

Chaos without exponential instability

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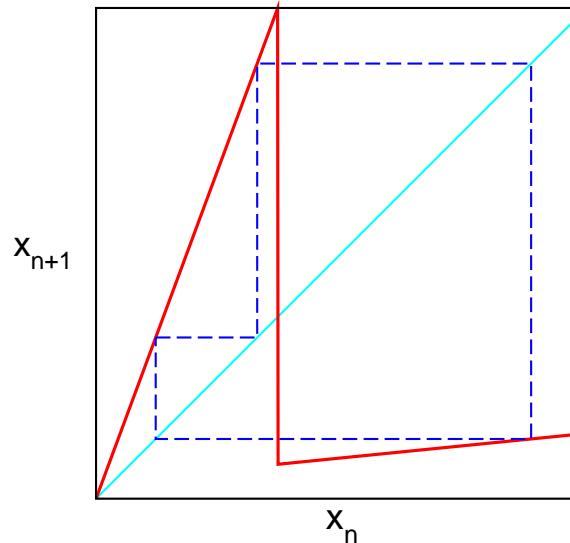
OUTLINE

- COUPLED MAP MODELS
- CHAINS OF FORCED OSCILLATORS
- RELATIONSHIP WITH CELLULAR AUTOMATA
- PROPAGATION OF PERTURBATIONS
- LEAKY INTEGRATE-AND-FIRE NEURONS
- CHAINS OF HARD-POINT PARTICLES

LINEARLY STABLE CHAOS IN COUPLED MAPS

$$x_{n+1}^i = F(\bar{x}_n^i)$$

$$\bar{x}_n^i = \frac{\varepsilon}{2}x_n^{i-1} + (1 - \varepsilon)x_n^i + \frac{\varepsilon}{2}x_n^{i+1}$$

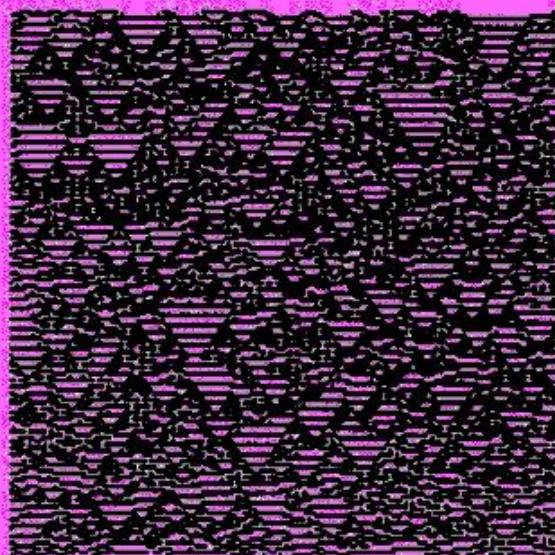


[J.P.Crutchfield, K.Kaneko, PRL, **60** 2715 (1988)]

[A.P., R.Livi, G.-L.Oppo, R.Kapral, Europhys. Lett. **22** 571 (1993)]

$\varepsilon = 2/3$

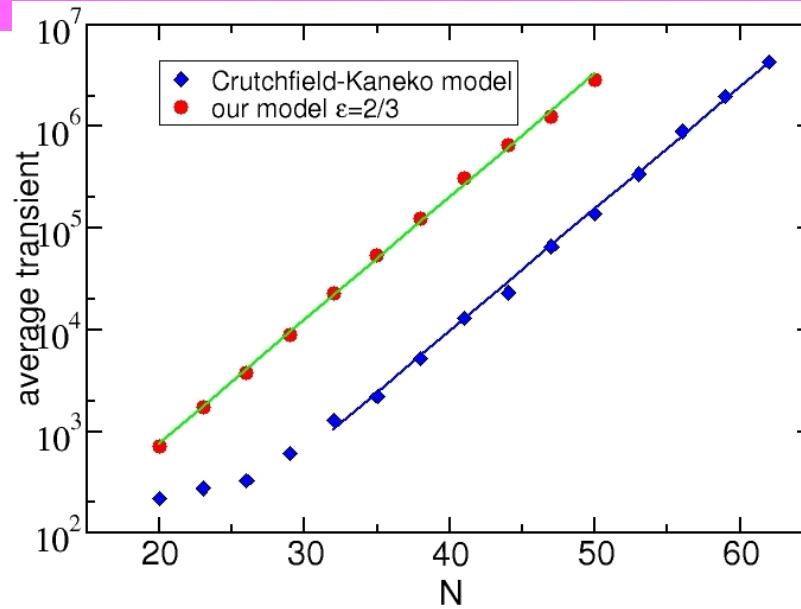
a)

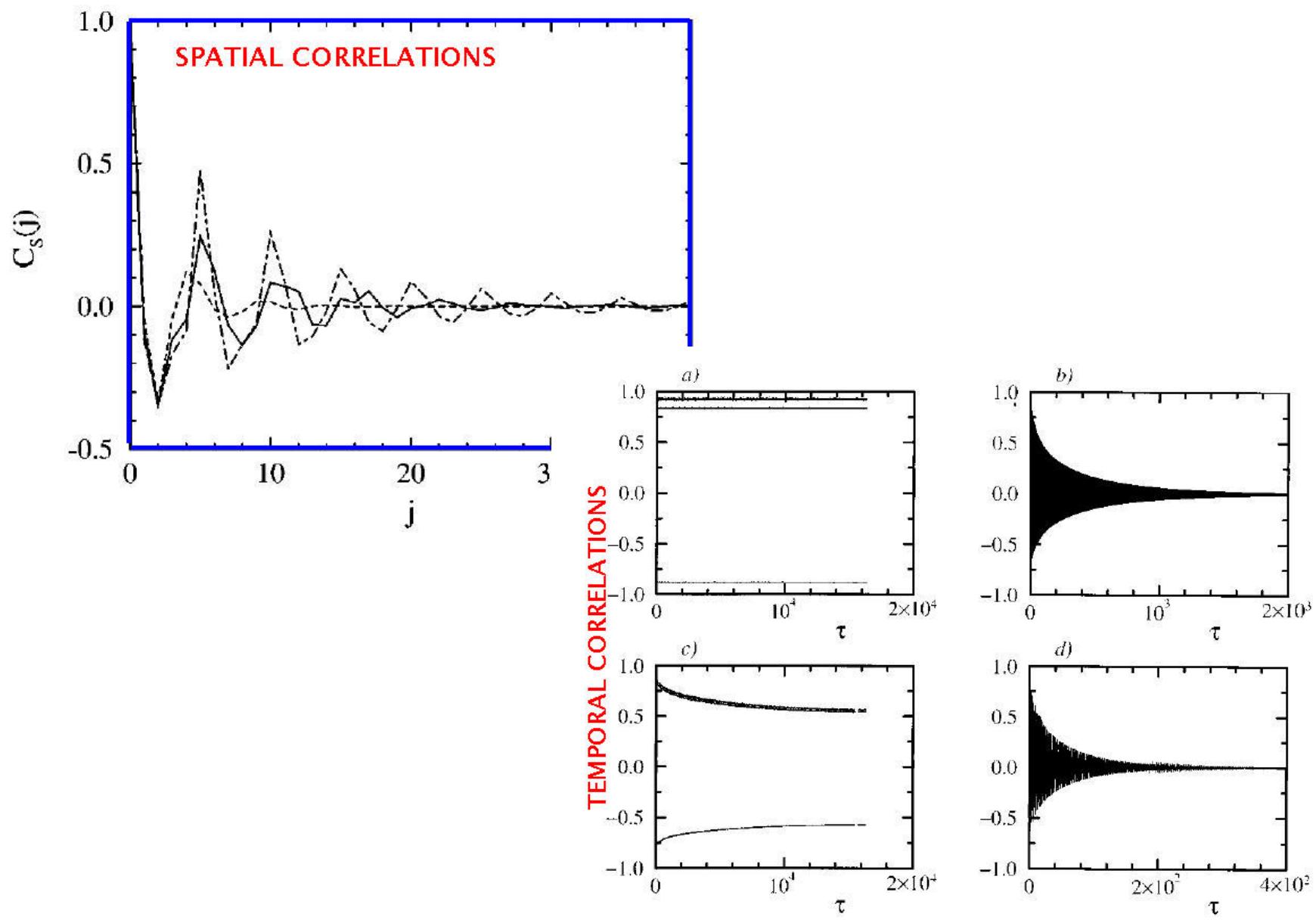


b)

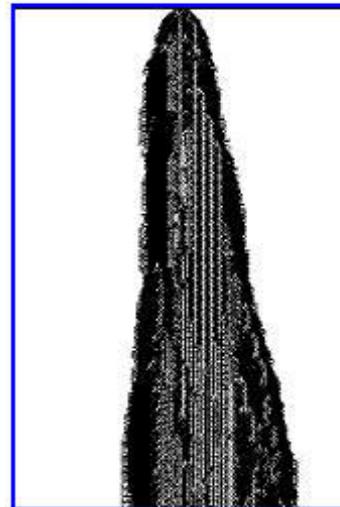
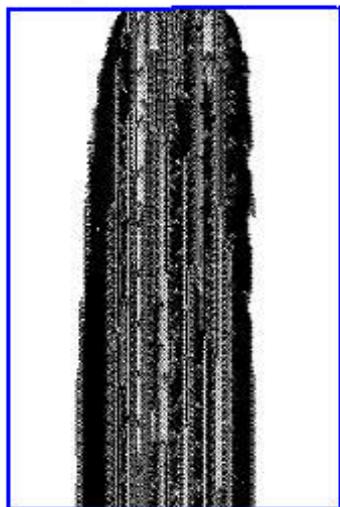


$\varepsilon = 0.76$

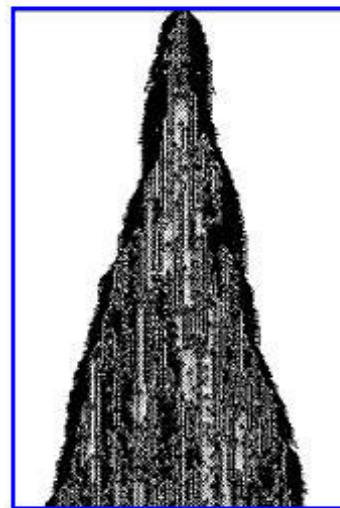
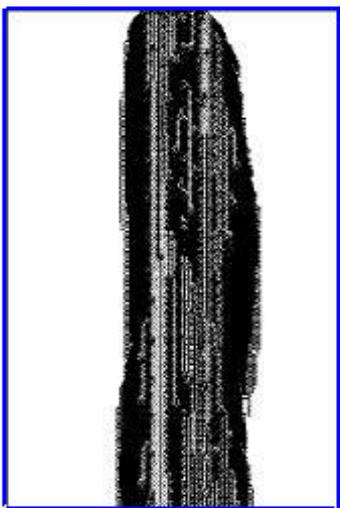




PERTURBATION PROPAGATION SPACE

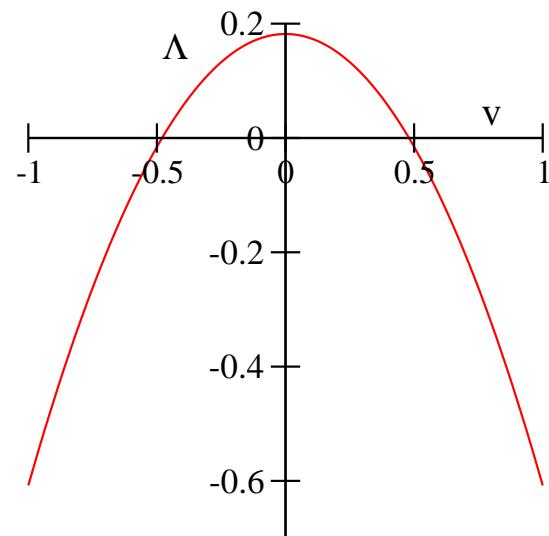
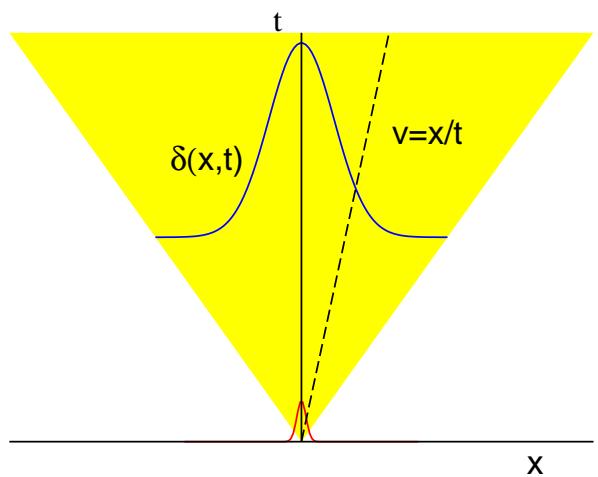


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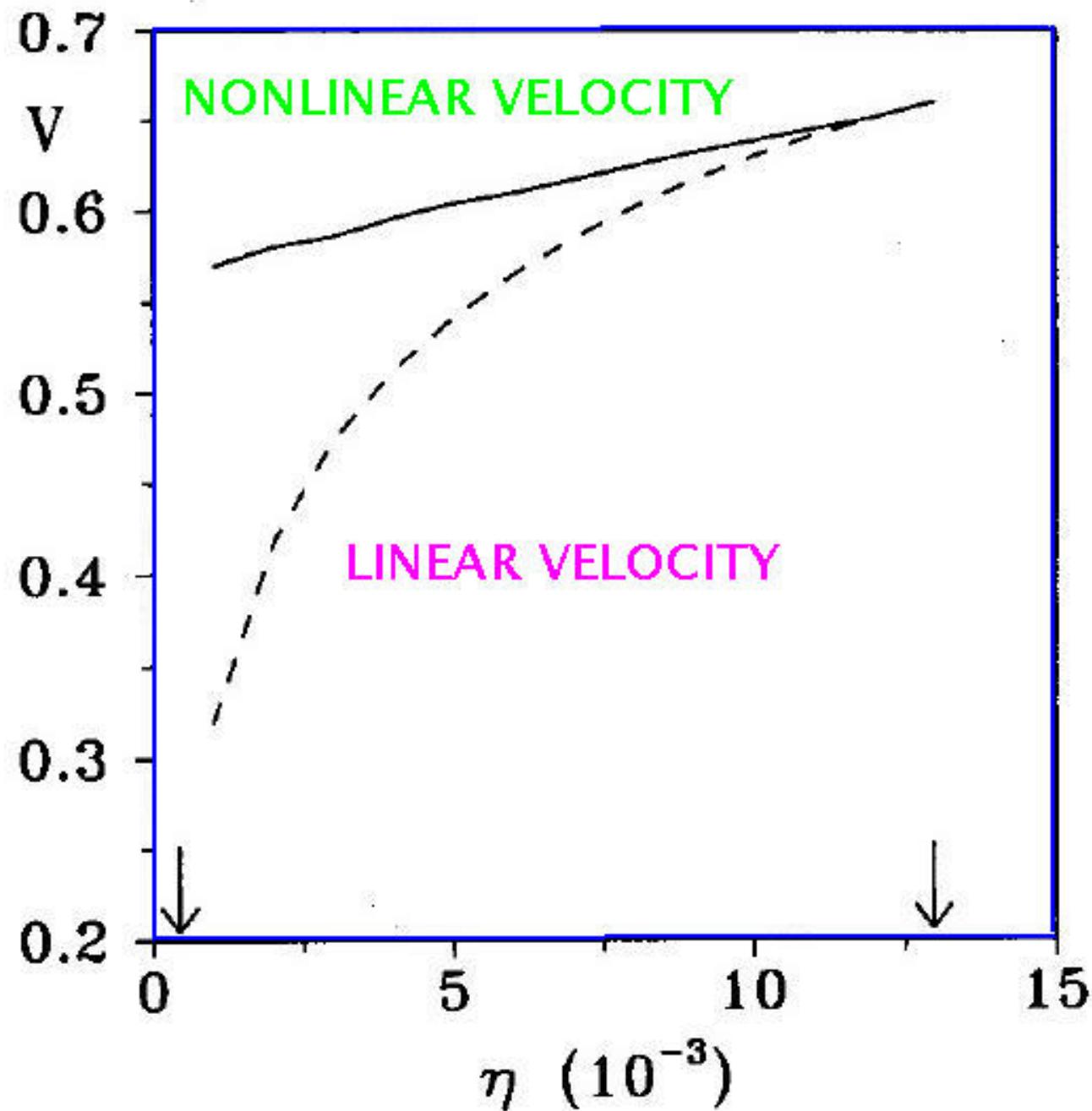
CONVECTIVE LYAPUNOV EXPONENTS



$$\delta(x, t) \approx \exp[\Lambda(x/t), t]$$

$$\Lambda(v = x/t) = \lim_{t \rightarrow \infty} \frac{1}{t} \ln \left(\frac{|\delta(x, t)|}{|\delta(0, 0)|} \right)$$

[R.J.Deissler, K. Kaneko **119** 397 (1987)]



NETWORKS OF LEAKY INTEGRATE-AND-FIRE NEURONS

$$\tau \dot{v}_j = L_j - v_j - \frac{\tau}{N} \sum_m \left[G_{j,i(m)}(v_j + E) \right] \delta(t - t_m)$$

[D.Z. Jin, PRL **89**, 208102 (2002)]

v_j, L_j

membrane, resting potential

$G_{i,j}$

inhibitory synapse conductance

t_m

time of the m th spike emitted by the $i(m)$ th neuron

Θ_j

threshold of j th neuron

R_j

resetting potential

$$\dot{x}_j = a - x_j - G_0 \sum_m F_{j,i(m)}(x_j + r) \delta(t - t'_m) \quad a = 2, r = 4/7$$

Dilution (a given fraction of links is cut)

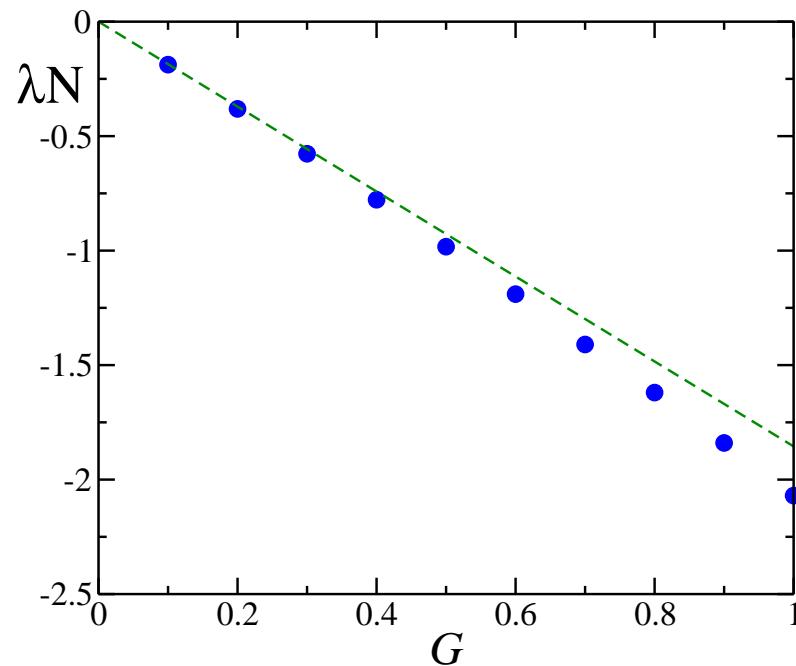
THE MAXIMUM LYAPUNOV EXPONENT

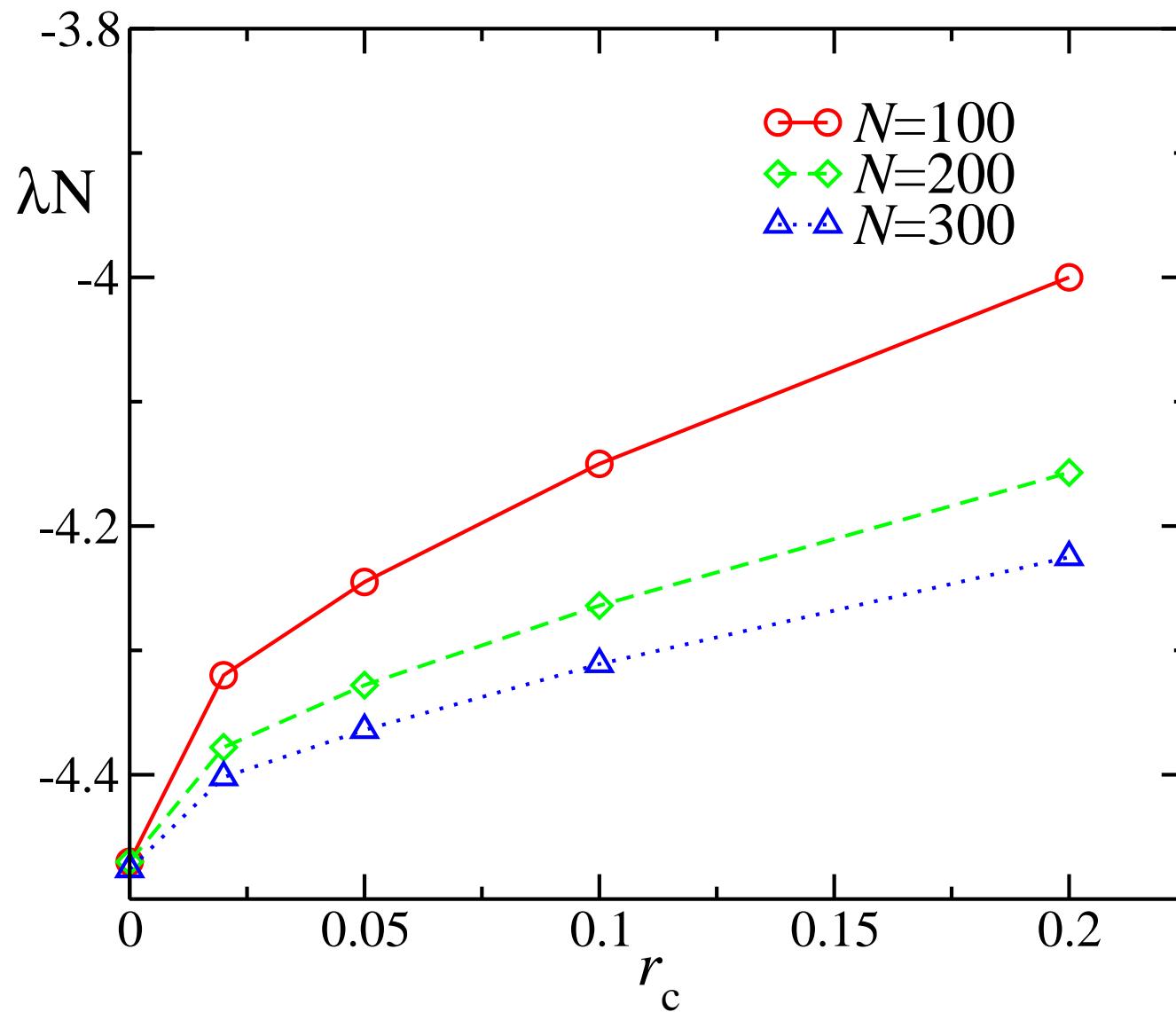
$$-\frac{\partial \Gamma}{\partial x} + \frac{\partial \Gamma}{\partial t} = -(\Gamma_\alpha + G)\Gamma + G \frac{c+w}{c-1}$$

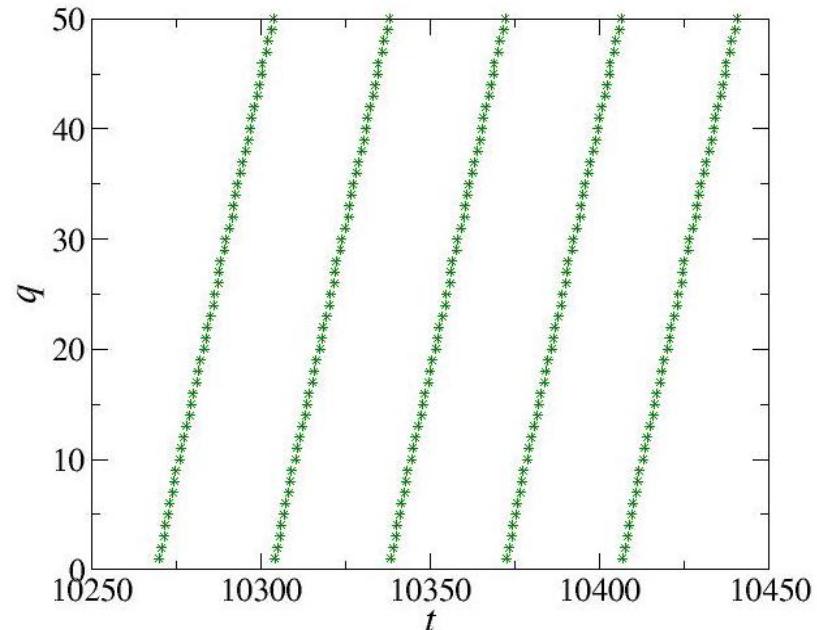
$$\Gamma = \exp(\tau)$$

$$\Gamma(0) = 1, \Gamma(1) = \frac{c}{c-1}$$

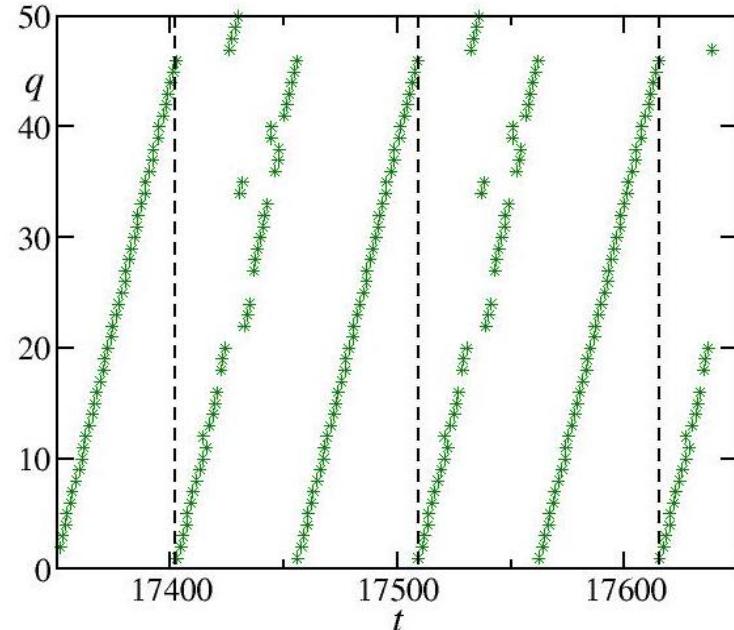
(τ = time to threshold with no coupling)



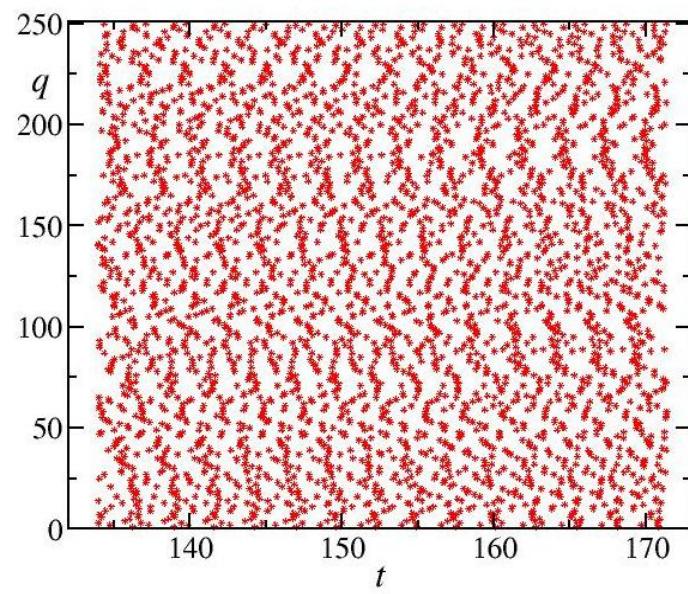




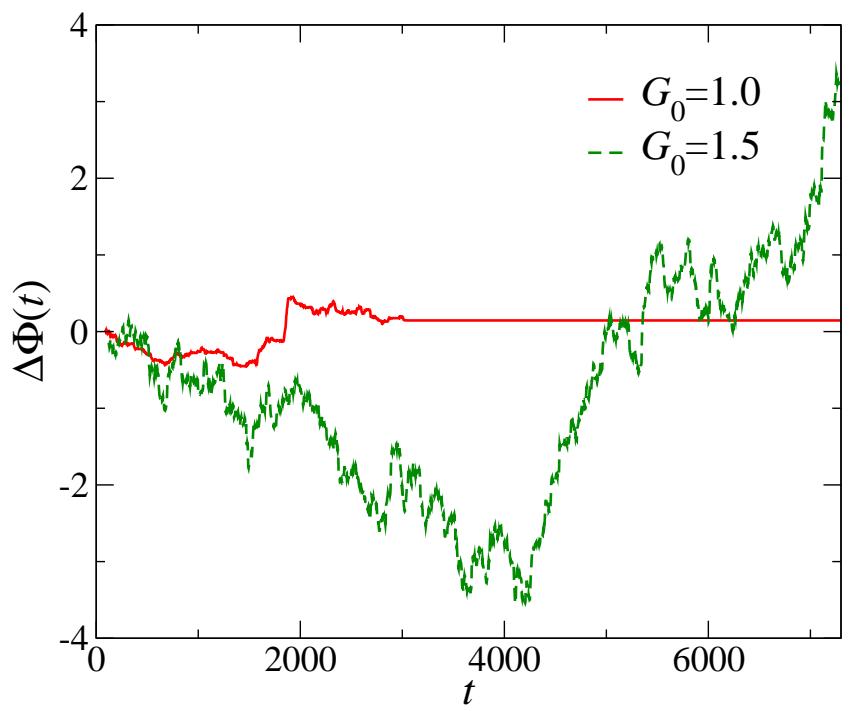
$G=1$



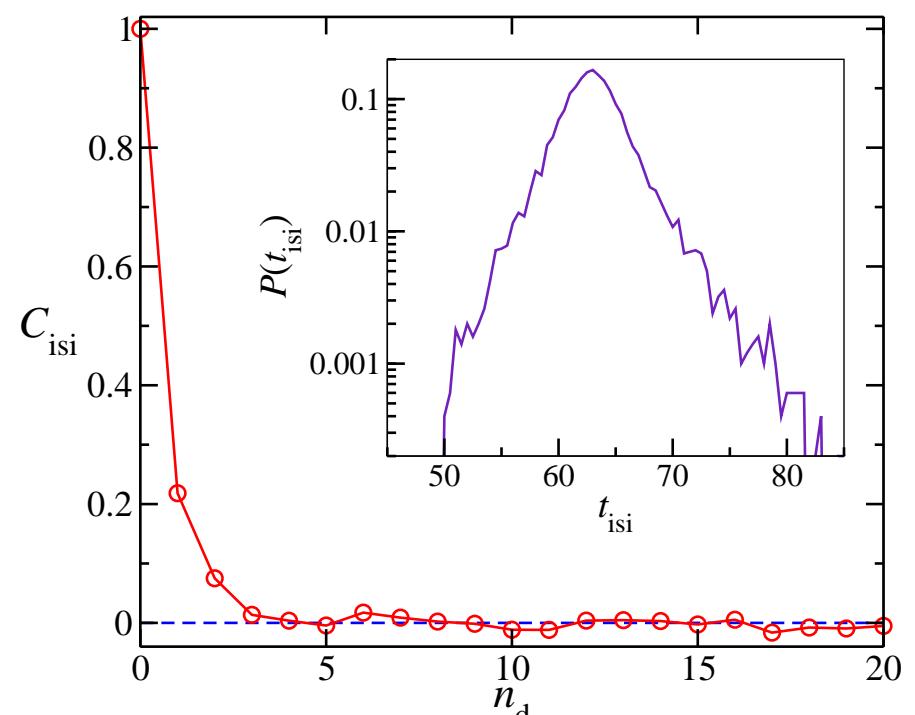
$G=2$.



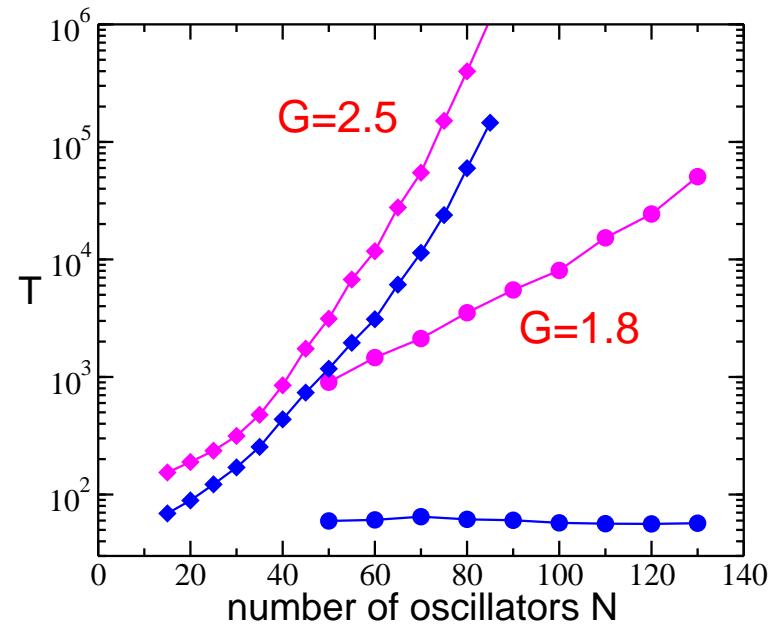
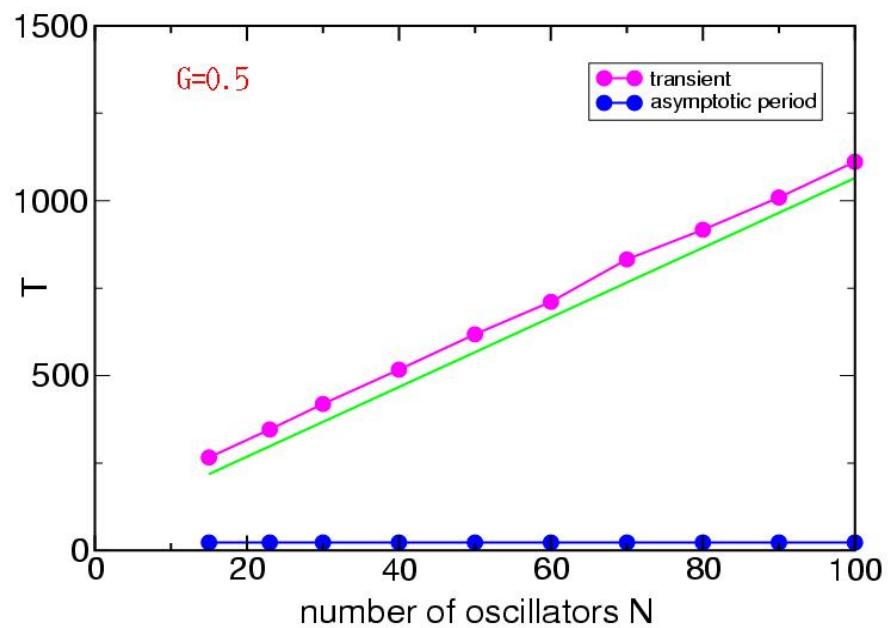
$G=1.7$



Integrated phase shift

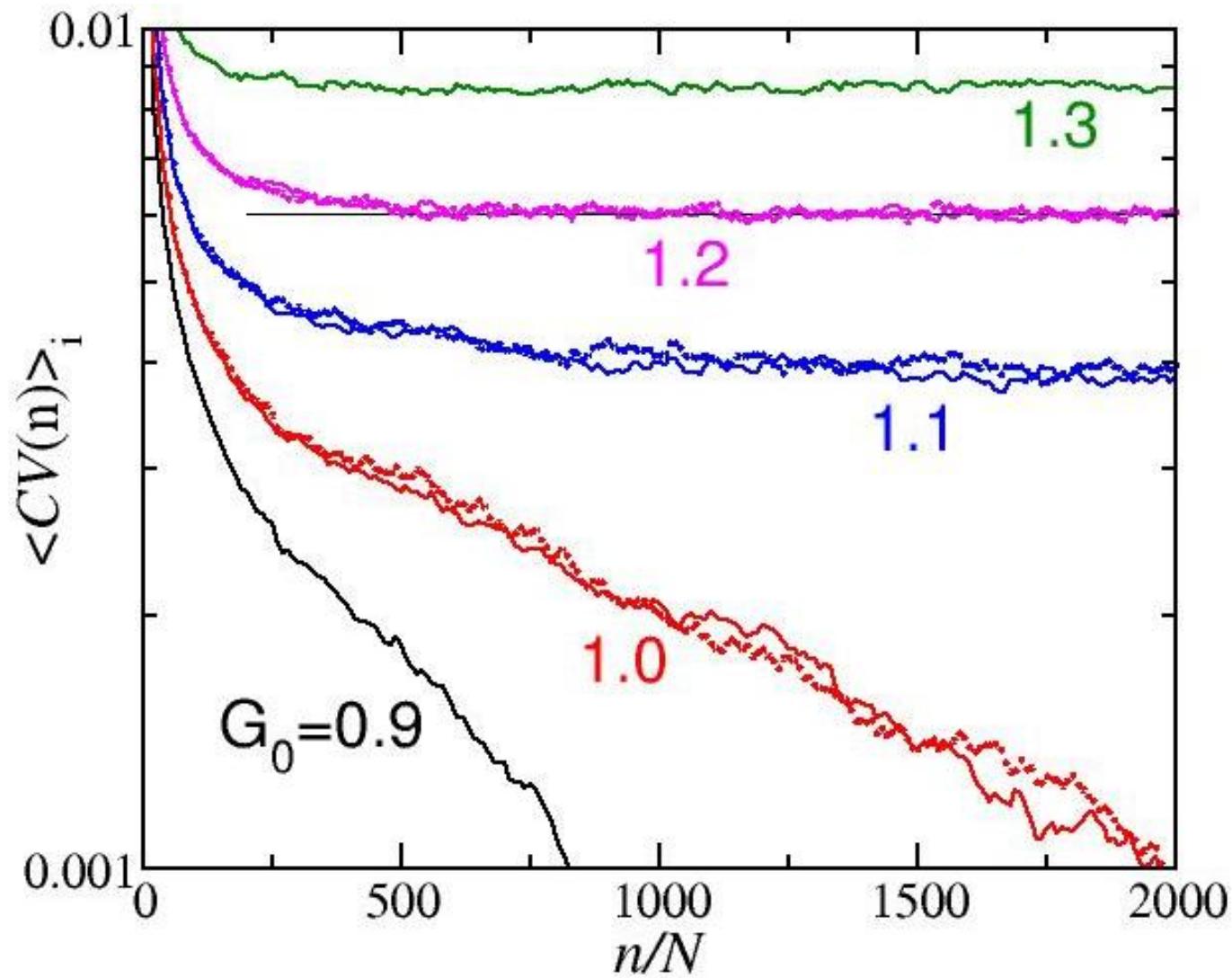


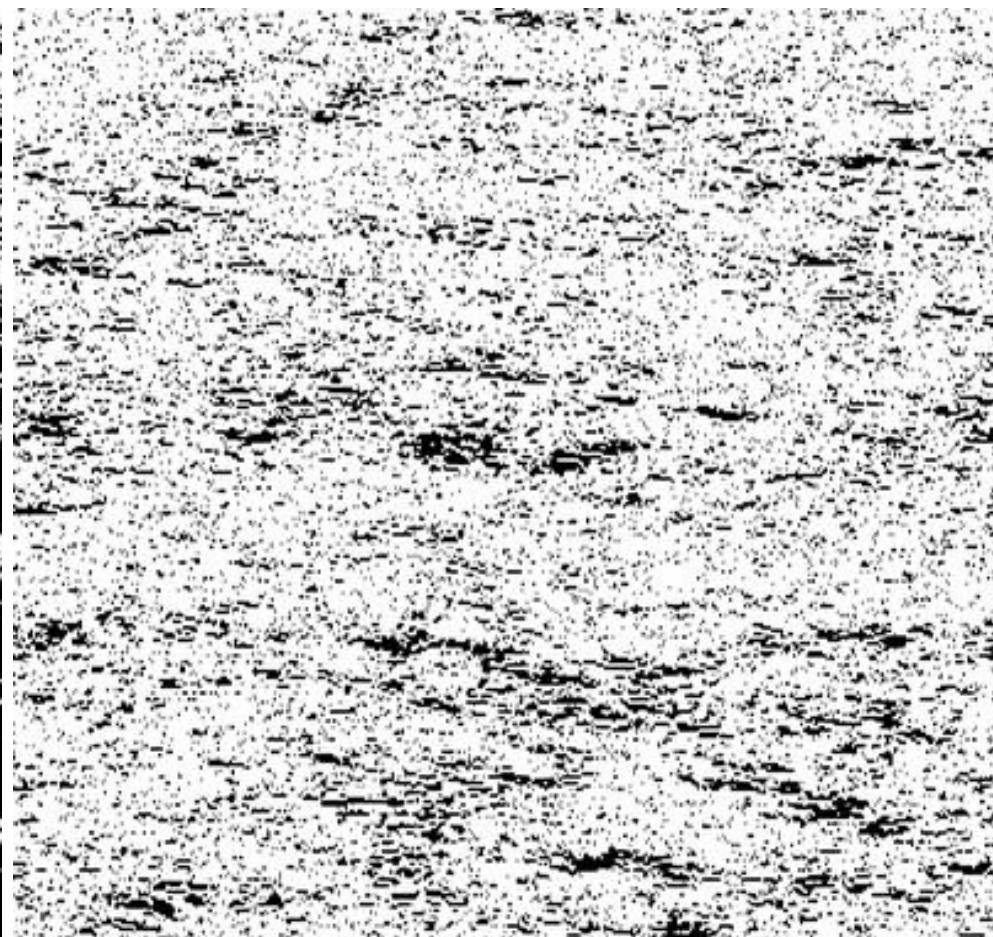
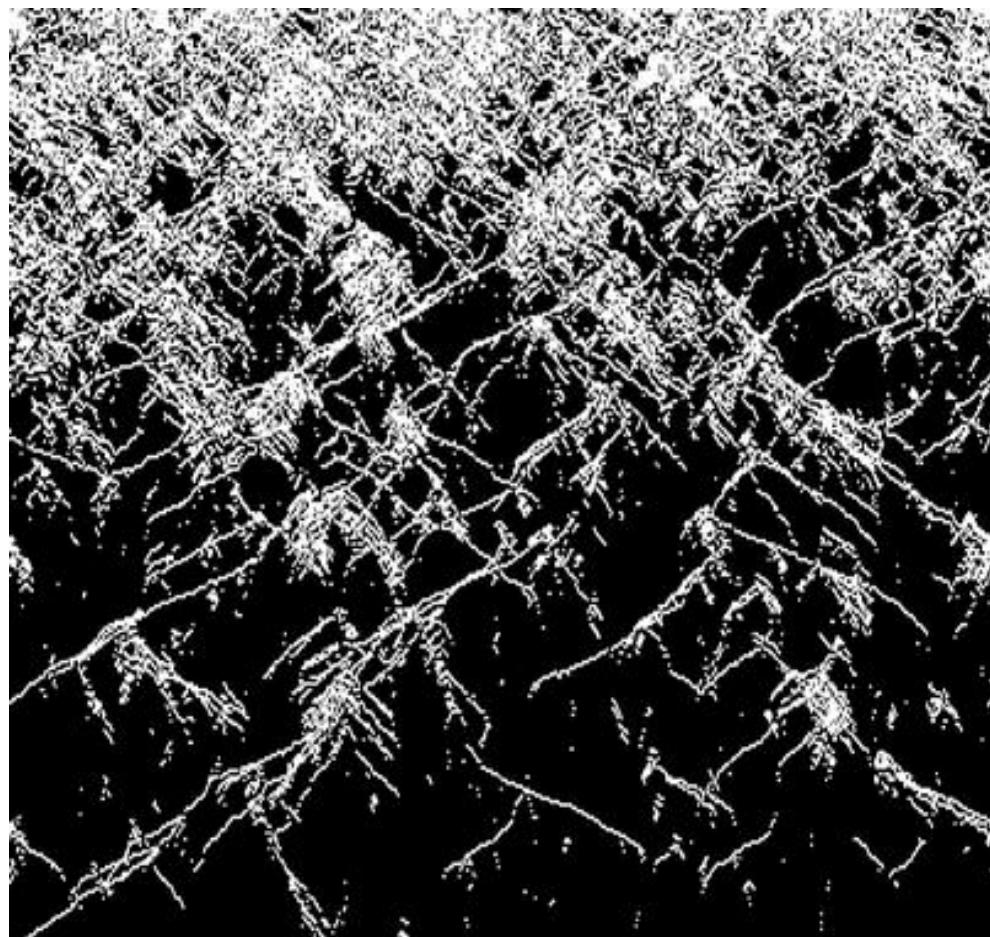
ISI-correlation function



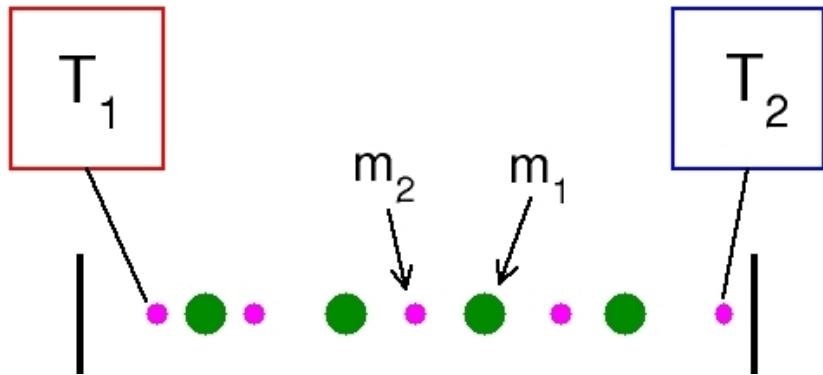
[A. Zündieck, M. Timme, T. Geisel, F. Wolf, PRL **93** 244103 (2004)]

positive Lyapunov exponent & delay





HARD-POINT GAS AND HARD-POINT CHAIN



$$v'_i = v_j \pm \frac{1-r}{1+r} (v_i - v_j)$$

$$r = \frac{m_1}{m_2} \quad \text{mass ratio}$$

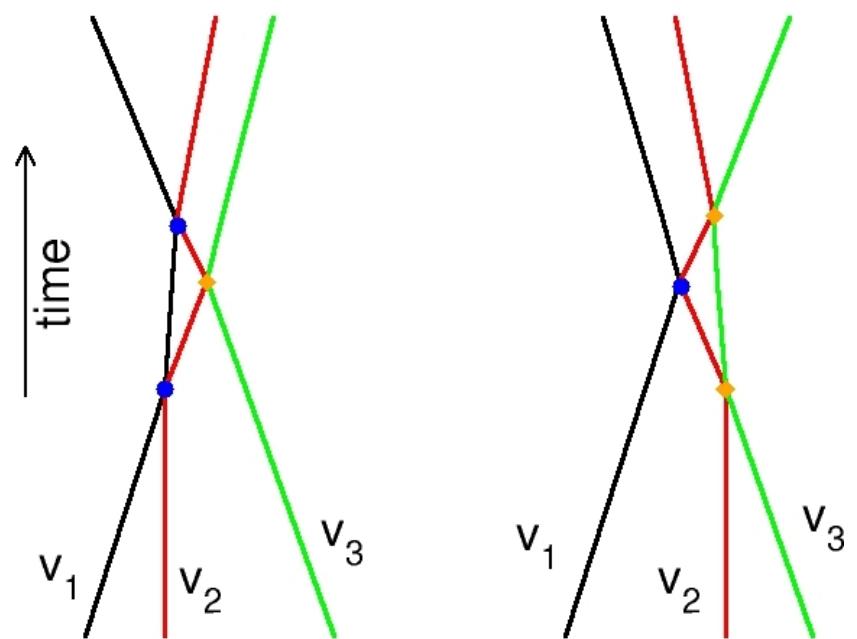
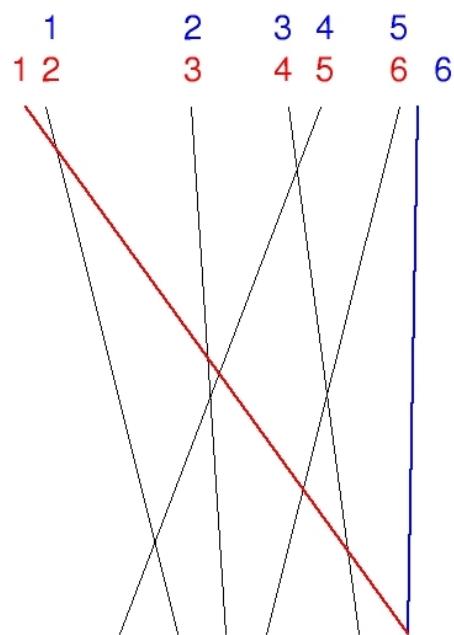
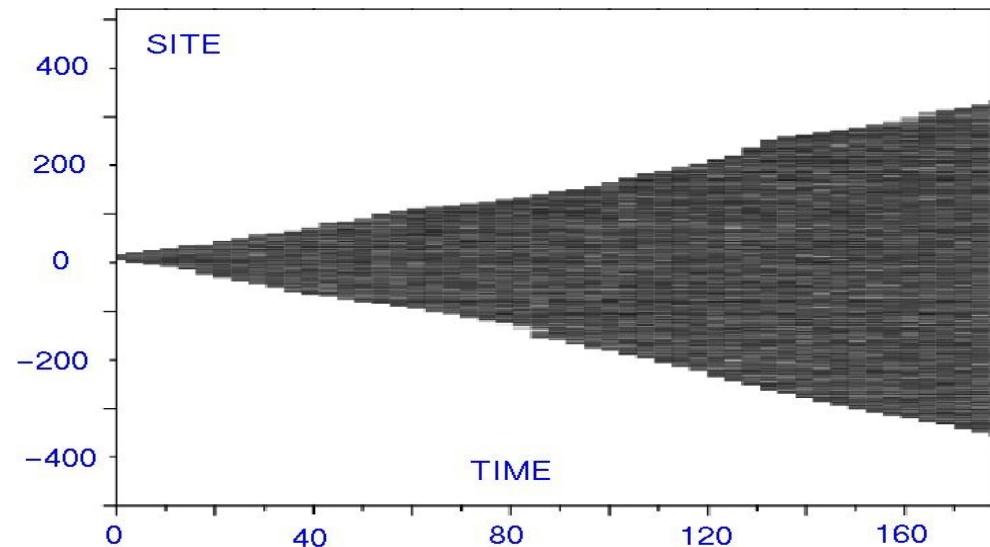
Evolution in tangent space equal to that in real space

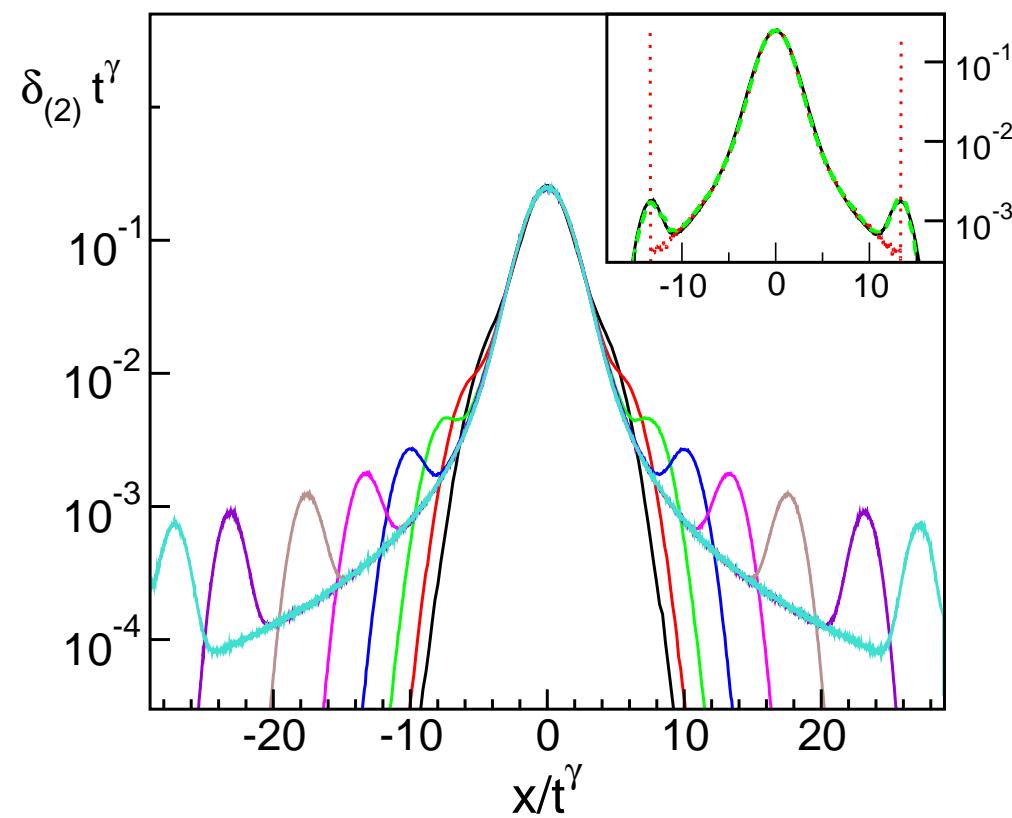
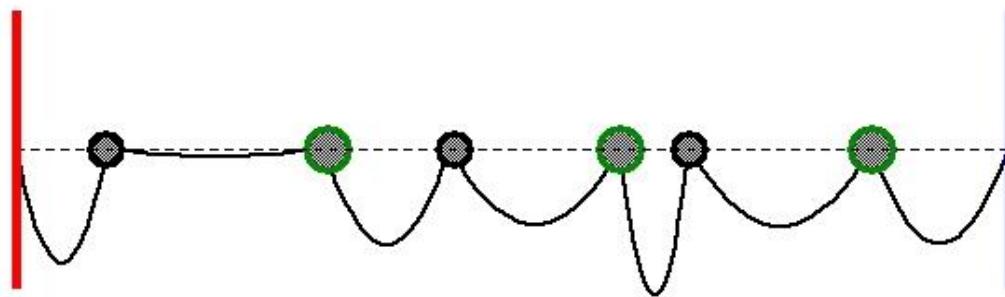


Conservation of the Euclidean norm of infin. perturbations



NO STANDARD CHAOS





FINITE PERTURBATIONS

