GMWBs

Tom Salisbury
Math & Stat - York University
Fields Finance Seminar, Oct 25, 3006

- *Financial Valuation of Guaranteed Minimum Withdrawal Benefits*,

  M.A. Milevsky & T.S. Salisbury.

  Ins. Math & Econ. 2006
- *Asset Allocation and the Transition to Income: The Importance of Product Allocation in the Retirement Risk Zone,*

  M.A. Milevsky & T.S. Salisbury

  IFID working paper, 2006 (www.ifid.ca)
- *Asset Allocation with GMWBs*, (in progress, 2006)

  H. Huang, M.A. Milevsky & T.S. Salisbury.

- A GMWB is a rider on a variable annuity contract. Deposit an amount $W(0)$ into a mutual fund (or portfolio of funds). The account value $W(t)$ then evolves dynamically.

  One is entitled to withdraw $g=G\cdot W(0)$ per year (eg. $G=7\%$) for $1/G$ years, even if this (coupled with market performance) drives the account value to zero.
- If it does, the firm selling the VA steps in and makes good on the guarantee.

  Typically this is one of several embedded options, eg. GMDB, ratchets, passport option.

- One may withdraw less than g (often there is a bonus if one withdraws nothing). One may leave the VA at any time (lapsation) and receive the account value less any deferred surrender charge (DSC).

  But if one withdraws more than g this typically resets the guaranteed amount or the withdrawal period, or both.
- GMWB-for-life: payment stream continues after time $1/G$. Increasingly seeing this.

- Funding: a yearly fee α (insurance charge) applied variously to the account value or remaining amount guaranteed ($W(0)$ less withdrawals).
- This is a complex product, because of the interaction of the various embedded options, but also because of the optionality of lapsation and withdrawals.
- In the U.S. variable annuities are tax-sheltered retirement savings plans, and represent a substantial face value ($1,125 billion at end of 2004), with about half protected by GMWB riders (NAVA factbook)
- Why??

FINANCIAL POST

FP1 NATIONAL POST, WEDNESDAY, JUNE 28, 2006

NORTEL REJIGS PENSION PLAN

ERASING PENSION CRISIS?

LATEST COST CUTTING

1,900 new job cuts, pension change to save US$275M

GM joins list of companies freezing pension schemes

By James Mackintosh in London

The demise of the guaranteed pension moved another step closer yesterday when General Motors became the third Dow Jones Industrial company in two months to lay out plans to freeze a pension scheme, affecting 26,000 white-collar employees.

The carmaker closed its defined benefit scheme for white-collar staff to new entrants four years ago, with employees taken on since then put in a hybrid cash balance plan.

But freezing the scheme for existing members cuts the company's exposure to the risk that the pension plan's investment fails to achieve its assumed return of 9 per cent.

IBM, the IT company, and Verizon, the second-largest US telephone company, froze pension entitlements for a total of 175,000 workers in December and January. GM pensioners and members of its much larger blue-collar scheme are unaffected.

Rick Wagoner, GM chairman and chief executive, said the moves, along with a cap on healthcare for white-collar pensioners, were needed to allow the carmaker to compete with overseas rivals that did not pay health and pension costs.

"We are now subject to global competition, we are running against people who do not have these costs associated with prior employees because they are picked up by the governments," he said. "Whether it is the auto sector or the tech sector I don't think there's any sector that isn't affected here."

He said details of the changes to the salaried pensions would be set out next month and involve either a defined contribution scheme, where employees bear the investment risk, or a cash balance plan. GM's hourly and salaried US pension plans, together with a $1bn a year interest bill for bonds it issued to fund the schemes, cost $2.2bn last year, or $1,718 per car made in North America. But the cost is forecast to fall to $1.6bn this year after two years of better than expected returns on its investments.

Mercer Human Resources, a US benefit consultant, estimated that 315 out of the more than 2,000 schemes it supports were at least partly frozen. But Richard McEvoy, a principal, said he expected a much slower move away from defined benefits for pension schemes covering unionised and public sector workers.

See Lex

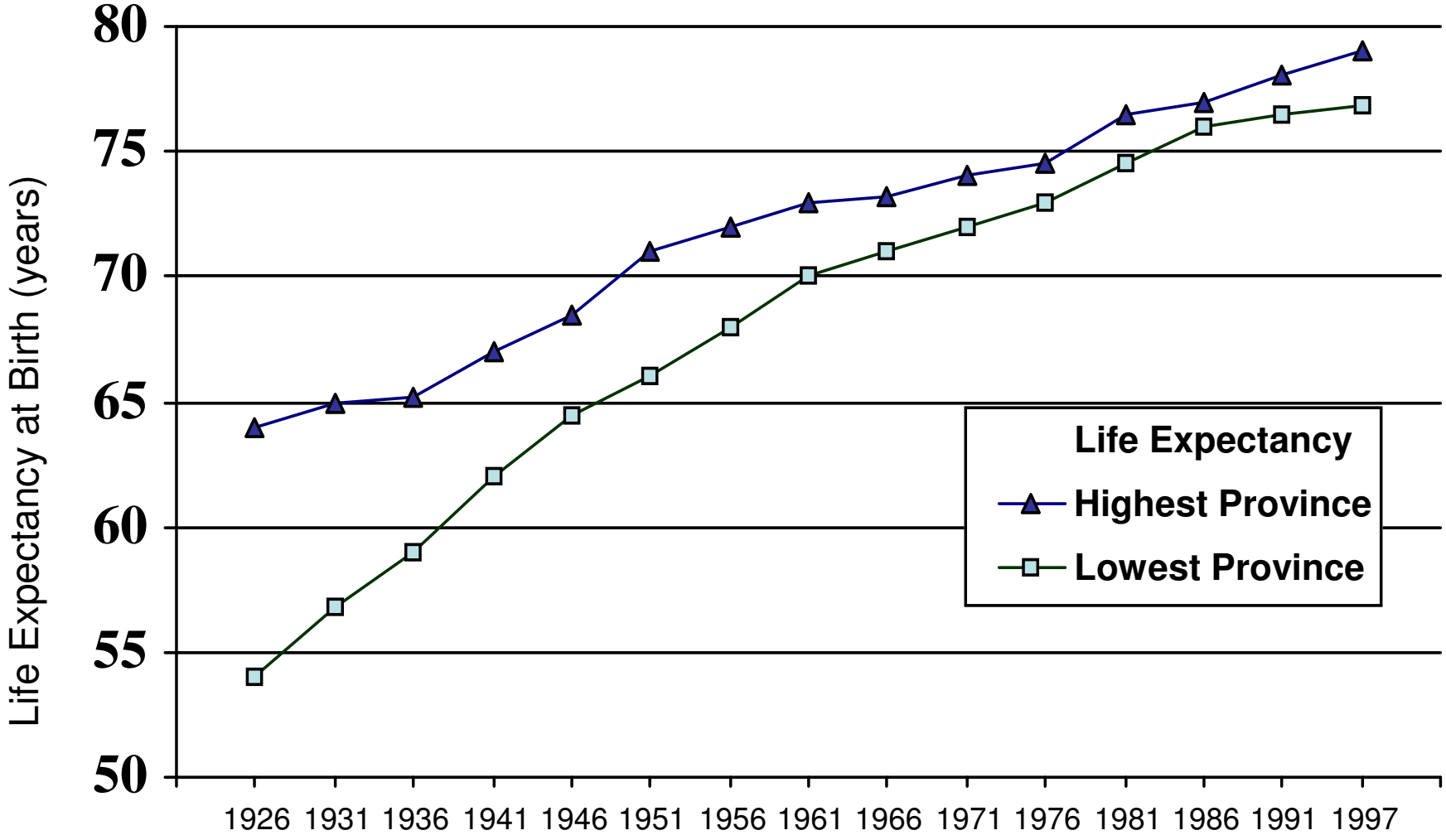
Number of Participants in DB and DC Plans

	1980		1990		2005	
Registered Pension Plan Type	**Number**	**% of Total**	**Number**	**% of Total**	**Number**	**% of Total**
Defined Benefit	4,194,283	**93.7**	4,633,587	**90.7**	4,604,775	**81.2**
Defined Contribution	231,275	**5.2**	430,561	**8.4**	885,840	**15.6**
All RPPs	4,475,429	**100**	5,109,363	**100**	5,669,858	**100**

Source: CANSIM II

- Increasingly, retirees are left to take responsibility for their own retirement planning. Firms transfer risk to individuals
- Risk factors - improving mortality leaves us the risk of outliving our money.
- Ageing of the population

Range of Life Expectancy at Birth for Both Sexes in the Canadian Provinces: 1921-1997



Data Sources: Institute for Clinical Evaluative Sciences; Atlas Reports, The Health of Ontarians
Organization for Economic Cooperation and Development Health Data 1998

Probability a 65 year-old Canadian Will Live to Age…

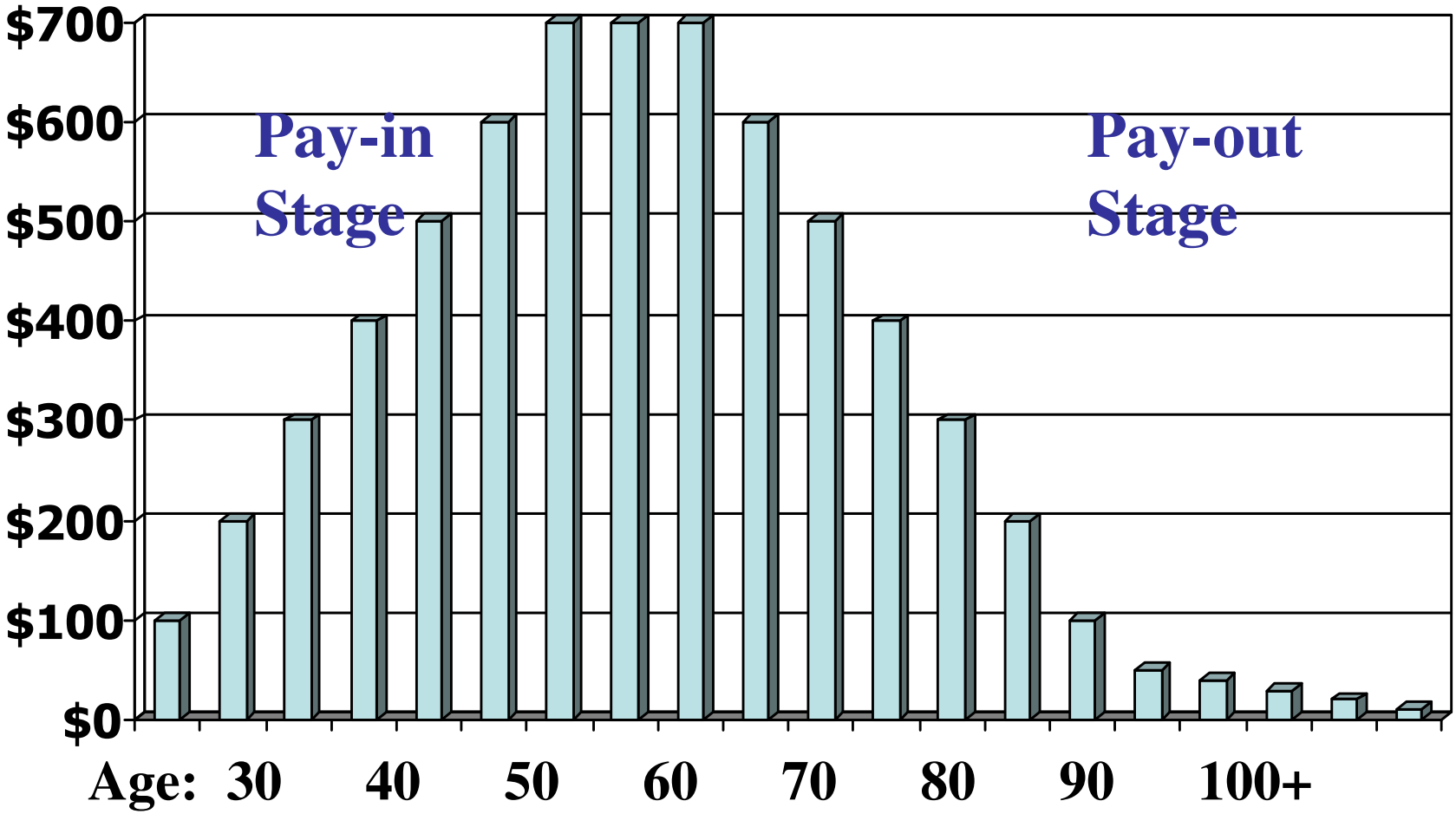
Age	Male	Female	Joint
70	89.13%	93.94%	99.34%
75	73.99%	84.90%	96.07%
80	54.76%	71.42%	87.07%
85	33.44%	52.61%	68.46%
90	15.14%	30.60%	41.11%
95	4.34%	12.03%	15.85%

Canada Pension Plan Retirement Beneficiaries

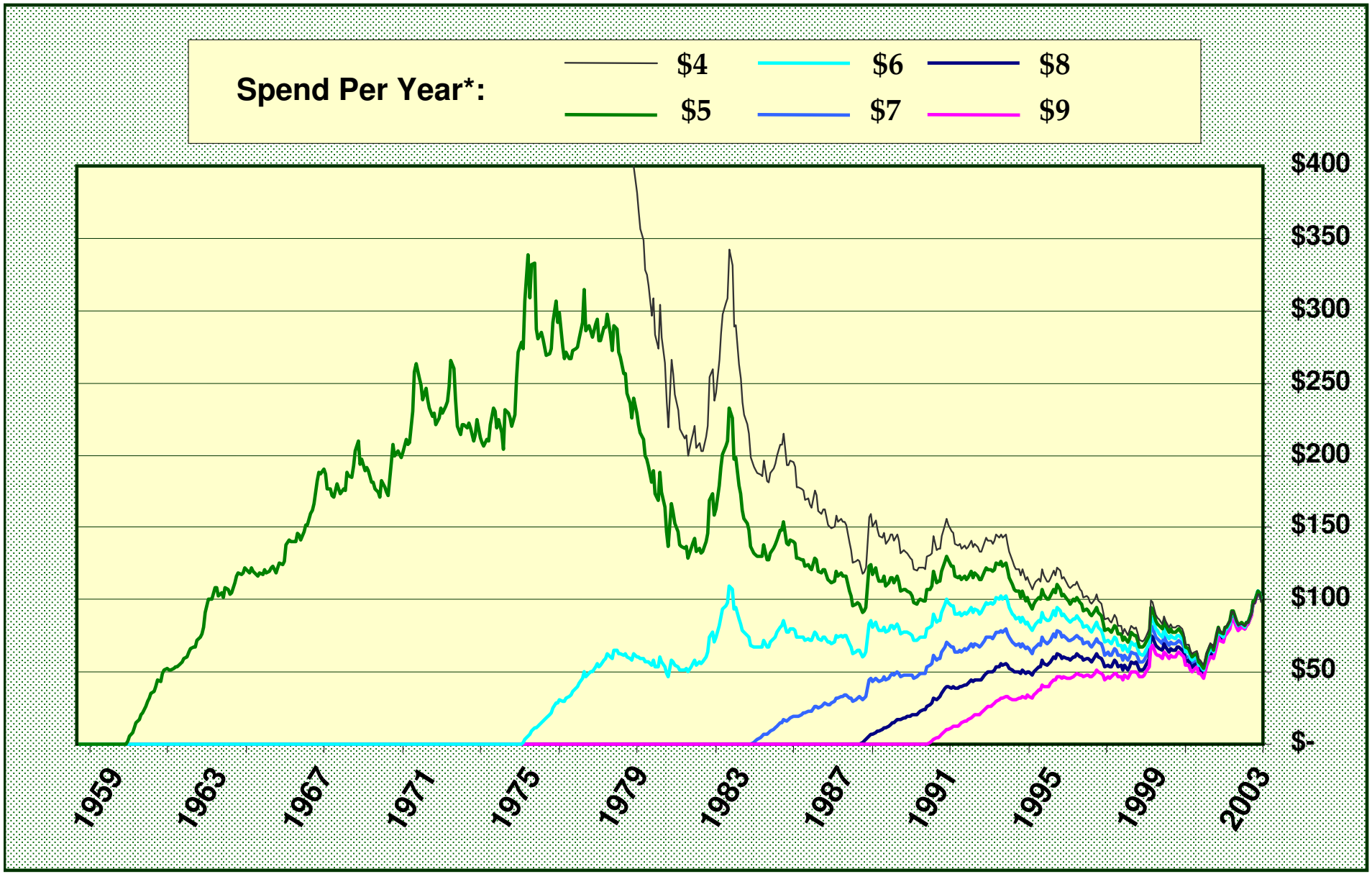
	1990		2001	
Age	**thousands**	**%**	**thousands**	**%**
60-64	231	13.7	416	15.1
65-69	567	33.5	748	27.2
70-74	407	24.1	635	23.1
75-79	281	16.6	479	17.4
80+	208	12.3	471	17.1

Source: Statistics Canada, Catalogue No. 74-507-XPE

The Annuity Life-Cycle



Wealth Path Under a Reversed Historical Returns Sequence



*Inflation Adjusted

A Canadian Investor Starts Withdrawing (Retires) at Age 65
What is the Probability of Retirement Ruin?

		Annual Inflation-adjusted Spending Rate, per $100 Initial Nest Egg						
% Equity	% Bonds	$4	$4.5	$5	$5.5	$6	$6.5	$7
		Probability of Ruin (%)						
100	**0**	**12.0**	**15.1**	**18.5**	**22.1**	**25.7**	**29.4**	**33.1**
80	**20**	**9.9**	**12.9**	**16.3**	**19.9**	**23.6**	**27.5**	**31.5**
60	**40**	**8.7**	**11.8**	**15.2**	**18.9**	**22.9**	**27.0**	**31.3**
50	**50**	**8.6**	**11.7**	**15.2**	**19.0**	**23.1**	**27.4**	**31.9**
40	**60**	**8.7**	**11.9**	**15.6**	**19.6**	**23.9**	**28.4**	**33.0**
20	**80**	**10.1**	**13.8**	**18.0**	**22.5**	**27.3**	**32.2**	**37.1**
0	**100**	**13.9**	**18.4**	**23.2**	**28.4**	**33.6**	**38.9**	**44.1**

Can Your Planned 30 Years of Retirement Income Survive Five Bad Years in the Market?

100% Invested in Equities

Spending Rate per $100*:	$4	$5	$6	$7
	Probability of Ruin (%)			
Investment Returns Earned as Planned:	**22.08**	**35.34**	**47.64**	**61.46**
1st Five Years Earned -5% Returns:	48.48	72.72	88.16	96.24
2nd Five Years Earned -5% Returns:	41.78	59.50	74.66	84.10
3rd Five Years Earned -5% Returns:	35.70	51.78	65.76	75.86
4th Five Years Earned -5% Returns:	30.96	45.74	57.78	69.60
5th Five Years Earned -5% Returns:	26.64	41.14	53.98	65.56
6th Five Years Earned -5% Returns:	24.22	37.36	49.76	62.74

*Inflation-Adjusted

Can Your Planned 30 Years of Retirement Income Survive Five Bad Years in the Market?

60% Invested in Equities

Spending Rate per $100*:	$4	$5	$6	$7
	Probability of Ruin (%)			
Investment Returns Earned as Planned:	**14.26**	**31.40**	**51.74**	**68.50**
1st Five Years Earned -5% Returns:	51.64	83.68	96.82	99.66
2nd Five Years Earned -5% Returns:	40.66	67.80	84.92	93.70
3rd Five Years Earned -5% Returns:	32.22	55.98	75.12	86.16
4th Five Years Earned -5% Returns:	24.90	46.34	66.08	80.18
5th Five Years Earned -5% Returns:	20.52	40.04	59.86	75.24
6th Five Years Earned -5% Returns:	15.42	33.44	54.46	70.42

*Inflation-Adjusted

Can Your Planned 30 Years of Retirement Income Survive Five Bad Years in the Market?

100% Invested in Bonds

Spending Rate per $100*:	$4	$5	$6	$7
	Probability of Ruin (%)			
Investment Returns Earned as Planned:	**21.68**	**56.66**	**84.84**	**95.88**
1st Five Years Earned -5% Returns:	83.38	99.24	100.00	100.00
2nd Five Years Earned -5% Returns:	66.56	93.90	99.20	99.96
3rd Five Years Earned -5% Returns:	52.58	86.76	97.44	99.60
4th Five Years Earned -5% Returns:	42.08	76.08	94.26	99.14
5th Five Years Earned -5% Returns:	32.86	68.86	90.84	97.68
6th Five Years Earned -5% Returns:	24.12	60.72	85.26	96.56

*Inflation-Adjusted

• GMWBs (esp. GMWBs-for-life) are intended to manage some of this risk. They step in where DB pension plans used to.

• Very popular in US, Japan, UK.

• First is currently being marketed in Canada - Manulife's *IncomePlus*

Static case - Price as a "Quanto Asian Put" plus annuity:

$$Y_t = \exp\{-(r - \alpha - 0.5\sigma^2)t - \sigma B_t\}$$

$$\mathsf{A} := \frac{1}{T}\int_0^T Y_t dt, \quad \mathsf{Y} := Y_T$$

$$\text{Option Payoff} = w_0 \frac{\max[1 - \mathsf{A}, 0]}{\mathsf{Y}}$$

What would Wall Street charge for a plain vanilla GMWB?

Withdrawal Rate	Cost in Basis Points
4%	23 – 60
5%	37 – 90
6%	54 – 123
7%	73 – 158
8%	94 –194
9%	117 – 232

***Assumption**: Stochastic pricing model with 5% discount rate and 20% to 30% volatility*

plus ***ZERO OPTIMAL LAPSATION….***

IME paper, 2006

- Simple case of the 1st paper: no passport, no ratchets, no GMDB.

$$dW(t) = (r-\alpha)W(t)\,dt + \sigma W(t)\,dB(t) - \gamma(t)\,dt$$

(under the risk neutral measure)

The guaranteed amount evolves as $dA(t) = -\gamma(t)\,dt$ (as long as $\gamma(t) < g$).

- With this model it would be optimal to sell and repurchase as soon as $W(t)>W(0)$, reestablishing the guarantee at a higher level. So we include a DSC at (flat) rate κ: In other words, as long as $W(t)>A(t)$, the account holder receives $f(\gamma(t))\,dt$

  where $f(\gamma(t)) = \gamma(t) - \kappa(\gamma(t)-g)_+$

- The seller hedges, and we assume that both the insurance charge and DSC are retained in the hedge (ie not used as trailers etc).
- We also assume that the seller is to be hedged against all choices of withdrawals/lapsation (worst case scenario). In that case this becomes an American-style option with extremal behaviour consisting of withdrawing gdt unless $W(t)$ exceeds some threshold, at which time the buyer lapses, pays the DSC and resets the guarantee.

- The hedge value $V(t)=v(W(t), A(t))$ satisfies

$$dV(t) = rV(t)dt + \mathrm{d}M(t) - f(\gamma(t))dt$$

$$V(t) \geq (1- \kappa)W(t)$$

- The resulting pde for v is parabolic (the a variable plays the role of time), and can be solved numerically by standard techniques.
- If $V(0) > W(0)$ then the initial capital is not sufficient to hedge completely, while if

  $V(0) < W(0)$ then there is a clear profit to the seller. If equality holds, the hedge is perfect: for given κ one can solve for the critical α making this so.

- In fact, being under-hedged is typical; the embedded options are sold below cost.

  Eg. g=7%, r=5%, σ=20%, κ=1% gives a critical α of about 160 b.p. whereas typically α is 30 - 50 b.p.
- How can this be sustained? The extremal $\gamma(t)$ is worse-case; if one assumes other behaviour for withdrawals/lapsation then a lower insurance charge will suffice.

Capital Market (Model) M&E Fee for a 7% GMWB

Investment Volatility:	Static* (Actuarial)	Dynamic (Financial)
$\sigma = 20\%$	73 b.p.	160 b.p.
$\sigma = 25\%$	113 b.p.	320 b.p.
$\sigma = 30\%$	158 b.p.	565 b.p.

Assuming 5% (risk free) pricing rate. Note that under a static model 28.53% of the initial premium deposit is used to purchase Q.A.P. and remaining 71.47% is used to purchase a 14.28 year term-certain annuity.

Details ….

$$dV_t = rV_t\,dt + dM_t - f(\gamma_t)\,dt$$

$$\begin{aligned} dv(A_t, X_t) &= v_a\,dA_t + v_x\,dX_t + \frac{1}{2}v_{xx}\,d\langle X\rangle_t \\ &= -v_a\gamma_t\,dt + (r-\alpha)X_t v_x\,dt + v_x\sigma X_t\,d\tilde{B}_t - v_x\gamma_t\,dt + \frac{\sigma^2 X_t^2}{2}v_{xx}\,dt. \end{aligned}$$

Equating gives

$$\left[(r-\alpha)X_t v_x + \frac{\sigma^2 X_t^2}{2}v_{xx} - rv\right]dt + \left[f(\gamma_t) - \gamma_t v_x - \gamma_t v_a\right]dt = dM_t - v_x\sigma X_t\,d\tilde{B}_t$$

where the RHS is a submartingale, and a martingale under the optimal choice of γ.

$$\left[(r-\alpha)xv_x + \frac{1}{2}\sigma^2x^2v_{xx} - rv\right] + \left[f(\gamma) - \gamma v_x - \gamma v_a\right] \leq 0$$

for every γ, with equality for some γ.

Because f is piecewise-linear, there are three critical cases, namely $\gamma=0$, $\gamma=g$, *and* $\gamma=\infty$. We arrive at the free boundary value problem

$$\left.\begin{aligned} (r-\alpha)xv_x + \frac{\sigma^2x^2}{2}v_{xx} - rv &\leq 0 \\ (r-\alpha)xv_x + \frac{\sigma^2x^2}{2}v_{xx} - rv + g[1 - v_x - v_a] &\leq 0 \\ (1-\kappa) - v_x - v_a &\leq 0 \end{aligned}\right\}$$

with equality in at least one case.

(*Hamilton-Jacobi-Bellman* equations)

- On the other hand Moody's special comment (October 2005) warns against this, quoting our numbers and rationale. Undercharging leaves sellers open to arbitrage, especially as a secondary market emerges for VA policies:

"most companies realize that policyholder inefficiency is a key driver of pricing."

"Increased efficiency in the markets only a matter of time: relying on policyholder inefficiencies is a losing long-term proposition."

"It is essential for insurance companies to monitor and guard against the arbitraging of actuarial assumptions."

www.NationalUnderwriter.com

LIFE & HEALTH

National Underwriter

THE WEEK IN LIFE & HEALTH

MARCH 29, 2004
VOL. 108, NO. 12
$5.00

Coventry Now Buying Unwanted VA Contracts

BY ALLISON BELL

COVENTRY FIRST L.L.C., Fort Washington, Pa., has started buying unwanted variable annuities.

The program appears to be the first well-publicized, systematic effort to buy contracts that are still in the accumulation phase.

"We've thought about doing this for several years," says Coventry CEO Alan Buerger. "We have registered reps and insurance agents asking about this on a regular basis."

Coventry, a life settlements company, prefers contracts in which investment performance has been so poor that the value of the death benefits now exceeds the value of the tax benefits.

Selling a VA contract to another party might be a better deal than letting the contract lapse, Coventry says, because the market value of the death benefits is often higher than the surrender value.

U.S. life settlement companies now buy about $5 billion in life policy face value each year, and VA purchase volume could amount to about 10% of life settlement volume, Buerger estimates.

VA resale prices will depend on the carrier, the contract terms and the seller's age, but typical holders might get the current value of their VA principal along with a price equal to about 13% of the death benefit value, Buerger says.

Further work or work in progress:

- *Ratchets* - Can finesse lapsation by building in ratchets. In continuous version, withdrawals are based on X^*= max observed value of X. Can then compute hedging cost (still expensive)
- *GMWBs for life* - the guaranteed withdrawals persist for the lifetime of the policyholder. One can approach this either with mortality fully diversified, or with some mortality risk retained and a partial hedge.

- The full product is more complex. But assuming constant w.d. at rate g (static case), it is amenable to Monte Carlo. In our eg, the critical dynamic α *is* 2-3 times that of the static one.
- *Optimal policyholder behaviour*: It isn't a 2-way market so utility maximizing behaviour may not be extremal for the seller. One can ask when a GMWB adds utility over the mutual fund, and for what α, κ it does so.
- An interesting question is the *asset allocation* within a *portfolio* protected by a GMWB-for-life. With only utility of consumption the simple answer is leverage. In practice - not allowed, but do people start to invest more agressively?

- Yes: LIMRA study - average allocation to large/midcap changes from 45% to 62%.
- Our analysis is that utility of consumption alone should predict a much bigger shift. Currently trying to model a combination of consumption and liquidity that reflects more realistic behaviour.

Other Literature

- No Arbitrage theory to value insurance (or pension) guarantees.
  - Brennan and Schwartz (1976)
  - Boyle and Schwartz (1977)
  - Pennachi (1999)
  - Mitchell, Lachance and Smetters (2003)
- Asian option pricing.
  - Milevsky and Posner (1998)
- Variable Annuity (VA) market.
  - Milevsky and Posner (2001)
  - Brown and Poterba (2004)
  - Boyle et al (2006) - lapsation modeling
- Actuarial work on Investment Guarantees
  - Hardy (2003), monograph