The Coordination of Pricing and Production Decisions

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Outline

- Ø **Pricing and Production Literature**
- Ø **Pricing and Production Model**
  - o **Three specific problems**
- Ø **Solvability of the Problems**
  - o **Complexity**
  - o **Optimal and approximation algorithms**
- Ø **Managerial Insights**
  - o **Compare uncoordinated, partially coordinated, and fully coordinated approaches**
- Ø **Pricing and Production: Research Agenda**

What is Pricing?

***Definition*:** setting prices for products and services.

From a *practical* perspective:

Pricing is a key business decision that directly adds value to other operational decisions.

From a *research* perspective:

Pricing is a research area within economics, deterministic operations research and stochastic operations research.

Why is Pricing an Active Research Area?

- History of poor communication between marketing and production departments.

- Increasing recognition of the importance of operational decisions in creating value for companies.

- Direct relationship between pricing and bottom line performance.

- Relationship with revenue management.

Pricing and Production Literature (1 of 5)

Surveys

Eliashberg and Steinberg (1993)

Yano and Gilbert (2004)

Two Types of Models

Stochastic – demand is a stochastic function of price

Deterministic – demand is a deterministic function of price

Pricing and Production Literature (2 of 5)

Stochastic Demand

Thomas (1974)

Production planning

Gallego and van Ryzin (1994)

Dynamic pricing of inventories

Duenyas and Hopp (1995)

Lead time quotation

Easton and Moodie (1999)

Uncertain bid outcomes

Pricing and Production Literature (3 of 5)

Deterministic Demand, Single Product

Zhao and Wang (2002)

Pricing and production in a two stage supply chain

Deng and Yano (2006)

Pricing and production under capacity constraints

Geunes *et al.* (2006)

Polynomial time algorithm for a generalized ELSP

Pricing and Production Literature (4 of 5)

Deterministic Demand, Multiple Products

Cheng (1990), Chen and Min (1994), Lee (1994)
Morgan *et al.* (2001), Dobson and Yano (2002)

Products are ordered with a common frequency

Gilbert (2000)

Different ordering frequencies are allowed
Extensive algorithmic development and sensitivity analysis

Charnsirisakskul *et al.* (2006)

Earliness – tardiness model solved by heuristics

Pricing and Production Literature (5 of 5)

All the existing models:

consider production decisions ***from an aggregate planning point of view*** where

- Detailed scheduling of each individual job is not considered
- Delivery of completed jobs occurs at the end of a planning time period
- Completed jobs can be held in inventory in order to satisfy future demand
- Aggregate planning costs – setup, production, finished product inventory holding costs – are considered

A Pricing and Scheduling Model

- Coordination of pricing and detailed job-by-job scheduling decisions
- Multiple products
- Deterministic demand functions
- Maximize net profit (i.e., revenue – scheduling costs)
- Short term planning model, with a single price for each product

Why Detailed Scheduling? (1 of 3)

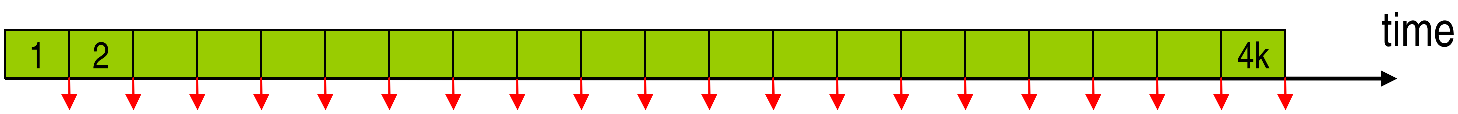
- Time-sensitive products (make-to-order, perishable)
  - Completed job delivered immediately after its completion
  - A significant penalty if delivered late
  - Little or no finished product inventory
  - Work-in-process inventory can be significant

Need to know the completion time of each individual job, since aggregate planning level measures of lateness and WIP costs may be inaccurate

Why Detailed Scheduling? (2 of 3)

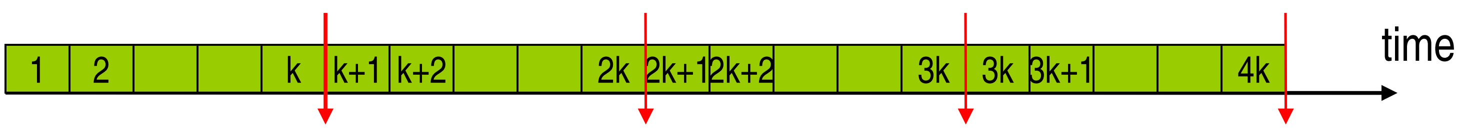
Example

A single product, processing time per unit = 1
Produce 4k units in time interval [0, 4k]



- Actual WIP cost, $\Sigma C_j = 1 + 2 + 3 + \ldots + 4k$
  $= 4k(4k+1)/2 = 8k^2 + 2k$

- But if estimated at an aggregate planning level with 4 time periods (each with k time units), then WIP cost, $\Sigma C_j = k*k + k*2k + k*3k + k*4k$
  $= 10k^2$



Why Detailed Scheduling? (3 of 3)

- Multi-stage production system
  - There may exist idle times at some stages
  - The idle times may depend on the job sequence

Capacity usage depends on job-by-job scheduling, therefore aggregate planning level estimates of capacity usage are inaccurate

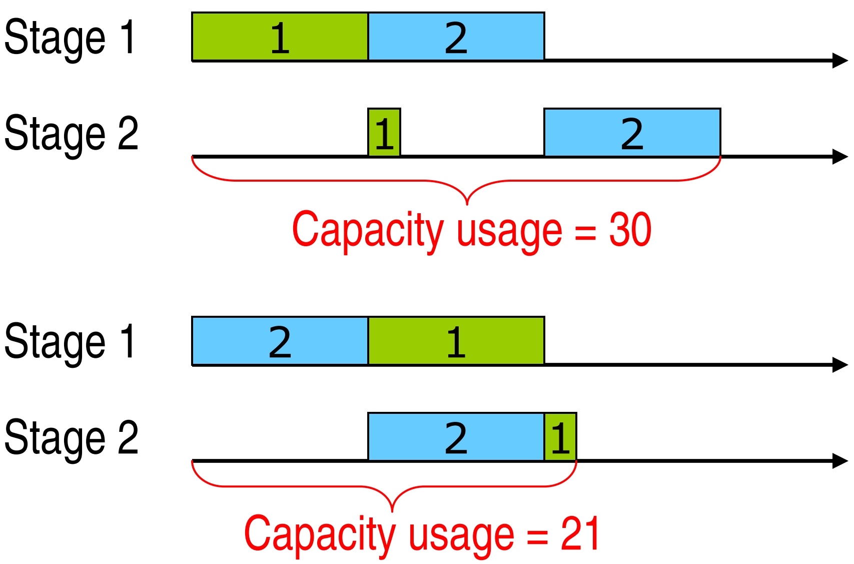
- Example:

  2 products, 1 unit each

  stage 1: $p_{11} = p_{12} = 10$

  stage 2: $p_{21} = 1$, $p_{22} = 10$

  The capacity usage cannot be estimated accurately at the aggregate planning level



Why a Deterministic Model? (1 of 3)

From a *practical* perspective:

Easier to implement.

Probability distributions are usually hard to find.

From a *research* perspective:

Optimization methods can be used more easily.

Allows accurate evaluation of tradeoffs, and sensitivity analysis.

Why a Deterministic Model? (2 of 3)

"In our supply chain operations planning, we do not have access to a probability distribution of future demand scenarios. Therefore, our forecast expected demand is often used ***deterministically*** *as the basis for planning."*

Scientist
Military Technology and Operations
Multinational Aerospace Manufacturer

Why a Deterministic Model? (3 of 3)

"For most of our clients, ***deterministic*** *models are preferred, in view of the unavailability of reliable probability distributions, and also the ease of implementation."*

Engineering Manager
Leading Supply Chain Solutions Provider

Model Description (1 of 2)

- Make to order
- n products, 1, 2, …, n. For product j
  - m_j allowable prices, $q_{1j} > q_{2j} > \ldots > q_{m_j j}$
  - demand $g_j(q_{ij})$ for price q_{ij}
  - incoming demand must be satisfied in full
  - due date d_j, weight w_j
- Production involves either a single stage with a single production line
  - processing time, p_j, for product j

  or two stages, each with a single production line
  - processing times, p_{1j} and p_{2j}, for product j

Model Description (2 of 2)

Two single-stage problems:

- Maximize: Total revenue – WIP cost ($\Sigma\Sigma w_j C_{ij}$)
- Maximize: Total revenue – Lateness penalty ($\Sigma\Sigma w_j U_{ij}$)
  - There is a penalty cost w_j when a unit of product j is delivered later than its due date d_j

One two-stage problem:

- Maximize: Total revenue – Capacity cost ($C_{\max}$)
  - Flowshop production configuration

Solvability of the Problems

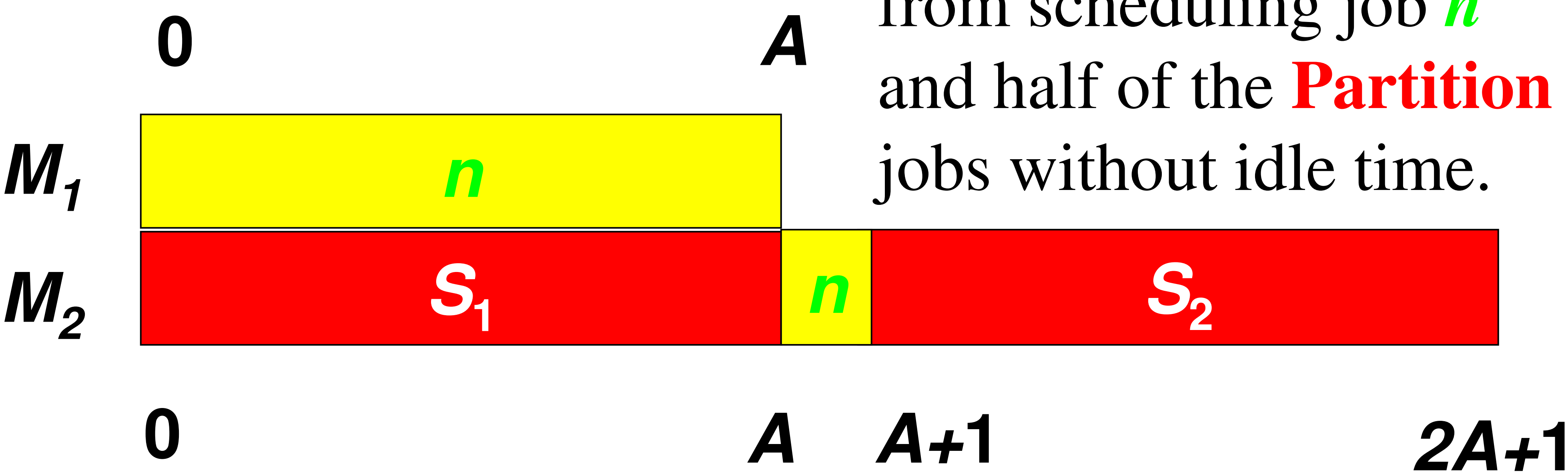
- Complexity: All three problems are binary *NP*-hard.
- Dynamic programming (DP) algorithm for finding an optimal solution for each problem
  - Based on optimality property that products are processed in certain sequence in an optimal solution
  - Pseudo-polynomial running time
- Fully polynomial time approximation scheme (FPTAS) for each problem
  - Based on the pseudo-polynomial time DP algorithm

Intractability

All three problems studied are binary *NP*-hard.

For example, consider minimizing makespan in a two machine flowshop.

Threshold profit comes from scheduling job *n* and half of the **Partition** jobs without idle time.



Therefore, the binary *NP*-hard problem Partition reduces to the pricing and scheduling problem.

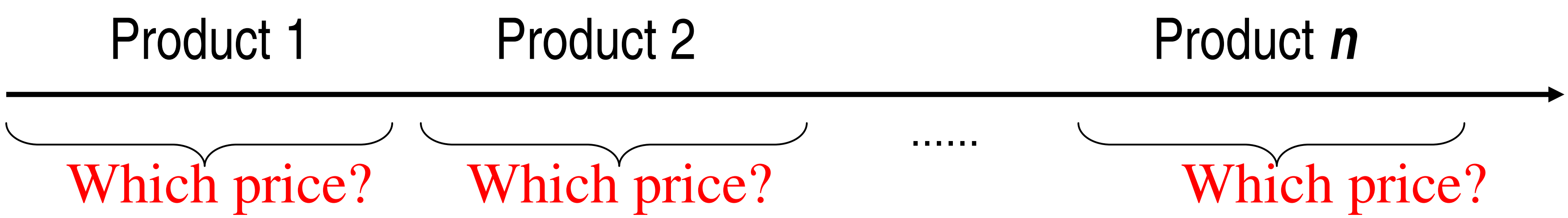
DP Algorithm for WIP Problem (1 of 3)

Optimality property: There exists an optimal schedule where

(i) the jobs of each product are scheduled consecutively;

(ii) the products are scheduled in SWPT order (i.e., nondecreasing order of p_j / w_j).

Reindex the products such that $p_1/w_1 \le p_2/w_2 \le \ldots \le p_n/w_n$



DP Algorithm for WIP Problem (2 of 3)

Value function:

$f(j, t)$ = maximum net profit from products **1, …, *j***, given that after product j is scheduled, the makespan of the schedule is t.

Initial condition: $f(0, 0) = 0$

Recurrence relation:

Total revenue of product j

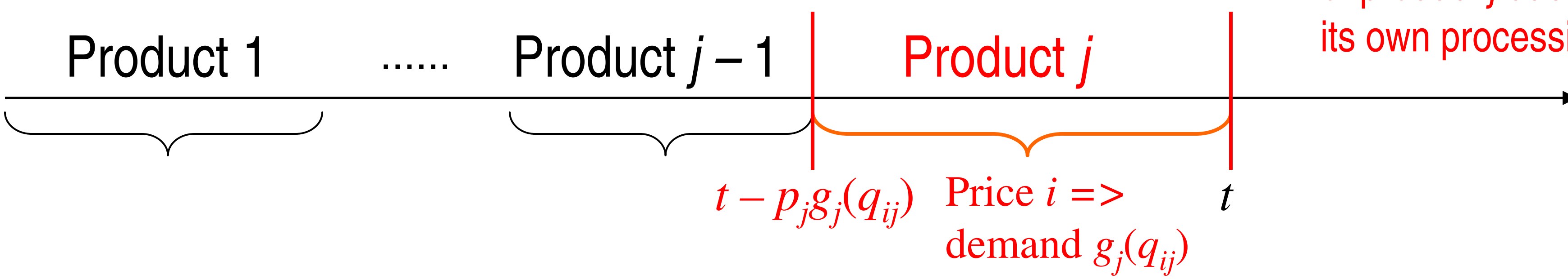
Total WIP cost of product j due to processing times of products 1, …, j-1

Net profit of product j

$$f(j, t) = \max_{1 \le i \le m_j} \{ \boxed{q_{ij} g_j(q_{ij}) - w_j g_j(q_{ij})[t - p_j g_j(q_{ij})] - w_j g_j(q_{ij})[g_j(q_{ij}) + 1]/2} + \boxed{f(j-1, t - p_j g_j(q_{ij}))} \}$$

Net profit of products 1, …, j-1

Total WIP cost of product j due to its own processing times



Optimal solution value: $\max \{ f(n, t) \mid 0 \le t \le \sum_{j=1}^{n} \bar{g}_j p_j \}$

DP Algorithm for WIP Problem (3 of 3)

The running time of the algorithm:

$$\mathrm{O}\left((\sum_{j=1}^{n} m_j)(\sum_{j=1}^{n} \bar{g}_j p_j) \right)$$

Number of allowable prices of product *j*

Maximum possible demand of product *j*

Unit processing time of product *j*

This time is pseudo-polynomial in the size of the input data

Formally, this is as efficient as possible, but in practice it can be time consuming

DP Algorithm for Lateness Penalty Problem

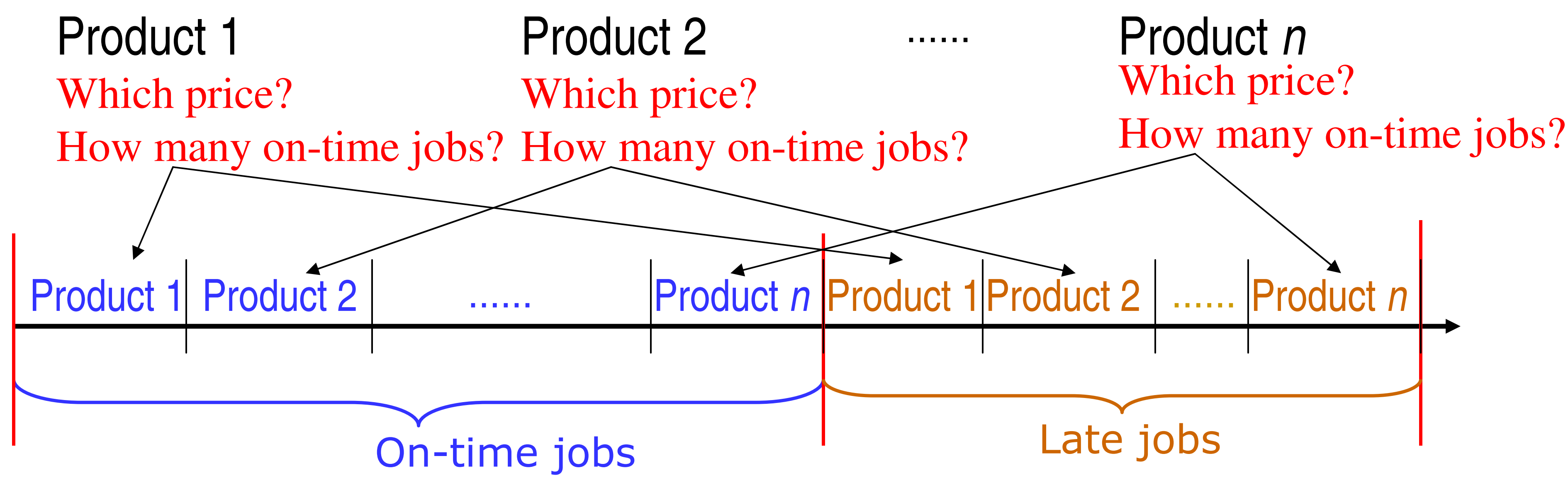
Optimality property: There exists an optimal schedule where

(i) the on-time jobs are scheduled before the late jobs;

(ii) the on-time jobs of each product are scheduled consecutively;

(iii) the on-time jobs are scheduled in EDD order (i.e., nondecreasing order of d_j).

Reindex the products such that $d_1 \leq d_2 \leq \ldots \leq d_n$



DP Algorithm for Capacity Problem

Optimality property: There exists an optimal schedule where

(i) the jobs of each product are scheduled consecutively;

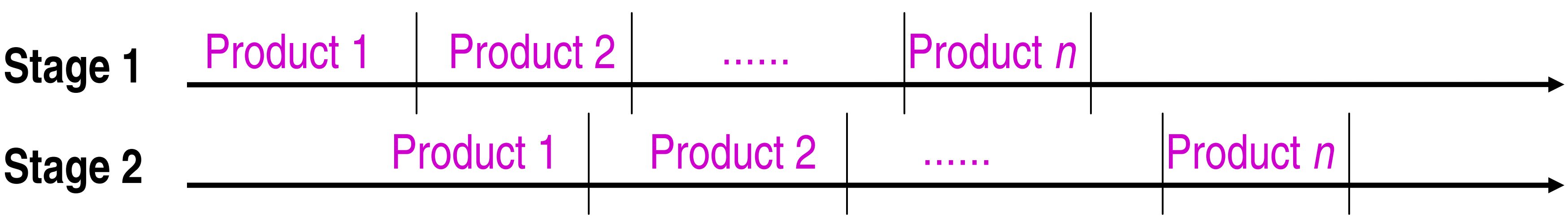
(ii) the products are scheduled according to Johnson's rule.

Johnson's rule:

1. **Divide products into two sets: S_1 with $p_{1j} \le p_{2j}$ and S_2 with $p_{1j} > p_{2j}$**
2. **Sequence the products in S_1 (S_2) in SPT (LPT) order**

Reindex the products according to Johnson's rule

Product 1	Product 2	......	Product n
Which price?	Which price?		Which price?



Approximation Schemes (1 of 3)

Goal: Reduce running time by sacrificing solution accuracy

Fully polynomial time approximation scheme (FPTAS):

For any given $\varepsilon > 0$, the algorithm finds a solution that is within a relative error ε from optimality, and has a running time that is polynomial in both the size of input data and $1/\varepsilon$.

For the WIP problem, FPTAS

Running time: $O(n^3 m_{\max} / \varepsilon)$,

where $m_{\max} = \max\{m_j \mid j = 1, \ldots, n\}$

- If $\varepsilon = 5\%$ (within 95% of optimality), then $O(20 n^3 m_{\max})$
- If $\varepsilon = 10\%$ (within 90% of optimality), then $O(10 n^3 m_{\max})$

Approximation Scheme (2 of 3)

Idea: State space trimming technique

Value function for WIP problem:

f*(*j*, *t*) =** maximum net profit from products 1, …, ***j, given that after product *j* is scheduled, the makespan of the schedule is ***t***.

$$0 \le \boldsymbol{t} \le \sum_{j=1}^{n} \bar{g}_j p_j$$

All possible values of ***t***

State space trimming

Polynomial number of ***t*** values

Approximation Schemes (3 of 3)

The running times of the fully polynomial time approximation schemes are:

- $\Sigma_j \, w_j C_j$ - R $\qquad O(n^3 m_{\max}/\varepsilon)$
- $\Sigma_j \, w_j U_j$ - R $\qquad O(n^3 m_{\max} \log \bar{g}_{max} \log(n\lambda)/\varepsilon^2)$
- $C_{\max}$ - R $\qquad O(n^2 m_{\max} log(n\lambda)/\varepsilon)$,

where $\lambda = \max_{1 \le j \le n,\ 1 \le i \le m_j} \{q_{ij} g_j(q_{ij})\}$.

Managerial Insights (1 of 4)

- What is the value of pricing and scheduling coordination?
- How does this value change with problem parameters?
- Is there a simple heuristic that can generate “good” solutions?

  - Compare four approaches
    1. Uncoordinated approach
       - Pricing first, followed by scheduling; independent decisions.
    2. Partially coordinated approach
       - Pricing first, followed by scheduling; the pricing decision partially considers the scheduling cost.
    3. Fully coordinated heuristic approach
    4. Optimally coordinated algorithm (DP)

Managerial Insights (2 of 4)

Uncoordinated Approach for WIP problem:

1. Pricing decision (without considering scheduling cost).

   For each product j, choose a price q_{kj} that maximizes product j's revenue, i.e. $q_{kj}g_j(q_{kj}) = \max \{ q_{ij}g_j(q_{ij}) \mid 1 \leq i \leq m_j \}$.

2. Scheduling decision (given the prices).

   Schedule the jobs in SWPT order to minimize total WIP cost.

Managerial Insights (3 of 4)

Partially Coordinated Approach for WIP problem:

1. Pricing decision (considering part of the scheduling cost).

   For each product j, choose a price q_{kj} that maximizes product j's net profit (revenue minus scheduling cost of product j),

   i.e. $q_{kj}g_j(q_{kj}) - z_j(g_j(q_{kj})) = \max \{ q_{ij}g_j(q_{ij}) - z_j(g_j(q_{ij})) \mid 1 \le i \le m_j \}$,

   where $z_j(g_j(q_{ij}))$ is the WIP cost of $g_j(q_{ij})$ jobs of product j if they are processed starting from time 0

2. Scheduling decision (given the prices).

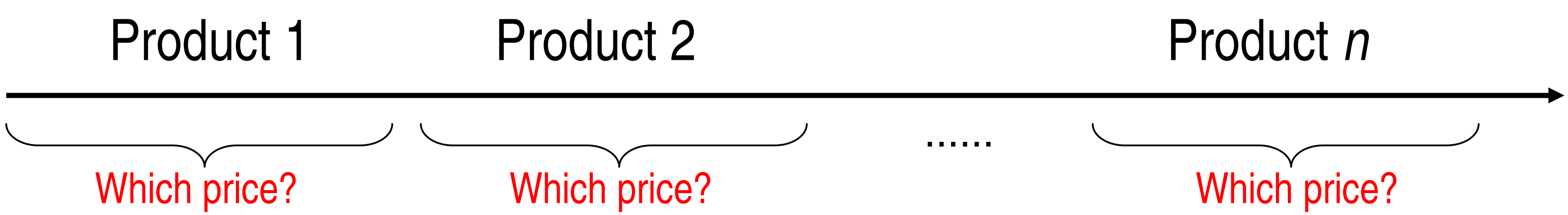
   Schedule the jobs in SWPT order to minimize the total WIP cost.

Managerial Insights (4 of 4)

Fully Coordinated Heuristic Approach:

Pricing and scheduling jointly

- Consider products in WSPT order
- Price and schedule the next product to maximize its net profit



Similar logic to optimal DP algorithm, but no state space enumeration

Computational Results (1 of 3)

Test instances for WIP problem:

- Linear demand function: $g_j(q_{ij}) = \max\{0, \lfloor \alpha_j - \beta_j q_{ij} \rfloor\}$
- $\alpha_j \in U[20, 40]$, $\beta_j \in U[0.0015, 0.0025]$, or
  $\alpha_j \in U[35, 65]$, $\beta_j \in U[0.0035, 0.0065]$
- # of products n = 10 or 50
- # of allowable prices $m_j \in U[2, 6]$, or $U[4, 12]$
- Allowable prices, $q_{ij} \in U[1000, 10000]$
- Processing time, $p_j \in U[1, 10]$
- Weight: $w_j \in U[1, 10]$ or $U[1, 20]$ when $n = 10$
  $w_j \in U[1, 3]$ or $U[1, 6]$ when $n = 50$

Computational Results (2 of 3)

WIP Problem	**Profit Gap**	**Demand Gap**
Uncoordinated	– 27.01%	28.49%
Partially Coordinated	– 21.95%	25.36%
Fully Coordinated Heuristic	– 3.54%	9.62%
Lateness Penalty Problem		
Uncoordinated	– 6.73%	15.52%
Partially Coordinated	– 4.95%	12.72%
Fully Coordinated Heuristic	– 1.79%	0.20%
Capacity Problem		
Uncoordinated	– 14.69%	21.16%
Partially Coordinated	– 4.28%	– 8.26 %
Fully Coordinated Heuristic	– 2.17%	– 1.12 %

Computational Results (3 of 3)

Main Insights

1. Pricing-scheduling coordination increases net profit substantially
   - If full coordination is not possible, at least partial coordination should be implemented
2. The simple fully coordinated heuristic generates near optimal solutions
   - This heuristic can be used if the optimal DP algorithm is too complex to implement
3. The value of pricing-scheduling coordination increases with demand sensitivity to price, and with number of products

Pricing and Scheduling Conclusions

- We consider three pricing and scheduling problems with a variety of scheduling costs.
- All three problems are formally intractable.
- We describe computationally efficient optimal algorithms.
- We describe fully polynomial time approximation schemes.
- We describe a simple heuristic that usually provides very close to optimal solutions for all three problems.
- Sensitivity analyses provide insights about when to coordinate pricing and scheduling decisions.

Future Research

- Consider problems where orders do not have to be accepted.
- Consider other measures of scheduling cost.
- Develop heuristics with good performance guarantees.
- Develop models with service level constraints.
- Coordinate other decisions, such as distribution, with pricing and scheduling.

Research Agenda (1 of 3)

- Interesting research in this area usually results from the integration of new issues with pricing.
- When integrating pricing with other decisions, there is a natural tradeoff that makes problems intractable (typically, binary *NP*-hard).
- There is a need to vary the assumption that demand is independent between time periods, either deterministically or stochastically.
- Models of customer choice are overly simplistic, relative to the marketing literature.

Research Agenda (2 of 3)

- Capacity constraints have not been convincingly integrated into pricing models.
- There is a need to integrate product life cycle planning into pricing models.
- There is a need to integrate the effects of different types of competition (oligopoly, perfect competition,…) into pricing decisions.
- There is a need to integrate pricing decisions that reveal demand curve information with production decisions.

Research Agenda (3 of 3)

- In some environments, there is a need to model production costs using individual job scheduling costs.
- In some environments, there is a need to model service levels using individual job scheduling completion times.
- In some environments, there is a need to model capacity usage using a detailed job schedule.
- Production issues need to be integrated into non-cooperative pricing games.

Research Paper and Coauthor

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Copies of the research paper
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Thank you for your attention!

Are there any questions?

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