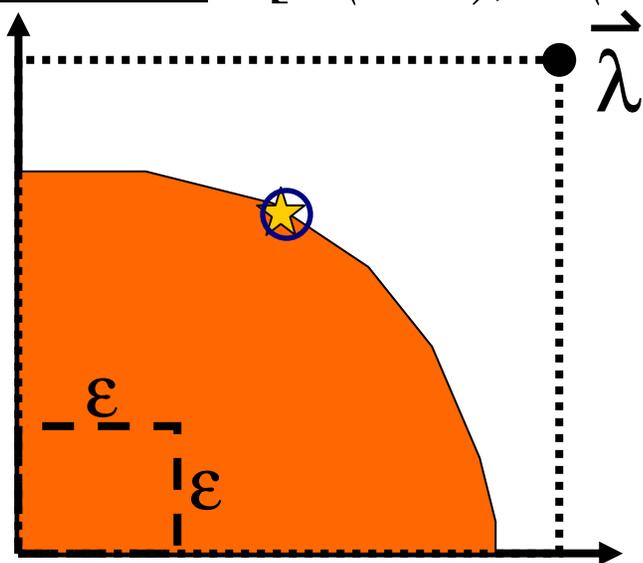
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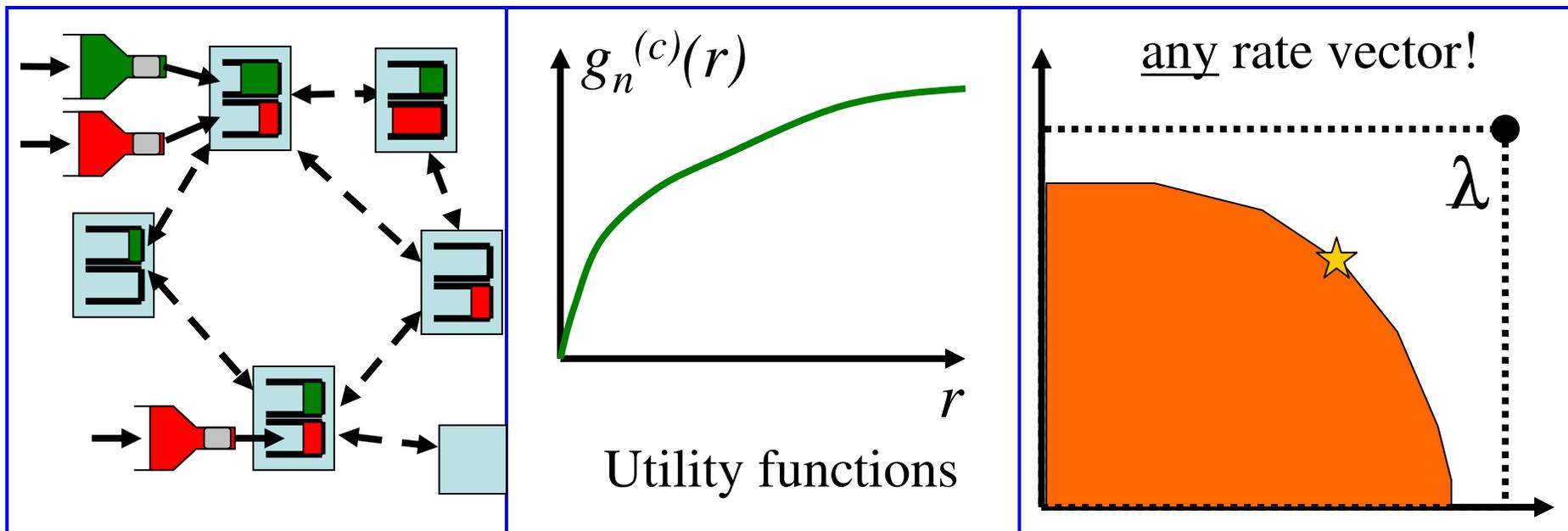
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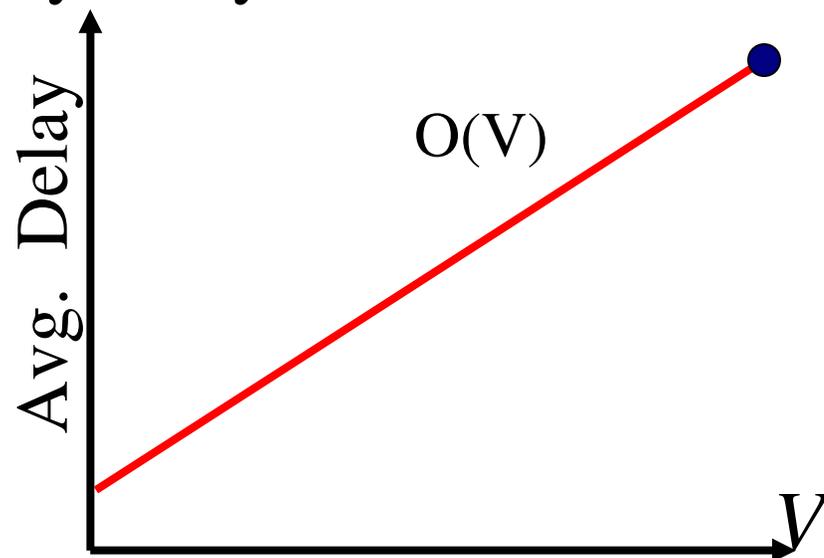
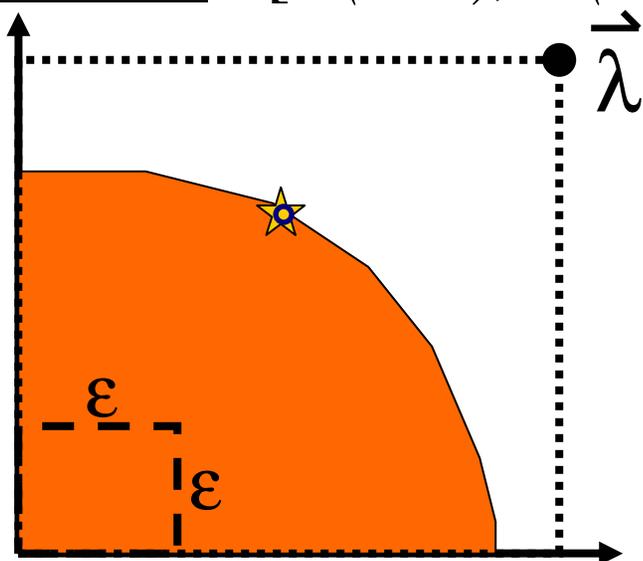
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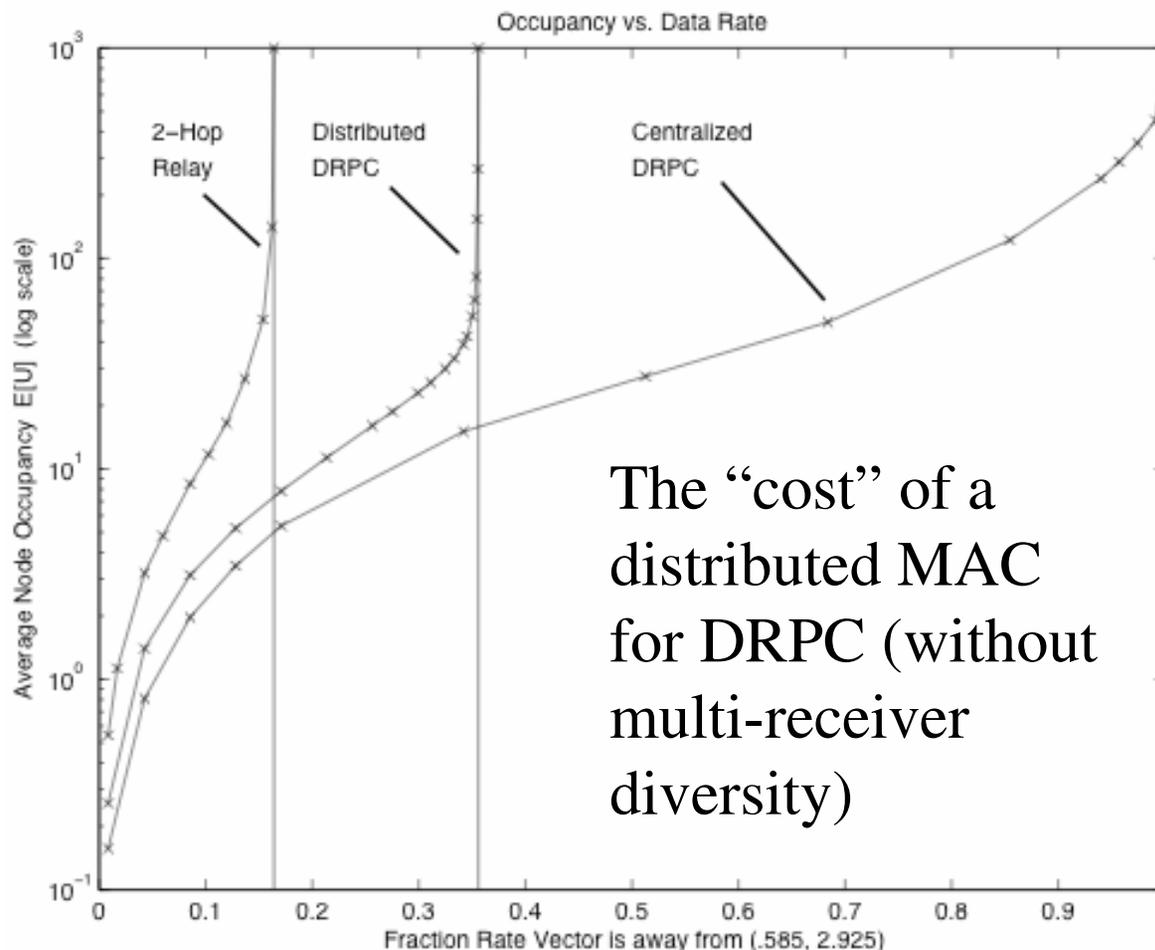


DIVBAR also works for:

-Non-i.i.d. arrivals and channel states

-Distributed MAC via Random Access

(similar to analysis in Neely 2003, JSAC 2005)



(DRPC  
Alg. Of  
JSAC 2005)