Numerical Methods for Image Registration

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Motivation

Image Registration

Given a reference image $\mathcal{R}$ and a template image $\mathcal{T}$,

find a reasonable transformation y, such that

the transformed image $\mathcal{T}[y]$ is similar to $\mathcal{R}$

reference $\mathcal{R}$

template $\mathcal{T}$

transformed template $\mathcal{T}[y]$

Motivation

Image Registration

Given a reference image $\mathcal{R}$ and a template image $\mathcal{T}$,

find a reasonable transformation y, such that

the transformed image $\mathcal{T}[y]$ is similar to $\mathcal{R}$

Questions:

- What is a transformed image $\mathcal{T}[y]$? $\rightsquigarrow$ image model $\mathcal{T}[y]$
- What is similarity of $\mathcal{T}[y]$ and $\mathcal{R}$? $\rightsquigarrow$ $\mathcal{D}[\mathcal{T}[y],\mathcal{R}]$
- What is reasonability of y? $\rightsquigarrow$ $\mathcal{S}[y]$

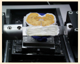
Image Registration: Variational Problem

$$\mathcal{D}[\mathcal{T}[y],\mathcal{R}] + \mathcal{S}[y - y_{\text{reg}}] \xrightarrow{y} \min, \qquad y_{\text{reg}}(x) = x$$

Outline

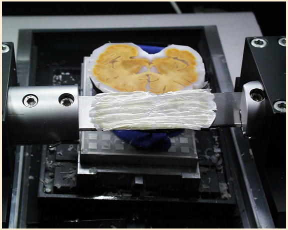
- Applications
- Variational formulation $\mathcal{D}[\mathcal{T}[y],\mathcal{R}]+\mathcal{S}[y]\xrightarrow{y}\min$
  - image models $T[y]$
  - distance measures $\mathcal{D}[T[y],R]$
  - regularizer $\mathcal{S}[y]$
- Numerical methods
- Constrained image registration
- Conclusions

Applications



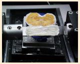
HNSP: Sectioning

with Oliver Schmitt,
Institute of Anatomy, University Rostock, Germany

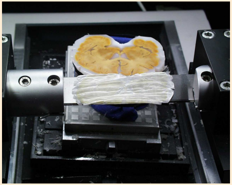


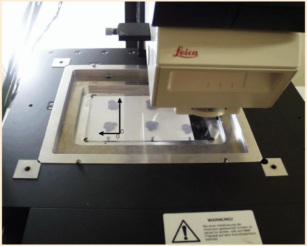
- sliced
- flattened
- stained
- mounted
- ...
- digitized

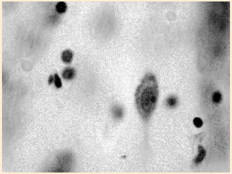
large scale digital images, up to 10.000×20.000 pixel

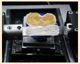


HNSP: Microscopy



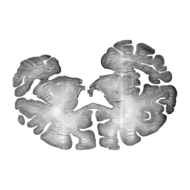




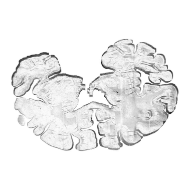
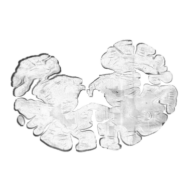


HNSP: Deformed Images

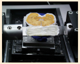
sections 3.799 and 3.800 out of about 5.000




human | affine linear | elastic

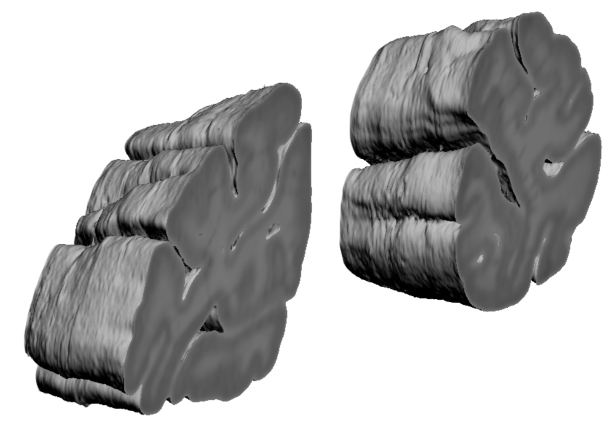



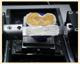

$|T_{\text{orig}} - R| = 100\%$ $|T_{\text{linear}} - R| = 72\%$ $|T_{\text{elastic}} - R| = 50\%$



HNSP: Results

3D elastic registration of a part of the visual cortex
(two hemispheres; 100 sections á 512×512 pixel)



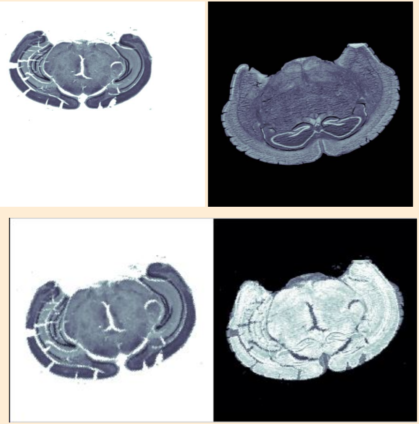


Multi-Modal Registration

with Stefan Heldmann, SAFIR, University of Lübeck

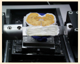
template

reference



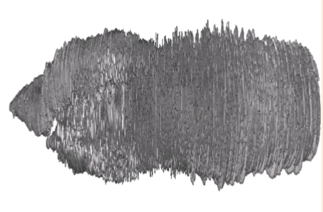
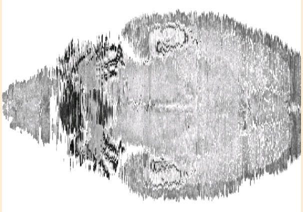
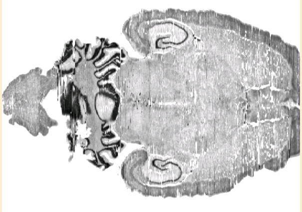
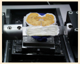
Curvature+MI

difference



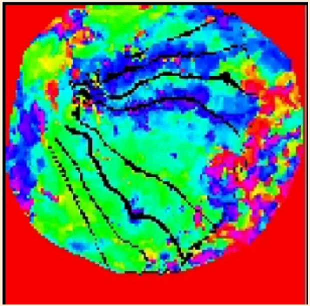
Registration of *Sprague-Dawley* Brain

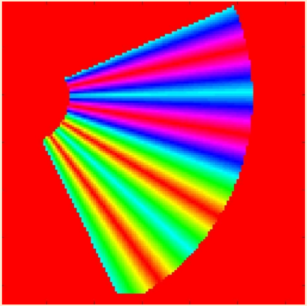
with Stefan Wirtz, SAFIR, University of Lübeck

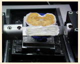
Neuroimaging (fMRI)

with Brian A. Wandell, Department of Psychology,
Stanford Vision Science and Neuroimaging Group



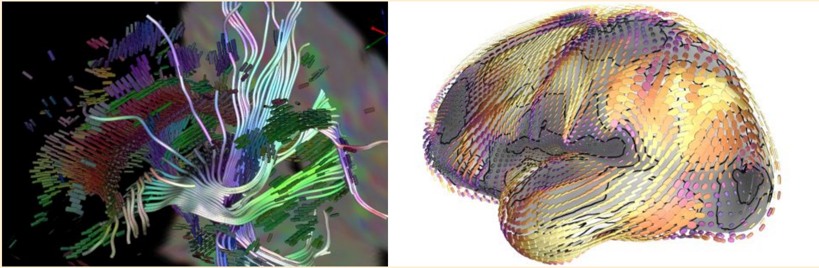


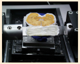
“flattened visual cortex”



DTI: Diffusion Tensor Imaging

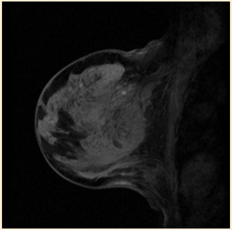
with Brian A. Wandell, Department of Psychology, Stanford Vision Science and Neuroimaging Group



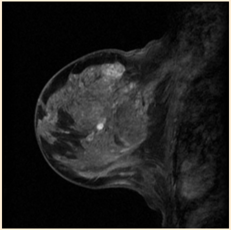


MR-mammography, biopsy (open MR)

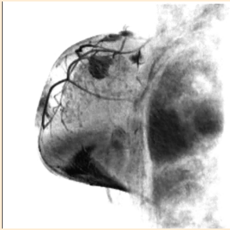
with Bruce L. Daniel,
Department of Radiology, Stanford University



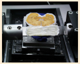
pre contrast



post contrast

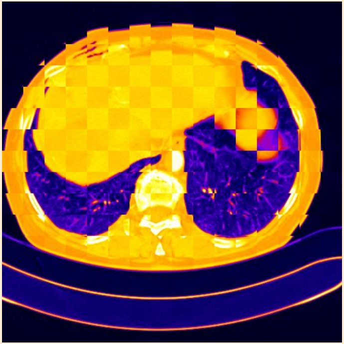


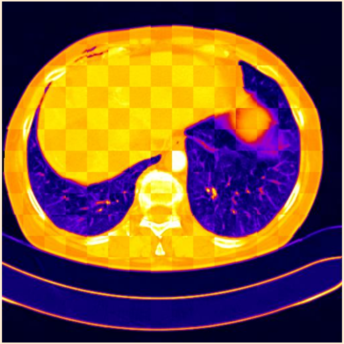
3D

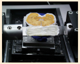


Liver Registration

with Stefan Wirtz, SAFIR, University of Lübeck
& Siemens Erlangen, Germany



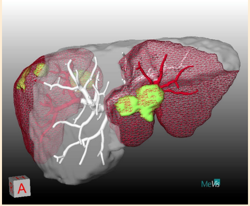


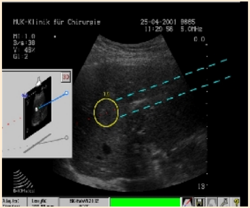


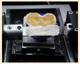
Virtual Surgery Planning

S. Bommersheim & N. Papenberg, SAFIR, BMBF/FUSION

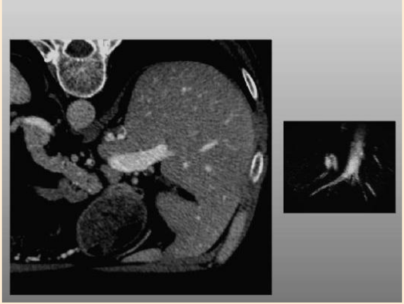
Future Environment for Gentle Liver Surgery Using Image-Guided Planning and Intra-Operative Navigation

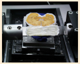






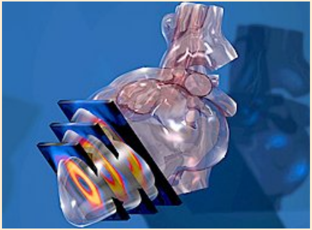
Results for 3D US/CT

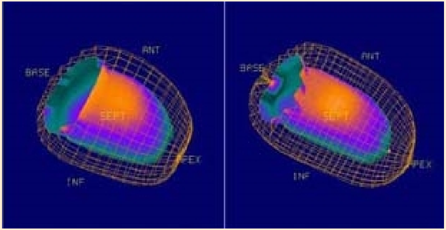


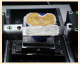


Cardiac Images

with Tracy Faber, Department of Radiology,
Emory University, Atlanta,USA

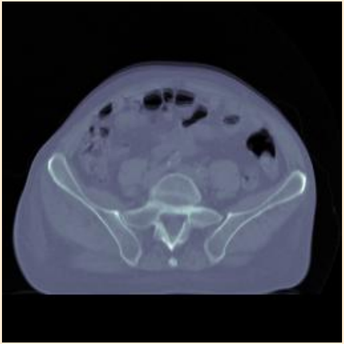


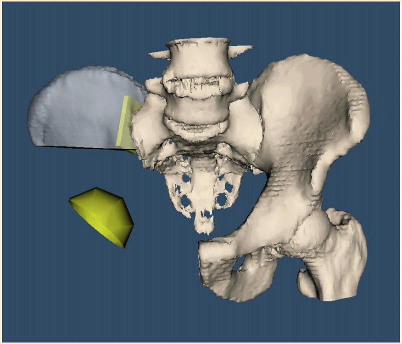


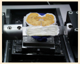


Virtual Surgery Planning

from Jan Ehrhardt,
Medical Computer Science, University of Hamburg

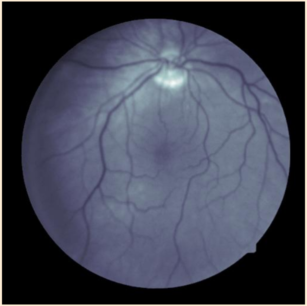




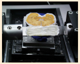


Laser Surgery

from Joachim Noack,
Medical Laser Center Lübeck

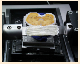


(Macula pucker, warped retina)



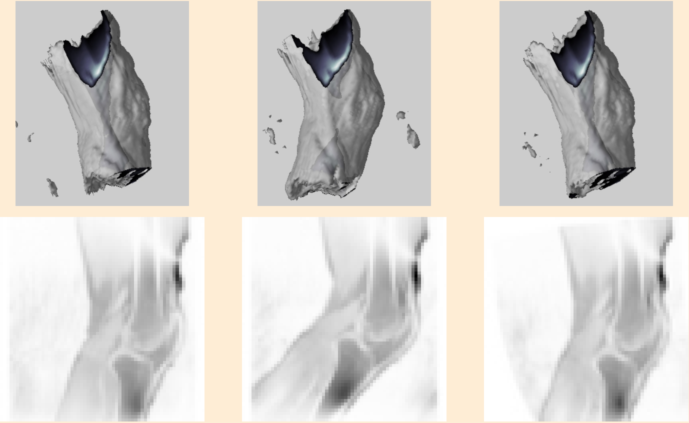
Motion Correction

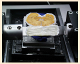
from Thomas Netsch,
Philips Research, Hamburg, Germany



Human Knee, 3D

from Astrid Franz, Philips Research, Hamburg, Germany

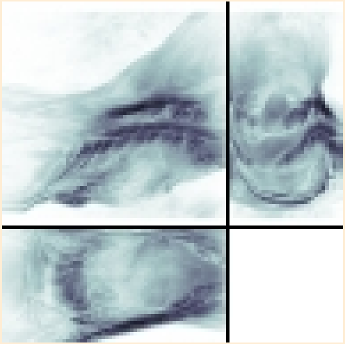




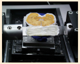
Human Knee, 3D

Maximum Intensity Projections

MIP$(R - T)$

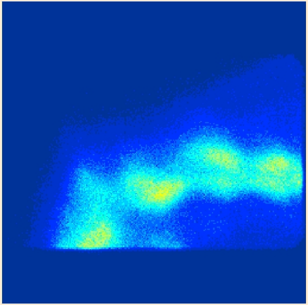


MIP$(R - T(y))$

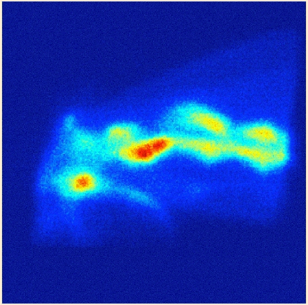


SPECT: Single Photon Emissions CT

with Oliver Mahnke, SAFIR, University of Lübeck
& MiE GmbH, Seth, Germany

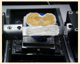


image



registered

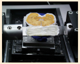
Lester



Registration in Medical Imaging

- Comparing/merging/integrating images from different
  - times, e.g., pre-/post surgery
  - devices, e.g., CT-images/MRI
  - perspectives, e.g., panorama imaging
  - objects, e.g., atlas/patient mapping
- Template matching, e.g., catheter in blood vessel
- Atlas mapping, e.g., find 2D view in 3D data
- Serial sectioning, e.g., HNSP
- . . .

Registration is not restricted to medical applications



Classification of Registration Techniques

- feature space
- search space
- search strategy
- distance measure

- dimensionality of images $(d = 2, 3, 4, \ldots)$
- modality (binary, gray, color, . . .)
- mono-/multimodal images
- acquisition (photography, FBS, CT, MRI, . . .)
- inter/intra patient

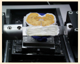
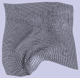


Image Registration

Transforming Images

$$\mathcal{D}[\mathcal{T}[y],\mathcal{R}] + \alpha\mathcal{S}[y - y_{\mathrm{reg}}] \xrightarrow{y} \min$$



Variational Approach for Image Registration

$$\mathcal{D}[\mathcal{T}[y], \mathcal{R}] + \alpha \mathcal{S}[y - y_{\text{reg}}] \xrightarrow{y} \min$$

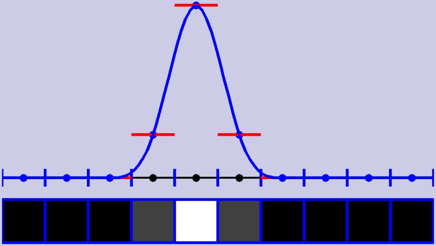
- Continuous models $\mathcal{R}, \mathcal{T}$ for reference and template:

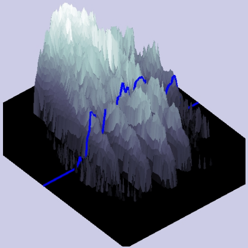
  discrete data $\mathtt{X}, \mathtt{T} \quad \rightsquigarrow \quad \mathcal{T}(x) = \text{interpolation}(\mathtt{X}, \mathtt{T}, x)$

- Transformation $y : \mathbb{R}^d \to \mathbb{R}^d$

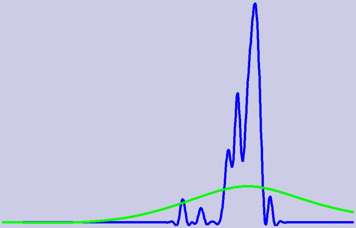
  $\mathcal{T}[y](x) = \mathcal{T}(y(x)) = \text{interpolation}(\mathtt{X}, \mathtt{T}, y(x))$

Interpolation

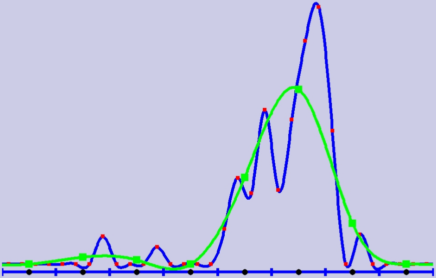




Multi-Scale



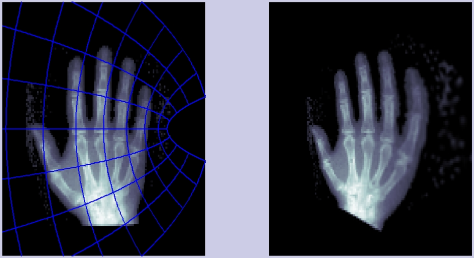
Multilevel



Transforming Images

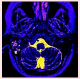
$$\mathcal{T}[y](x) = \mathcal{T}(y(x)) = \text{interpolation}(\mathtt{X}, \mathtt{T}, y(x))$$

non-linear



Distance Measures

$$\mathcal{D}[\,\mathcal{T}[y],\mathcal{R}\,] + \alpha\mathcal{S}[\,y - y_{\text{reg}}\,] \xrightarrow{y} \min$$



Distance Measures

- Feature Based (Markers / Landmarks / Moments / Localizer)

- L_2-norm, *Sum of Squared Differences (SSD)*

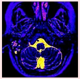
$$\mathcal{D}^{\mathrm{SSD}}[\,\mathcal{T}[y],\mathcal{R}\,] = \frac{1}{2}\int_\Omega [\,\mathcal{T}(y(x)) - \mathcal{R}(x)\,]^2\,dx,$$

- correlation

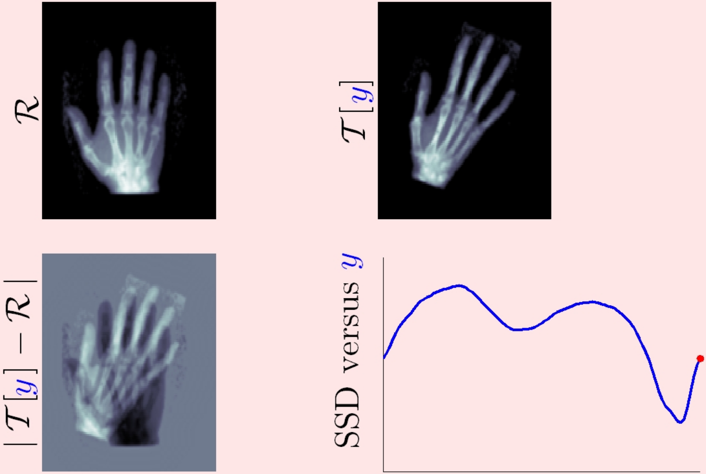
- Mutual Information (multi-modal images)

- Normalized Gradient Fields

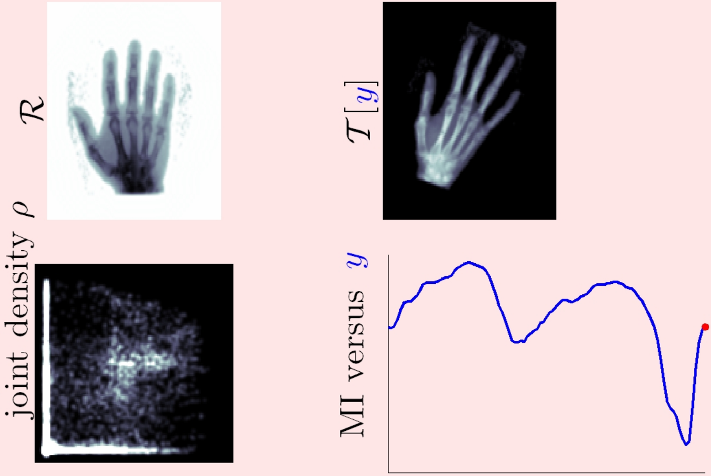
- ...

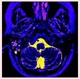


Sum of Squared Differences



Mutual Information

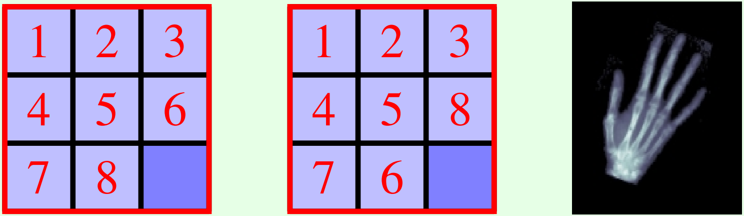




Regularization

$$\mathcal{D}[\mathcal{T}[y],\mathcal{R}] + \alpha\mathcal{S}[y - y_{\text{reg}}] \xrightarrow{y} \min$$

Transformation y



- Registration is severely ill-posed
- Restrictions onto the transformation y needed
- Goal: implicit physical restrictions

Implicit versus Explicit Regularization . . .

Registration is ill-posed $\rightsquigarrow$ requires regularization

- Parametric Registration
  - restriction to (low-dimensional) space (rigid, affine linear, spline,. . .)
  - regularized by properties of the space (implicit)
  - not physical or model based
- Non-parametric Registration
  - regularization by adding penalty or likelihood (explicit)
  - allows for a physical model
  - $\rightsquigarrow y$ is no longer parameterizable

. . . implicit versus explicit regularization

registration is ill-posed ⇝ requires regularization

- parametric registration

parametric registration

$\mathcal{D}[R,T;y] \overset{y}{=} \min \quad \text{s.t.} \quad y \in \mathcal{Q} = \{x + \sum w_j q_j,\ w \in \mathbb{R}^m\}$

- non-parametric registration

non-parametric registration

$\mathcal{D}[R,T;y] + \alpha \mathcal{S}[y - y_{\mathrm{reg}}] \overset{y}{=} \min$

References for Well-Posedness

- M. Droske and M. Rumpf.
  A variational approach to non-rigid morphological registration.
  SIAM Appl. Math., 64(2):668–687, 2004.
- B. Fischer and J. Modersitzki.
  A unified approach to fast image registration and a new curvature based registration technique.
  Linear Algebra and its Applications, 380:107–124, 2004.
- J. Weickert and C. Schnörr.
  A theoretical framework for convex regularizers in PDE-based computation of image motion.
  Int. J. Computer Vision, 45(3):245–264, 2001.
- ...

Regularizer $\mathcal{S}$

$y(x) = x + u(x)$, displacement $u : \mathbb{R}^d \to \mathbb{R}^d$

- "elastic registration" $\mathcal{S}^{\text{elas}}[u] =$ elastic potential of u
- "fluid registration" $\mathcal{S}^{\text{fluid}}[u] =$ elastic potential of $\partial_t u$
- "diffusion registration" $\mathcal{S}^{\text{diff}}[u] = \frac{1}{2}\sum_{\ell=1}^{d} \int_\Omega \|\nabla u_\ell\|_{\mathbb{R}^2}^2 \; dx$
- "curvature registration" $\mathcal{S}^{\text{curv}}[u] = \frac{1}{2}\sum_{\ell=1}^{d} \int_\Omega (\Delta u_\ell)^2 \, dx$
- ...

Elastic Registration

Transformation/displacement, $y(x) = x + u(x)$

$$\mathcal{S}^{\text{elas}}[u] = \text{elastic potential of } u$$

$$= \int_\Omega \frac{\lambda+\mu}{2} \| \nabla \cdot u \|^2 + \frac{\mu}{2} \sum_{i=1}^{d} \| \nabla u_i \|^2 \, dx$$

image painted on a rubber sheet

C. Broit.
Optimal Registration of Deformed Images.
PhD thesis, University of Pensylvania, 1981.

Bajcsy & Kovačič 1986, Christensen 1994, Bro-Nielsen 1996, Gee et al. 1997, Fischer & M. 1999, Rumpf et al. 2002, . . .

Fluid Registration

Transformation/displacement, $y(x,t) = x + u(x,t)$

$$\mathcal{S}^{\text{fluid}}[u] \quad = \quad \text{elastic potential of } \partial_t u$$

image painted on honey

- GE. Christensen.
  Deformable Shape Models for Anatomy.
  PhD thesis, Sever Institute of Technology, Washington University, 1994.
- Bro-Nielsen 1996, Henn & Witsch 2002, . . .

Diffusion Registration

Transformation/displacement, $y(x) = x + u(x)$

$$\begin{aligned} \mathcal{S}^{\text{diff}}[\,u\,] &= \text{oszillations of } u \\ &= \tfrac{1}{2}\sum_{\ell=1}^{d} \int_\Omega \|\nabla u_\ell\|_{\mathbb{R}^2}^2 \; dx \end{aligned}$$

heat equation

B. Fischer and J. Modersitzki.
Fast diffusion registration.
AMS Contemporary Mathematics, Inverse Problems, Image Analysis, and Medical Imaging, 313:117–129, 2002.

Horn & Schunck 1981, Thirion 1996, Droske, Rumpf & Schaller 2003, . . .

Curvature Registration

Transformation/displacement, $y(x) = x + u(x)$

$$\mathcal{S}^{\text{curv}}[u] = \text{oscillations of } u$$
$$= \frac{1}{2}\sum_{\ell=1}^{d} \int_\Omega \|\Delta u_\ell\|_{\mathbb{R}^2}^2 \, dx$$

bi-harmonic operator

B. Fischer and J. Modersitzki.
Curvature based image registration.
J. of Mathematical Imaging and Vision, 18(1):81–85, 2003.

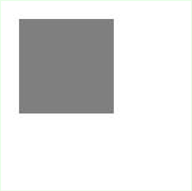
Stefan Henn.
A multigrid method for a fourth-order diffusion equation with application to image processing.
SIAM J. Sci. Comput., 2005.

Registration of a ■

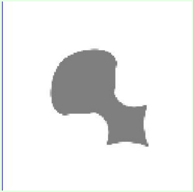
Curvature Registration

- Goal: do not penalize affine linear transformations
  $\mathcal{S}[Cx+b] \stackrel{!}{=} 0$ for all $C \in \mathbb{R}^{d\times d}$ and $b \in \mathbb{R}^d$
- But: $\mathcal{S}^{\text{diff,elas,fluid,}\ldots}[Cx+b] \neq 0$!
- Idea: $\mathcal{S}^{\text{curv}}[y] = \sum_\ell \int_\Omega (\Delta y_\ell)^2 \, dx \;\Rightarrow\; \mathcal{S}^{\text{curv}}[Cx+b] = 0$

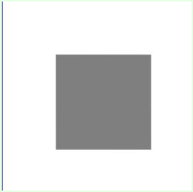
reference



fluid



curvature



Summary Regularization

- Registration is ill-posed $\rightsquigarrow$ requires regularization
- Regularizer controls reasonability of transformation
- Application conform regularization
- Enabling physical models
  (linear elasticity, fluid flow, ...)
- $\rightsquigarrow$ high dimensional optimization problems

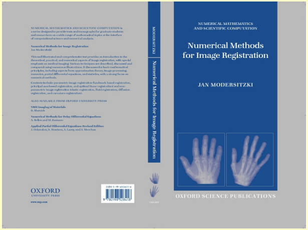
Numerical Methods for Image Registration

Optimize $\leftrightarrow$ Discretize

Image Registration

$$\mathcal{D}[\mathcal{T}[y],\mathcal{R}] + \alpha\mathcal{S}[y - y_{\text{reg}}] \xrightarrow{y} \min, \qquad y_{\text{reg}}(x) = x$$

Numerical Approaches:



- Optimize $\rightarrow$ Discretize
- Discretize $\rightarrow$ Optimize
- relatively large problems:
  2.000.000 – 500.000.000 unknowns

Optimize $\longrightarrow$ Discretize: ELE

Image Registration

$$\mathcal{J}[y] = \mathcal{D}[\mathcal{T}[y], \mathcal{R}] + \alpha \mathcal{S}[y - y_{\text{reg}}] \xrightarrow{y} \min, \qquad y_{\text{reg}}(x) = x$$

- Euler-Lagrange eqs. (ELE) give necessary condition:
  $\mathcal{D}_y + \alpha \mathcal{S}_y = 0 \iff f[y] + \alpha \mathcal{A} y = 0$
  system of non-linear partial differential eqs. (PDE)
- outer forces f, drive registration
- inner forces $\mathcal{A}y$, tissue properties
- ELE $\rightsquigarrow$ PDE: balance of forces

Optimize → Discretize: Summary

Continuous Euler-Lagrange equations

$$f[y] + \alpha \mathcal{A}\, y = 0, \quad f[y^k] + \alpha \mathcal{A}\, y^{k+1} = 0, \quad f[y] + \alpha \mathcal{A}\, y = y_t$$

- all difficulties dumped into right hand side f
- spatial discretization straightforward
- efficient solvers for linear systems
- small controllable steps ($\rightsquigarrow$ movies)
- moderate assumptions on f and $\mathcal{A}$ (smoothness)
- no optimization problem behind
- non-linearity only via f
- small steps
- software: `http://www.math.uni-luebeck.de/SAFIR`

Discretize → Optimize: Summary

Discretization $\rightsquigarrow$ finite dimensional problem: $y^h \approx y(x^h)$

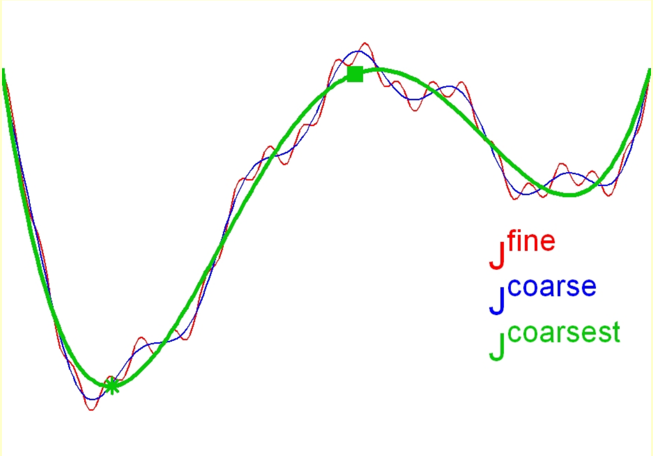
$$D(y^h) + \alpha S(y^h) \xrightarrow{y^h} \min, \quad y^h \in \mathbb{R}^n, \qquad h \longrightarrow 0$$

- efficient optimization schemes (Newton-type)
- linear systems of type $H\,\delta_y = -\mathrm{rhs}$,

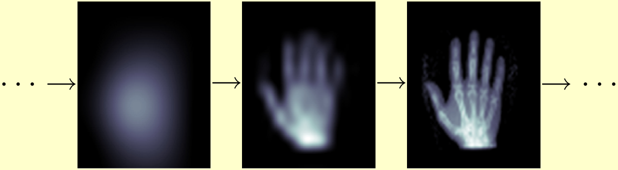
$$H = M + \alpha B^\top B, \quad M \approx D_{yy}, \quad \mathrm{rhs} = D_y + \alpha(B^\top B)y^h$$

- efficient multigrid solver for linear systems
- large steps
- discretization not straightforward (multigrid)
- all parts have to be differentiable (data model)

Multilevel



Multilevel



$\cdots \quad h^{\ell-1} \quad h^{\ell} \quad h^{\ell+1} \quad \cdots$

for $\ell = 1 : \ell_{\max}$ **do**

transfer images to level ℓ

approximately solve problem for y

prolongating y to finer level $\rightsquigarrow$ perfect starting point

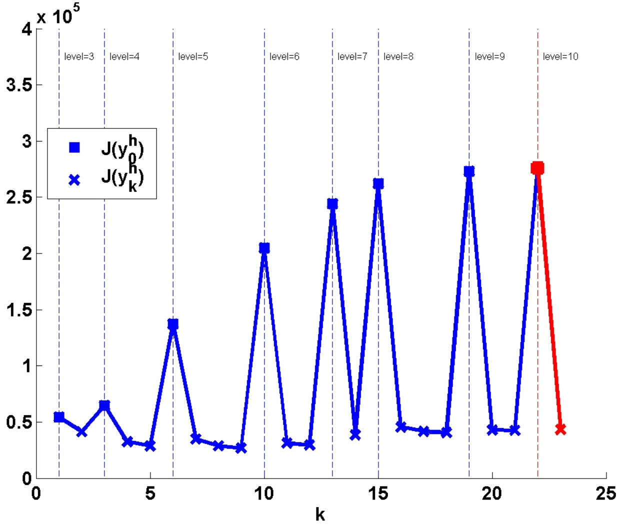
end for

Advantages of Multilevel Strategy

- Regularization
- Focusses on essential minima
- Creates extraordinary starting value
- Reduces computation time

- ?

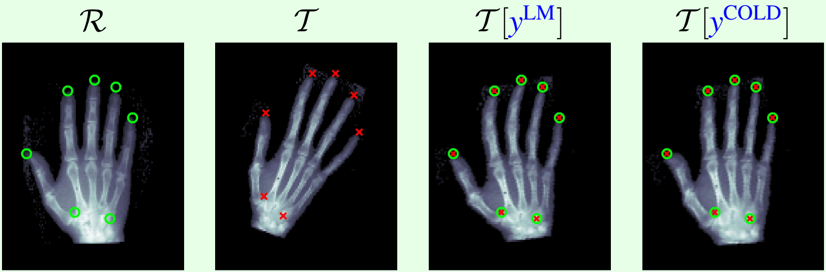
Example: Multilevel Iteration History



Constrained Image Registration

Example: COLD

Combining Landmarks and Distance Measures



Patent AZ 10253 784.4; Fischer & M., 2003

Adding Constraints

Constrained Image Registration

$$\mathcal{D}[\mathcal{T}[y],\mathcal{R}] + \alpha\mathcal{S}[y - y_{\text{reg}}] + \beta \int_\Omega \psi\left(\mathcal{C}^{\text{soft}}[y]\right)\, dx \xrightarrow{y} \min$$

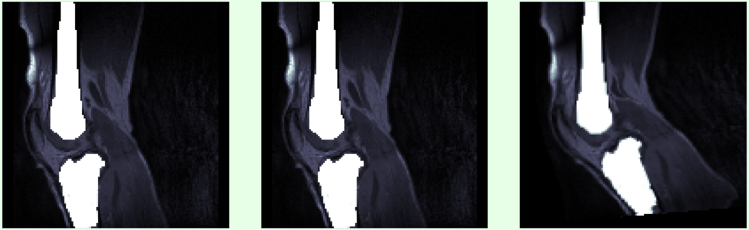
subject to $\mathcal{C}^{\text{hard}}[y](x) = 0$ for all $x \in \Omega_{\mathcal{C}}$

Example: landmarks/volume preservation

$$\begin{aligned} \mathcal{C}_i^{\text{LM}}[y] &= \|y(r_i) - t_i\|, & \psi(\mathcal{C}) &= 0.5\|\mathcal{C}\|^2 \\ \mathcal{C}^{\text{VP}}[y](x) &= \det(\nabla y(x)), & \psi(\mathcal{C}) &= \log \mathcal{C} \end{aligned}$$

- soft constraints (penalty)
- hard constraints
- both constraints

Rigidity Constraints



Soft Rigidity Constraints

FAIR with Soft Rigidity

$$\mathcal{D}[\,\mathcal{T}[y],\mathcal{R}\,] + \alpha\mathcal{S}[\,y - y_{\mathrm{reg}}\,] + \beta\mathcal{C}[\,y\,] \xrightarrow{y} \min$$

$\mathcal{C}$ soft constraints / penalty:

$$\mathcal{C}[\,y\,] = \tfrac{1}{2}\|\underbrace{r^{\mathrm{linear}}(y)}_{\text{linear}}\|^2_{\mathcal{Q}} + \tfrac{1}{2}\|\underbrace{r^{\mathrm{orth}}(y)}_{\text{orthogonal}}\|^2_{\mathcal{Q}} + \tfrac{1}{2}\|\underbrace{r^{\mathrm{det}}(y)}_{\text{orientation}}\|^2_{\mathcal{Q}}$$

$$r^{\mathrm{linear}}(y) = [\partial_{1,1}y_1,\ldots,\partial_{d,d}y_1,\ \partial_{1,1}y_2,\quad \ldots\,]$$

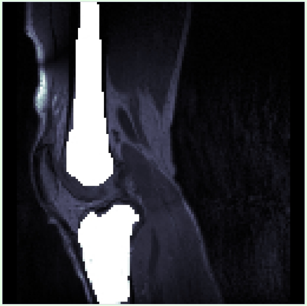
$$r^{\mathrm{orth}}(y) = \nabla y^{\top}\nabla y - I_d$$

$$r^{\mathrm{det}}(y) = \det(\nabla y) - 1$$

y rigid $\iff [\; r^{\mathrm{linear}} = 0 \;\wedge\; r^{\mathrm{orth}} = 0 \;\wedge\; r^{\mathrm{det}} = 0\;]$

The Weight $\mathcal{Q}$

- only locally rigid
- use weight function $\mathcal{Q}$
- regions to be kept rigid move with y



$$\| f \|_{\mathcal{Q}}^2 = \int_\Omega f(x)\ \mathcal{Q}(y(x))^2\, dx$$

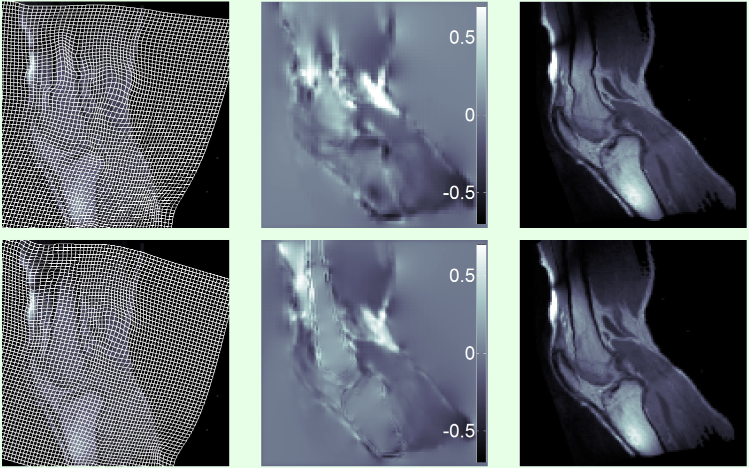
Numerical Scheme

- $Q(y^h) \approx \mathcal{Q}(y(x^h))$
- $r(y^h) = [\mathrm{diag}(Q(y^h))\, r_1(y^h), \ldots, \mathrm{diag}(Q(y^h))\, r_{\mathrm{end}}(y^h)]$
- $C(y^h) = \frac{1}{2} r(y^h)^\top r(y^h)$
- $C_y(y^h) =$ lengthy formula
- $D(y^h) + \alpha S(y^h) + \beta C(y^h) \stackrel{y^h}{\longrightarrow} \min$
- Optimizer: Gauß-Newton type approach,

$$H \approx \text{“}\nabla^2 \mathcal{D}\text{”} + \alpha B^\top B + \beta {r_y}^\top r_y$$

Example: Knee

T & grid $\qquad \det(\nabla y) - 1 \qquad T(y)$



not penalized

penalized

Summary of Soft Rigidity Constraints

- Results are OK
- Implementation is straightforward
- Constraints are not fulfilled
- How to pick penalty (β, ψ)?

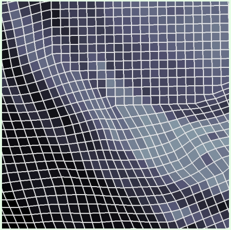
Hard Rigidity Constraints

FAIR with Hard Rigidity

$\mathcal{D}[\mathcal{T}[y],\mathcal{R}]+\alpha\mathcal{S}[y-y_{\text{reg}}] \xrightarrow{y}$ min subject to y rigid on $\mathcal{Q}$

Eulerian $\to$ Lagrangian

- computations of $\mathcal{D}$ and $\mathcal{S}$ involve $\det(\nabla y)$
- rigidity in $\mathcal{T}$ domain $\rightsquigarrow$ linear constraints



$$y(x) = D_k x + t_k, \quad k = 1 : \#\text{segments}$$

Lagrangian Model of Rigidity (2D)

- rigid on segment i

$$y(x) = Q(x)w^i = \begin{pmatrix} \cos w_1^i & -\sin w_1^i \\ \sin w_1^i & \cos w_1^i \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} w_2^i \\ w_3^i \end{pmatrix}$$

- $w = (w^1, \dots, w^m), \quad \mathcal{C} = (\mathcal{C}^1, \dots, \mathcal{C}^m), \quad m = \#\text{segments}$

$$\mathcal{C}^i[y, w] = y(x) - Q(x)w^i, \qquad i = 1, \dots, m$$

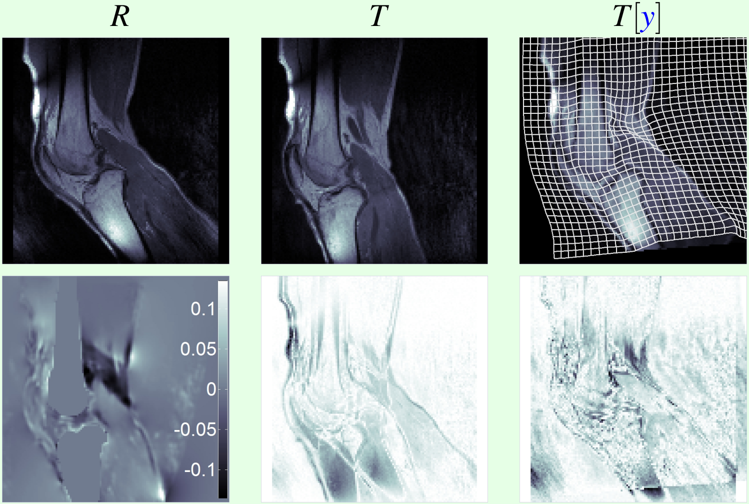
- Lagrangian:

$$L(y, w, p) = \mathcal{D}[y] + \alpha\mathcal{S}[y] + p^\top \mathcal{C}[y, w]$$

- Numerical Scheme:

Sequential Quadratic Programming

Rigidity as a Hard Constraint



Summary of Hard Rigidity Constraints

- Results are OK
- Implementation is interesting
- Constraints are fulfilled
- No additional Parameters

Summary

Summary

- Introduction to image registration:
  important, challenging, interdisciplinary
- General framework based on a variational approach:
  $\mathcal{D}[\,\mathcal{T}[y],\mathcal{R}\,] + \alpha\mathcal{S}[\,y - y_{\mathrm{reg}}\,] \xrightarrow{y} \min$
- Discussion of various building blocks:
  - image model $\mathcal{T}[y]$
  - distance measures $\mathcal{D}$
  - regularizer $\mathcal{S}$
- Numerical methods:
  multilevel, optimize $\leftrightarrow$ discretize
- Constraints $\mathcal{C}$:
  landmarks, local rigidity, intensity correction, ...

Solutions and Algorithms For Image Registration