A Calibratable Dynamic Model for CDO's: Application to Leveraged-Super-Senior Tranche Valuation

- Rapid growth of credit derivatives market.
- Valuation of CDO's in terms of a calibrated static model.
- Risk-neutral pricing, i.e. pricing relative to market prices
- Development of a dynamic model built onto the static model.
- Example of valuation using the dynamic model: LSS-LT's

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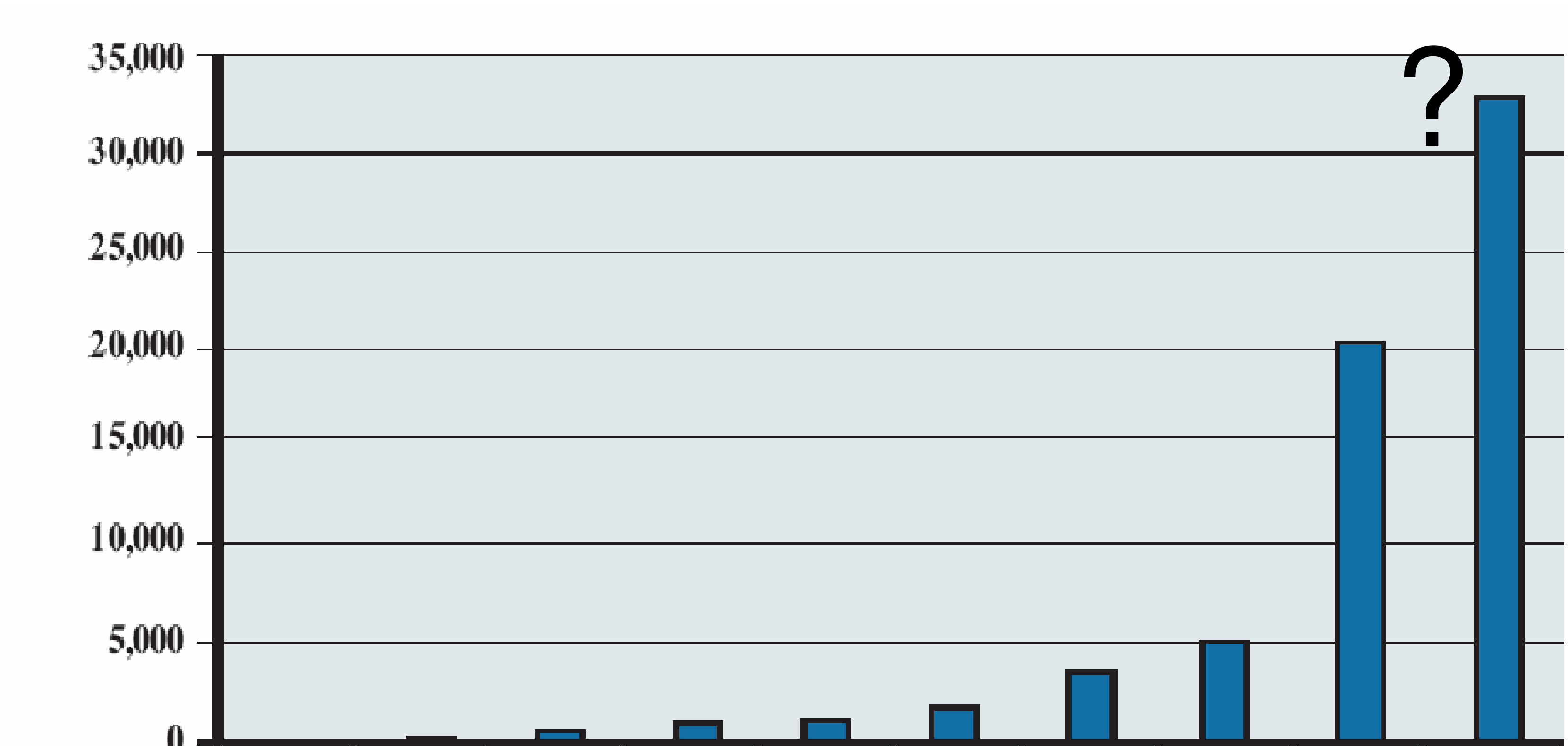
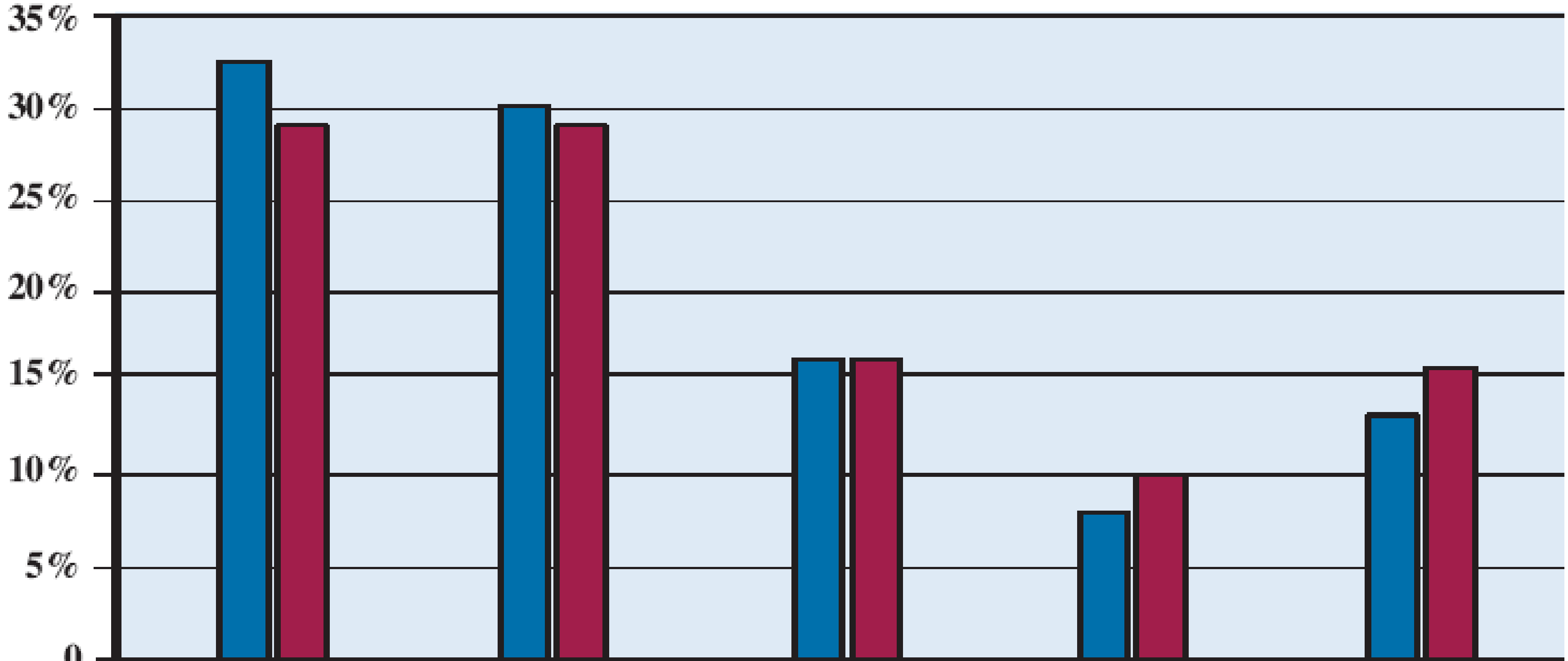
Global Credit Derivatives Market US$ bn (from BBA Credit Derivatives Report 2006)



	1996	1998	1999	2000	2001	2002	2003	2004	2006	2008 (est.)
USD (bn)	180	350	586	893	1,189	1,952	3,548	5,021	20,207	33,120

Recent report-Isda: 38% rise in CD notional, 1st half of 2007

Credit Derivatives Products

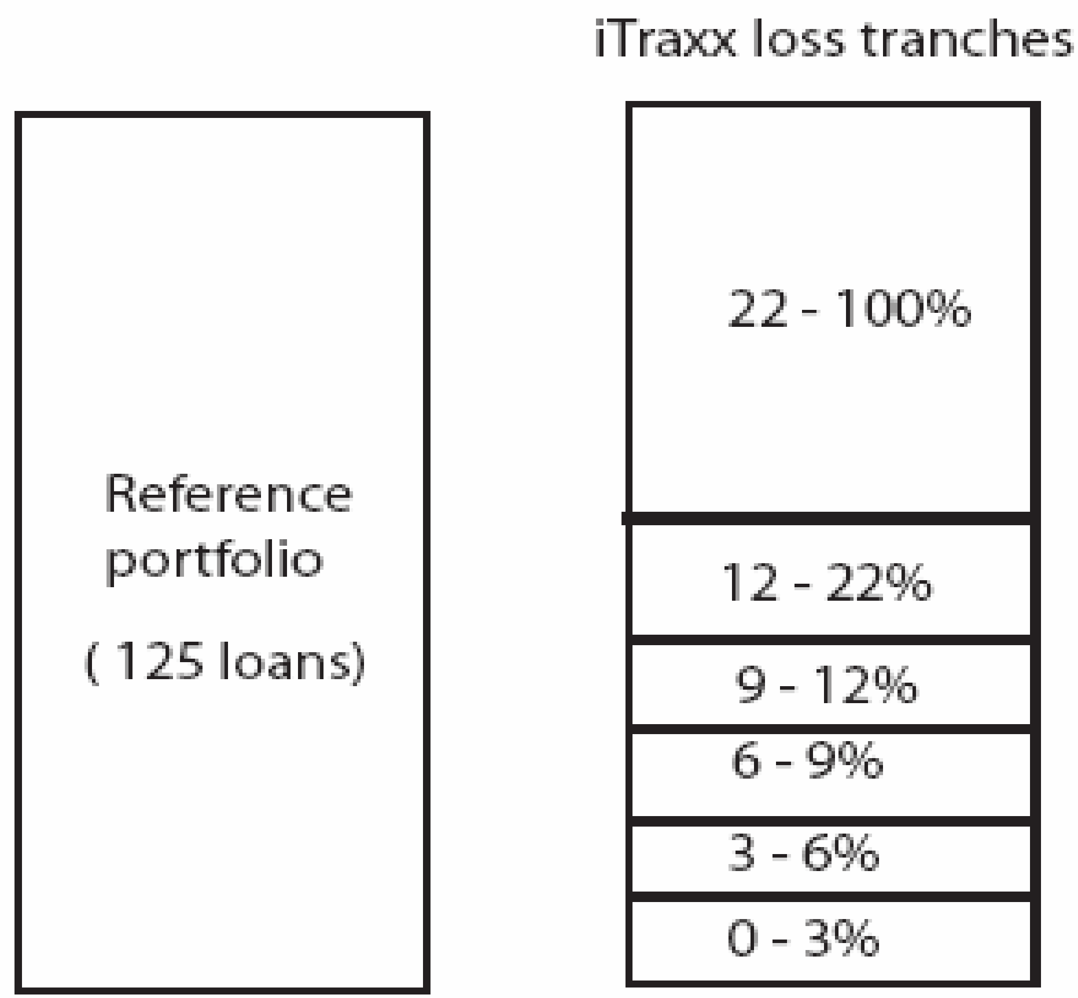


	Single-name credit default swaps	Full index trades	Synthetic CDOs	Tranched index trades	Others
End 06	33%	30%	16%	8%	13%
End 08	29%	29%	16%	10%	16%

Further remarks

- CDO’s have been prominent players in the global credit crunch that began in July-August
- Leveraged super-senior tranches (my last example) are probably the most common asset underlying the Canadian non-bank ABCP market (which is currently frozen)
- The development of methods of valuing portfolio credit derivatives is still very much an ongoing process.

CDO's – a simplistic view



CDO contracts provide insurance against tranche losses

- e.g. consider 3-6% tranche
- Protection buyer buys insurance against all losses from 3 to 6% of total notional.
- Protection buyer pays a regular quarterly premium to an investor
- Investor pays any losses lying between 3% and 6% to the protection buyer

iTraxx Tranche Quotes - 21 June 2005				
Tranche	3 year	5 year	7 year	10 year
1: 0-3%	7.3	27.38	43.8	53.25
2: 3-6%	26	91	245	455
3: 6-9%	7.4	31.5	60	123
4: 9-12%	2.4	19	30	56
5: 12-22%	2.3	12.5	19.5	35.5
0-100%	23.25	39.25	49.75	60.5

0-3% quoted as upfront %; remaining in bps per year
(data from Julien Houdain and Fortis Investments)

Focus – The calibration problem

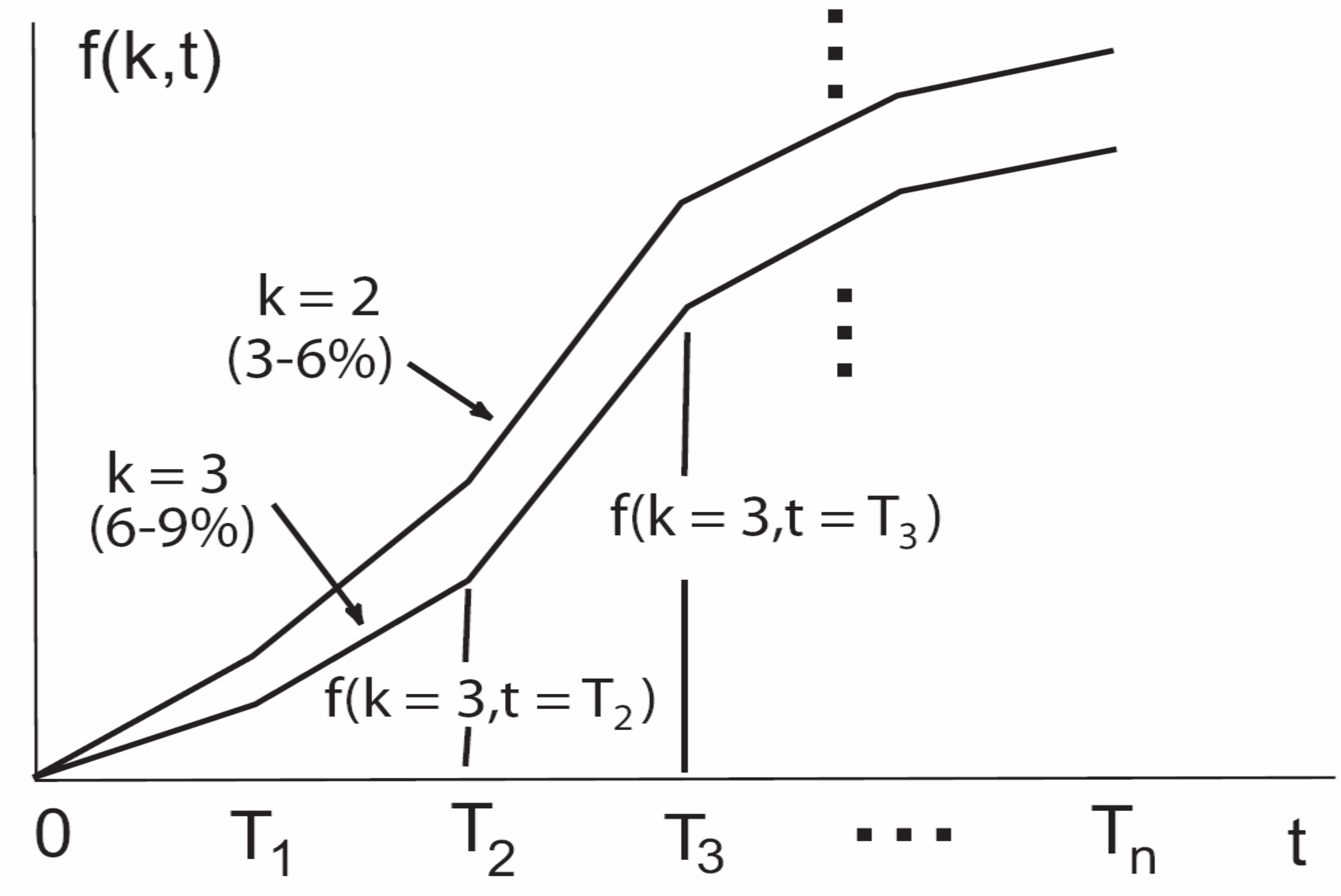
- Risk –neutral pricing is pricing relative to market prices
- There are typically 24 different CDO contracts (differing in maturity and loss tranche) on the market that reference the same underlying portfolio.
- The problem is to find a risk-neutral measure that can be calibrated to reproduce all available market prices – a central problem of much recent research
- The accurate marking of tranches to market, and the pricing of forward CDO's and dynamics-sensitive derivatives such as options and leveraged super-senior swaps requires simultaneous calibration to all available market prices.

The Basic Pricing Equation

- For a CDO contract on a given tranche and for a given maturity, a fair premium requires that:

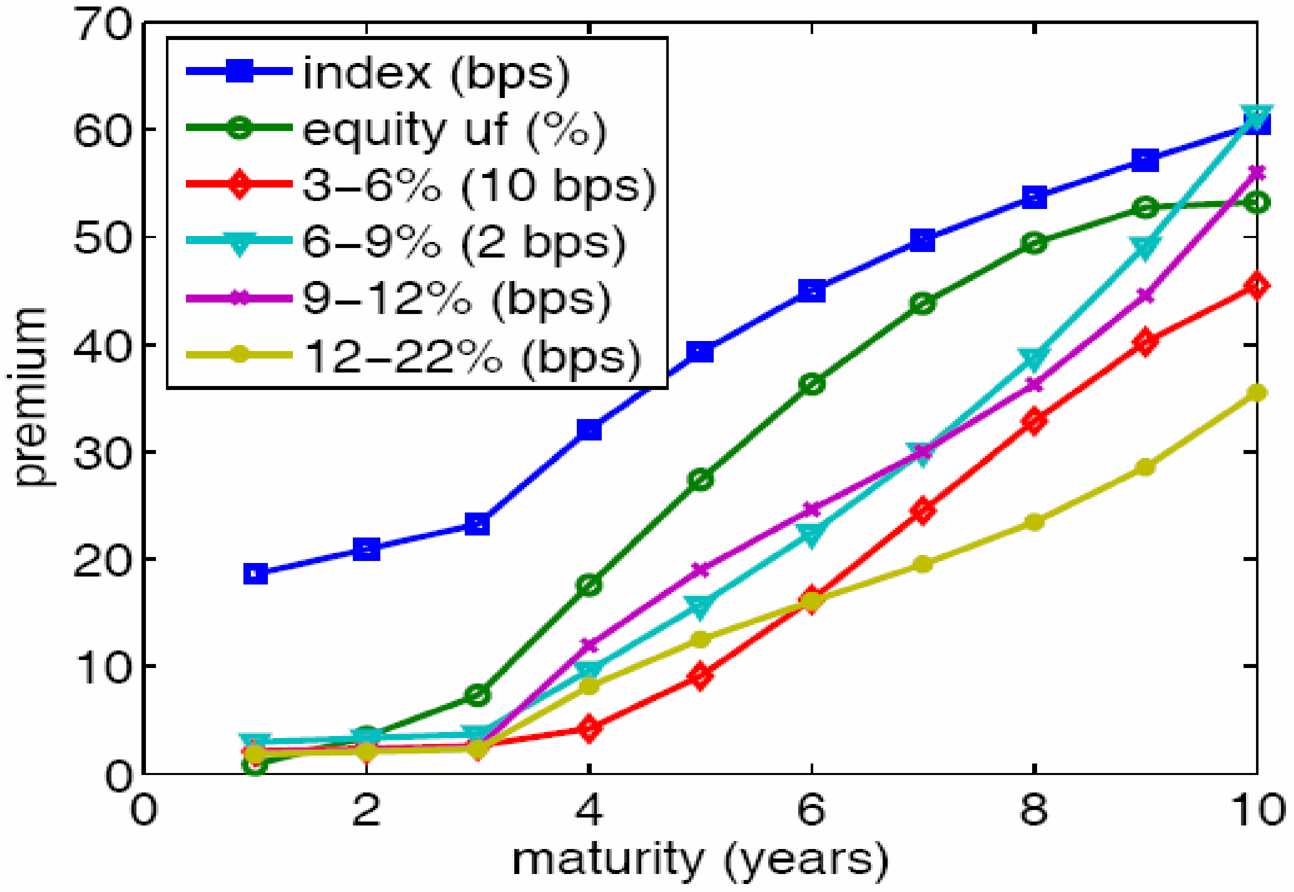
  PV(Expected tranche losses) = w X T_{eff}
- Define f(k,t) = expected loss per unit tranche notional for tranche k at time t
- A knowledge of f(k,t) defines a static model for the valuation of CDO’s

Expected loss for tranche k



Tranche term structures

iTraxx data for 21 June 2005 from Julien Houdain and Fortis Investments



Calibration procedure has been extensively tested

- Walker (2006) – successful calibration to 97 daily sets of iTraxx quotes
- Torresetti et al (2006) – successful calibration to 616 sets of CDX quotes and to 473 sets of iTraxx quotes (Nov 03 to May 06) – calibrated model reproduces the tranche prices to within the bid-ask spread

Applications of Static Model

- Marking to market of tranches as accurately as possible
- Valuing long-short forward tranches as accurately as possible
- For details, see Walker (2005, 2006).

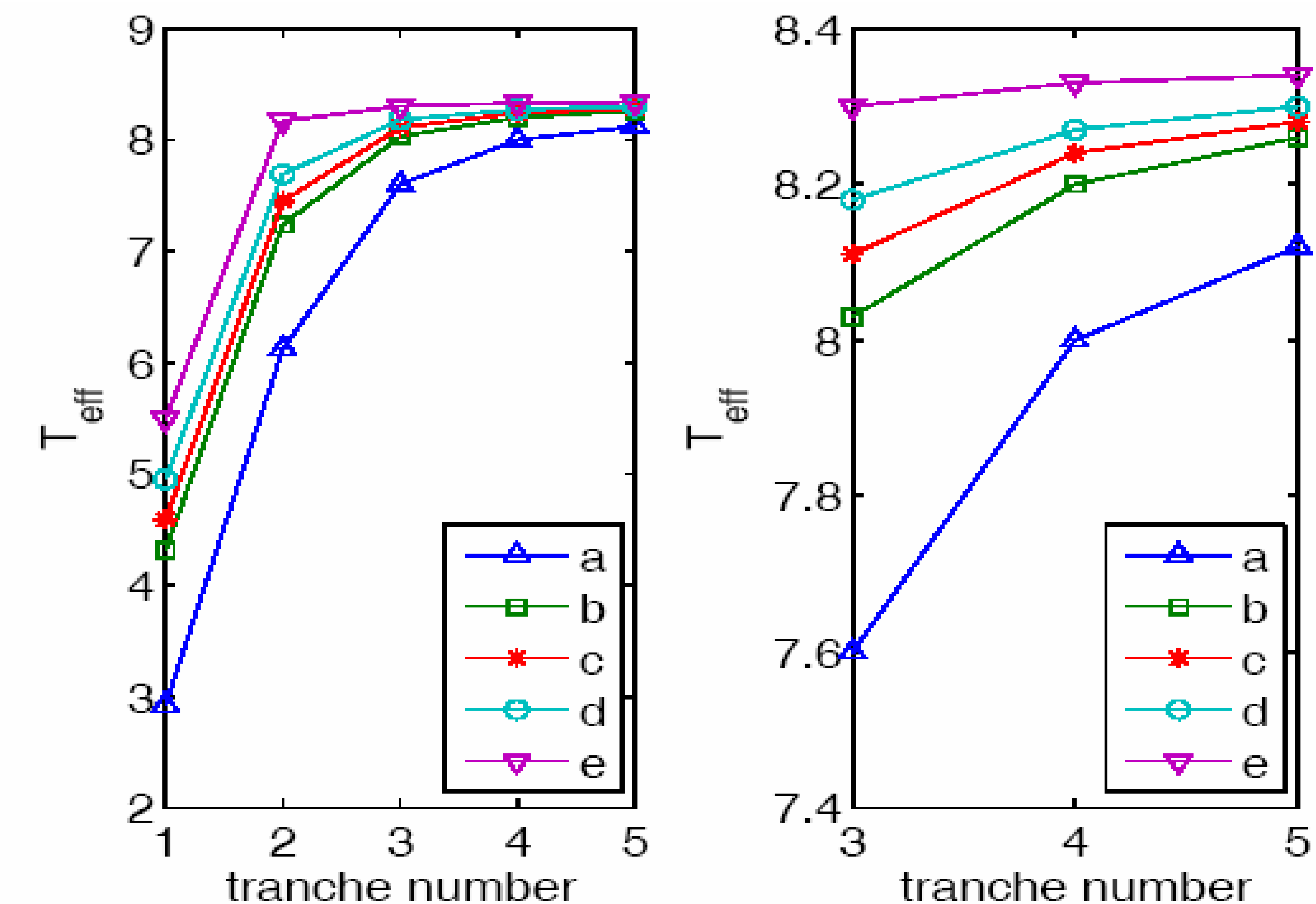
Consider the 10 yr equity tranche, priced in terms of an upfront percentage u plus 500 bps per year

- Suppose that 23 of 24 tranche-maturity spreads are unchanged after one day, but that there is no market for the 10-yr equity tranche. All upfront u's consistent with

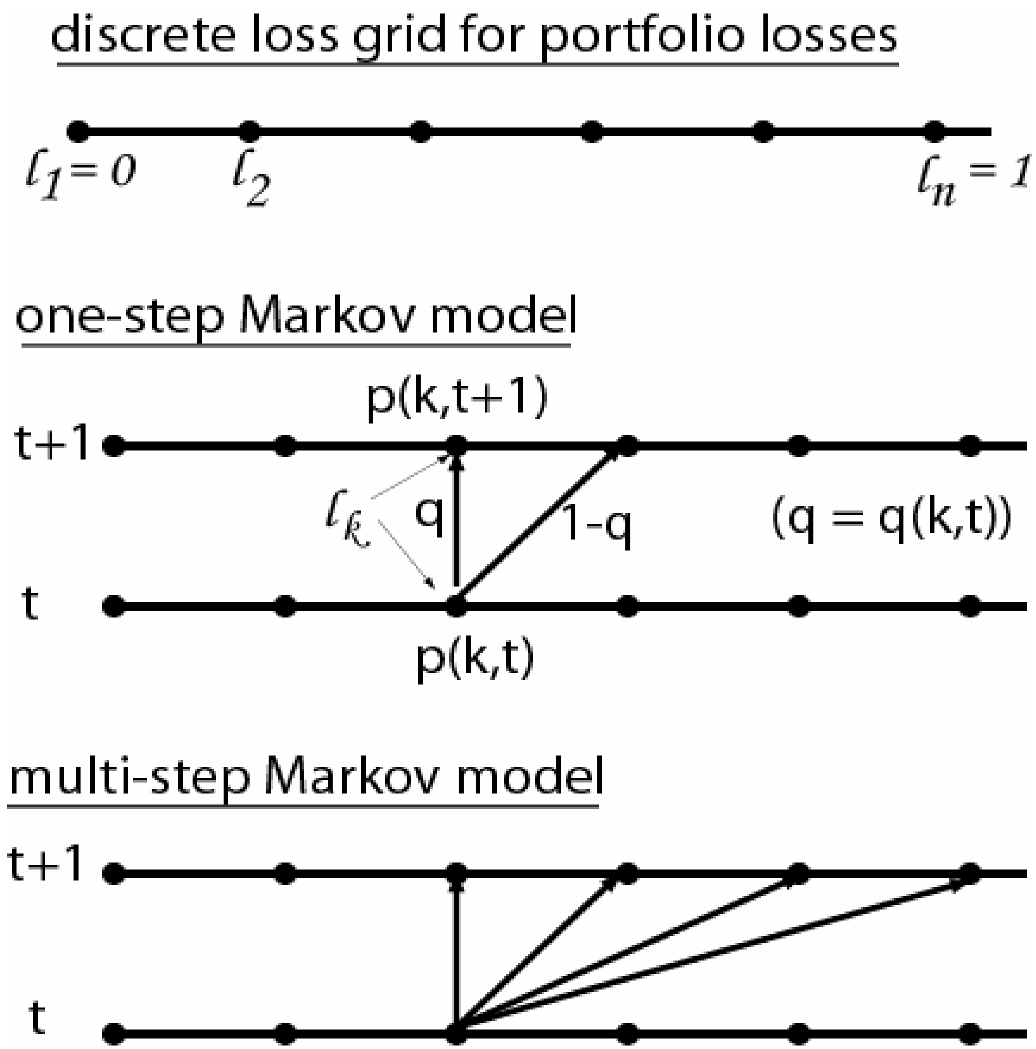
  $40\% < u < 65\%$ are arbitrage free
- For bespoke portfolio, calibration of index to corresponding portfolio of single names, but no market for tranches:

  $-41\% < u < 72\%$
- Tranche prices are market prices: no tranche market = no tranche price (from risk-neutral pricing)

Mark-to-market 10 yr maturity



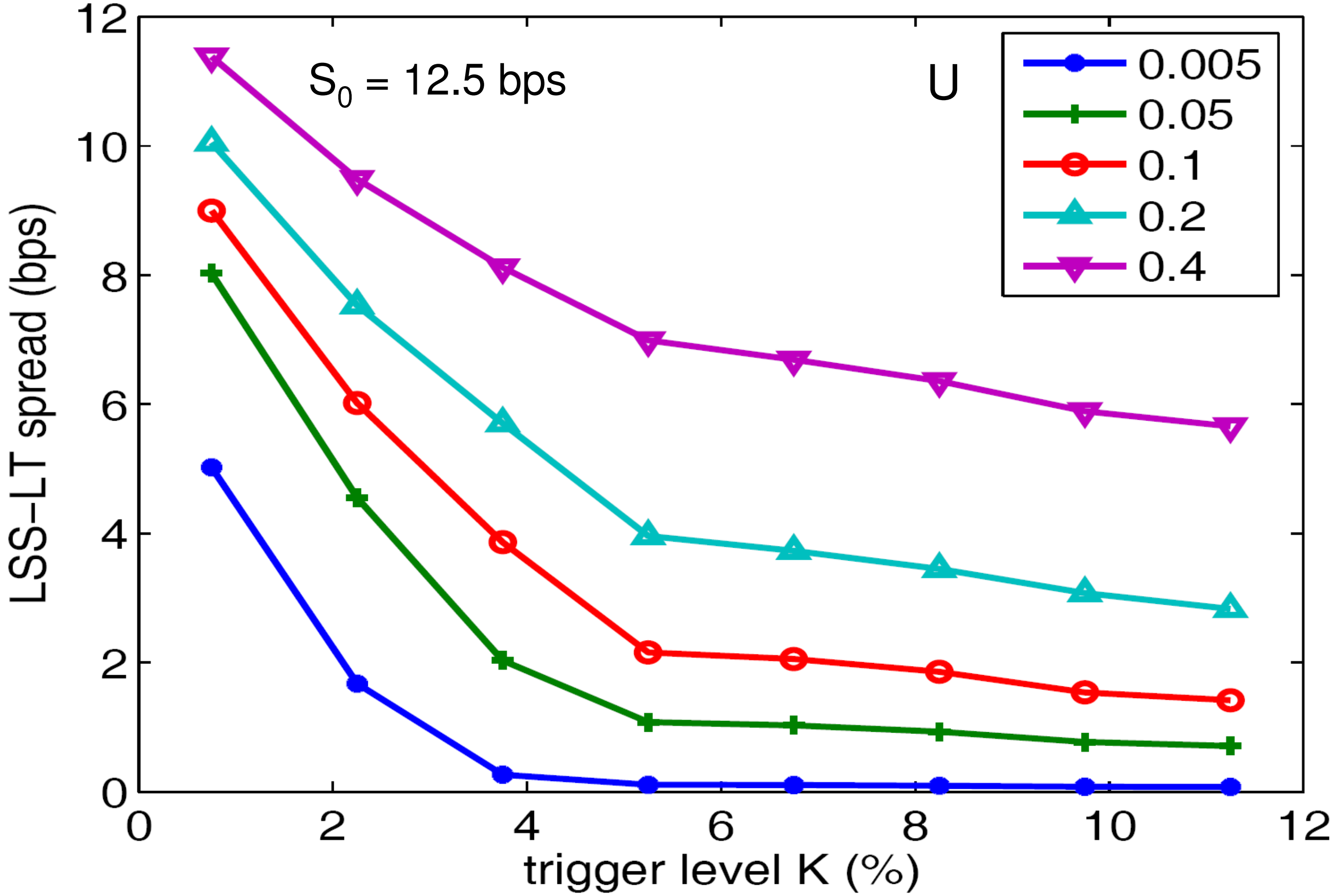
A Dynamic Model



Leveraged super-senior swaps with loss triggers

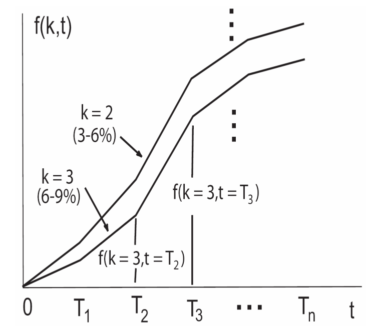
- Exotic, path-dependent, challenging to value
- Return on notional for SS's is low
- For LSS's investor is liable only for a posted collateral that is a fraction $U < 1$ (say $U = 1/10$) of the full notional
- Loss trigger protects protection buyer against losses exceeding U
- The first time the basket loss exceeds the trigger value K(t), the contract is wound up on a mark-to-market basis

LSS-LT fair spreads
(Calibration to 21 June 2005 CDO Market prices)

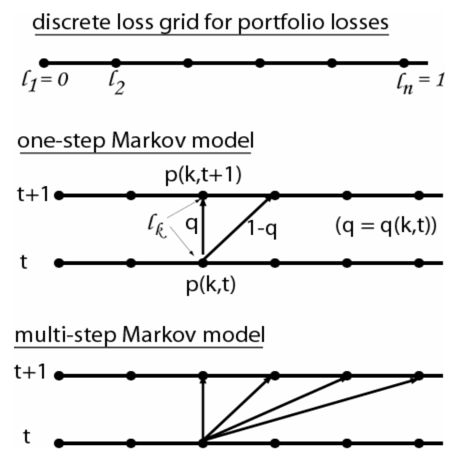


Two Main Technical Ideas

- The introduction of a static risk-neutral measure that allows easy and complete calibration to CDO spreads. Marking-to-market and long-short forwards.



- The introduction of a dynamic multi-step Markov model that allows complete calibration to CDO spreads and easy calibration to prices of dynamics-sensitive derivatives



Conclusions: cont'd

LSS-LT's (exotic, path-dependent) were discussed as an example

References to related work can be found in:

1) Michael Walker, 2006, "CDO Valuation: Term Structure, Tranche Structure and Loss Distributions,"
http://www.physics.utoronto.ca/~qocmp/nextLongB.2006.09.22.pdf
2) Roberto Torresetti, Damiano Brigo and Andrea Pallavicini, 2006, "Implied Expected Tranched Loss Surface from CDO Data,"
http://damianobrigo.it
3) Michael Walker, 2007, "Simultaneous Calibration to a Range of Portfolio Credit Derivatives with a Dynamic Discrete-Time Multi-Step Markov Loss Model, "
http://www.physics.utoronto.ca/~qocmp/FCDO2.pdf