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Traffic flow modelling with safety embedded notions

María Elena Lárraga

Instituto de Ingeniería

UNAM

México

mlarragar@iingen.unam.mx

Joint work with: L. Álvarez-Icaza

On leave from Facultad de Ciencias, UAEM

Why to model traffic flow?



Traffic jam is one of the most serious issues in modern societies

Introduction

- Ecological considerations, space and budget constraints limit solving traffic congestion by upgrading and constructing new roadway systems.

Solutions are now oriented to a better management of existing systems and thus to

- solve traffic congestion,
- decrease environmental issues and
- improve traffic safety.

Traffic flow

Traffic = macroscopic system of interacting particles

Various approaches:

- hydrodynamic
- gas-kinetic
- car-following
- cellular automata (CA)

Why are CA able to model traffic flow?

Cellular automata are able to reproduce many aspects of highway traffic (despite their simplicity):

- Spontaneous jam formation
- Metastability, hysteresis
- Existence of 3 phases: Free, synchronized and jam flow)

Advantage: Simulations of networks faster than real-time possible

- Online simulation
- Forecasting

What are the specific properties that should be considered by a realistic traffic model?

- (i) Velocity anticipation:* the anticipation of the leaders velocity avoids abrupt braking of the traffic behind and therefore reduces the probability to form jams.
- (ii) Retarded acceleration:* Comfortable driving also implies that cars do not accelerate immediately in case of a larger gap ahead if they observe slow downstream traffic.
- (iii) Timely braking:* Finally, timely braking suppresses another mechanism of jam formation: When the velocity adjustment is only based on the distance to the next car ahead, jams often emerge in the layer between free-flow and synchronized traffic.

-The drivers to adjust their speed to the vehicles ahead.

Realistic traffic models

- To anticipate the actions of other cars in the next timestep.
- This implies that drivers need more information about the next car ahead, not just its distance.

Different models

Nagel-Schreckenberg model (NaSch)

1. **acceleration** (up to maximal velocity)
2. **braking** (avoidance of accidents)
3. **randomization** (“dawdle”)
4. **motion**

plus:

Safety distances

velocity anticipation

smaller cells

Realistic acceleration and deceleration
capabilities

Emergency braking

...

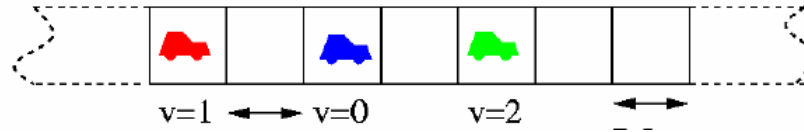


LA model

Lárraga-Álvarez, 2007

Our model

The model



- The street is divided into L cells of length Δx m
- Δx can take values of 1.25, 2.5 or 5m that is a finer discretization than the used in NaSch model.
- For all cases, the car length is 5m, i.e., a car occupies $l_c = 5 / \Delta x$ cells.
- Vehicles can only have integer velocity values, $v_i = 0, 1, \dots, v_{\max}$.
- We used $v_{\max} = 6 \cdot l_c$, which is equivalent to 108 km/h in real world units (24, 12 and 6 cells).
- This corresponds to time steps $\Delta t = 1$ s.

Variables

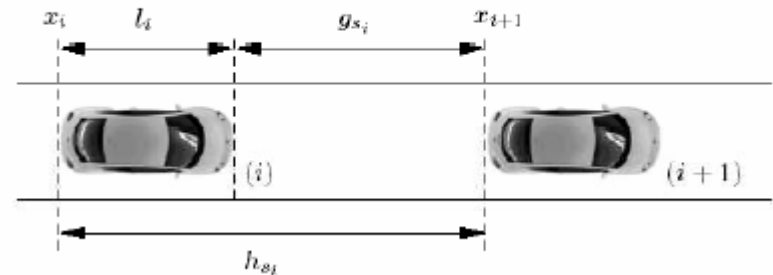
- The n th car is characterized by its position $x_n(t)$ and velocity $v_n(t)$ at time t .

- Cars are numbered in the driving direction, i.e. vehicle $(n + 1)$ precedes vehicle n .

- The gap between consecutive cars (where s is the length of the cars in cells) is

$$d_n = x_{n+1} - x_n - s$$

–It is assumed that a vehicle position denotes the cell that contains its rear bumper



Dynamic

R0: Read its safety distances $d_{decc}(t)$, $d_{acc}(n)$ and $d_{keep}(t)$ from matrixes D, A and K respectively

R1: Acceleration.

If $d_n(t) \geq d_{accn}(t)$, the velocity of the car n is increased by one, i.e.,
$$v_n(t+1) \rightarrow \min(v_n(t) + 1, v_{max})$$

R2a: Cruising.

If $d_{accn}(t) > d_n(t) \geq d_{keepn}(t)$, velocity of vehicle n is kept equal with probability $1 - R$, i.e.,
$$v_n(t+1) \rightarrow v_n(t) \text{ with probability } 1 - R.$$

R2b: Random braking.

If $d_{accn}(t) > d_n(t) \geq d_{keepn}(t)$ and $v_n(t) > 0$, velocity of vehicle n is reduced by one with probability R :
$$v_n(t+1) \rightarrow \max(v_n(t) - 1, 0) \text{ with probability } R.$$

R3: Braking.

If $d_{keepn}(t) > d_n(t) \geq d_{deccn}(t)$ and $v_n(t) > 0$, velocity of vehicle n is reduced by one:

$$v_n(t) \rightarrow \max(v_n(t) - 1, 0)$$

R4: Emergency braking.

If $v_n(t) > 0$ and $d_n(t) < d_{deccn}(t)$, velocity of vehicle n is reduced by M , provided it does not go below zero:

$$v_n(t+1) \rightarrow \max(v_n - M, 0)$$

R5: Vehicle movement.

Each vehicle is moved forward according to its new velocity determined in rules 1-4:

$$x_n(t+1) \rightarrow x_n(t) + v_n(t+1)$$

Safety distances

$$\begin{aligned}
 d_{acce_n}(t) = & \frac{M}{2} [(v_n(t) + 1) \text{ Div } M] + 1 [(v_n(t) + 1) \text{ Div } M] \\
 & + [(v_n(t) + 1) \text{ Mod } M] [(v_n(t) + 1) \text{ Div } M] + 1 \\
 & - \frac{M}{2} [(v_{n+1}(t) - M) \text{ Div } M] + 1 [(v_{n+1}(t) - M) \text{ Div } M] \\
 & - [(v_{n+1}(t) - M) \text{ Mod } M] [(v_{n+1}(t) - M) \text{ Div } M] + 1
 \end{aligned} \tag{1}$$

$$\begin{aligned}
 d_{keep_n}(t) = & \frac{M}{2} [v_n(t) \text{ Div } M] + 1 [v_n(t) \text{ Div } M] \\
 & + [v_n(t) \text{ Mod } M] [v_n(t) \text{ Div } M] + 1 \\
 & - \frac{M}{2} [(v_{n+1}(t) - M) \text{ Div } M] + 1 [(v_{n+1}(t) - M) \text{ Div } M] \\
 & - [(v_{n+1}(t) - M) \text{ Mod } M] [(v_{n+1}(t) - M) \text{ Div } M] + 1
 \end{aligned} \tag{2}$$

$$\begin{aligned}
 d_{dec_n}(t) = & \frac{M}{2} [(v_n(t) - 1) \text{ Div } M] + 1 [(v_n(t) - 1) \text{ Div } M] \\
 & + [(v_n(t) - 1) \text{ Mod } M] [(v_n(t) - 1) \text{ Div } M] + 1 \\
 & - \frac{M}{2} [(v_{n+1}(t) - M) \text{ Div } M] + 1 [(v_{n+1}(t) - M) \text{ Div } M] \\
 & - [(v_{n+1}(t) - M) \text{ Mod } M] [(v_{n+1}(t) - M) \text{ Div } M] + 1
 \end{aligned} \tag{3}$$

M=time-steps to reach the minimum allowable deceleration

Three fixed tables are generated: $A_{(v_{\max}+1) \times (v_{\max}+1)}$, $K_{(v_{\max}+1) \times (v_{\max}+1)}$, $D_{(v_{\max}+1) \times (v_{\max}+1)}$

Distances meaning

Safety distances

- In each term in the r.h.s. of the equations (1)-(3), we incorporate implicit anticipation effects and reduced acceleration and deceleration capabilities by means of relative distances (according to cars' velocities).
- Each term on the r.h.s. is derived based on the kinematics of two neighbor vehicles, assuming that in the next time step the leader vehicle will apply full brakes.
- The equations predict the traveled distance by each vehicle. The difference between these two distances yield the safety gap.

Interpretation of the Rules

R1. Acceleration: Drivers want to move as fast as possible (or allowed)

R2a Cruising: Drivers will try to keep their velocities if they perceive the distance with the vehicle in front as sure.

R2b Random braking

- a) overreactions at braking
- b) delayed acceleration
- c) psychological effects (fluctuations in driving)
- d) road conditions

R3 Braking: In normal situations driver decelerates to expand the gap to reach his/her desired gap and keep safety.

R4 Emergency braking: Avoid accidents when the leader brakes suddenly or follower approaches a stopped vehicle

R5 Driving: Motion of cars

Example: Safety distances

$$\Delta x = 2.5 \text{ m} \rightarrow s=2, \quad M=2$$

Maximum acceleration= 2.50 m/s^2 (0-100 km/h in 12 s)

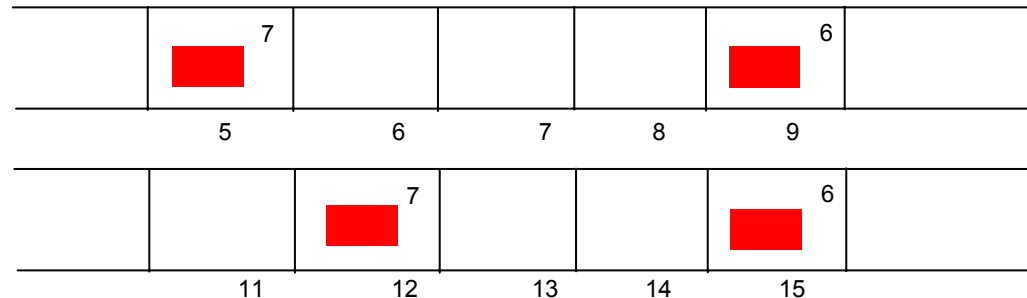
Emergency braking= -5.00 m/s^2

$v_{\max} = 12 \text{ cells} = 108 \text{ km/h} \rightarrow d_{\text{acc}} = 24, d_{\text{dec}} = 19, d_{\text{keep}} = 14$

Configuration at time t:



Motion (state at time t+1):



Aim

- To reproduce macroscopic and microscopic traffic flow behavior.
- To guarantee that microscopic vehicle behavior follows capabilities of real ones by incorporating implicit anticipations effects and reduced acceleration and deceleration capacities.
 - To reproduce a real driver behavior.
- To preserve the simplicity of CA models.

Real values

- Emergency braking in all cases of Δx will have a value of -5m/s^2 and will be reached in one time-step.
- Maximum acceleration will be 5 m/s^2 , 2.5 m/s^2 and 1.25 m/s^2 for cell lengths of 5.00 , 2.50 and 1.25 m , respectively.

Fundamental diagram

$$R = 0.15$$

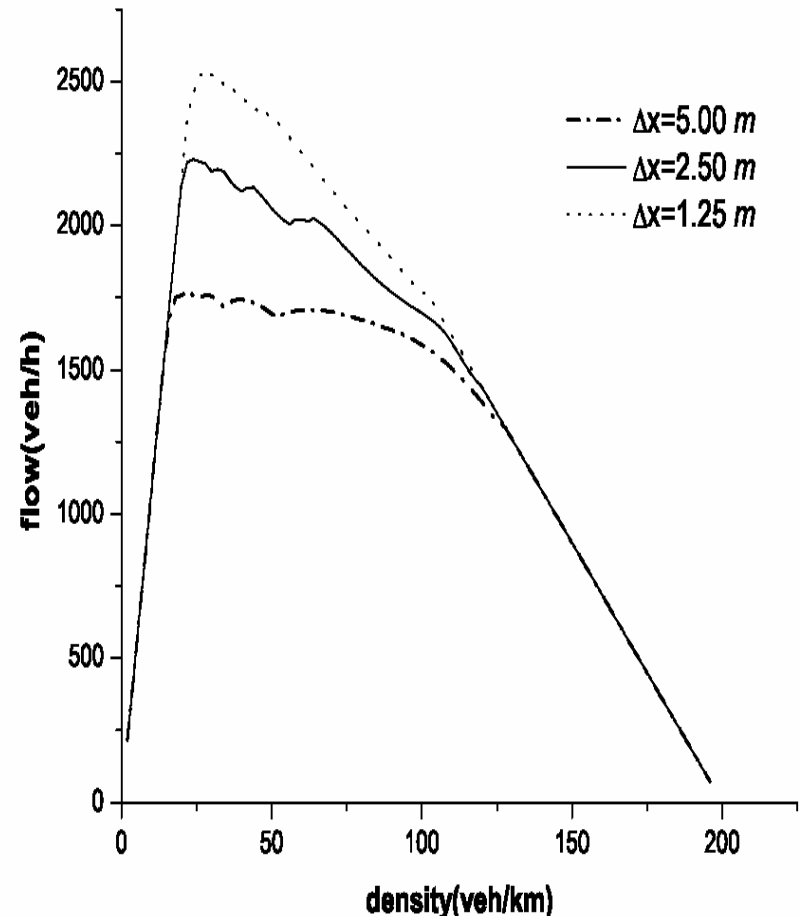
$$L = N_{\max} * s, \quad N_{\max} = 10,000$$

Density $\rho = N/L$ varying from 2 to 200 veh/km in steps of 2 veh/km in real units.

Smaller values of Δx , that is lower acceleration levels, imply larger flows.

$$T = 15 * N_{\max} \text{ time-steps}$$

- Data are averaged over the final $5 * N_{\max}$ time-steps.



Traffic flow organization

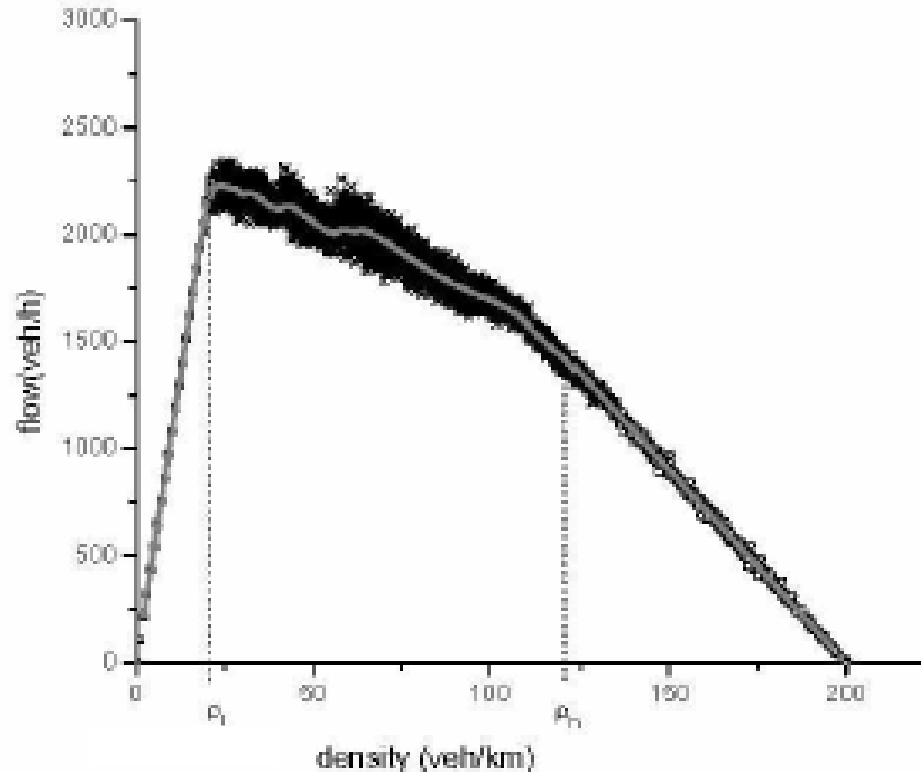
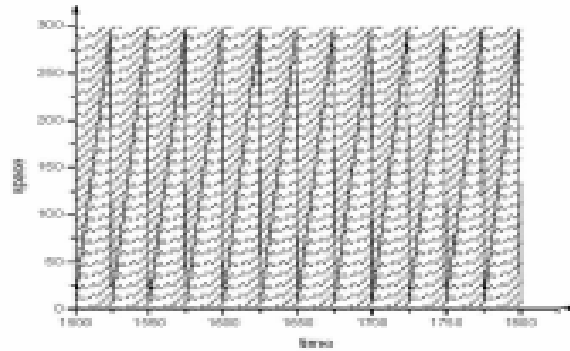
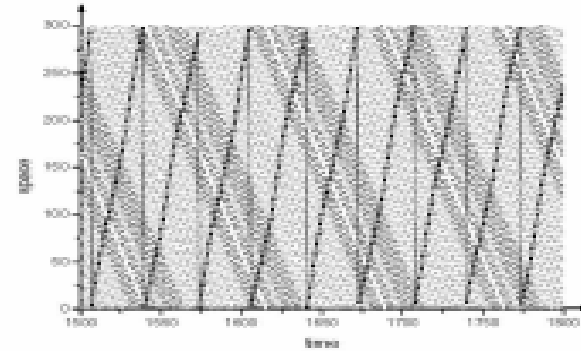


Figure 3. Fundamental diagram resulting from random initial conditions. The simulations are performed on a ring with a length of 20000 cells, each corresponding to 2.5 m. The parameters of the model are $v_{max} = 108 \text{ km h}^{-1} = 12 \text{ cells s}^{-1}$, $R = 0.15$, $\Delta x = 2.5 \text{ m}$, and $M = 2$

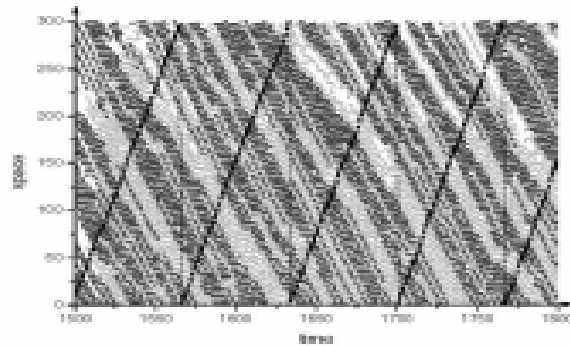
Space diagrams



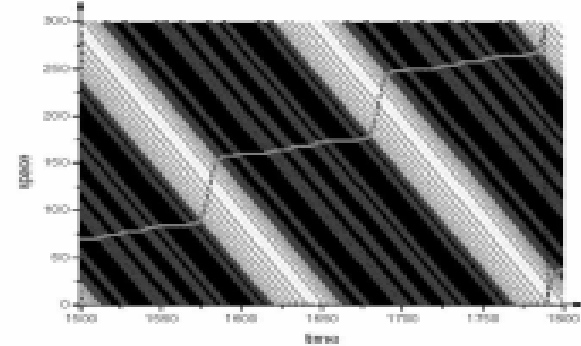
(a)



(b)



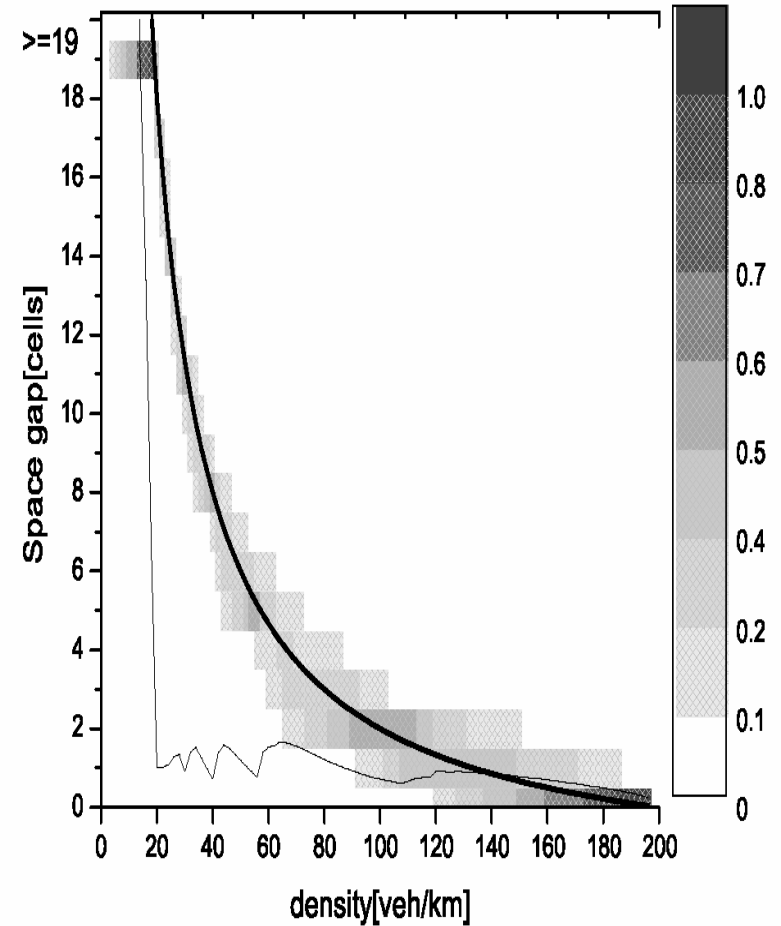
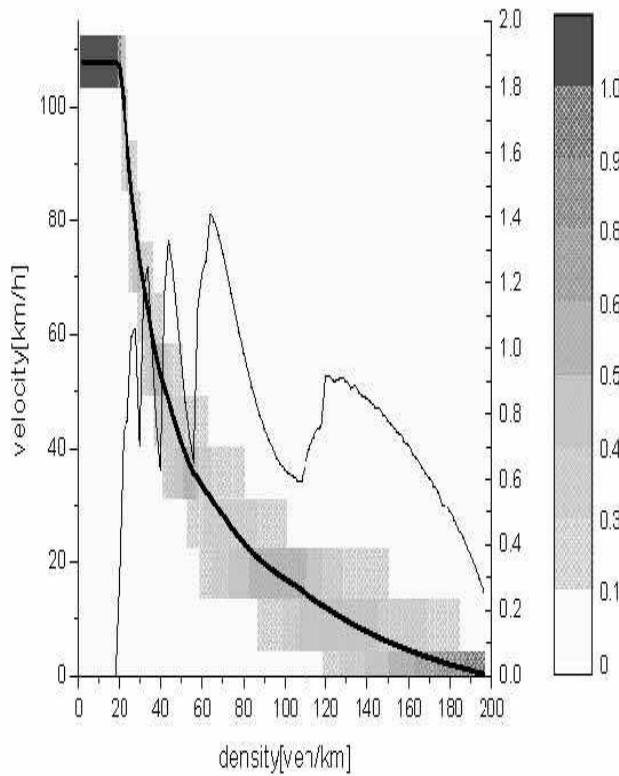
(c)



(d)

Fig. 1. Space-time diagram for $R = 0.15$, $\Delta x = 2.5 \text{ m}$ and different values of density. The solid lines in diagrams correspond to the trajectory for a specific car at different time-steps. (a) corresponds to a low density range, for $\rho = 16 \text{ veh/km}$. (b)-(c) correspond to an intermediate density range for $\rho = 30$ and $\rho = 50 \text{ veh/km}$, respectively. (d) corresponds to $\rho = 140 \text{ veh/km}$.

Speed and gap distribution



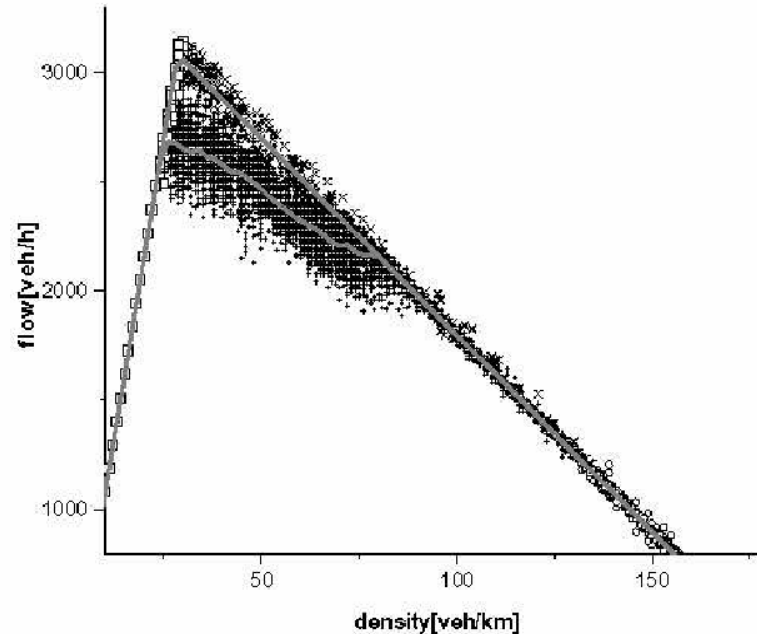
Metastability

Homogeneous state:

$$g_{ini} = \frac{L}{N} - s$$

v_{ini} = maximum speed
such that

$$g_{ini} \geq d_{keep}$$



System initialized homogeneously → A metastable high-flow branch exists

Figure 7. Fundamental diagram resulting from two different initial conditions for $R = 0$. The upper curve was calculated by starting from an homogeneous state, whereas the lower curve was obtained from random initial distribution

$$\rho_{max} = \frac{L}{v_{max} + s}$$

$$\rho_{max} = 28.57 \text{ veh/km}$$

$$q_{max} = 3085 \text{ veh/km}$$

Summary

- Due to simplicity and easy implementation on computers for numerical investigations, the CA traffic flow models developed very quickly in the last years.
- However it have not obtained the best traffic CA model which should be both realistic as well as simple.
- The aim of the proposed model is to describe more faithfully the behavior of real drivers and the macroscopic behavior of traffic flow observed.

Summary

- A simple and natural set of rules to better capture driver reactions allowed to describe the three states of flow observed in the reality: Free, synchronized and jam flow.
- Moreover, the model still preserved simplicity of CA model and at the same time, as vehicles' behavior was based on a safety analysis to determine the most appropriate action for a vehicle to take, made results closer to real highway behavior.
- Besides, there is an intuitive explanation for all rules in the model.

Thank you