# General Characteristics of Stratospheric Singular Vectors

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# Motivation

1. We now have the ability to compute SVs with the NASA model, so what can we do that is new and interesting?

2. What can SVs tell us about the stratosphere?

3. Can we learn some new things about SSW or about stratospheric/tropospheric interactions?

4. In what ways are SVs determined for the stratosphere similar or different than those for the troposphere.

# Singular Vectors

Given a linearized model

$$\mathbf{y} = \mathbf{M}\mathbf{x}$$

Find the $\mathbf{x}$ that maximizes

$$L_2 = \mathbf{y}^{\mathrm{T}}\mathbf{L}_2\mathbf{y}$$

Given

$$L_1 = \mathbf{x}^{\mathrm{T}}\mathbf{L}_1\mathbf{x}$$

The solution:

$$\mathbf{x} = \mathbf{L}_1^{-\frac{1}{2}}\mathbf{z}$$

where

$$\mathbf{L}_1^{-\frac{1}{2}\mathrm{T}}\mathbf{M}^{\mathrm{T}}\mathbf{L}_2\mathbf{M}\mathbf{L}_1^{-\frac{1}{2}}\mathbf{z} = \lambda^2\mathbf{z}$$

$\lambda$ is a singular value of $\mathbf{L}_2^{\frac{1}{2}}\mathbf{M}\mathbf{L}_1^{-\frac{1}{2}}$, $\mathbf{z}$ is the corresponding right singular vector, and $\lambda^2$ may be interpreted as $L_2/L_1$.

# The Energy Norm

(Talagrand 1981 *Tellus*; Errico 2000 *QJRMS*)

$$E = \frac{1}{2} \sum_{i.j.k} \left( \frac{\Delta p}{p_s - p_t} \right)_{i,j,k} (\delta A)_j \left[ u'^2 + v'^2 + \frac{C_P}{T_r} T'^2 + \frac{R T_r}{p_{sr}} p_s'^2 \right]_{i,j,k}$$

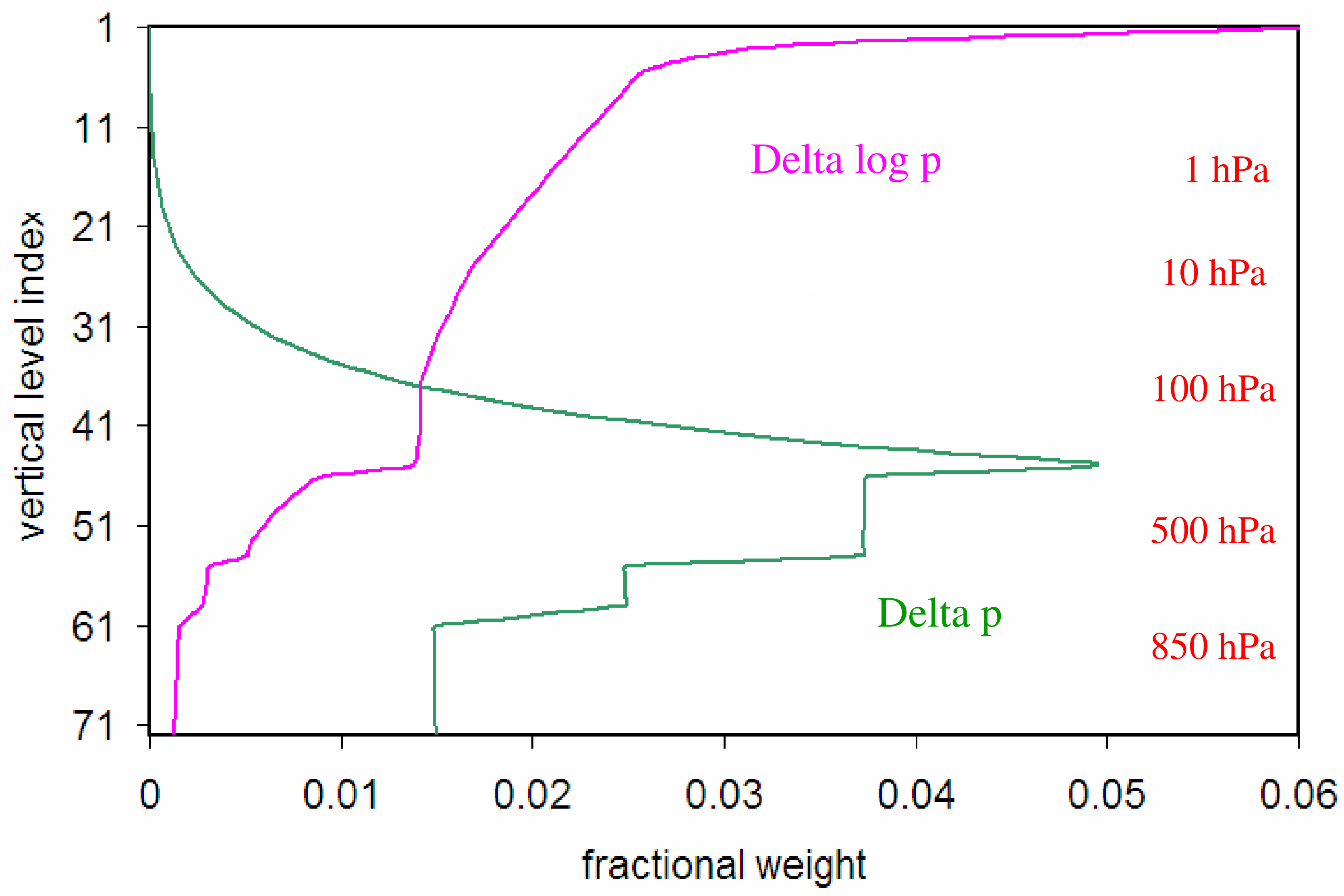
KE + APE

For tropospheric SVs, $T_r = 300\text{K}$ and $p_{sr} = 10^5\text{Pa}$

# NASA's GEOS-5

1. Finite-Volume dynamical core (Lin and Rood 1996)
2. Full physics package
3. Resolution 1.25 x 1 degrees on 72 levels
4. Top at 1 Pa
5. 40 levels at $p<150$ hPa

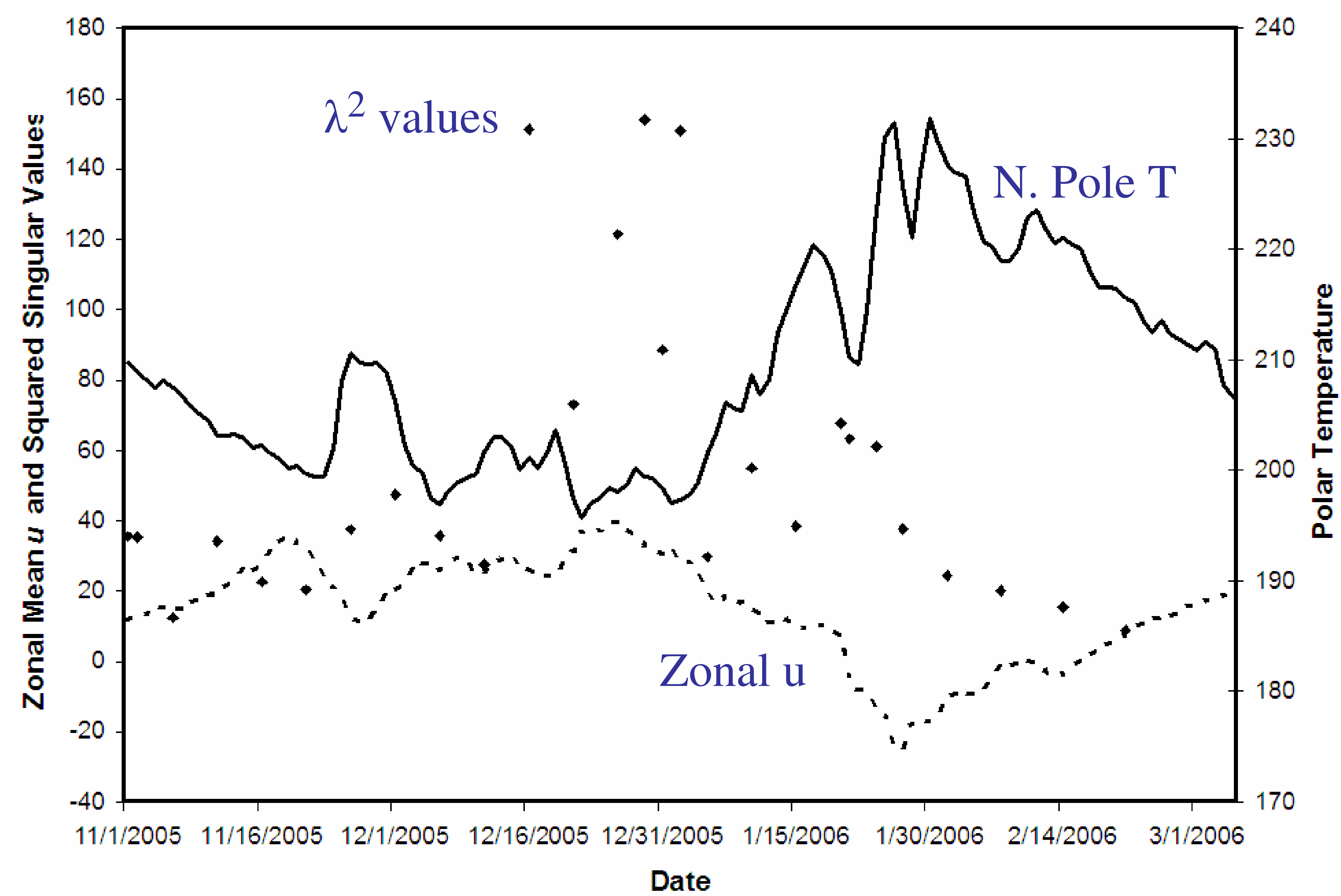
# Spacing of NASA GEOS-5 vertical grid



# Adjoint version of NASA's GEOS-5

1. Exact adjoint of dynamical core
2. Simplified physics:
   a. Del**4 horizontal diffusion
   b. vertical diffusion with K-coefficients specified by NLM
   c. "sponge" replaces radiation etc. high in atmosphere
   d. surface drag
3. Resolution 5x4 degrees on 72 levels

*E* at final time measured only N. of 30°N for 10h Pa < $p$ < 90 hPa
*E* at initial time measured globally
10-20 iterations of Lanczos algorithm to solve for SVs.
Results presented for only leading SVs


λ² values
N. Pole T
Zonal u
Zonal Mean u and Squared Singular Values
Polar Temperature
Date
180
160
140
120
100
80
60
40
20
0
-20
-40
240
230
220
210
200
190
180
170
11/1/2005
11/16/2005
12/1/2005
12/16/2005
12/31/2005
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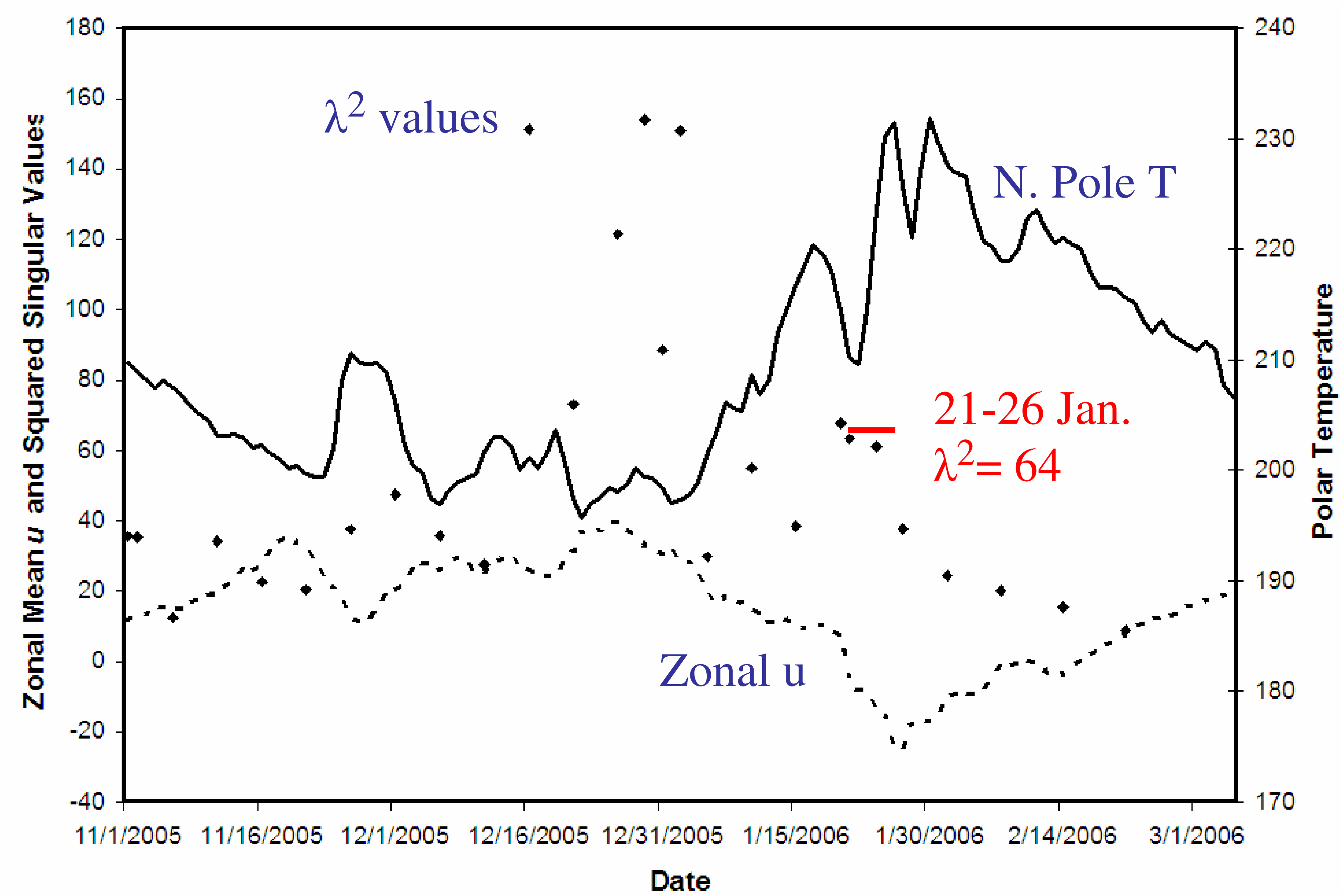
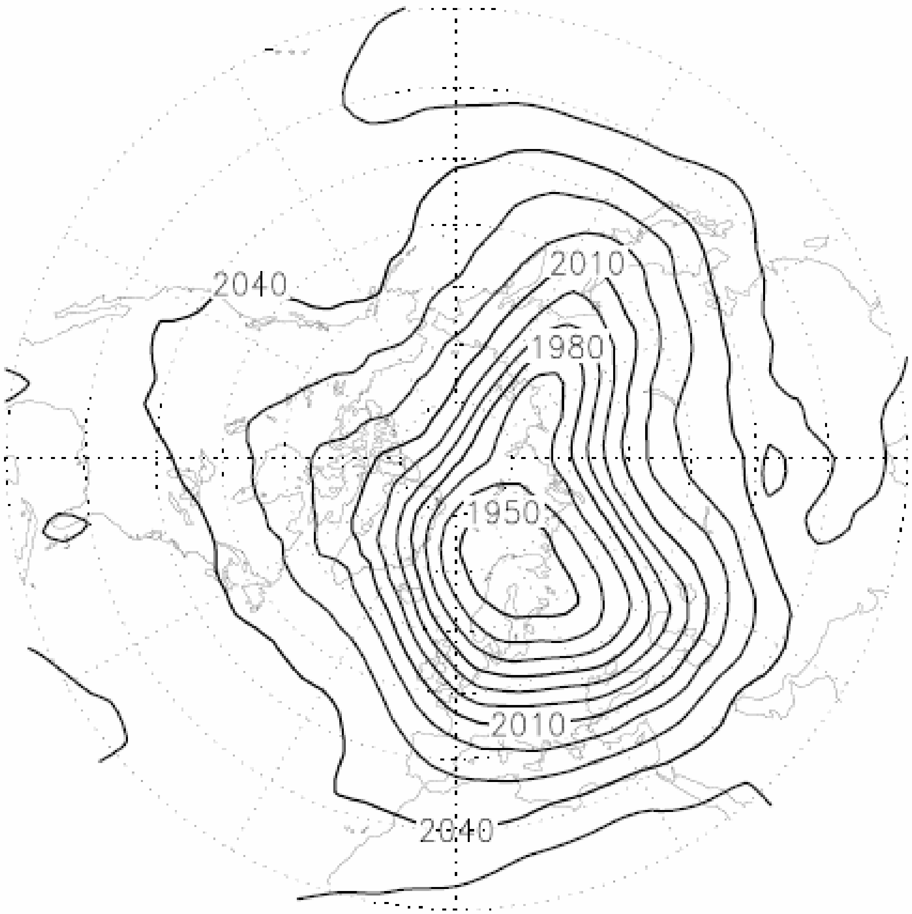
1/30/2006
2/14/2006
3/1/2006

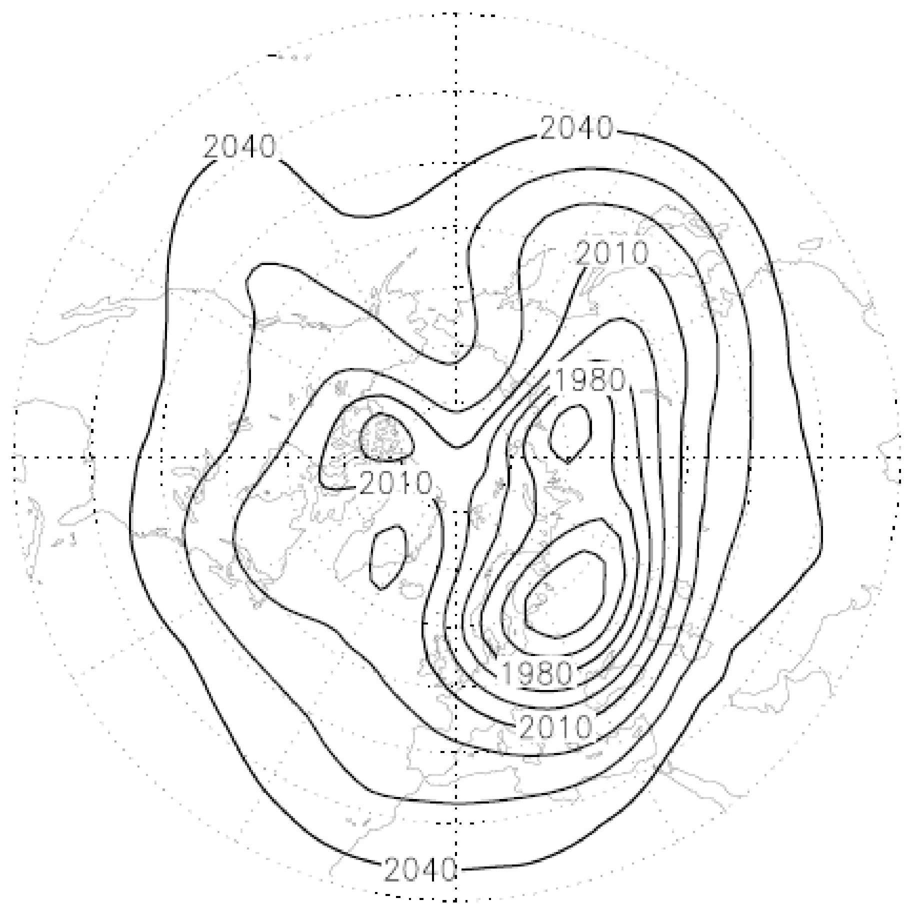


λ2 values
N. Pole T
21-26 Jan.
λ2= 64
Zonal u
Zonal Mean u and Squared Singular Values
Polar Temperature
Date
180
160
140
120
100
80
60
40
20
0
-20
-40
240
230
220
210
200
190
180
170
11/1/2005
11/16/2005
12/1/2005
12/16/2005
12/31/2005
1/15/2006
1/30/2006
2/14/2006
3/1/2006

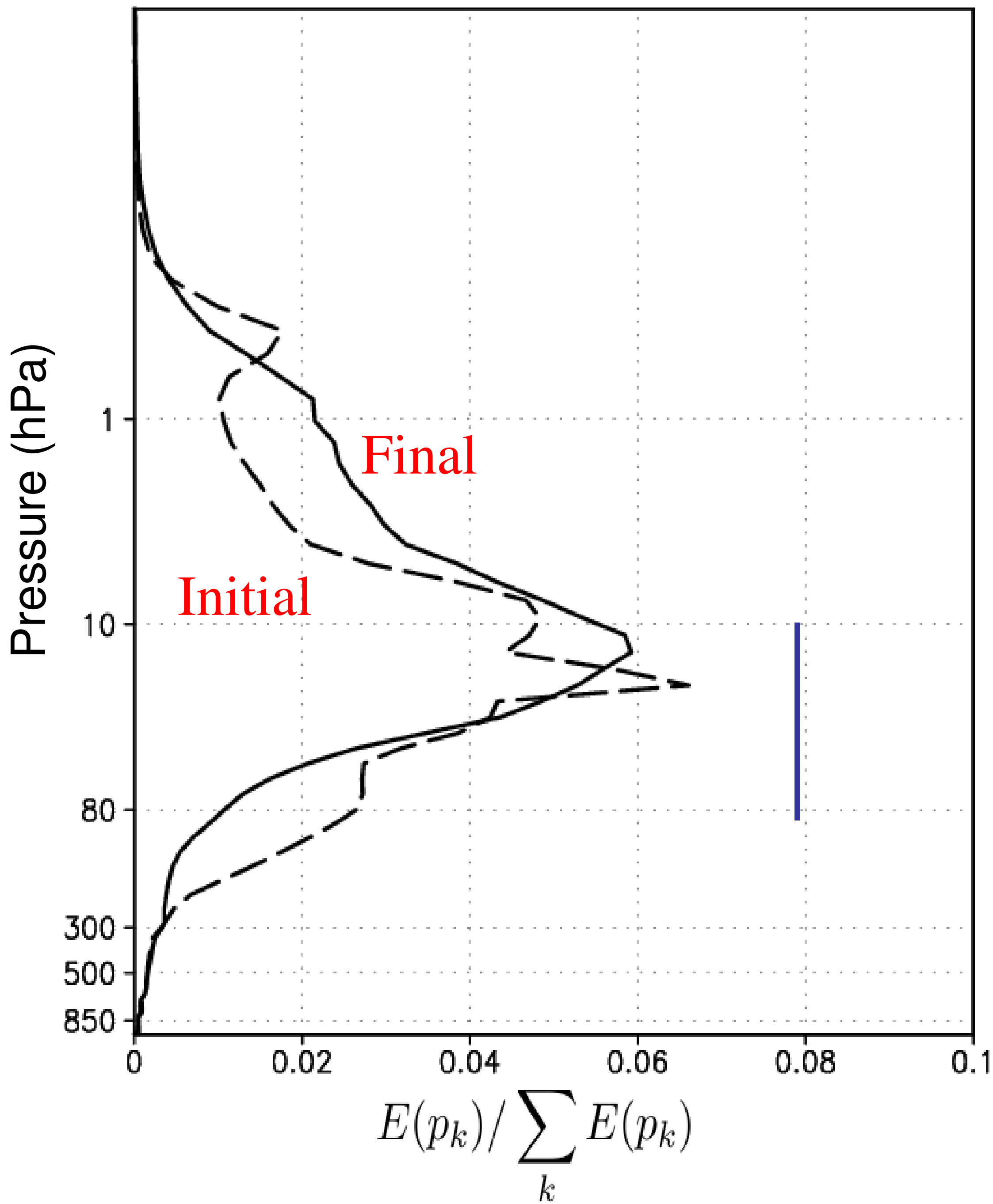
# 50 hPa Geopotential Height




21 January 2006

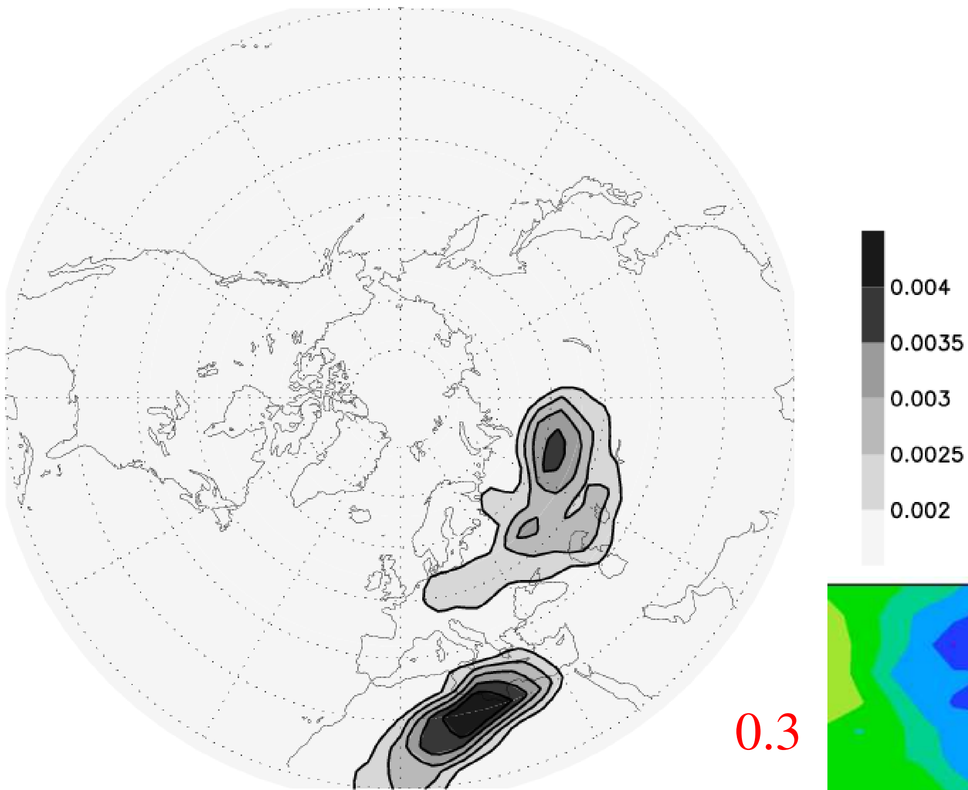



26 January 2006

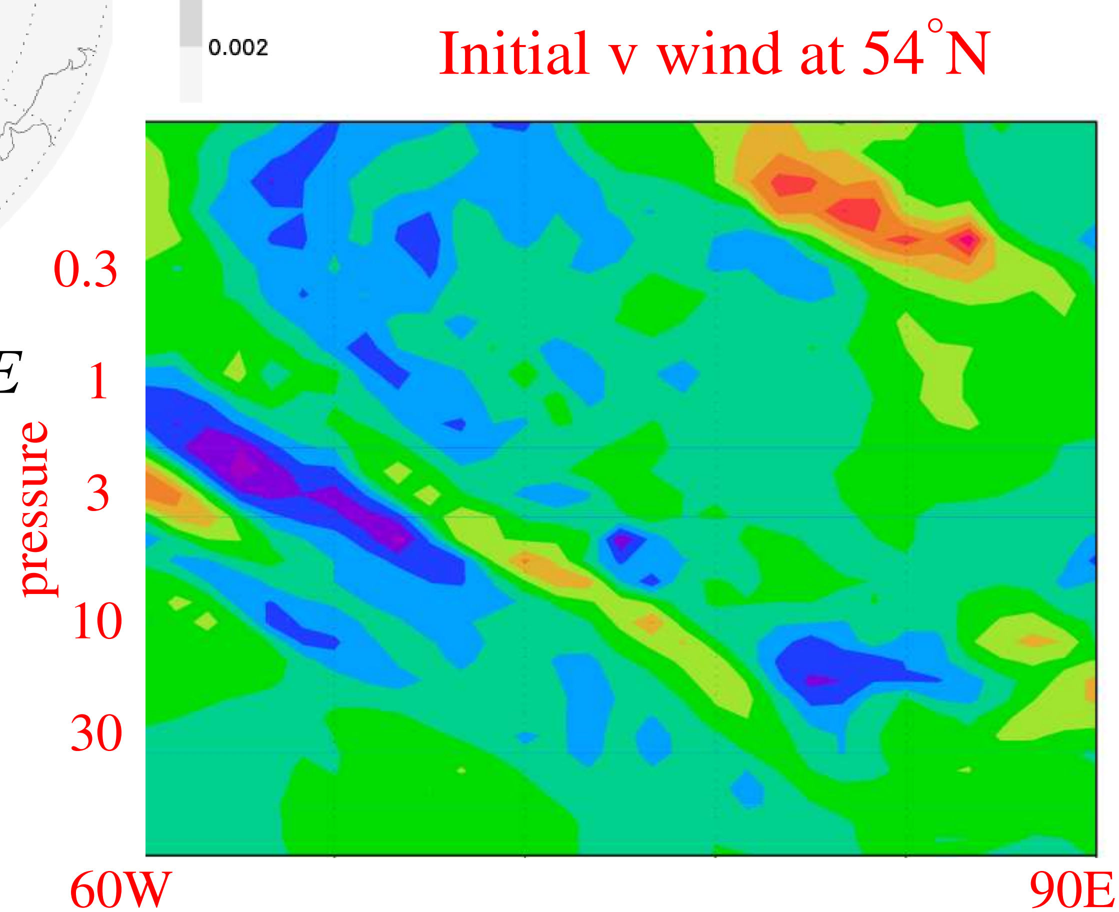


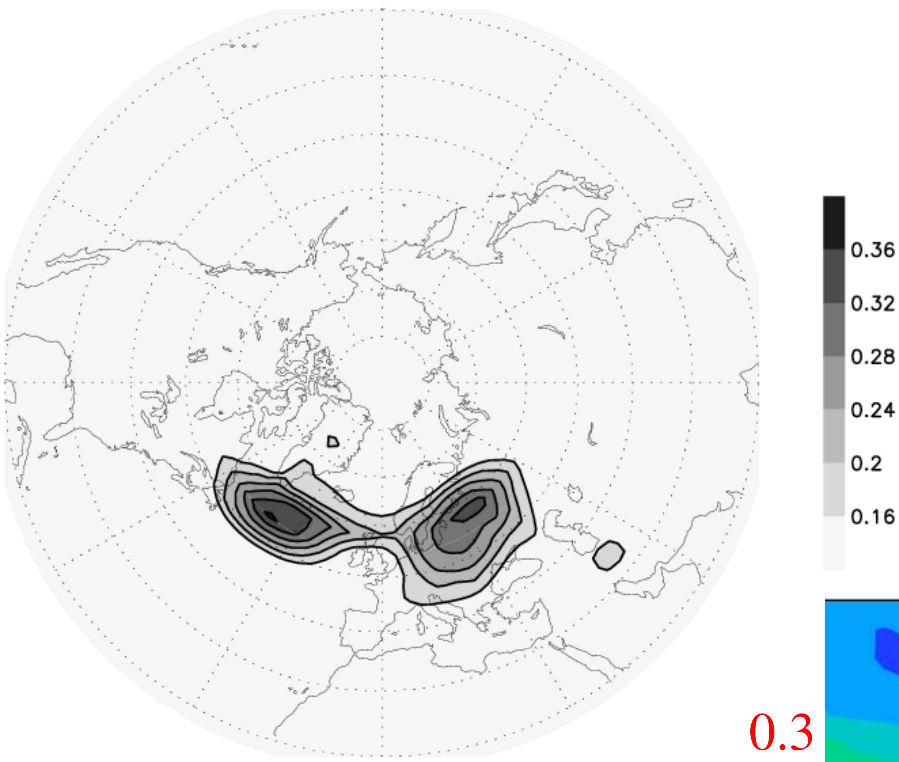

Perturbation *E* distribution in the vertical

Initial KE/APE = 1.0
Final KE/APE= 2.0



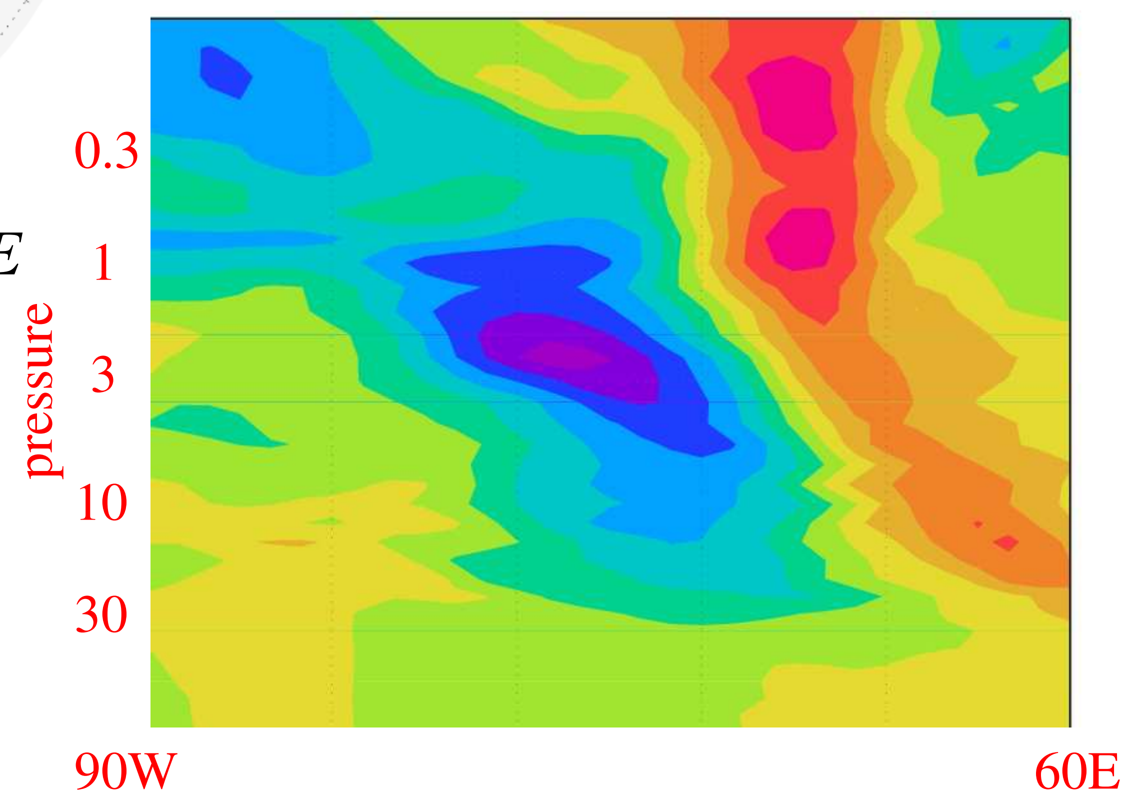

Initial vertically integrated $E$



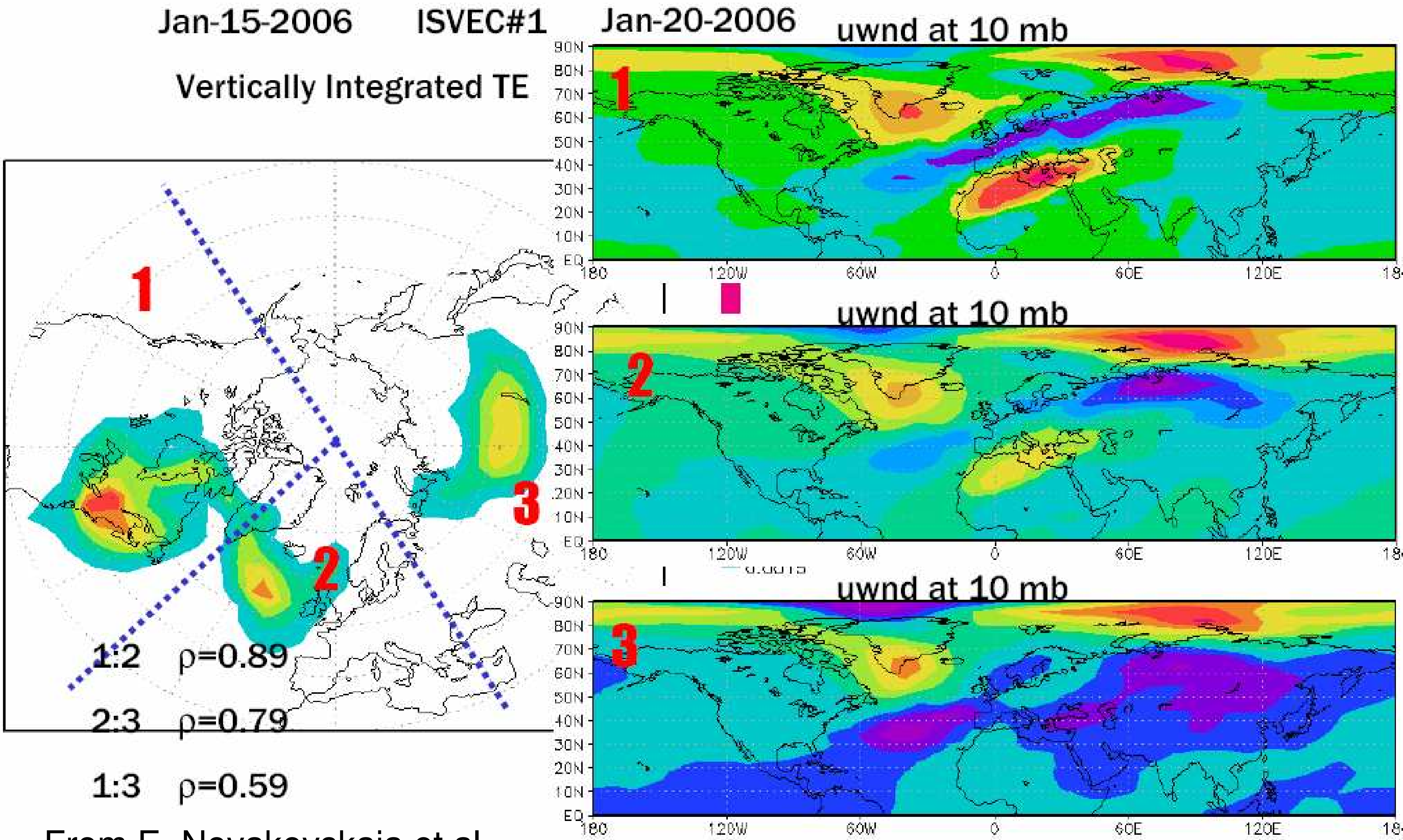



Final vertically integrated $E$

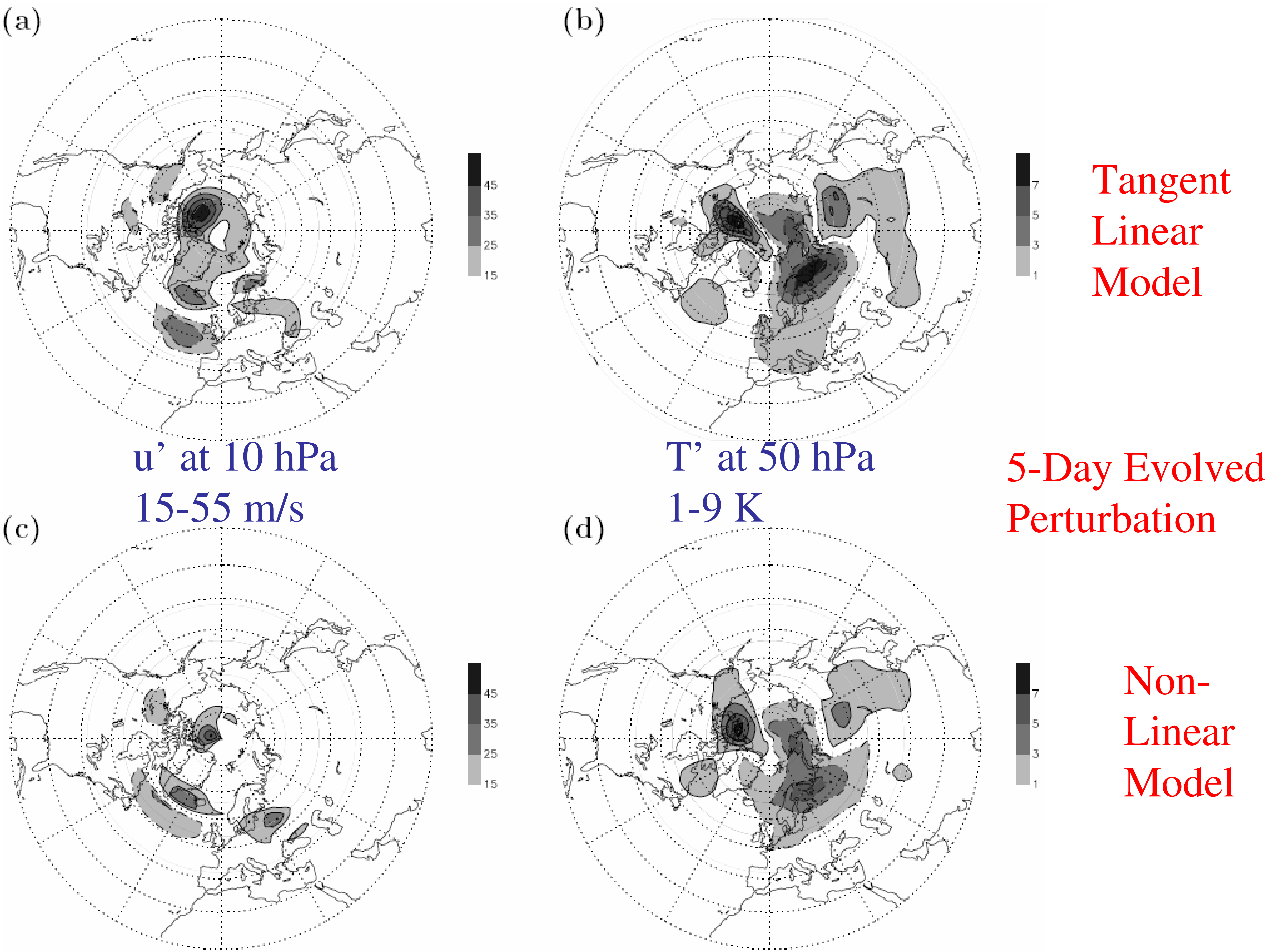
Final v wind at 62°N



# NON-LOCAL INITIAL STRUCTURES




From E. Novakovskaia et al.



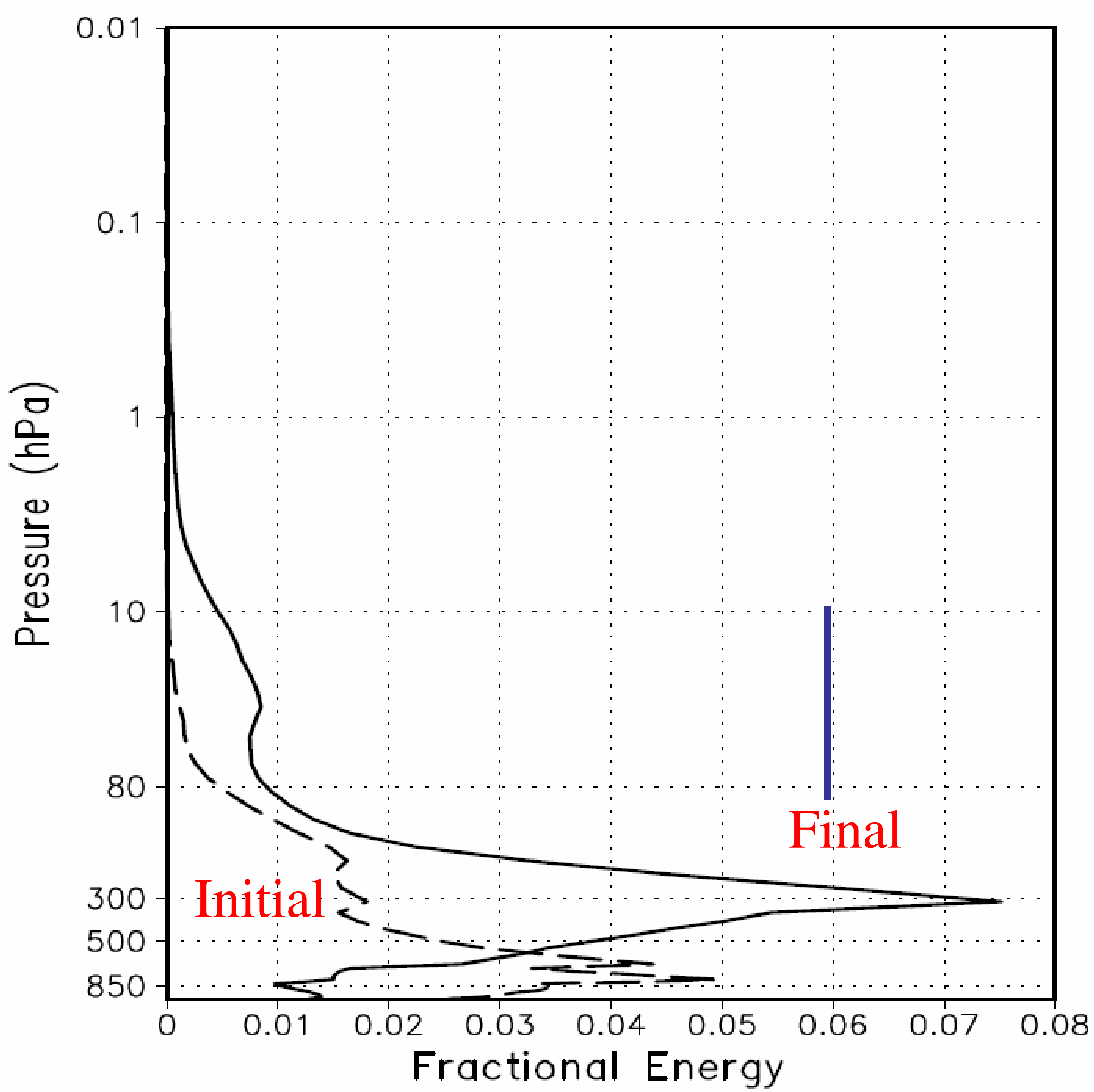
(a)
(b)
(c)
(d)
45
35
25
15
7
5
3
1
Tangent Linear Model
u’ at 10 hPa
15-55 m/s
T’ at 50 hPa
1-9 K
5-Day Evolved Perturbation
Non-Linear Model

# The V-norm

$$V = \frac{1}{2}\sum_{i.j.k}\left(\frac{\Delta \ln p}{\ln p_s - \ln p_t}\right)_{i,j,k}(\delta A)_j\left[u'^2 + v'^2 + \frac{C_P}{T_r}T'^2 + \frac{RT_r}{p_{sr}}p_s'^2\right]_{i,j,k}$$

V=1 applied globally as initial constraint

E norm, restricted as before, applied at final time




Perturbation *E* distribution in the vertical

Leading SV for 21-26 Jan. using initial V-norm, but final E-norm in restricted vol.

# Conclusions

## 5-day stratospheric SVs are like tropospheric ones:

1. Singular values much smaller in winter than summer.
2. Perturbation shapes change strongly over time.
3. Rotational winds strongly dominate divergent ones.
4. The initial *E*-norm SVs exhibit a strong westward tilt with increasing height, which becomes less pronounced as the SV evolves.

# Conclusions

5-Day stratospheric SVs are unlike tropospheric ones:

1. Their singular values are much less than tropospheric ones.
2. Their structures are nonlocal.
3. The agreement between TLM and NLM evolution remains close up to 5 days for initially large perturbations.
4. The interpretations are strongly dependent on the vertical weighting in the choice of norm.

# Conclusions

SVs are potentially useful for understanding stratospheric behavior and interactions with the troposphere.

Errico et al., Meteorologische Zeitschrift, late 2007 or early 2008.

Hooghoudt and Barkmeijer, same issue.