Application of Potential Theory to MR Elastography

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Magnetic Resonance Elastography (1)

Problem: Finding material parameters of biological materials **in vivo** and map them in an anatomically meaningful image that provides useful clinical information (elastogram).

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Imaging modalities:

- ultrasound (sonoelastography or sonoelasticity).
- MRI (magnetic resonance elastography).
- optics (optical elastography).

Magnetic Resonance Elastography (2)

- We can employ different tissue excitations:
  - the tissue is compressed (static elastography) (Ophir *et al.* 1991, Cespedes *et al.* 1993, Krouskop *et al.* 1998).
  - a time harmonic excitation made on the boundary creates a time harmonic wave in the tissue (dynamic elastography) (Muthupillai *et al.* 1995, Lorenzen *et al.* 2003, Sinkus *et al.* 2000, Plewes *et al.* 2000, McKnight *et al.* 2002, Dresner *et al.* 2001).
  - a time dependent pulse on the boundary creates a propagating wave in the tissue (transient elastography) (Catheline *et al.* 1999,Sandrin *et al.* 2003).
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- Present work: dynamic MR elastography, elastogram of the shear wave speed.

Magnetic Resonance Elastography (3)

Dynamic MR Elastography:

- MR images are recorded while a vibrating plate placed on the skin propagates mechanical shear waves of a known frequency in the tissue.
- From the images of the motion we estimate the wavelength of the shear wave.
- The MRI signal contains both amplitude and phase information. In the linear regime, the phase-difference field is directly proportional to the displacement field.

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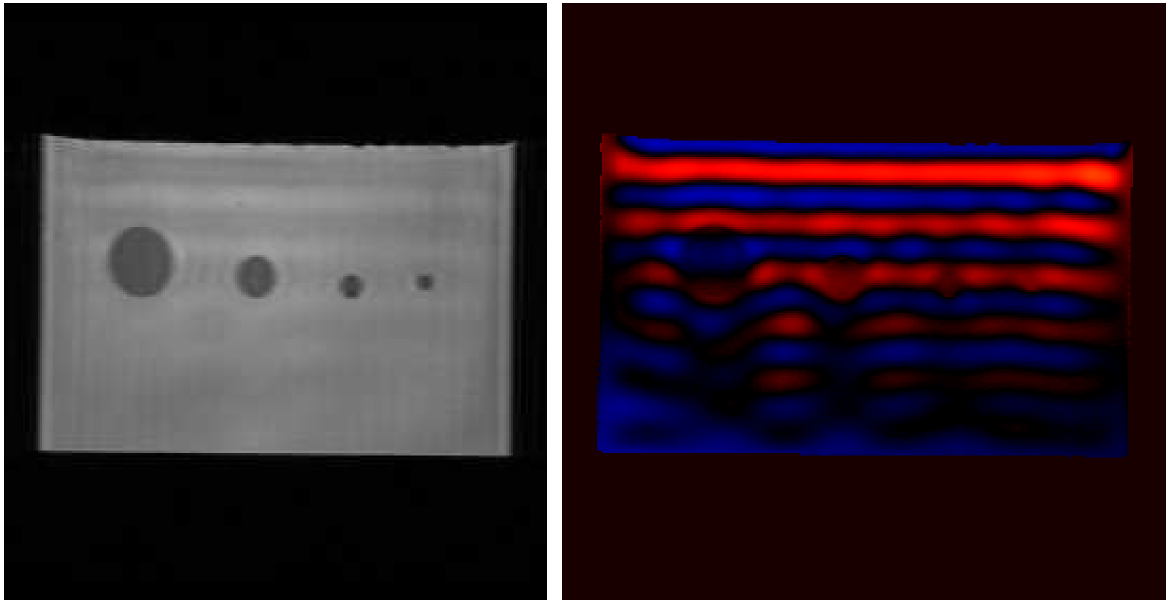

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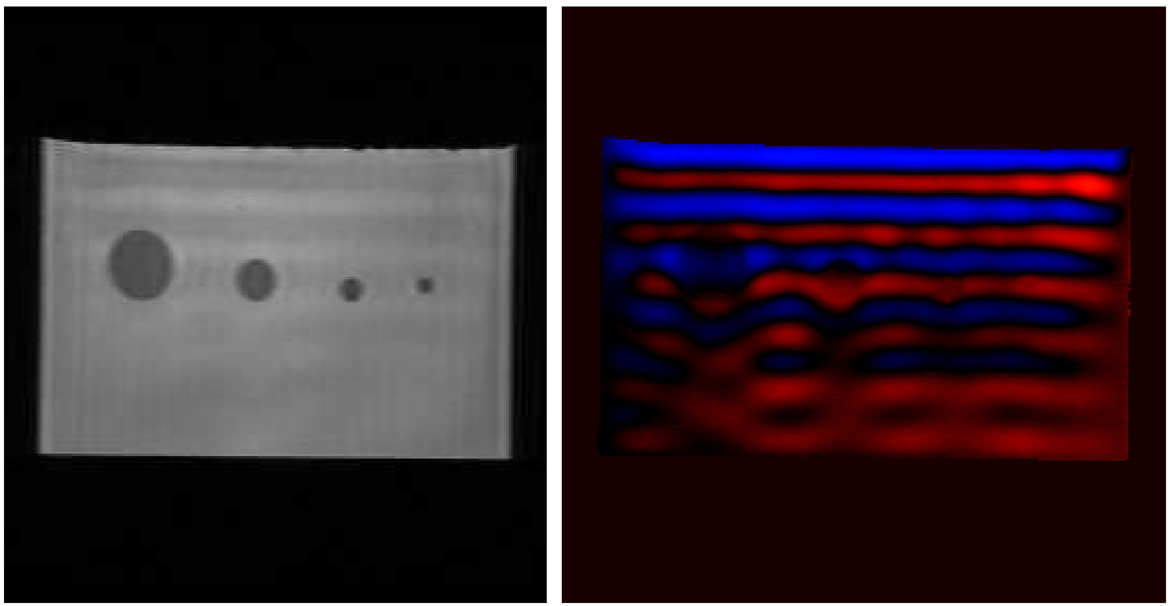

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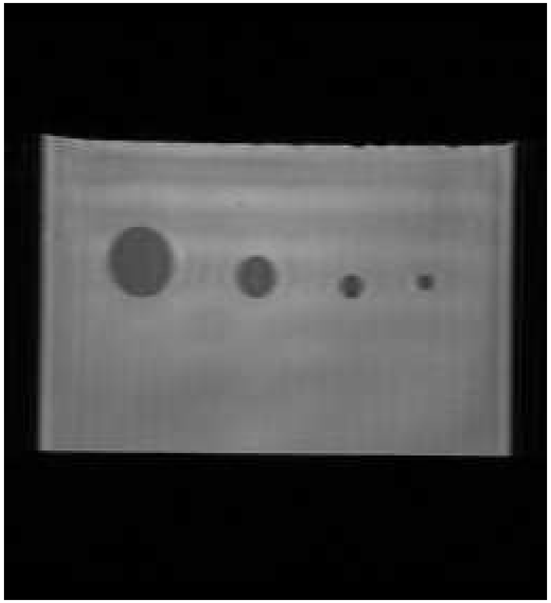

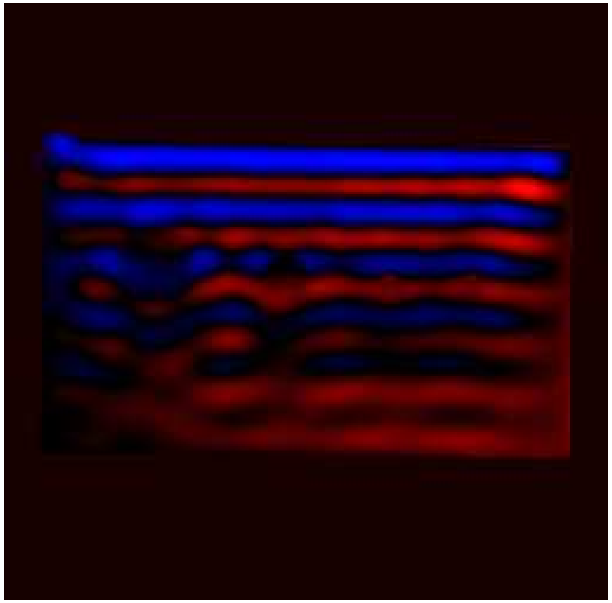

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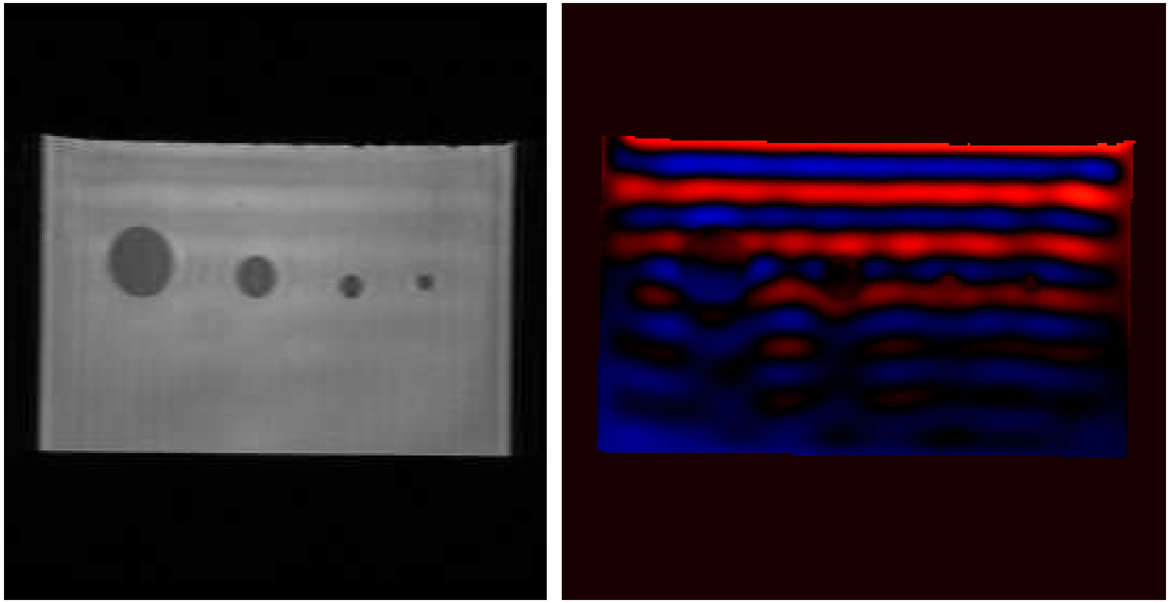

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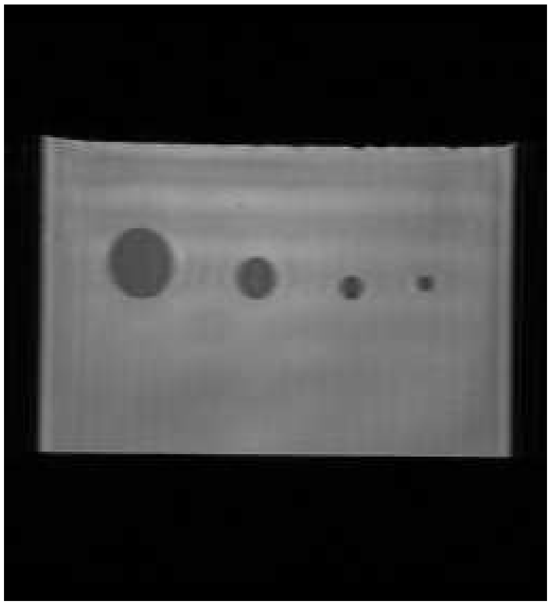

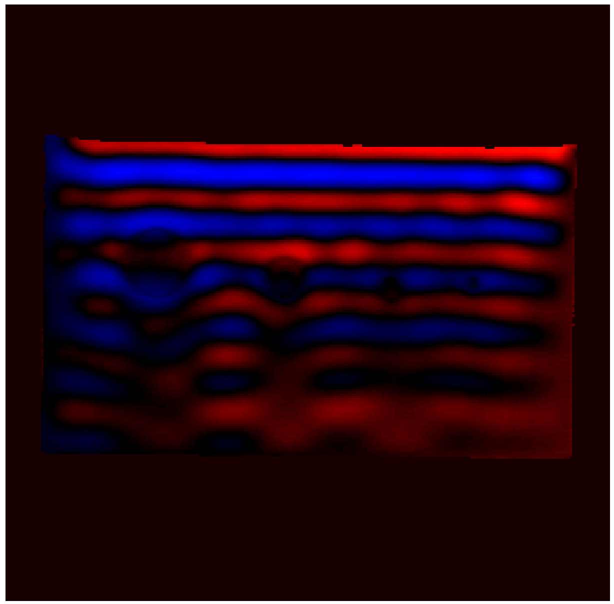

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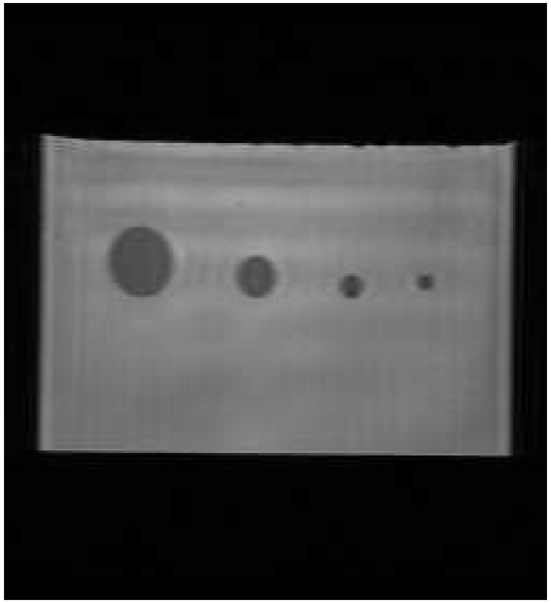

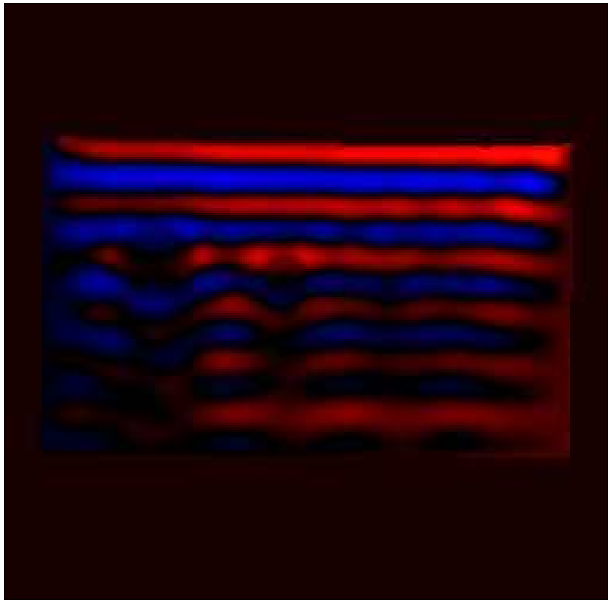

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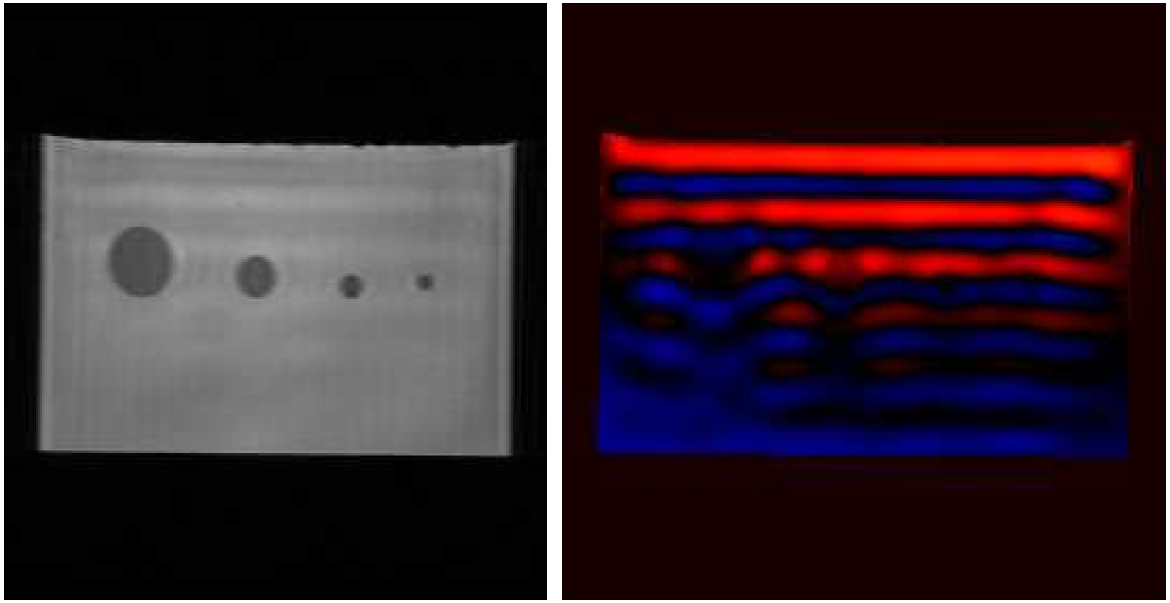

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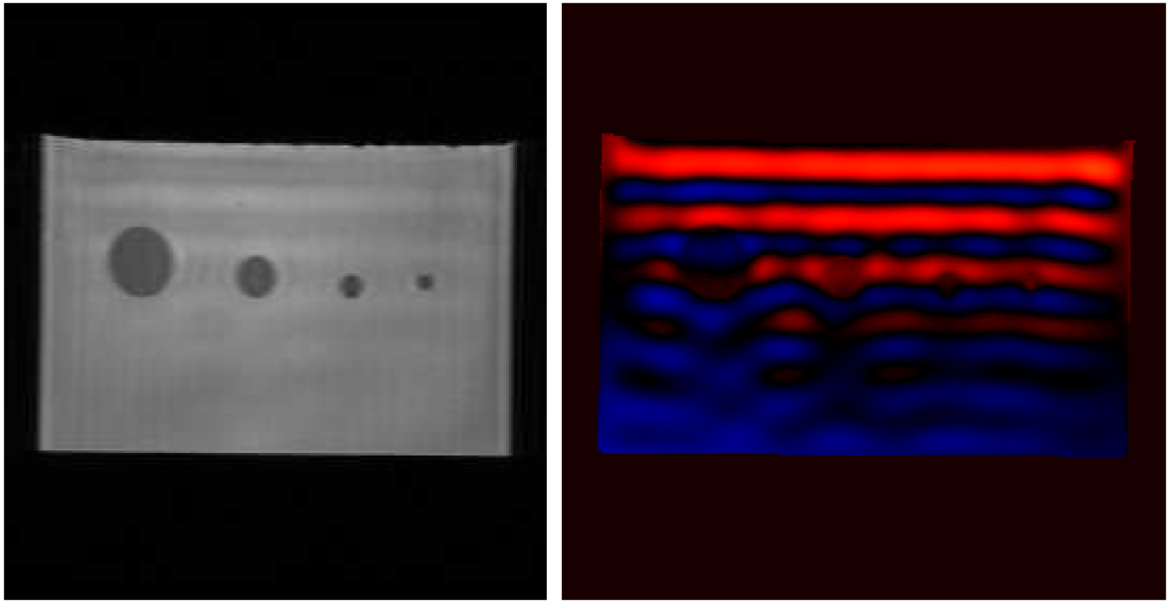

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- Once the displacement field is known, we need a constitutive model for the biological tissues from which to extract the material parameters (the shear modulus).

Magnetic Resonance Elastography (4)

Constitutive Assumptions: Biological tissues are locally homogeneous, isotropic, almost incompressible, linear viscoelastic materials of density ≈ 1 g/cm^3 (Burelew *et al.*, 1980).

- Navier equations for displacements in the frequency domain are:

$$(\Lambda(\omega) + M(\omega))\, \nabla \left(\nabla \cdot \vec{U}(\vec{x}, \omega) \right) + M(\omega) \nabla^2 \vec{U}(\vec{x}, \omega) = -\omega^2 \vec{U}(\vec{x}, \omega)$$

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- For biological tissues, $\Lambda >> M$ ($\Lambda = \mathcal{O}(10^9)$ Pa, $M = \mathcal{O}(10^2 \div 10^7)$ Pa), M varies strongly with tissue pathology.
- Two reduced forms of Navier equations:
  - DI equation: $M\nabla^2\vec{U} = -\omega^2\vec{U}$
  - Curl-DI equation: $M\nabla^2(\nabla \times \vec{U}) = -\omega^2(\nabla \times \vec{U})$
- M can be found using the method of algebraic inversion of differential equations AIDE (Oliphant *et al.*, 2001).

Magnetic Resonance Elastography (5)

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- Motivation comes from the Helmholtz decomposition method:
  - Allows the breakup of a vector wave field into its longitudinal and shear components.
  - One can then process only the shear component of the displacement field.
  - Mathematical proof is based on the concept of a potential vector field.

Magnetic Resonance Elastography (6)

- Replace the displacement field by its corresponding potential field:

$$\vec{V}(\vec{x}) = -\frac{1}{4\pi}\int\int\int \frac{1}{|\vec{x}-\vec{y}|}\vec{U}(\vec{y})d\vec{y} \ \ (\text{2D case})$$

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- The potential field satisfies the Navier equations and thus:
  - PDI equation (analogous to DI equation):

$$M\nabla^2\vec{U} \ = \ -\omega^2\vec{U} \Rightarrow \underbrace{M\vec{U} \ = \ -\omega^2\vec{V}}_{\text{no derivatives}}$$

  - Curl-PDI equation (analogous to Curl-DI equation):

$$M\nabla^2(\nabla\times\vec{U}) \ = \ -\omega^2(\nabla\times\vec{U}) \Rightarrow \underbrace{M(\nabla\times\vec{U}) \ = \ -\omega^2(\nabla\times\vec{V})}_{\text{only first order derivatives}}$$

Magnetic Resonance Elastography (7)

Results: We compare DI and Curl-PDI.

- The elastogram is made of the square of the shear wave speed values, c_s^2, where:

$$c_s(f) = \sqrt{\frac{2(Re(M)^2 + Im(M)^2)}{Re(M) + \sqrt{Re(M)^2 + Im(M)^2}}}$$

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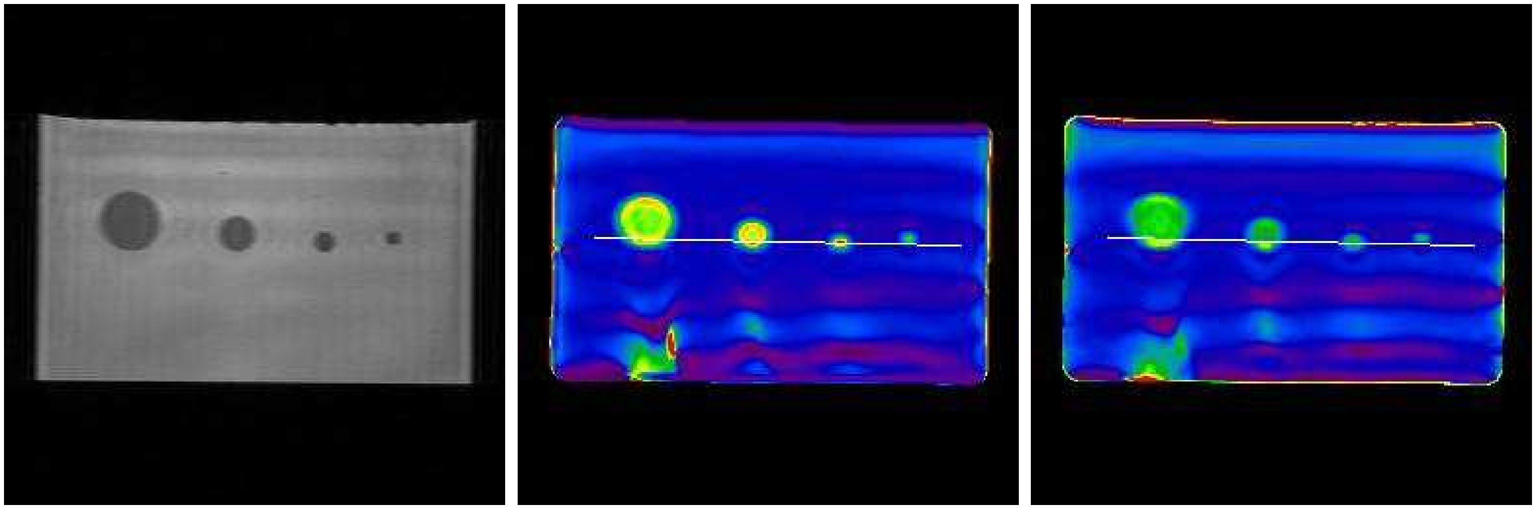
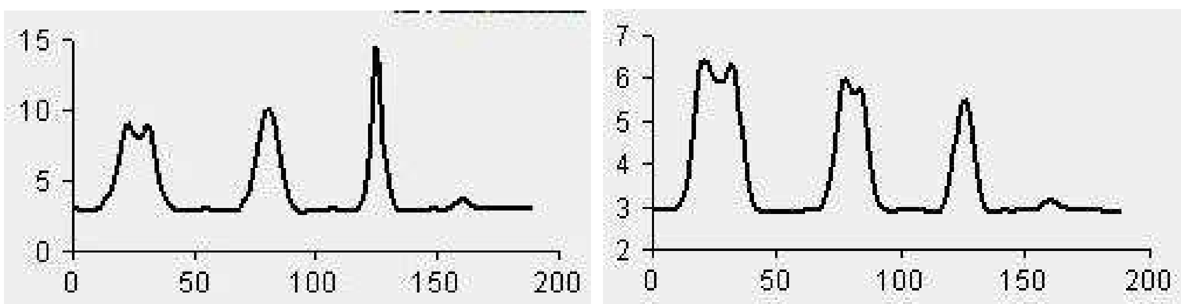


Up: Magnitude (left), DI (center), Curl-PDI (right); Down: DI values (left), Curl-PDI values (right)



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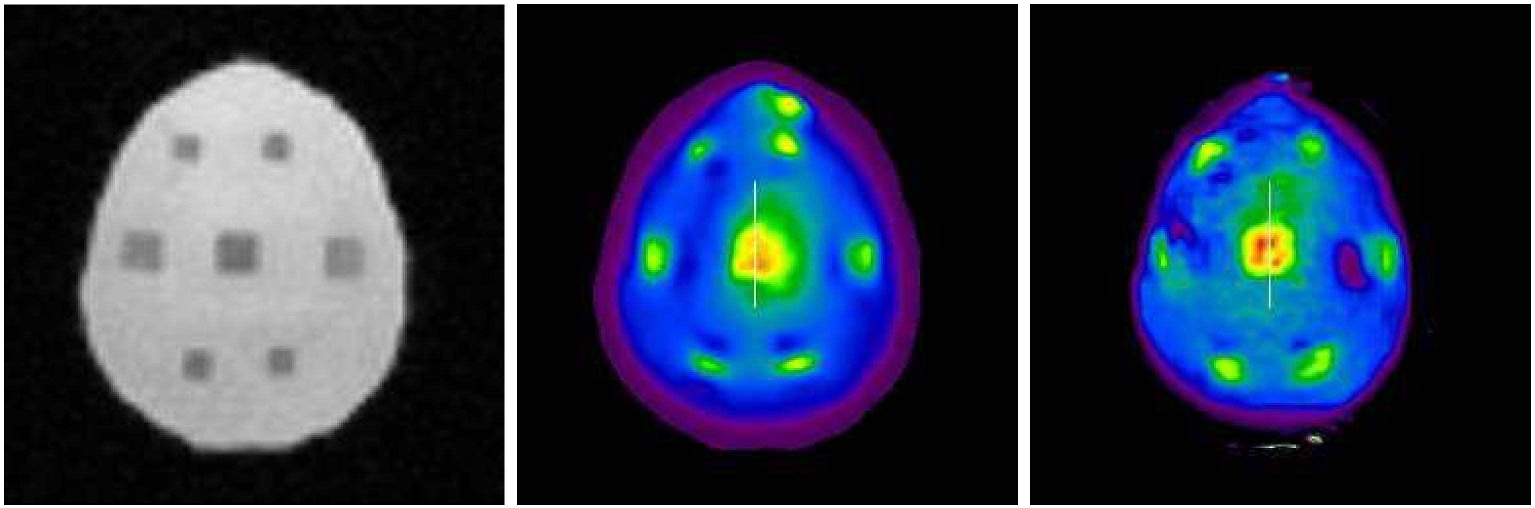

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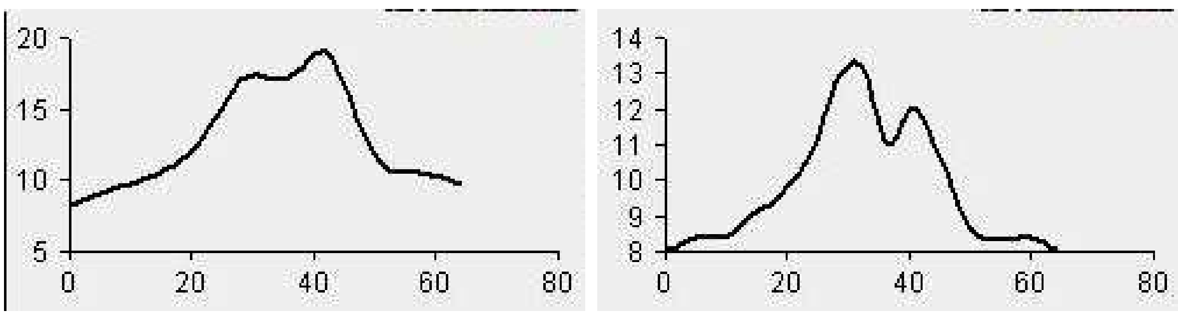

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Discussion:

- By using the potential of the displacement field we reduced the order of numerical differentiation:
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THANK YOU!