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Bowling Alone

Some empirical tests of the Normal Copula model.

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Motivation

Structural models and the use of asset correlation proxies suggest that the Normal Copula model should do a reasonable job.

Other models are sometimes not so different, due to the infectious nature of copulas.

Models use for Equity portfolio management often imply Normal Copula dependence even when other bells and whistles are added.

"Portfolio Selection With Higher Moments" Harvey, Liechty, Liechty, Muller - 2004

Portfolio Credit Analytics

Pragmatism:

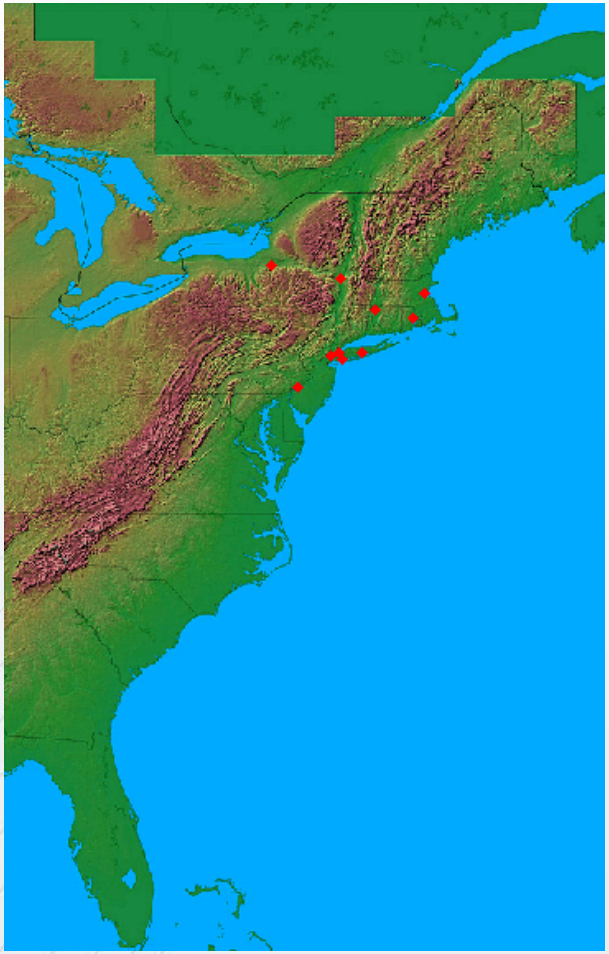
- One factor models are a pragmatic alternative to three factor models
- Three-factor models provide an approximation to full-rank models.
- Full rank models provide an approximation to reality

It makes sense to exploit conditional independence in analytic computations for portfolio credit derivatives

Questions

Hold on a minute… *has the model ever been tested on a repeatable system?* Or is our intuition good enough?

- *How many factors* does it take to model rainfall in ten different locations on the East Coast?
- *How many factors* does it take to model Equity price falls amongst the top ten cap stocks?



Study #1: Rain

LGA	New York [La Guardia Airport], NY, US
JFK	New York [John F Kennedy Intl], NY, US
EWR	Newark [Newark Liberty Intl], NJ, US
ISP	Islip [Long Island Mac Arthur Airport], NY, US
BDL	Windsor Locks [Bradley Intl], CT, US
BOS	Boston [Logan Intl], MA, US
ALB	Albany [Intl], NY, US
PVD	Providence, RI, US
PHL	Philadelphia [Intl], PA, US
SYR	Syracuse [Syracuse Hancock Intl], NY, US

1-Jan-1997 through 12-31-2007 (4019 days)

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Rain Correlation (binary)

	LGA	JFK	EWR	ISP	BDL	BOS	ALB	PVD	PHL	SYR
LGA	1.00	0.83	0.81	0.74	0.63	0.54	0.50	0.61	0.68	0.34
JFK	0.83	1.00	0.84	0.73	0.65	0.57	0.53	0.61	0.69	0.36
EWR	0.81	0.84	1.00	0.70	0.62	0.54	0.53	0.59	0.70	0.36
ISP	0.74	0.73	0.70	1.00	0.63	0.56	0.47	0.63	0.60	0.32
BDL	0.63	0.65	0.62	0.63	1.00	0.68	0.62	0.70	0.52	0.40
BOS	0.54	0.57	0.54	0.56	0.68	1.00	0.59	0.73	0.44	0.37
ALB	0.50	0.53	0.53	0.47	0.62	0.59	1.00	0.57	0.44	0.53
PVD	0.61	0.61	0.59	0.63	0.70	0.73	0.57	1.00	0.51	0.35
PHL	0.68	0.69	0.70	0.60	0.52	0.44	0.44	0.51	1.00	0.30
SYR	0.34	0.36	0.36	0.32	0.40	0.37	0.53	0.35	0.30	1.00

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Rain correlation (gauss)

	LGA	JFK	EWR	ISP	BDL	BOS	ALB	PVD	PHL	SYR
LGA	1.00	0.97	0.96	0.92	0.84	0.77	0.72	0.83	0.89	0.53
JFK	0.97	1.00	0.97	0.92	0.86	0.79	0.75	0.83	0.89	0.564
EWR	0.96	0.97	1.00	0.90	0.83	0.76	0.75	0.81	0.90	0.559
ISP	0.92	0.92	0.90	1.00	0.84	0.78	0.69	0.84	0.82	0.501
BDL	0.84	0.86	0.83	0.84	1.00	0.88	0.83	0.90	0.74	0.606
BOS	0.77	0.79	0.76	0.78	0.88	1.00	0.81	0.92	0.65	0.57
ALB	0.72	0.75	0.75	0.69	0.83	0.81	1.00	0.79	0.65	0.749
PVD	0.83	0.83	0.81	0.84	0.90	0.92	0.79	1.00	0.73	0.537
PHL	0.89	0.89	0.90	0.82	0.74	0.65	0.65	0.73	1.00	0.467
SYR	0.53	0.56	0.56	0.50	0.61	0.57	0.75	0.54	0.47	1

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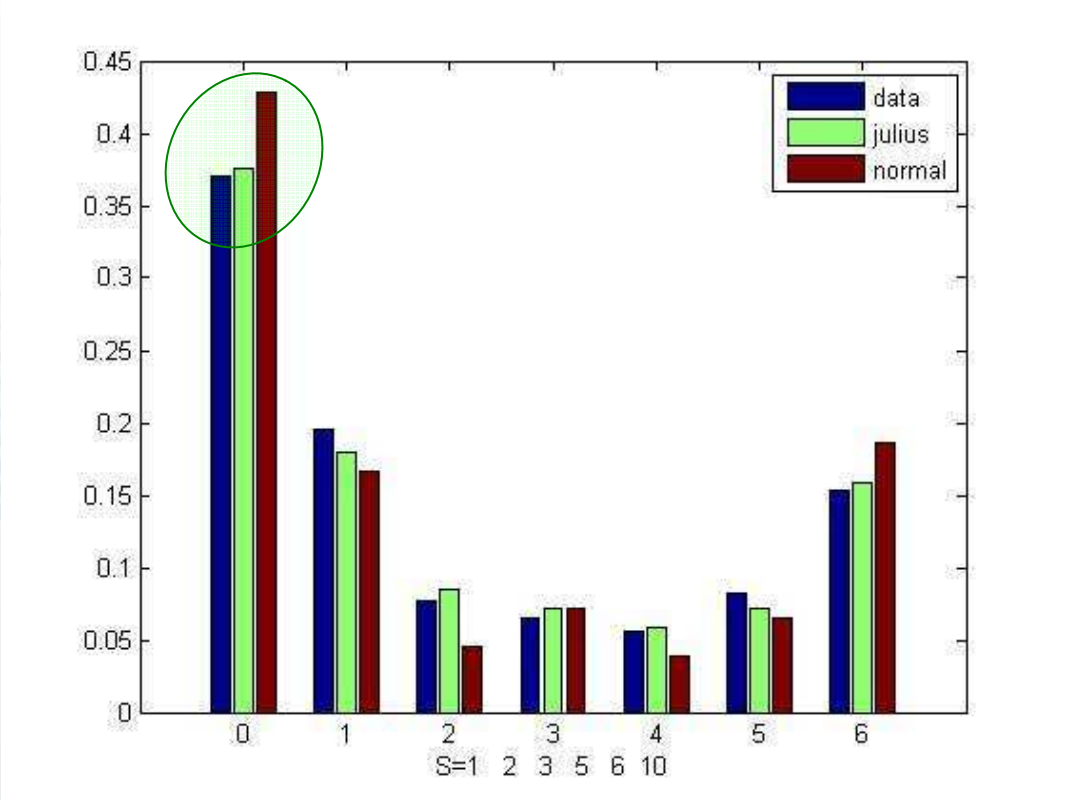
Three factor rain model



Probability that rain occurs in 0,1,2,…6 separate locations (out of 6)

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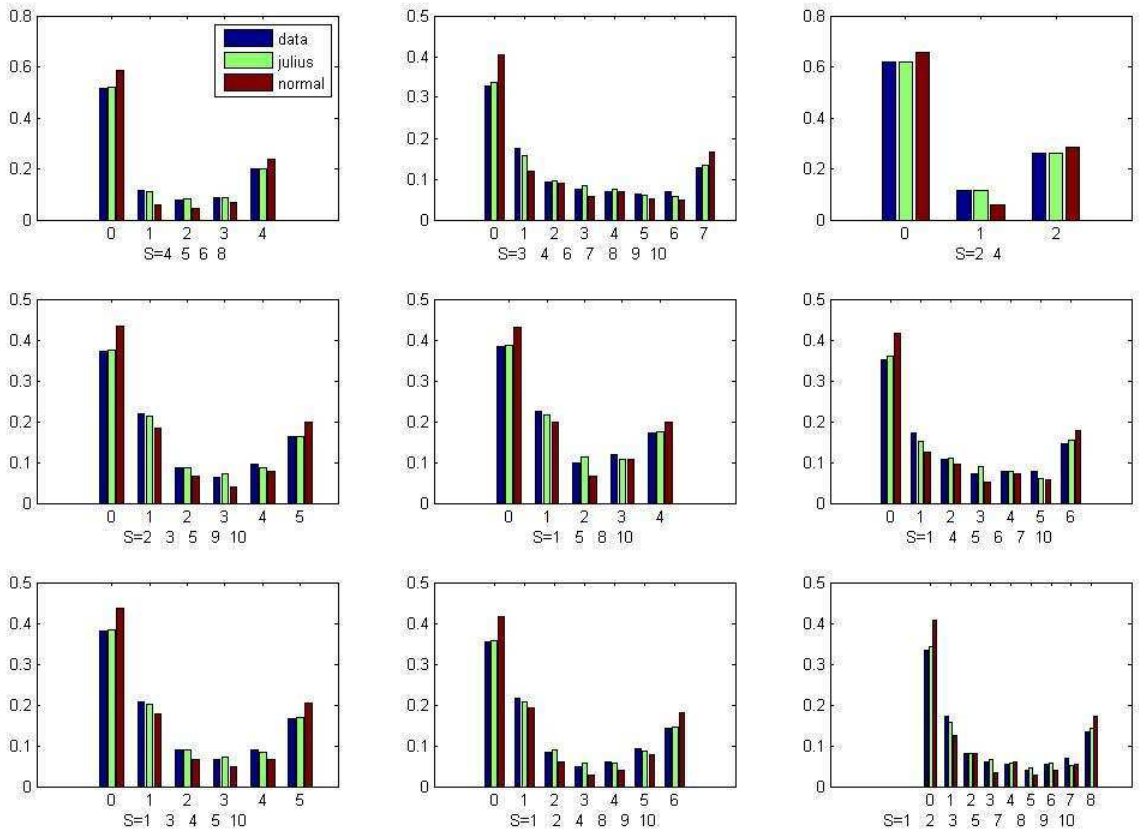
Three factor rain model



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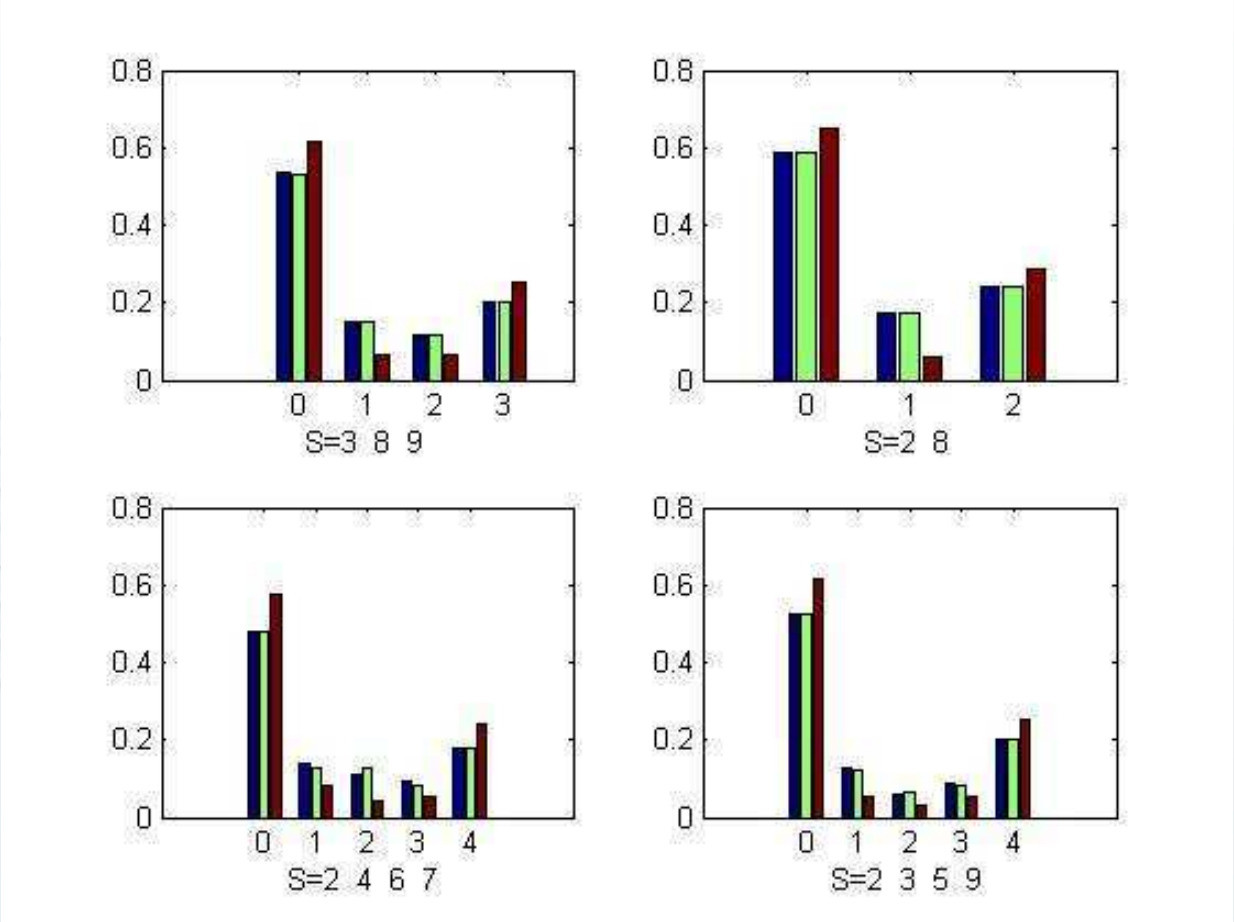
One factor rain model



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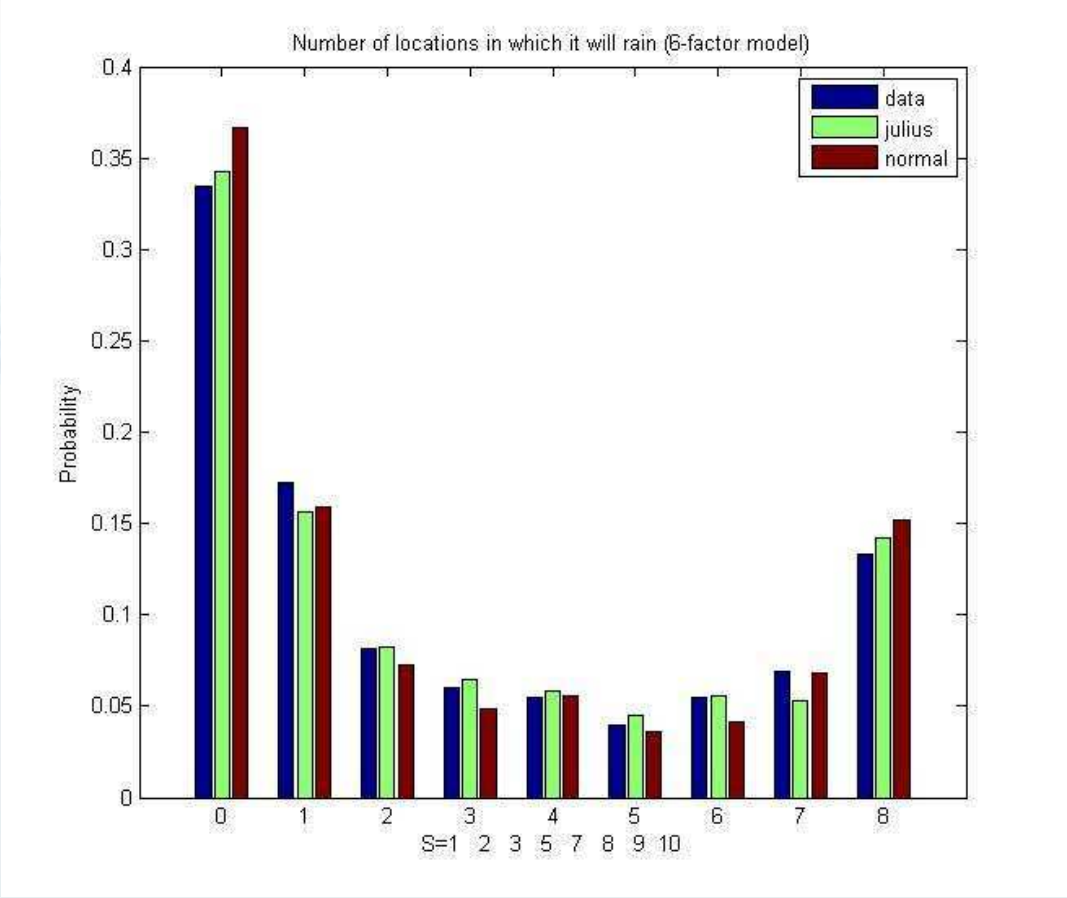
Six factor rain model



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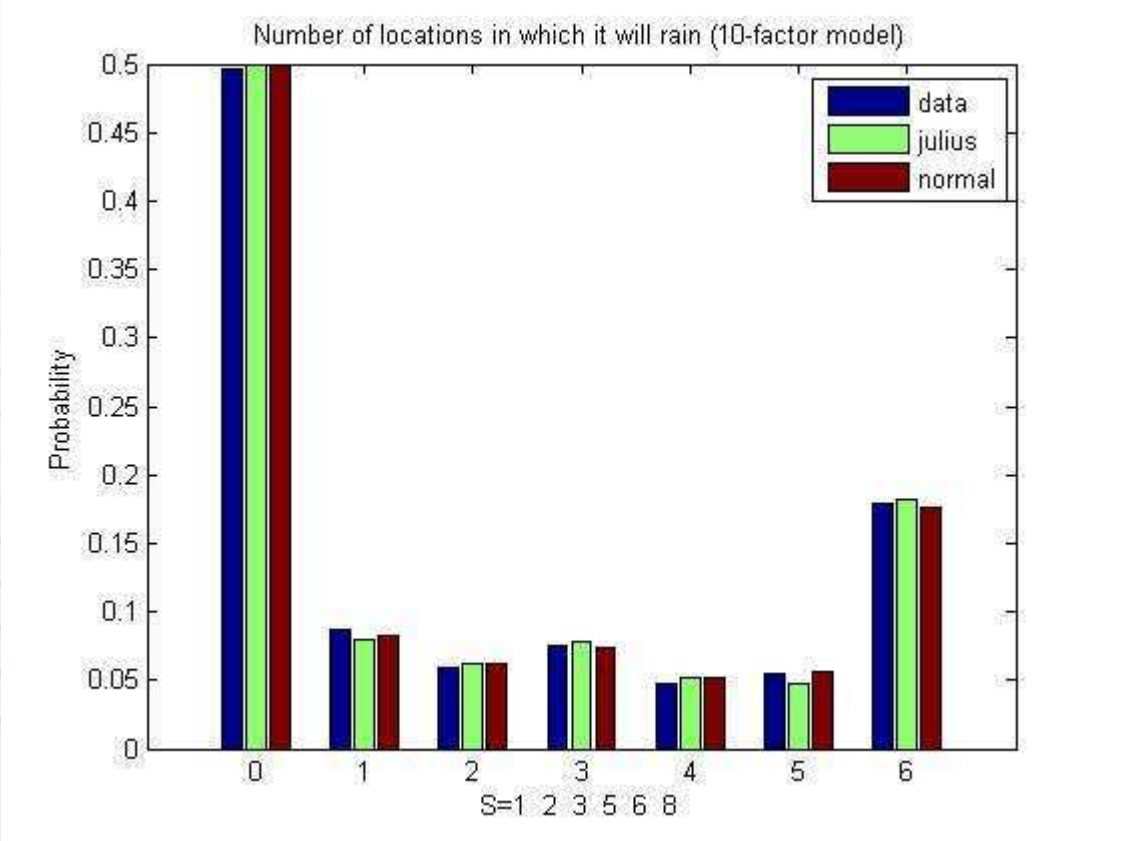
Ten factor rain model



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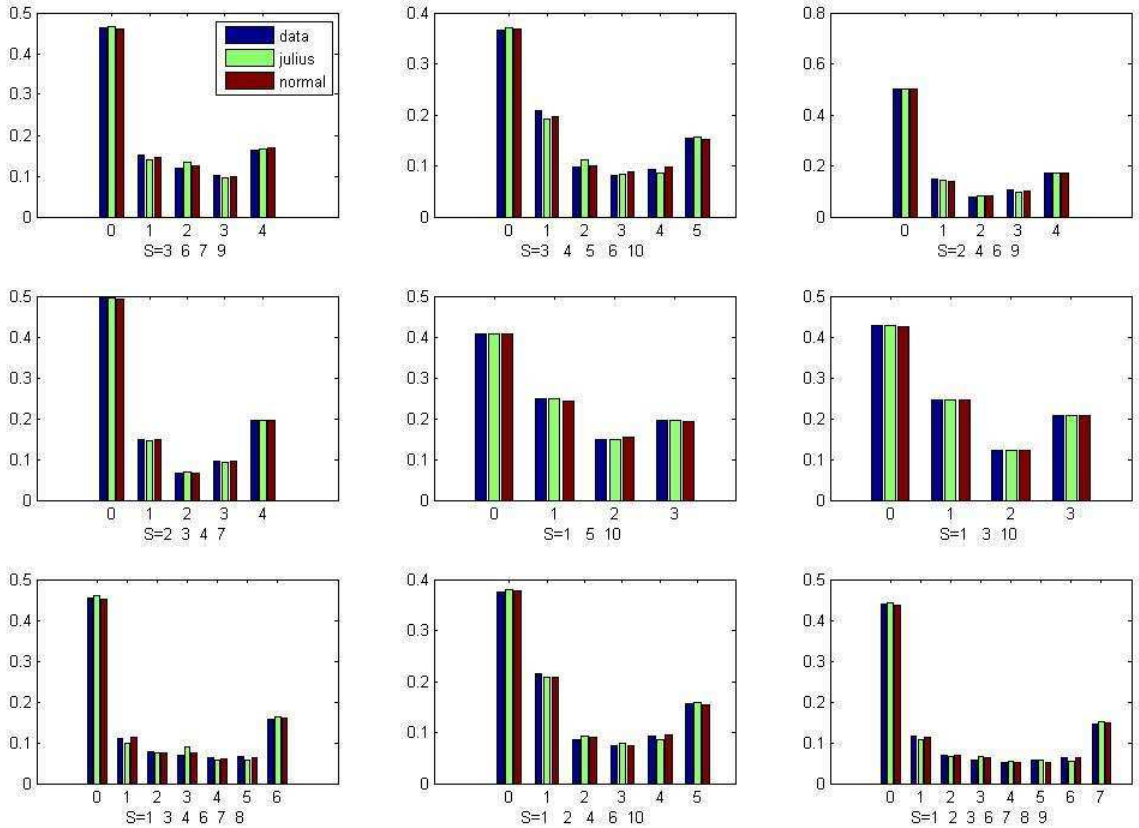
Ten factor rain model



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How many rain factors?

Average *percentage* pricing errors *amongst worst percentile*

	Number of locations in which it rained					
Model	0	1	2	3	4	5
1-factor	21%	146%	181%	180%	93%	31%
2-factor	17%	128%	171%	136%	109%	24%
3-factor	15%	132%	113%	101%	79%	20%
4-factor	15%	118%	90%	81%	72%	17%
5-factor	11%	100%	72%	64%	71%	16%
6-factor	9%	63%	54%	50%	58%	16%
7-factor	7%	43%	51%	45%	32%	13%
8-factor	4%	14%	34%	40%	14%	8%
9-factor	**3%**	**10%**	**14%**	**19%**	**10%**	**4%**
10-factor	**2%**	**11%**	**11%**	**12%**	**8%**	**3%**
Julius	**1%**	**13%**	**15%**	**13%**	**20%**	**3%**

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Tentative conclusions

- The Normal Copula model performs very well in predicting the number of locations in which rain falls on a given day.
- Approximation by lower rank correlation matrices is fraught with peril.
- Snow in Canada left as exercise for the reader

Study #2: Equity Prices

Daily stock price falls of one percent or more (1960-present)

DECATUR EVENING HERALD

WALL STREET IN NEW PANIC

LAKE FREIGHTER SINKS CLAIMING LIVES OF EIGHT

SENATOR BURTON, LONG LEADER OF OHIO POLITICS AND FRIEND OF PRESIDENT HOOVER IS DEAD

FEAR SAFETY OF AIR LINER WITH FIVE ON BOARD

JOHN ROACH STRATON, NOTED FUNDAMENTALIST AND PASTOR OF CALVARY CHURCH IS DEAD

Bankers Finally Bring Selling to Halt Three Minutes Before Closing

Senator Watson Ordered to Take Rest in Florida

NOKOMIS TRIP RIDER KILLED

AUTO LICENSES SET NEW TOTAL

SNEED LEAVES STATE UNION

ROUTE 40 BOOSTERS GATHER IN PANA TO PUSH HIGHWAY

WEST NOT CONCERNED WITH TARIFF FIGHT, SAYS GRUNDY

Taylorville A. C's Booster Fund, Raised to Secure New Industries, Deposited in Three Closed Banks

GE	**General Electric**	**12.62%**
HPQ	**Hewlett-Packard**	**17.62%**
IBM	**International Business Machines**	**13.41%**
KO	**Coca-Cola**	**12.01%**
DIS	**Walt Disney**	**16.29%**
BA	**Boeing**	**16.73%**
MCD	**McDonalds**	**14.33%**
PG	**Procter & Gamble**	**10.55%**
MMM	**3M Co.**	**11.34%**
XOM	**Exxon Mobil**	**10.84%**

Data compiled by Nicholas de Jong

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Equity Correlation (gauss)

	GE	HPQ	IBM	KO	DIS	BA	MCD	PG	MMM	XOM
GE	1.00	0.52	0.59	0.56	0.55	0.52	0.54	0.54	0.59	0.53
HPQ	0.52	1.00	0.60	0.45	0.48	0.44	0.50	0.44	0.48	0.417
IBM	0.59	0.60	1.00	0.48	0.49	0.47	0.51	0.49	0.54	0.489
KO	0.56	0.45	0.48	1.00	0.50	0.43	0.52	0.55	0.47	0.453
DIS	0.55	0.48	0.49	0.50	1.00	0.50	0.49	0.46	0.47	0.439
BA	0.52	0.44	0.47	0.43	0.50	1.00	0.47	0.41	0.47	0.444
MCD	0.54	0.50	0.51	0.52	0.49	0.47	1.00	0.48	0.49	0.447
PG	0.54	0.44	0.49	0.55	0.46	0.41	0.48	1.00	0.49	0.444
MMM	0.59	0.48	0.54	0.47	0.47	0.47	0.49	0.49	1.00	0.488
XOM	0.53	0.42	0.49	0.45	0.44	0.44	0.45	0.44	0.49	1

Normal Copula correlation matrix implied by 1% daily price drops

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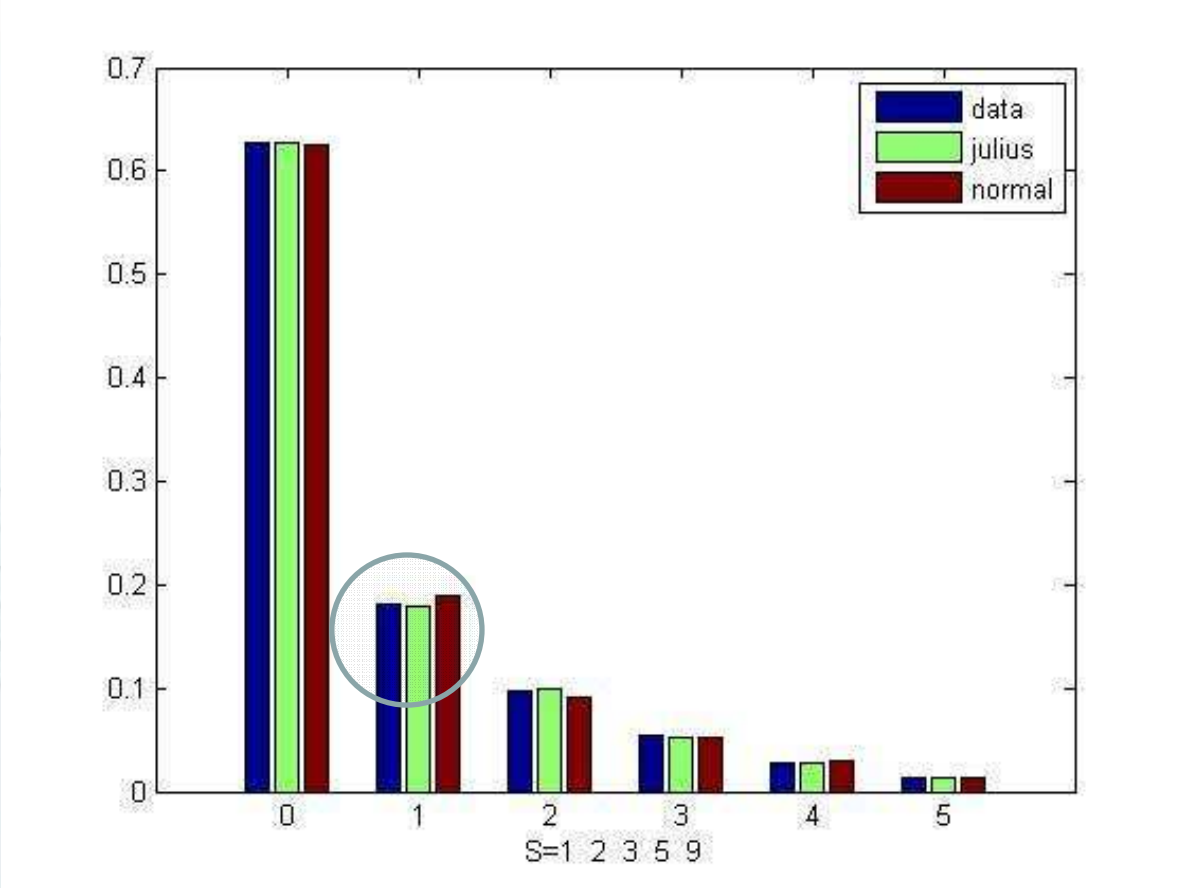
Equity example



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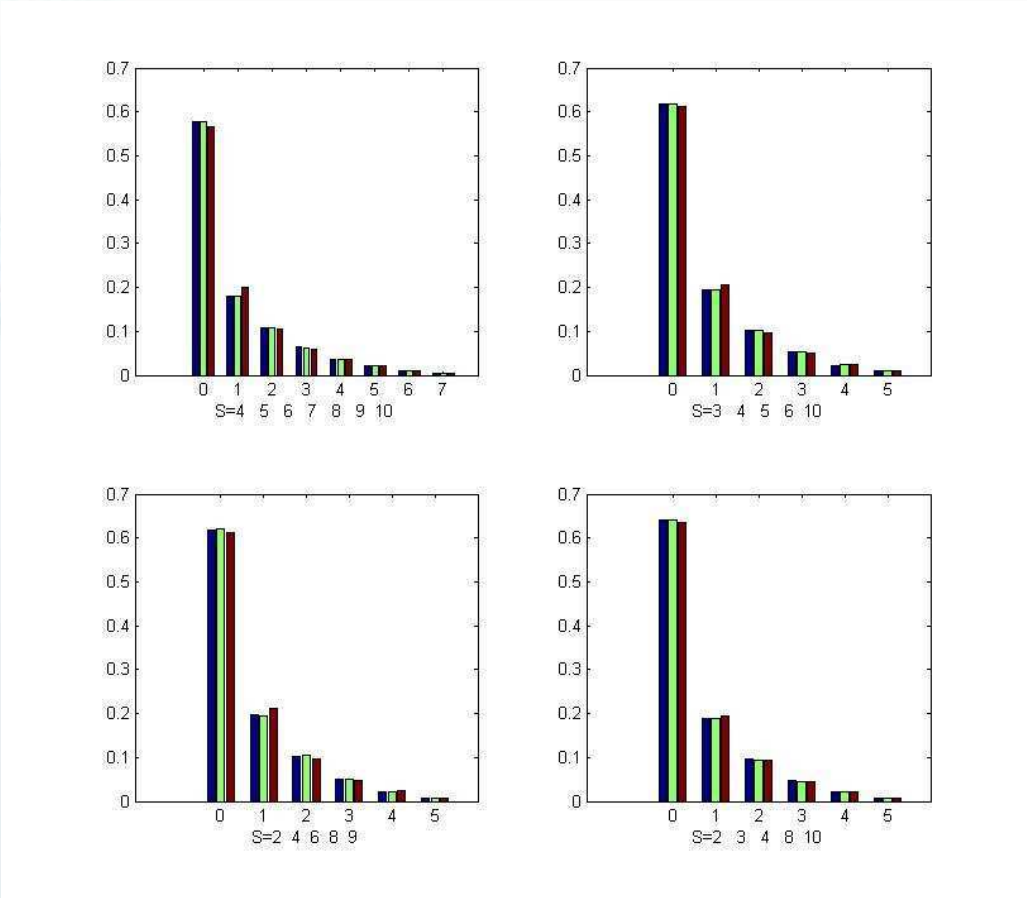
Equity (10-factor)



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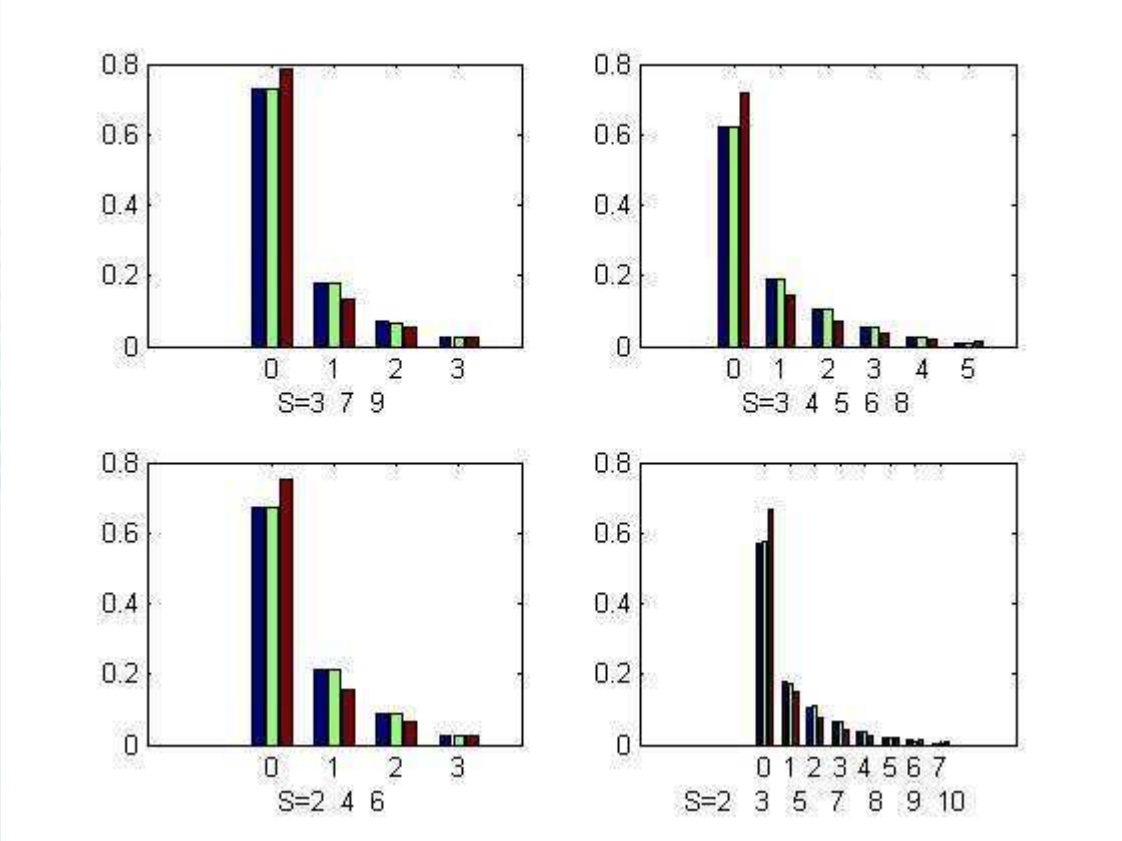
Equity (6-factor)



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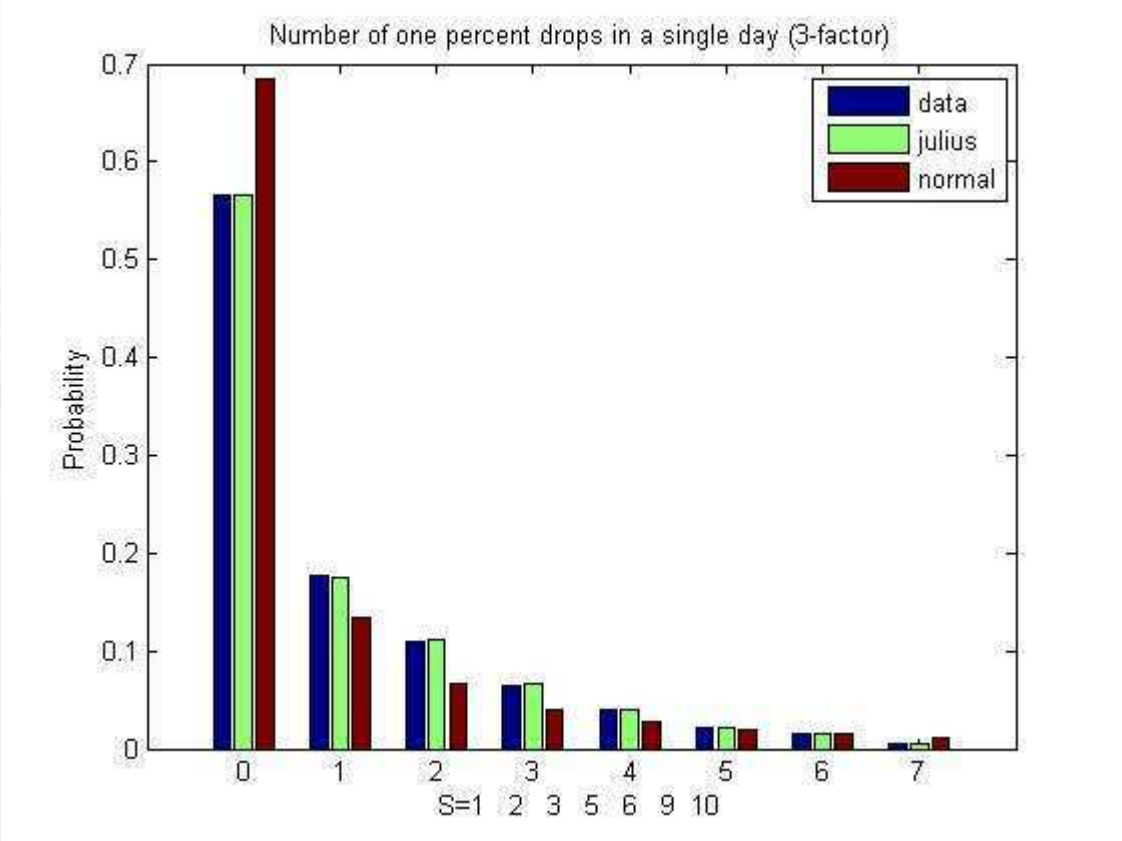
Equity (3-factor)



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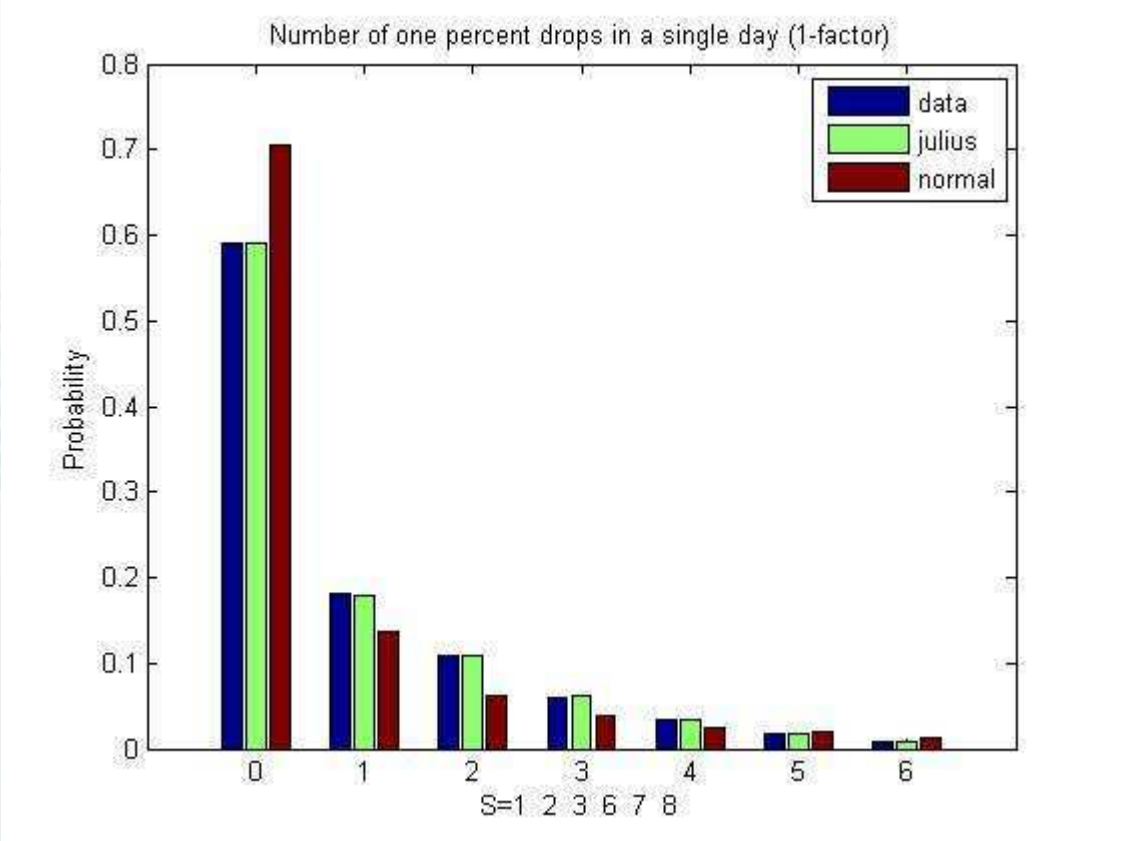
Equity (1-factor)



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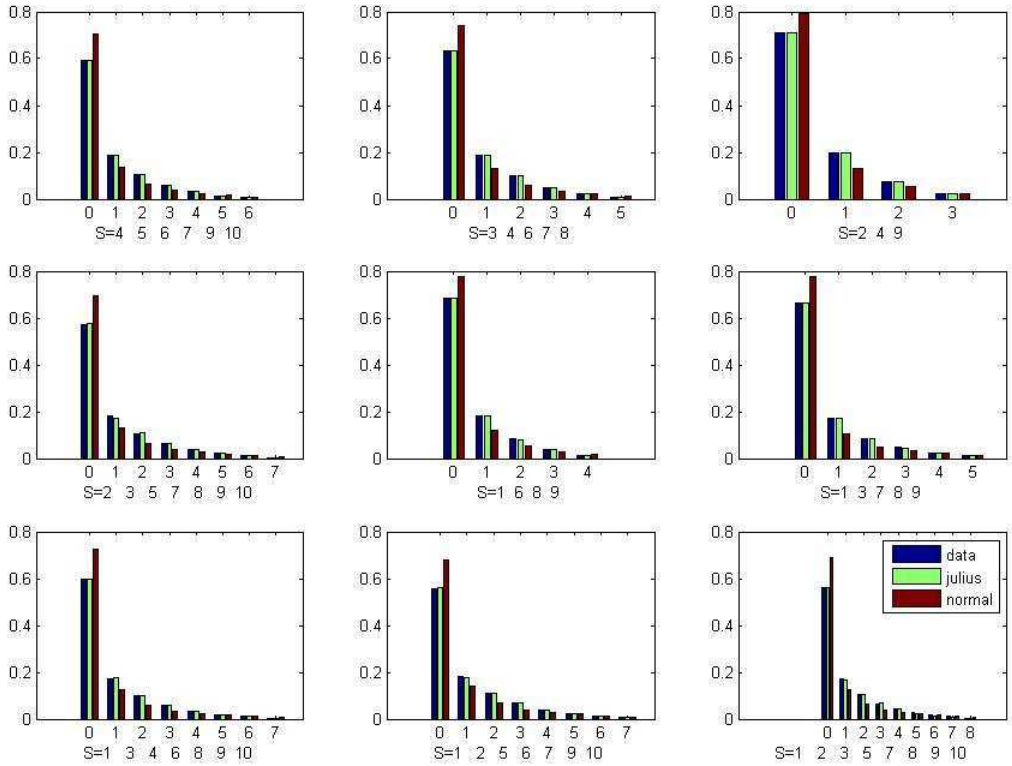
Equity (1-factor)



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How many equity factors?

Average absolute pricing error amongst worst percentile

	Number of stocks which fell one percent on the day							
Model	0	1	2	3	4	5	6	7
1-factor	0.126	0.053	0.049	0.028	0.014	0.005	0.003	0.006
2-factor	0.125	0.053	0.048	0.028	0.013	0.005	0.003	0.005
3-factor	0.124	0.051	0.047	0.027	0.013	0.006	0.003	0.005
4-factor	0.123	0.049	0.046	0.027	0.014	0.006	0.002	0.005
5-factor	0.118	0.045	0.047	0.027	0.012	0.005	0.003	0.005
6-factor	0.110	0.040	0.043	0.025	0.012	0.004	0.003	0.005
7-factor	0.085	0.028	0.037	0.022	0.011	0.004	0.003	0.004
8-factor	0.054	0.016	0.030	0.017	0.007	0.003	0.003	0.003
9-factor	0.020	0.022	0.019	0.013	0.006	0.003	0.002	0.003
10-factor	0.014	0.026	0.010	0.007	0.003	0.002	0.002	0.002
Julius	0.003	0.007	0.006	0.006	0.004	0.003	0.002	0.001

Average taken over all 120 portfolios containing 7 companies

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How many equity factors?

Average *percentage* pricing error amongst worst percentile

	Number of stocks which fell one percent on the day							
Model	0	1	2	3	4	5	6	7
1-factor	17.8%	43.8%	79.7%	71.6%	51.7%	28.4%	22.3%	53.1%
2-factor	17.8%	44.1%	77.3%	70.3%	48.1%	27.1%	19.7%	50.9%
3-factor	17.9%	41.6%	74.4%	68.9%	49.5%	29.5%	19.3%	50.9%
4-factor	17.3%	39.1%	74.7%	69.6%	50.2%	30.0%	18.6%	51.8%
5-factor	16.9%	35.6%	73.5%	65.7%	43.5%	23.3%	21.0%	51.0%
6-factor	16.2%	29.9%	63.4%	62.5%	42.5%	22.1%	18.9%	49.9%
7-factor	12.7%	18.9%	51.7%	51.1%	36.1%	18.0%	20.6%	42.8%
8-factor	8.5%	9.0%	38.4%	35.0%	21.4%	12.1%	19.7%	39.7%
9-factor	3.3%	11.2%	21.4%	25.9%	17.4%	13.5%	15.9%	35.0%
10-factor	**2.6%**	**13.1%**	**9.6%**	**11.0%**	8.8%	11.2%	13.3%	25.9%
Julius	**0.4%**	**3.9%**	**6.0%**	**8.3%**	8.9%	15.9%	16.5%	15.5%

Average taken over all 120 portfolios containing 7 companies

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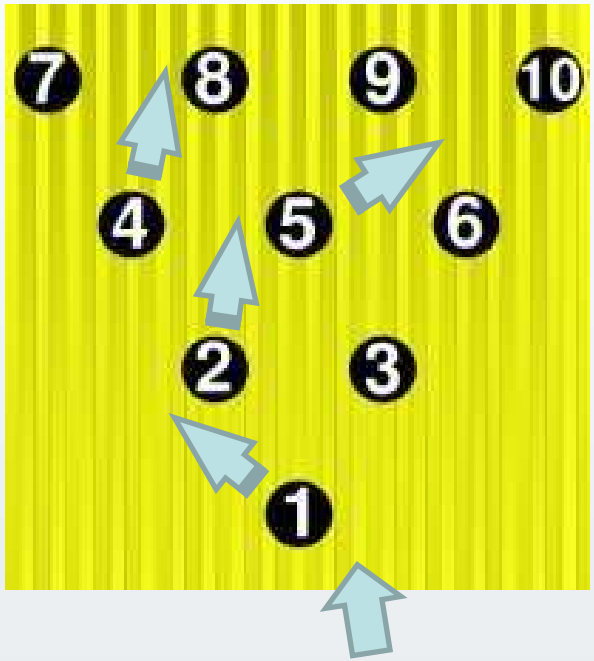
Tentative conclusions

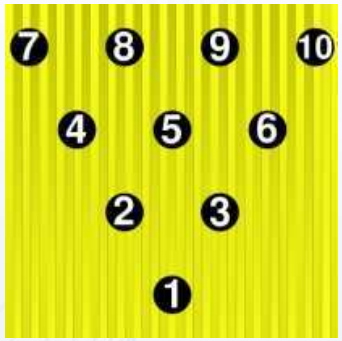
- The Normal Copula model misprices combinatorial bets on the number of price falls in a group of companies even when correlations are known precisely.
- Tail dependence is *not* the issue
- Approximating the correlation matrix can significantly exacerbate pricing errors

Study #3: Ten Pin Bowling



Using the normal copula model to predict residual pin formations
(Working paper available at www.financialmathematics.com)

Ten-pin example



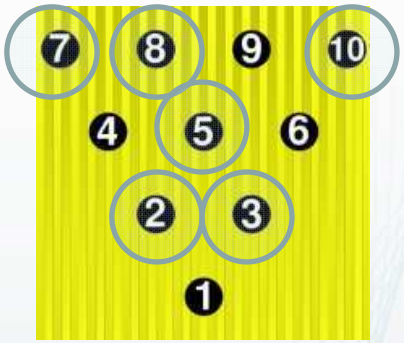
	1	2	3	4	5	6	7	8	9	10
1	1.0	0.7	0.7	0.4	0.7	0.2	-0.1	0.5	0.5	-0.2
2	0.7	1.0	0.4	0.6	0.7	-0.1	0.5	0.8	0.0	-0.1
3	0.7	0.4	1.0	0.0	0.8	0.8	-0.3	0.1	0.8	0.3
4	0.4	0.6	0.0	1.0	0.5	-0.3	0.8	0.9	-0.3	-0.8
5	0.7	0.7	0.8	0.5	1.0	0.6	0.3	0.5	0.4	0.0
6	0.2	-0.1	0.8	-0.3	0.6	1.0	-0.5	-0.4	0.8	0.5
7	-0.1	0.5	-0.3	0.8	0.3	-0.5	1.0	0.8	-0.7	-0.5
8	0.5	0.8	0.1	0.9	0.5	-0.4	0.8	1.0	-0.3	-0.5
9	0.5	0.0	0.8	-0.3	0.4	0.8	-0.7	-0.3	1.0	0.3
10	-0.2	-0.1	0.3	-0.8	0.0	0.5	-0.5	-0.5	0.3	1.0

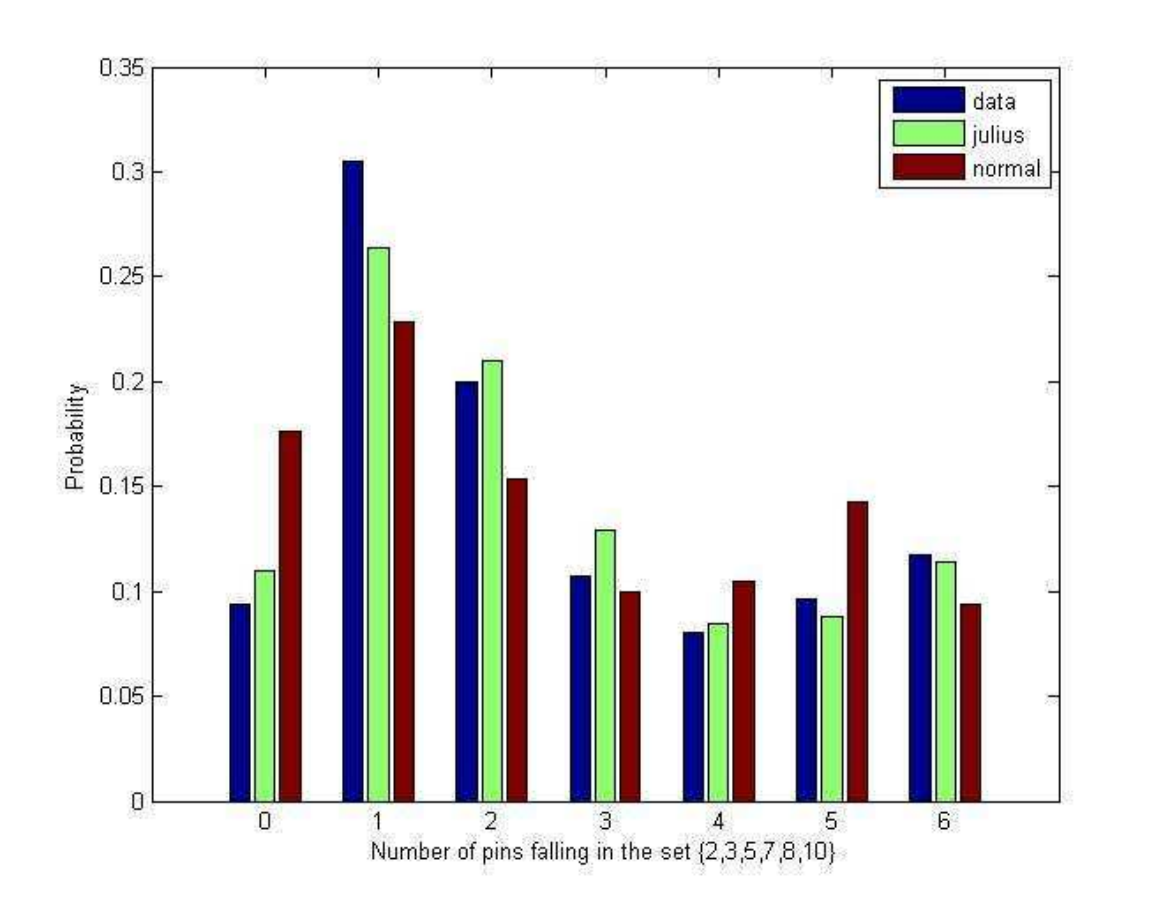
Portfolio {1,4,6,7,9,10}							
Pins	0	1	2	3	4	5	6
Data	0.05	0.17	0.37	0.26	0.09	0.02	0.04
Model	0.08	0.2	0.26	0.29	0.14	0.03	0.01

Underestimates Catastrophic risk by factor of four

Misprices most likely outcome by 50%

Bowling Example





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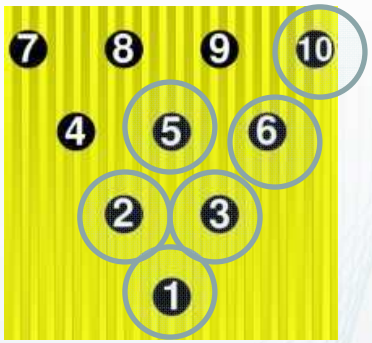
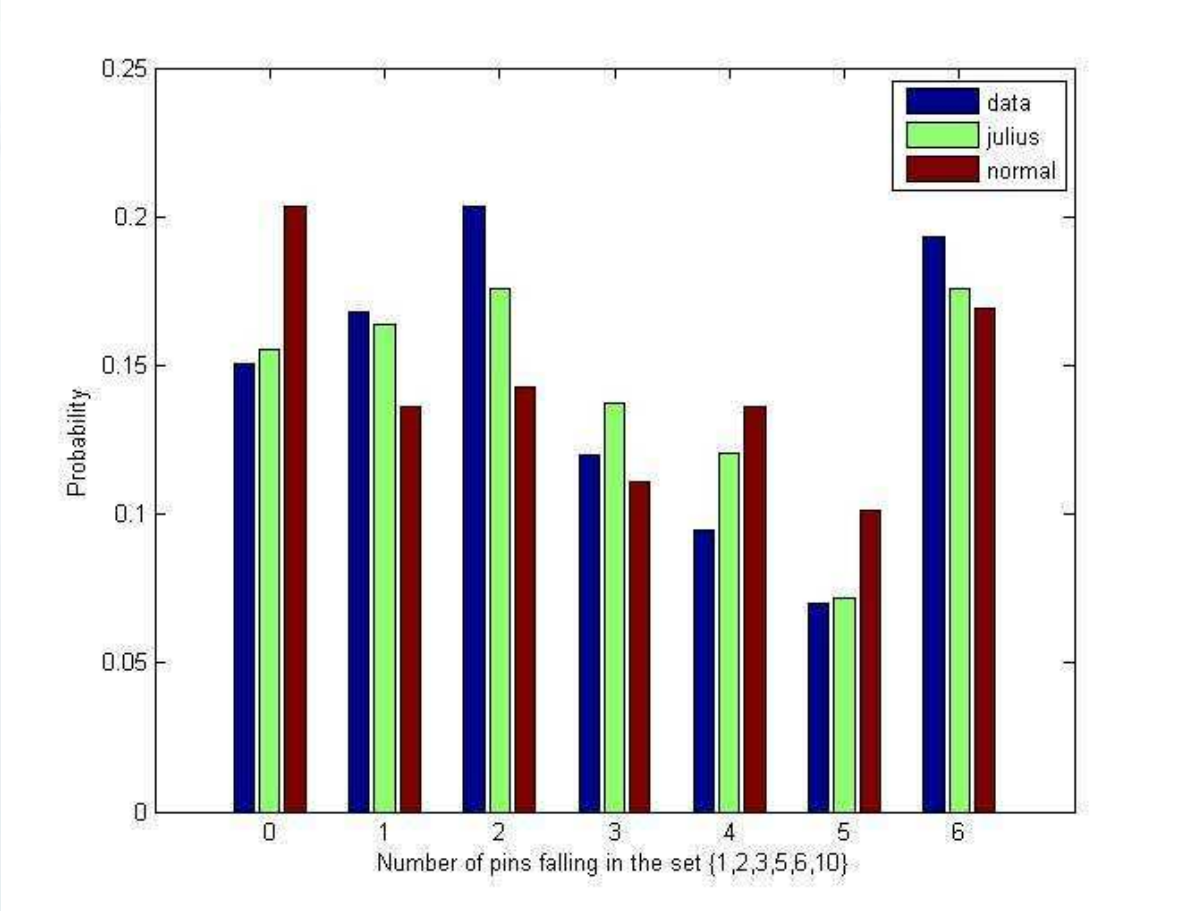
Bowling Example





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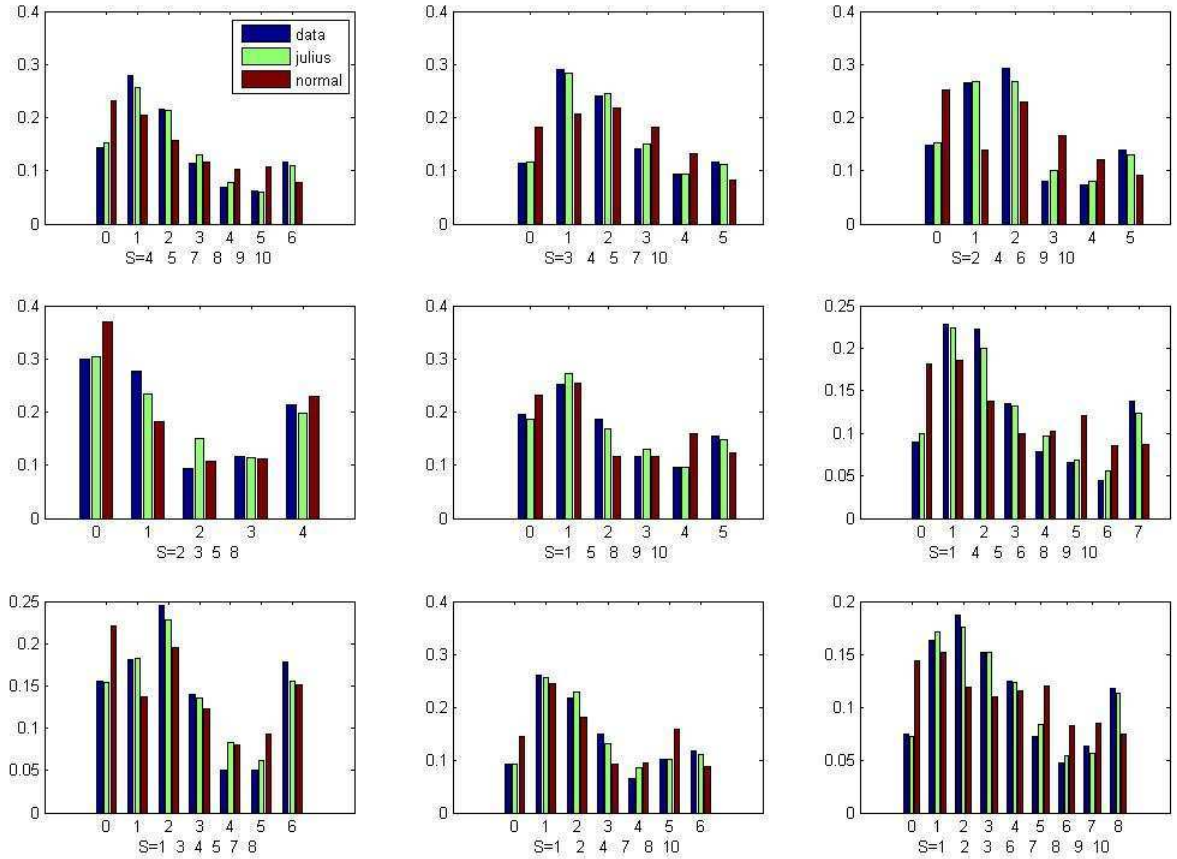
More bowling examples



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Pricing errors

Average over all 210 choices of 6 pin portfolios

Mean percentage pricing error							
Pins	0	1	2	3	4	5	6
Copula approach	33%	33%	41%	19%	31%	41%	34%
Julius approach	7.5%	9%	13%	12%	22%	27%	19%

Averaged across all 210 possible choices of 6-pin "portfolios"

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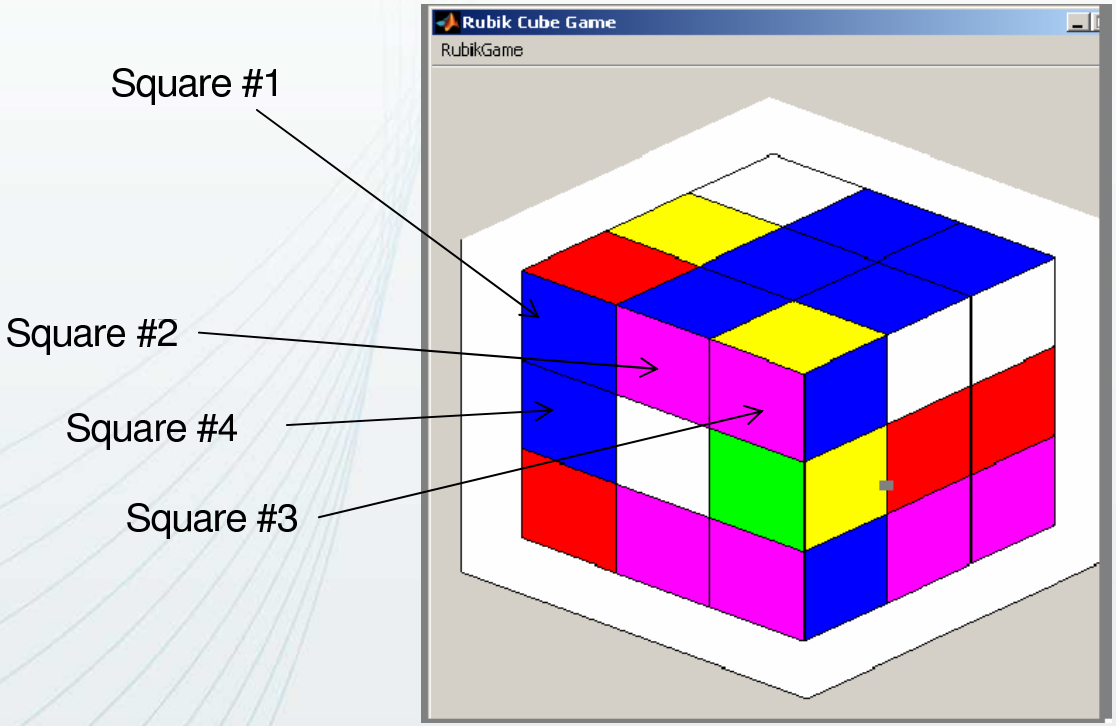
Conclusions

- The Normal Copula model is misprices combinatorial bets on the number of pins in a group even when correlations are known precisely.
- The errors are *large*. Model rejected out of hand.
- It should not be assumed that the Normal Copula can capture "knock-on effects" or "contagion".

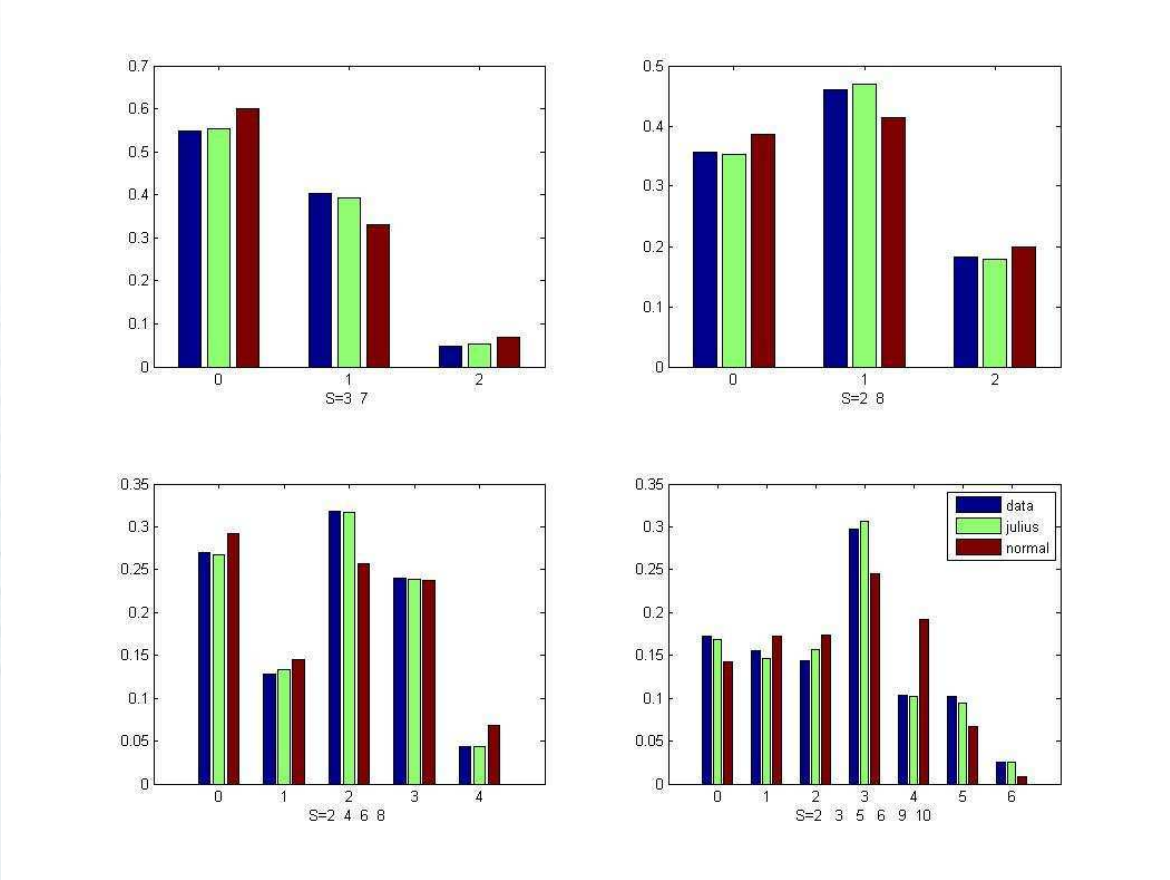
Study #4: Rubik's Cube



	Square 1	Square 2	Square 3	Square 4
Orig. Colour	Pink	Pink	Pink	Pink
New. Colour	Blue	Pink	Pink	Blue
Outcome	0	1	1	0

Using the normal copula model to predict the number of squares whose colour remains unchanged after five random moves

Rubik's Cube (1-factor)



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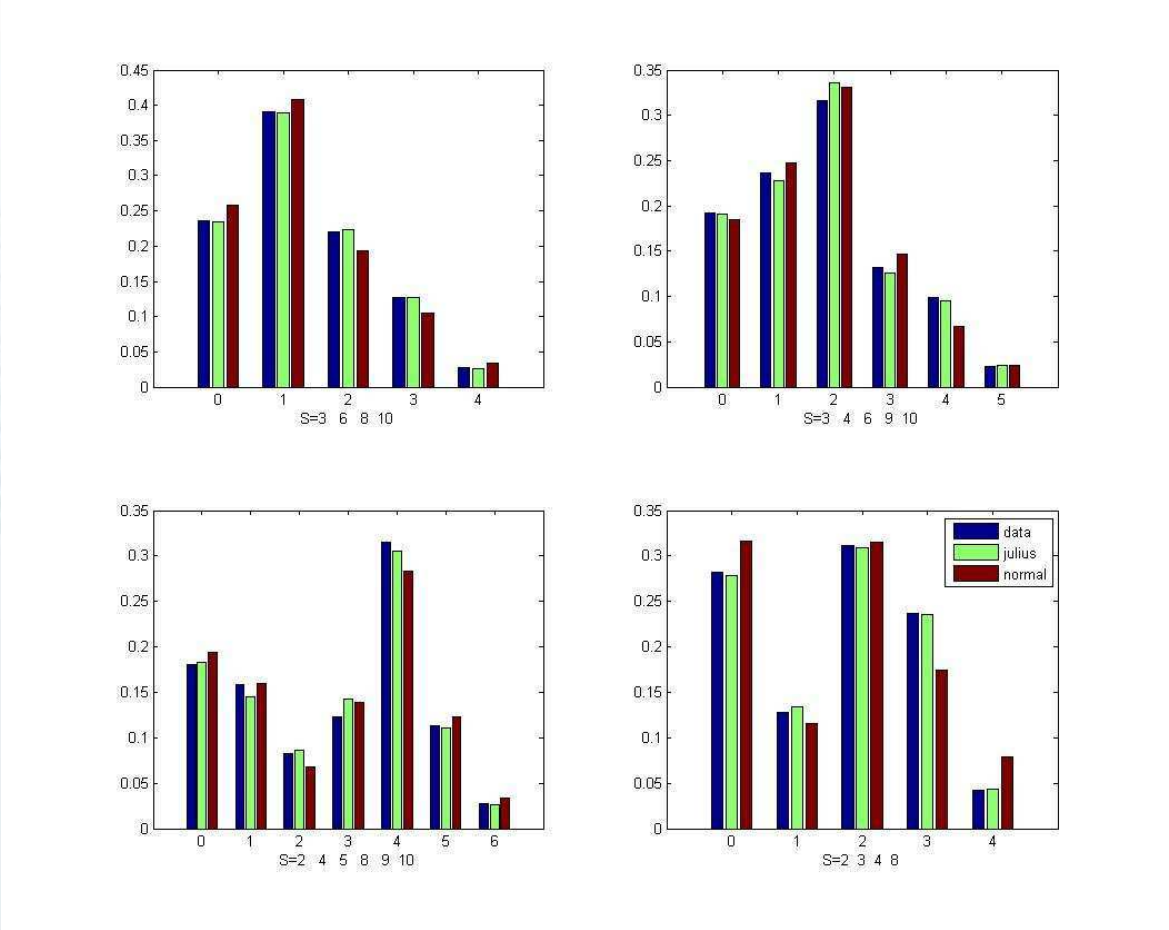
Rubik's Cube (2-factor)



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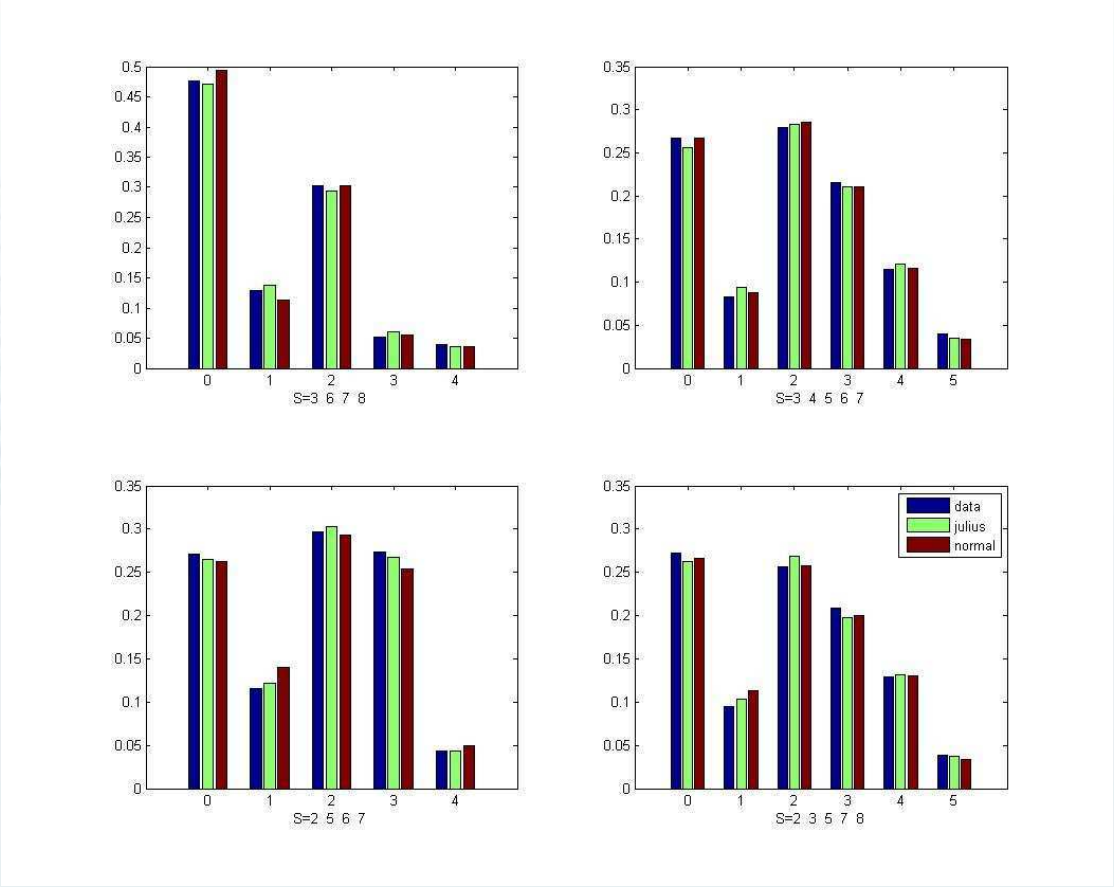
Rubik's Cube (3-factor)



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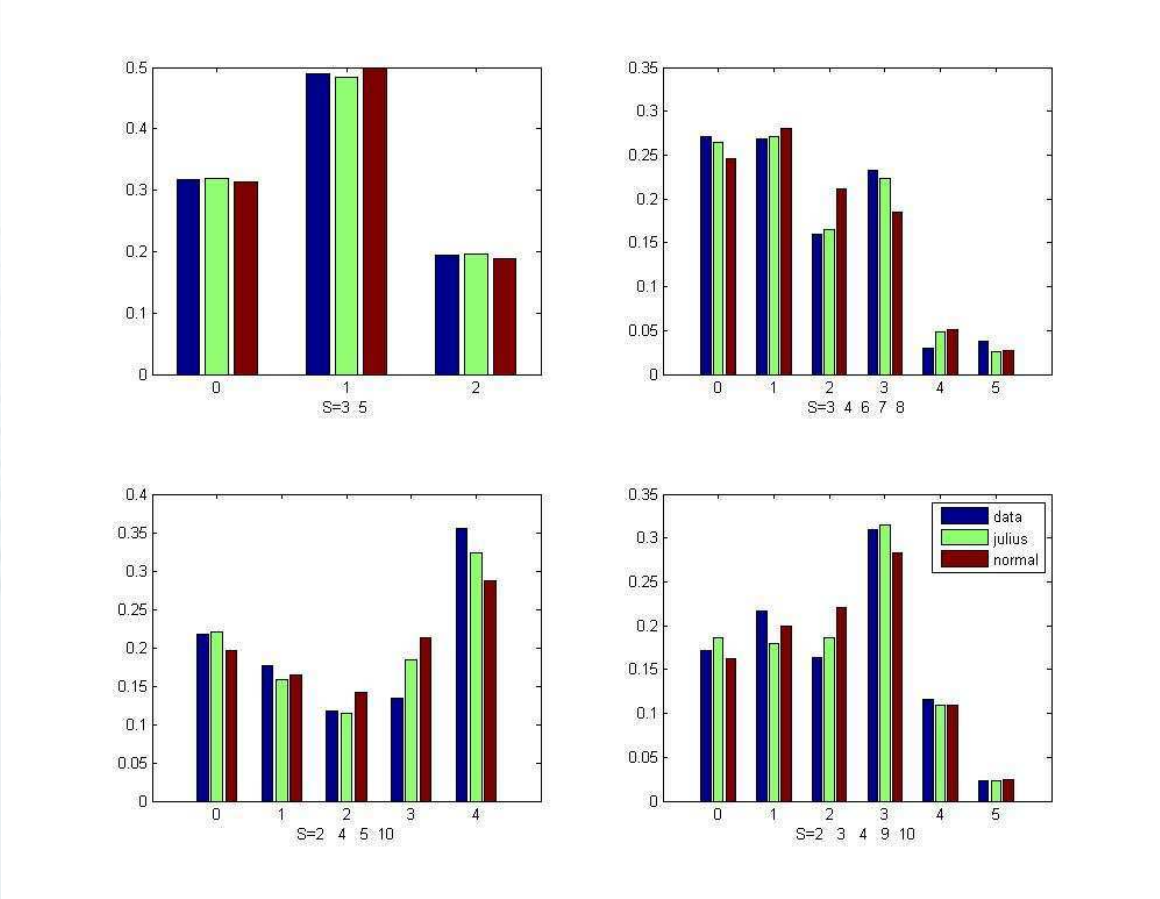
Rubik's Cube (10-factor)



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JULIUS FINANCE

How many Rubik's factors?

Mean percentage pricing error amongst worst percentile

	Squares remaining on their original colour							
Model	0	1	2	3	4	5	6	7
1-factor	33.4%	71.1%	60.0%	79.1%	55.1%	57.8%	77.7%	930.7%
2-factor	12.6%	51.6%	52.0%	44.0%	81.5%	79.0%	71.1%	58.0%
3-factor	10.0%	37.9%	34.0%	26.2%	27.7%	28.4%	60.1%	66.0%
4-factor	28.0%	56.3%	36.3%	39.5%	34.0%	34.1%	57.1%	99.5%
5-factor	32.4%	57.1%	40.1%	42.2%	37.4%	34.1%	56.3%	99.6%
6-factor	31.3%	57.4%	38.5%	37.7%	39.3%	41.0%	61.0%	110.4%
7-factor	33.6%	58.6%	40.2%	38.3%	40.1%	43.6%	64.6%	114.1%
8-factor	44.5%	60.5%	45.0%	47.5%	39.9%	42.3%	63.5%	95.3%
9-factor	43.4%	60.9%	45.6%	48.2%	40.2%	42.3%	63.3%	107.0%
10-factor	44.2%	61.0%	45.9%	48.4%	40.5%	42.8%	63.2%	97.4%
Julius	7.1%	35.3%	26.4%	23.3%	17.9%	24.3%	49.9%	40.3%

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Part 1. Tentative Conclusions

The Normal copula works quite well in one example (rainfall data)

Approximation by factor models *has not been found to work* for *less than three factors* in any real world system we have examined. The results are generally very bad.

We have *failed in every case* where a "nearby" correlation matrix is required – but it may just be that this part of the algorithm is lousy.

(Part 2 …)