

Pricing of Credit Derivatives

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Pricing of Credit Derivatives

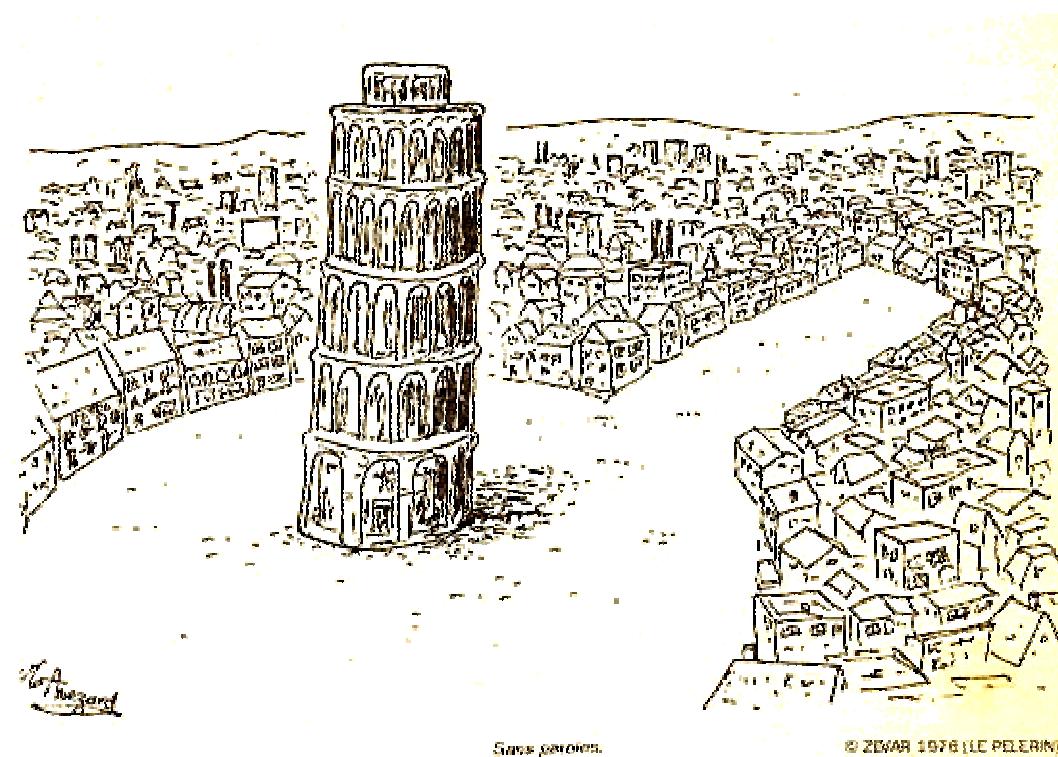
Alternative Investments



„You know, money is not enough to make you happy.
You should also have some stocks, gold and real estate.“
Danny Kaye, American actor

Pricing of Credit Derivatives

Rare but Severe Events



Pricing of Credit Derivatives

Overview

▪ Market Information

- Yield Curve Behaviour
US Treasury Strips
- Credit Spread Behaviour
US Industrials AA, A2, BBB1
- Economic Behaviour
US Gross Domestic Product (GDP)
- Credit Default Swaps
US Industrials AA, A2, BBB1



- The Generalized Model of SZ
- Model Performance
- Pricing of Credit Derivatives

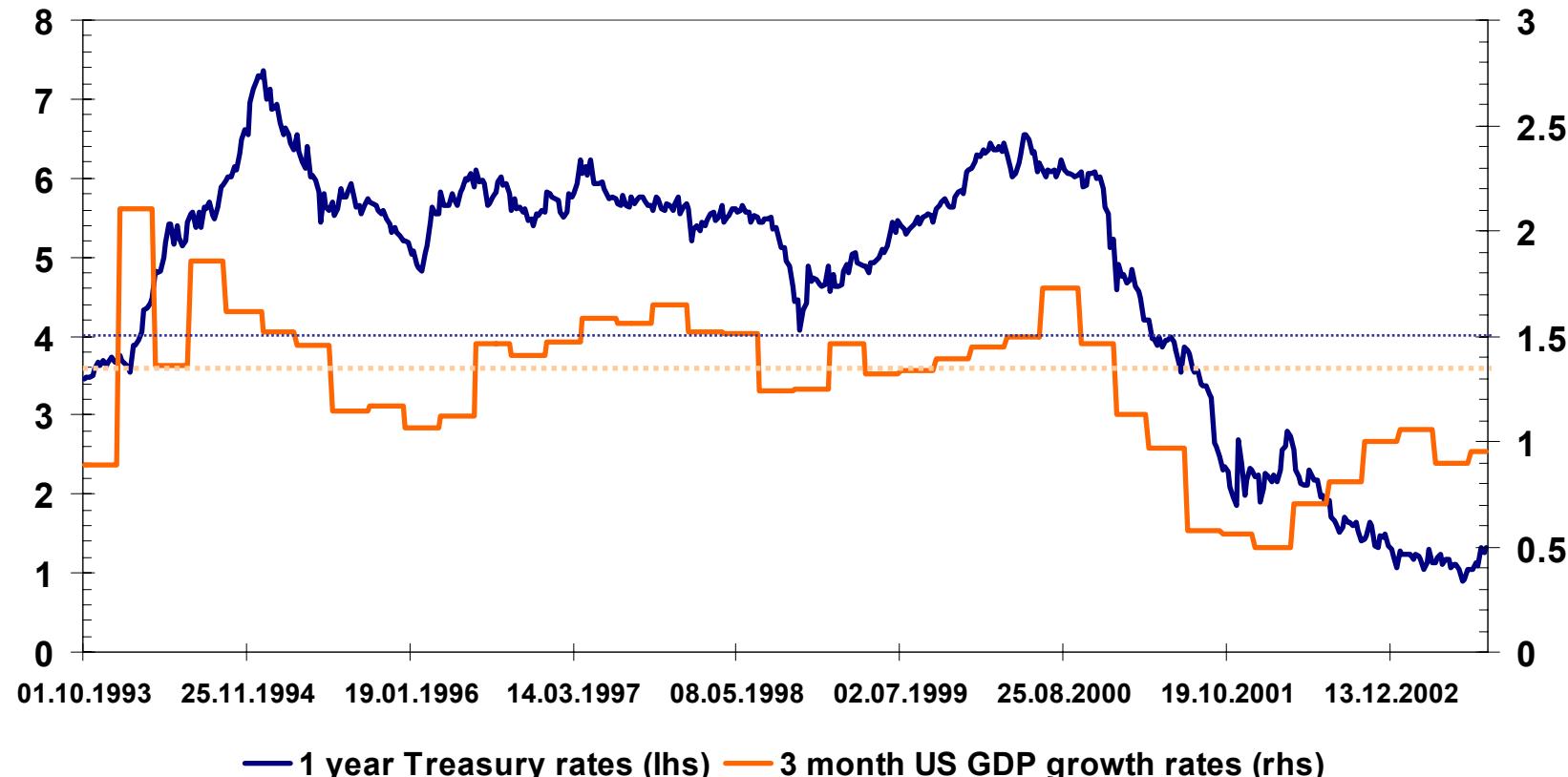
Treasury, Spread, and CDS Data

US Market 1993 - 2004

- American Treasury Strips
- AA, A2, and BBB1 rated American Industrials
- Maturities between 3 months and 30 years
- Time series of weekly bond prices from October 8, 1993 to December 31, 2004
- 3-month US GDP growth rates from 4th quarter 1993 to 4th quarter 2004
- Monthly prices on 5-year CDS for Kimberly Clark (AA), Caterpillar (A2), and Masco Corporation (BBB1) from December 31, 2003 to December 31, 2004
- All prices in US Dollar, i.e. no currency risk in credit spreads
- Parameter estimation using Kalman filters

US Treasury Strips and Industrials vs. US Gross Domestic Product

Time Period: 1993 – 2004

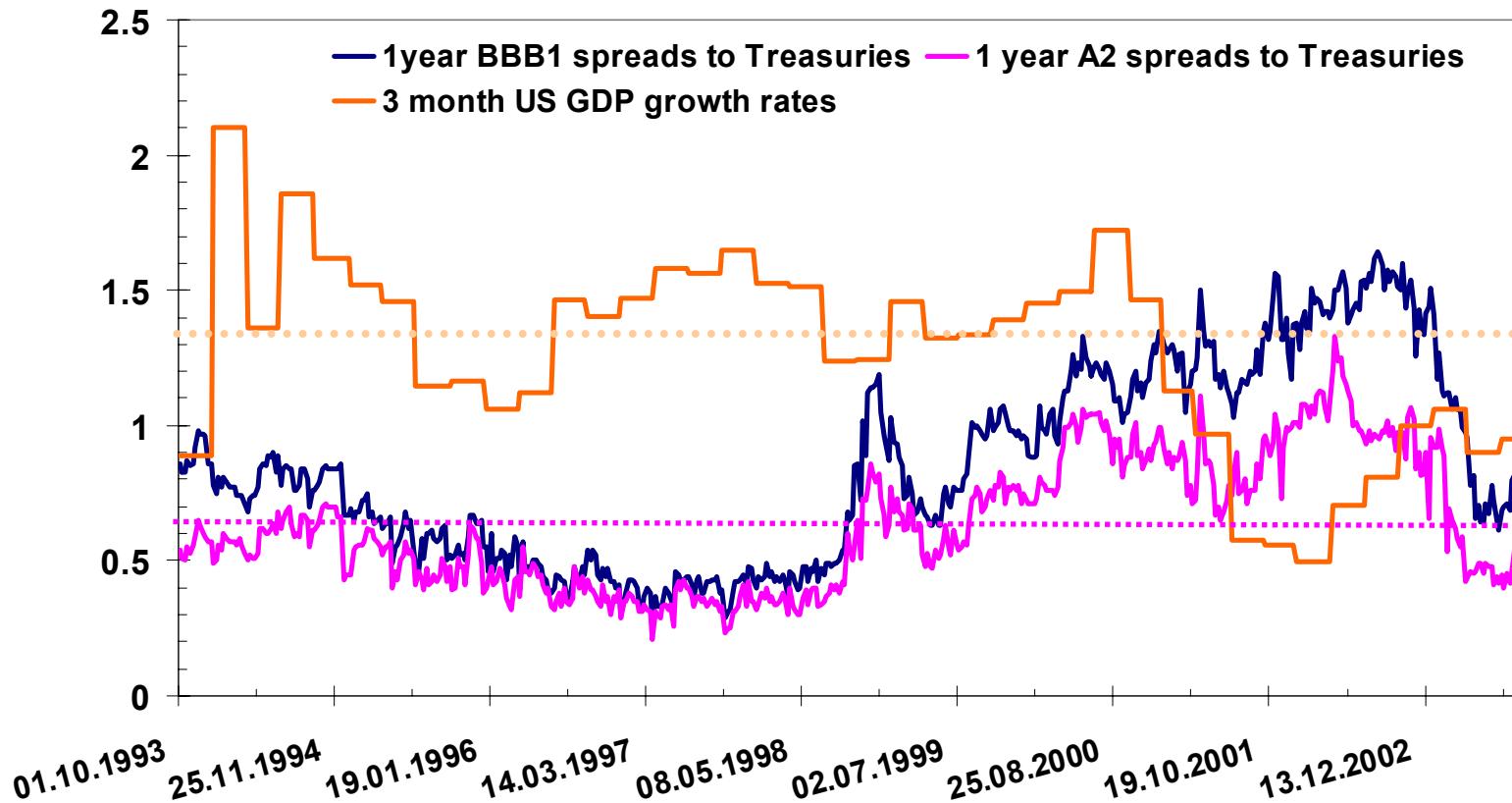


Source: Bloomberg

Linear Regression: $R(t+0.25, t+1.25) = -0.62 + 0.84 \cdot R(t, t+1) + 1.03 \cdot w(t) + \varepsilon$
 Correlation (1993-2001): $\rho(R(t, t+1), w(t)) = 0.08$

US Credit Spreads vs. US Gross Domestic Product

Time Period: 1993 – 2004



Linear Regression:

$$S^{A2}(t+0.25, t+1.25) = 0.25 + 0.84 \cdot S^{A2}(t, t+1) - 0.10 \cdot w(t) + \varepsilon$$

$$S^{BBB1}(t+0.25, t+1.25) = 0.24 + 0.77 \cdot S^{BBB1}(t, t+1) - 0.08 \cdot w(t) + \varepsilon$$

Source: Bloomberg

Correlation (1993-2001):

$$\rho(S^{A2}(t, t+1), w(t)) = -0.11, \rho(S^{BBB1}(t, t+1), w(t)) = -0.15$$

$$\rho(w(t), \text{default probability BBB}) = -0.49$$

Pricing of Credit Derivatives

Overview

- **Market Information**
- **The Generalized Model of Schmid and Zagst**

- **Reduced-Form Models:**

Duffee [1999]: „Estimating the Price of Default“, Review of Financial Studies 12, 197-226

- **Structural Models:**

Bakshi, Madan, Zhang [2006]: „Investigating the Role of Systematic and Firm-Specific Factors in Default Risk: Lessons from Empirically Evaluating Credit Risk “, J. of Business 79, 1955-1987

- **Hybrid Models:**

a) Schmid and Zagst [2000]:

„A Three-Factor Defaultable Term Structure Model“, Journal of Fixed Income 10, No. 2, 63-79

b) Zagst and Roth [2004]: „Three-Factor Defaultable Term Structure Models“, International Journal of Pure and Applied Mathematics 17, No.2, 249-285

c) Schmid, Zagst, Antes, Ilg [2006]:

„Empirical Evaluation of Hybrid Defaultable Bond Pricing Models“, Working Paper

- **Model Performance**

- **Pricing of Credit Derivatives**

The Generalized Model of Schmid and Zagst [2005]

Modelling of the Stochastic Processes Under the Martingale Measure Q

- **Dynamics of the yield curve (non-defaultable short rate)**

$$dr(t) = [\theta_r(t) + b_{r\omega} \cdot \omega(t) - a_r \cdot r(t)] dt + \sigma_r dW_r(t), \quad t \in [0, T^*], \quad a_r > 0, \quad \sigma_r > 0$$

- **Dynamics of the economy index**

$$d\omega(t) = [\theta_\omega - a_\omega \cdot \omega(t)] dt + \sigma_\omega dW_\omega(t), \quad t \in [0, T^*], \quad \theta_\omega \geq 0, \quad a_\omega > 0, \quad \sigma_\omega > 0$$

- **Dynamics of the uncertainty index**

$$du(t) = [\theta_u - a_u \cdot u(t)] dt + \sigma_u dW_u(t), \quad t \in [0, T^*], \quad \theta_u \geq 0, \quad a_u > 0, \quad \sigma_u > 0$$

- **Dynamics of the yield spread (short-rate credit spread)**

$$ds(t) = [\theta_s + b_{su} \cdot u(t) - b_{s\omega} \cdot \omega(t) - a_s \cdot s(t)] dt + \sigma_s dW_s(t), \quad t \in [0, T^*], \quad \begin{matrix} \theta_s \geq 0, \\ b_{s\omega} > 0, \\ a_s > 0, \\ \sigma_s > 0 \end{matrix}$$

- **The Wiener processes W_r , W_ω , W_u , and W_s are uncorrelated**

The Generalized Model of Schmid and Zagst [2005]

Pricing of Non-Defaultable Bonds

Theorem 1.

The price $P(t, T) = P(r, \omega, t, T)$ of a non-defaultable zero-coupon bond with maturity T at time $t < T$ is given by

$$P(t, T) = e^{A(t, T) - B(t, T)r - E(t, T)\omega}$$

with

$$B(t, T) = \frac{1}{a_r} \cdot \left(1 - e^{-a_r \cdot (T-t)}\right), \quad E(t, T) = \frac{b_{r\omega}}{a_r} \cdot \left(\frac{1 - e^{-a_\omega \cdot (T-t)}}{a_\omega} + \frac{e^{-a_\omega \cdot (T-t)} - e^{-a_r \cdot (T-t)}}{a_\omega - a_r}\right),$$

and

$$A(t, T) = \int_t^T \frac{\sigma_r^2}{2} \cdot B^2(\tau, T) + \frac{\sigma_\omega^2}{2} \cdot E^2(\tau, T) - \theta_r(\tau) \cdot B(\tau, T) - \theta_\omega \cdot E(\tau, T) \, d\tau.$$

The Generalized Model of Schmid and Zagst [2005]

Pricing of Defaultable Bonds

Theorem 2.

The price $P^d(t, T) = P^d(r, s, u, \omega, t, T)$ of a defaultable zero-coupon bond with maturity T at time $t < \min\{T, T^d\}$ is given by

$$\begin{aligned} P^d(t, T) &= e^{A^d(t, T) - B(t, T) \cdot r - C(t, T) \cdot s - D(t, T) \cdot u - E^d(t, T) \cdot \omega} \\ &= P(t, T) \cdot e^{A^*(t, T) - C(t, T) \cdot s - D(t, T) \cdot u + E^*(t, T) \cdot \omega} \end{aligned}$$

systematic
 firm specific
short-term
 firm specific
long-term
 systematic

where

$$C(t, T) = \frac{1}{a_s} \cdot \left(1 - e^{-a_s \cdot (T-t)}\right), \quad D(t, T) = \frac{b_{su}}{a_s} \cdot \left(\frac{1 - e^{-a_u \cdot (T-t)}}{a_u} + \frac{e^{-a_u \cdot (T-t)} - e^{-a_s \cdot (T-t)}}{a_u - a_s}\right),$$

$$E^d(t, T) = E(t, T) - E^*(t, T) \quad \text{with} \quad E^*(t, T) = \frac{b_{s\omega}}{a_s} \cdot \left(\frac{1 - e^{-a_\omega \cdot (T-t)}}{a_\omega} + \frac{e^{-a_\omega \cdot (T-t)} - e^{-a_s \cdot (T-t)}}{a_\omega - a_s}\right),$$

$$A^d(t, T) = A(t, T) + A^*(t, T) \quad \text{with}$$

$$\begin{aligned} A^*(t, T) &= \int_t^T \frac{\sigma_s^2}{2} \cdot C^2(\tau, T) + \frac{\sigma_u^2}{2} \cdot D^2(\tau, T) + \frac{\sigma_\omega^2}{2} \cdot E^{*2}(\tau, T) - \sigma_\omega^2 \cdot E(\tau, T) \cdot E^*(\tau, T) d\tau \\ &\quad - \int_t^T \theta_s \cdot C(\tau, T) + \theta_u \cdot D(\tau, T) - \theta_\omega \cdot E^*(\tau, T) d\tau \end{aligned}$$

The Generalized Model of Schmid and Zagst [2005]

Affine Term Structure of Interest Rates

- **Treasury Rates**

The time t value $R(t,T) = R(r, \omega, t, T)$ of the Treasury Rate with maturity $T > t$ is given by

$$R(t,T) = -\frac{\ln P(t,T)}{T-t} = -\frac{A(t,T)}{T-t} + \frac{B(t,T)}{T-t} \cdot r + \frac{E(t,T)}{T-t} \cdot \omega$$

- **Credit Spreads**

The time t value $R_s(t,T) = R_s(r, s, u, \omega, t, T)$ of the credit spread with maturity $T > t$ is given by

$$\begin{aligned} R_s(t,T) &= -\left(\frac{\ln P^d(t,T)}{T-t} - \frac{\ln P(t,T)}{T-t} \right) =: R^d(t,T) - R(t,T) \\ &= -\frac{A^*(t,T)}{T-t} + \frac{C(t,T)}{T-t} \cdot s + \frac{D(t,T)}{T-t} \cdot u - \frac{E^*(t,T)}{T-t} \cdot \omega \end{aligned}$$

The Generalized Model of Schmid and Zagst [2005]

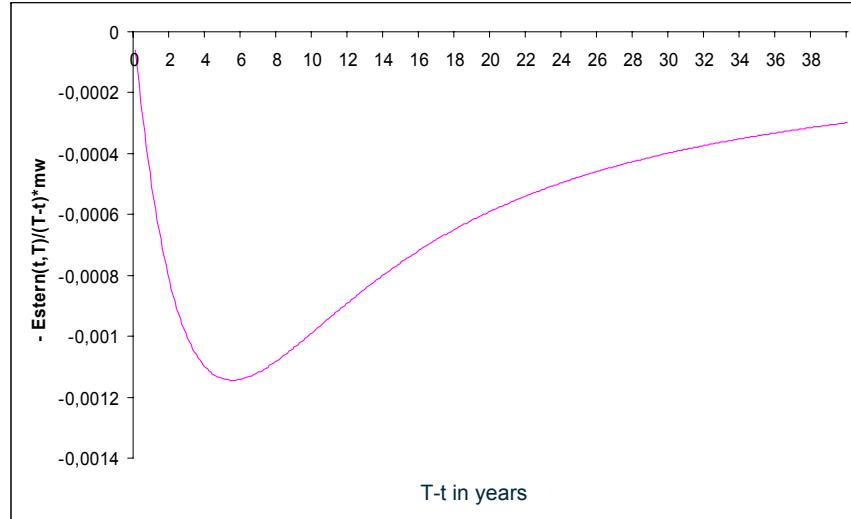
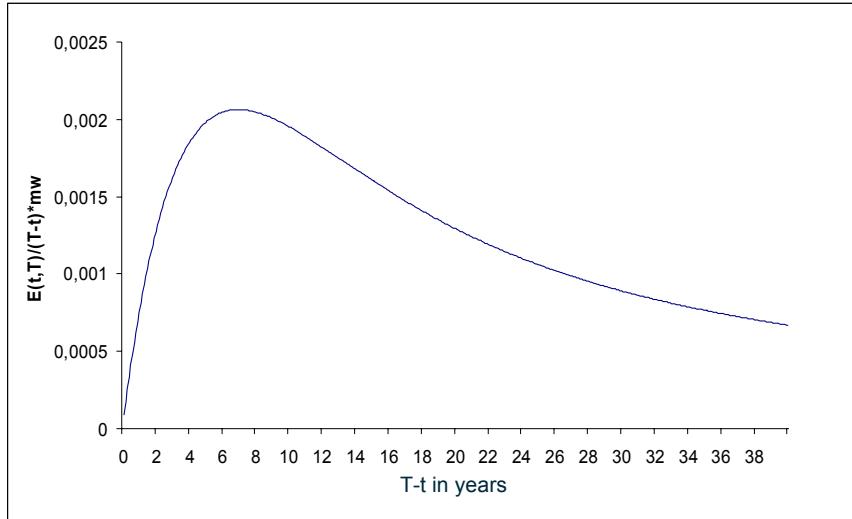
Parameter Estimation

Parameter	Estimation Treasury
a_r ($a_{r,\text{real}}$)	0.24782 (0.37867)
b_{rw}	0.13315
σ_r (%)	1.49553
real-world mean reversion level for r (%)	4.01 (emp. 3.96)
θ_w (%)	1.58331
a_w ($a_{w,\text{real}}$)	0.26847 (1.18532)
σ_w (%)	0.60146
real-world mean reversion level for w (%)	1.34 (emp. 1.28)

Parameter	Estimation AA	Estimation A2	Estimation BBB1
a_s ($a_{s,\text{real}}$)	0.59696 (2.96099)	0.46618 (2.80727)	0.39133 (2.41739)
θ_s (%)	0.27474	0.25139	0.23778
σ_s (%)	0.38855	0.30875	0.28679
b_{sw}	0.07373	0.07634	0.09369
real-world mean reversion level for s (%)	0.42 (emp. 0.43)	0.65 (emp. 0.62)	0.76 (emp. 0.76)
θ_u (%)	0.12173	0.18566	0.18980
a_u ($a_{u,\text{real}}$)	0.06878 (0.11269)	0.06341 (0.11057)	0.06670 (0.110480)
σ_u (%)	0.45920	0.47604	0.48862
real-world mean reversion level for u (%)	1.08020	1.67916	1.71793

The Generalized Model of Schmid and Zagst [2005]

Sensitivity of Treasury Strips and Credit Spreads with Respect to ω

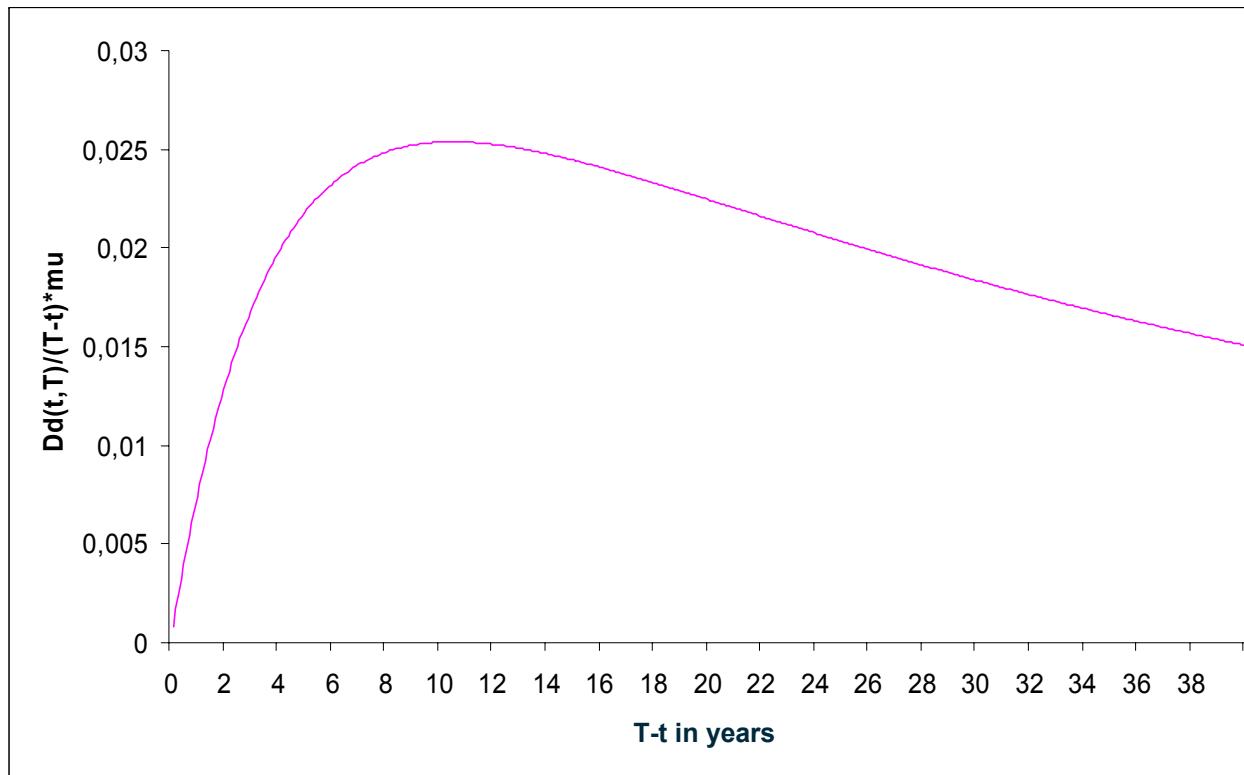


Correlation	R(t,t+1)	R(t,t+10)	R(t,t+30)
GDP growth rate w(t)	0.08	0.28	0.26

Correlation	S(t,t+1)	S(t,t+10)	S(t,t+30)
GDP growth rate w(t)	- 0.15	- 0.32	- 0.25

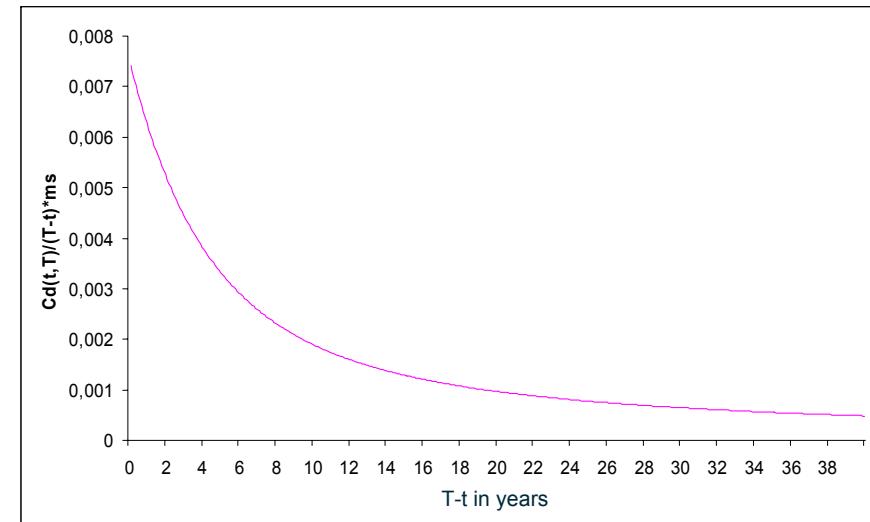
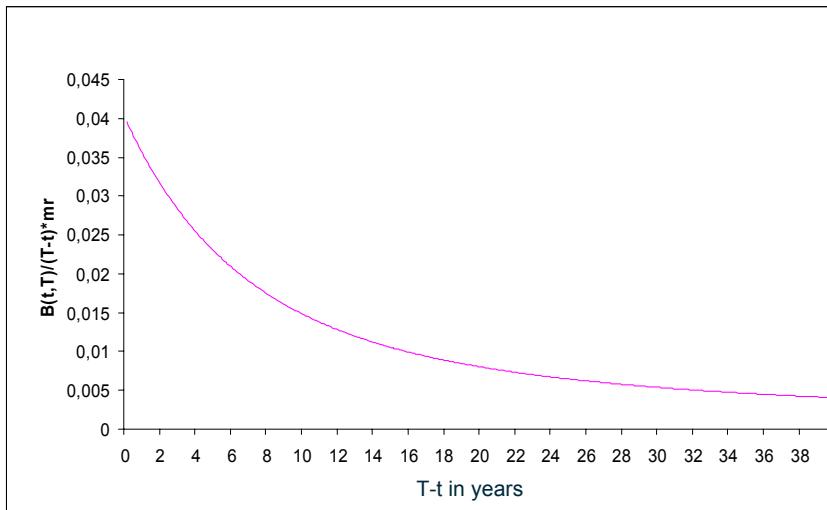
The Generalized Model of Schmid and Zagst [2005]

Sensitivity of Credit Spreads with Respect to u



The Generalized Model of Schmid and Zagst [2005]

Sensitivity of Credit Spreads with Respect to r and s



Pricing of Credit Derivatives

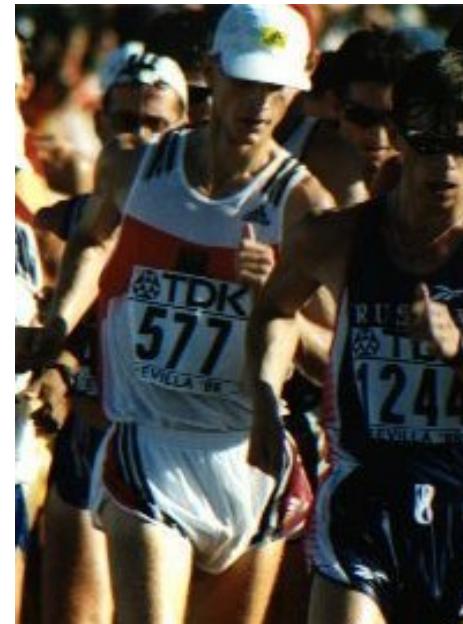
Overview

- Market Information
- The Generalized Model of SZ

▪ Model Performance

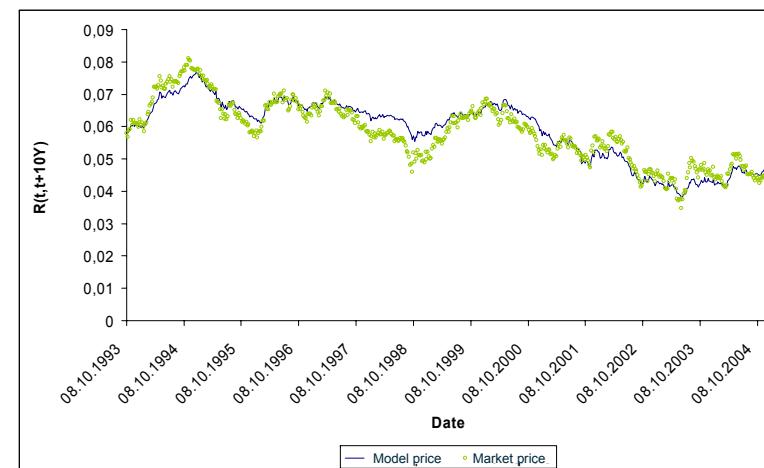
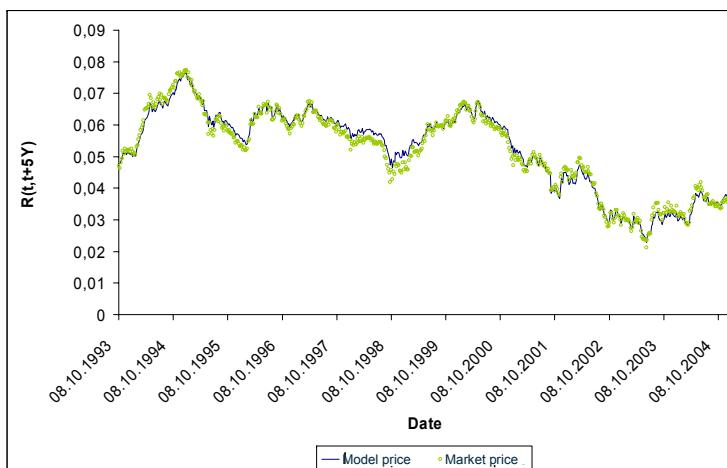
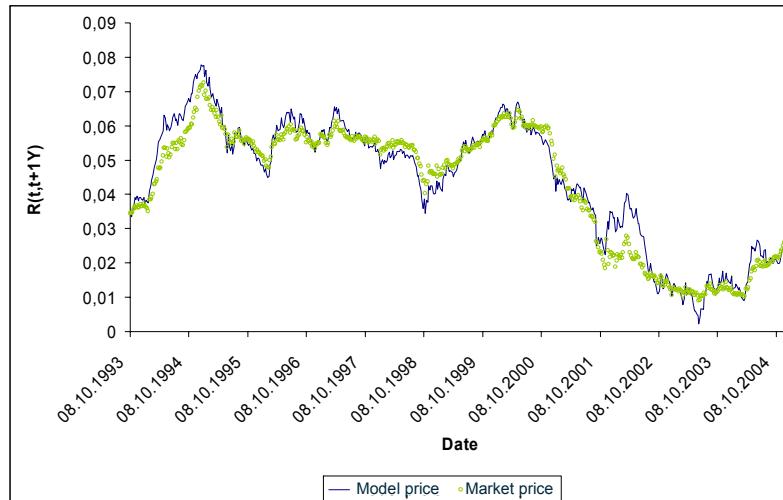
- Comparison of Model and Market Prices
- Default Probabilities

▪ Pricing of Credit Derivatives



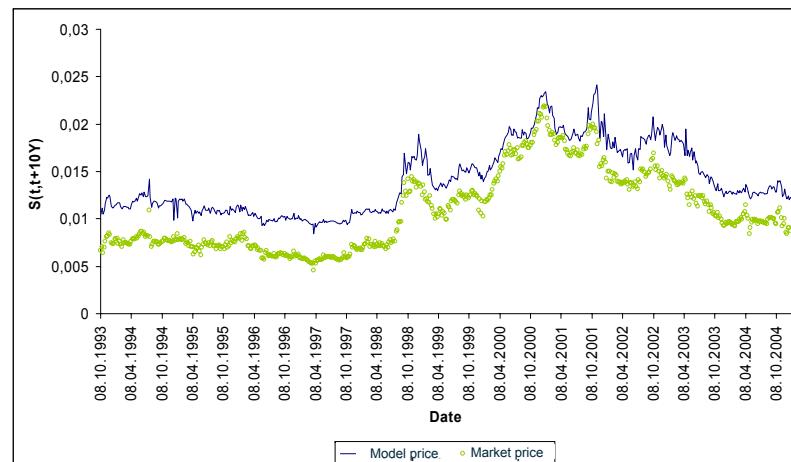
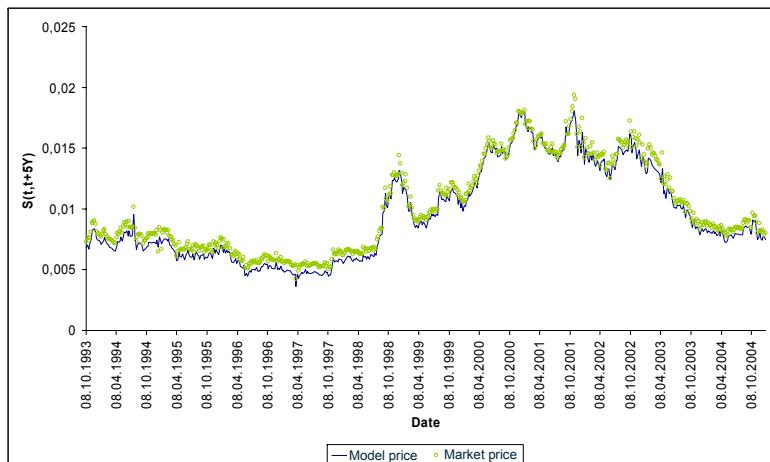
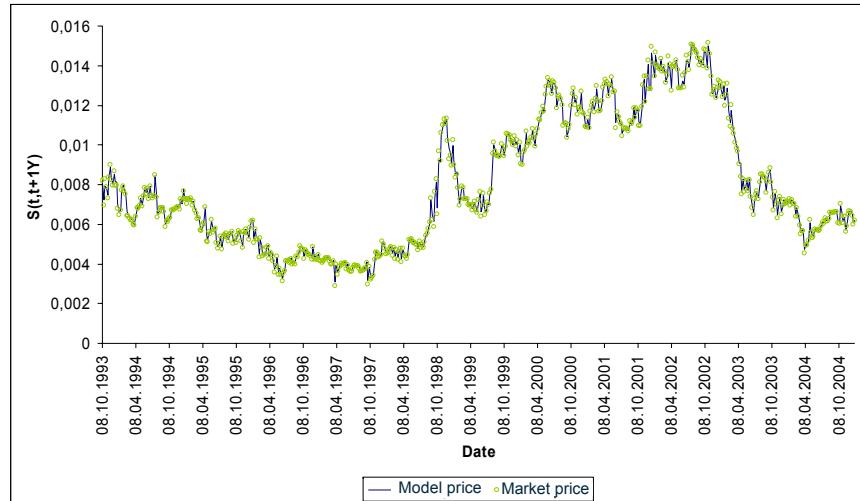
Model Performance

Comparison of Model and Market Treasury Yields



Model Performance

Comparison of Model and Market Credit Spreads (BBB1)



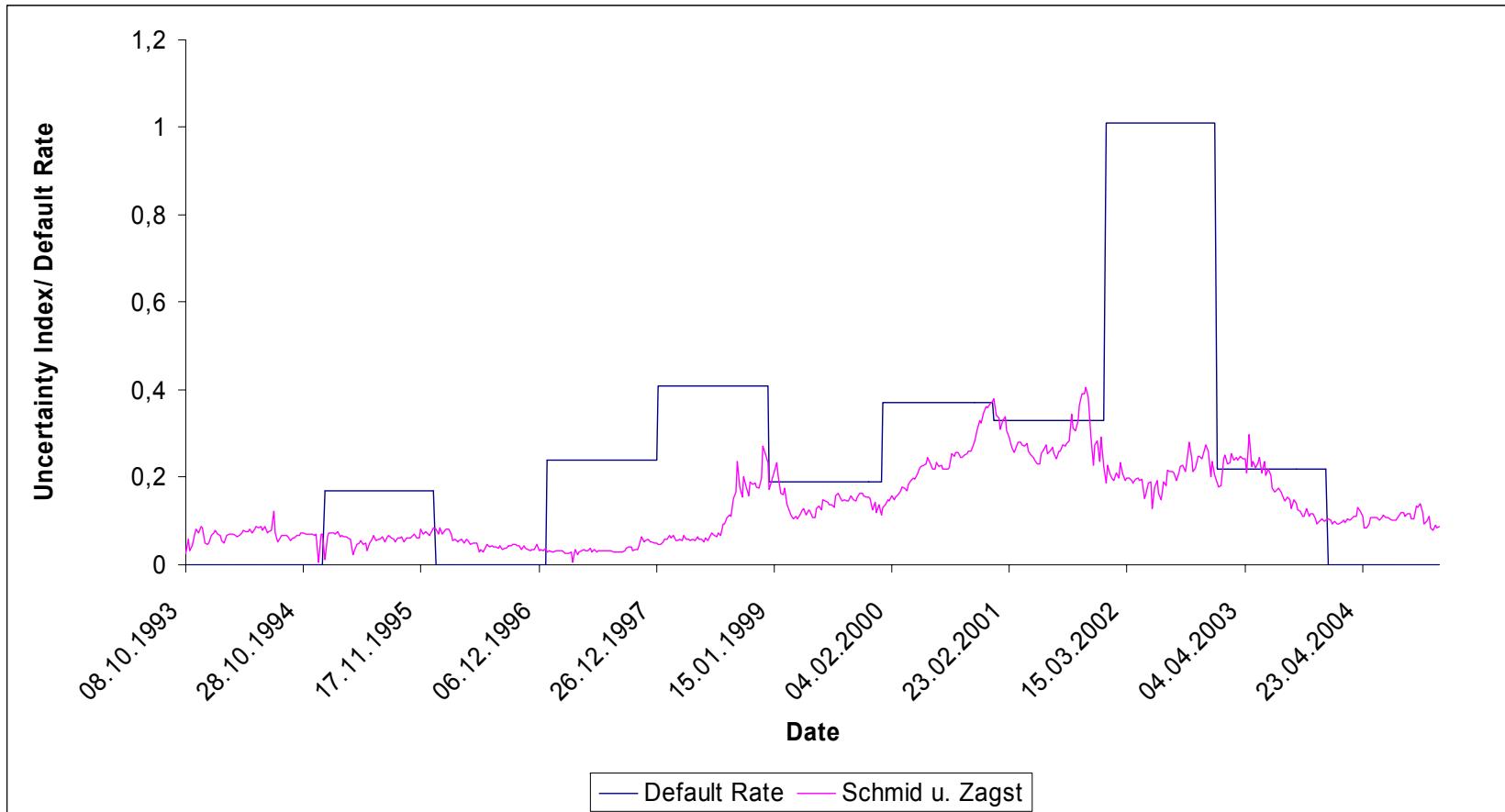
Model Performance

Uncertainty Index vs. Default Probability for Rating A2



Model Performance

Uncertainty Index vs. Default Probability for Rating BBB1



Pricing of Credit Derivatives

Overview

- Market Information
- The Generalized Model of SZ
- Model Performance
- Pricing of Credit Derivatives
 - Yield Spreads under Zero Recovery
 - Defaultable Bonds with Zero Recovery
 - Default Digital Options
 - Default Options
 - Default Swaps



Pricing Credit Derivatives

Yield Spreads Under Zero Recovery

- **Short-rate credit spread under zero recovery**

$$s(t) = (1 - z) \cdot s^{\text{zero}}(t), t \in [0, T^*]$$

with s denoting the previously modeled short-rate credit spread corresponding to a partial recovery of market value with constant recovery rate z .

- **Link to intensity-based approach**

$$s^{\text{zero}}(t) = \lim_{\varepsilon \rightarrow 0, \varepsilon > 0} \frac{Q(t < T^d \leq t + \varepsilon)}{\varepsilon}, t \in [0, T^*]$$

with T^d denoting the exogenous stopping time for the event of default in case of positive spreads. In our setup, s^{zero} is supposed to be a good approximation to the default intensity.

Pricing Credit Derivatives

Pricing of Defaultable Bonds with Zero Recovery

Theorem 3.

Let $E_Q[|Y|^q] < \infty$ for some $q > 1$ and $t \in (0, T]$. Then, we have

$$E_Q \left[e^{-\int_t^T r(l) dl} Y \cdot 1_{\{T^d > T\}} \mid \mathcal{I}_t \right] = 1_{\{T^d > t\}} \cdot E_Q \left[e^{-\int_t^T [r(l) + s^{zero}(l)] dl} Y \mid \mathcal{I}_t \right].$$

- A **defaultable bond with zero recovery** pays 1 if there has been no default up to maturity time T and 0 else. The price at time $t \in (0, T]$ is therefore given by

$$E_Q \left[e^{-\int_t^T r(l) dl} 1_{\{T^d > T\}} \mid \mathcal{I}_t \right] = 1_{\{T^d > t\}} \cdot \underbrace{E_Q \left[e^{-\int_t^T [r(l) + s^{zero}(l)] dl} \mid \mathcal{I}_t \right]}_{=: P^{d,zero}(t, T)}.$$

Pricing Credit Derivatives

Pricing of Defaultable Bonds with Zero-Recovery

Theorem 4.

The price $P^{d,\text{zero}}(t, T) = P^{d,\text{zero}}(r, s^{\text{zero}}, u, \omega, t, T)$ of a defaultable zero-coupon bond with zero recovery and maturity T at time $t < \min\{T, T^d\}$ is given by

$$P^{d,\text{zero}}(t, T) = e^{A^{d,\text{zero}}(t, T) - B(t, T) \cdot r - C^{\text{zero}}(t, T) \cdot s^{\text{zero}} - D^{\text{zero}}(t, T) \cdot u - E^{d,\text{zero}}(t, T) \cdot \omega}$$

where $C^{\text{zero}}(t, T)$, $D^{\text{zero}}(t, T)$, $E^{d,\text{zero}}(t, T)$, and $A^{d,\text{zero}}(t, T)$ are given by the corresponding formulas for $C(t, T)$, $D(t, T)$, $E^d(t, T)$, and $A^d(t, T)$ with θ_s , b_{su} , $b_{s\omega}$, and σ_s substituted by

$$\theta_s^{\text{zero}} = \frac{\theta_s}{1-z}, \quad b_{su}^{\text{zero}} = \frac{b_{su}}{1-z}, \quad b_{s\omega}^{\text{zero}} = \frac{b_{s\omega}}{1-z}, \quad \text{and} \quad \sigma_s^{\text{zero}} = \frac{\sigma_s}{1-z}.$$

Pricing Credit Derivatives

Pricing of Default Digital Options

- A **default digital option** pays 1 if the underlying defaults before or at maturity time T and 0 else.

Lemma 5.

The price at time $t \in (0, T]$ of a default digital option which pays 1 **at maturity** is given by

$$\begin{aligned} V_T^{\text{ddp}}(t) &= E_Q \left[e^{-\int_t^T r(l) dl} 1_{\{T^d \leq T\}} \mid \mathcal{I}_t \right] = E_Q \left[e^{-\int_t^T r(l) dl} \left(1 - 1_{\{T^d > T\}} \right) \mid \mathcal{I}_t \right] \\ &= P(t, T) - 1_{\{T^d > t\}} \cdot P^{\text{d,zero}}(t, T). \end{aligned}$$

Pricing Credit Derivatives

Pricing of Default Digital Options

Theorem 6.

Let $H(u) = 1_{\{T^d \leq u\}}$. Then, the price at time $t < \min\{T, T^d\}$ of a default digital option which pays 1 **at default** is given by

$$V_{T^d}^{ddp}(t) = E_Q \left[\int_t^T e^{-\int_t^u r(l) dl} dH(u) \mid \mathcal{I}_t \right] = \int_t^T E_Q \left[e^{-\int_t^u [r(l) + s^{zero}(l)] dl} s^{zero}(u) \mid \mathcal{I}_t \right] du .$$

Pricing Credit Derivatives

Pricing of Default Digital Options

Theorem 7.

Let $t < T$. Then,

$$E_Q \left[e^{-\int_t^T [r(l) + s^{zero}(l)] dl} s^{zero}(T) | \mathcal{I}_t \right] = P^{d,zero}(t, T) \cdot [G(t, T) + H(t, T) \cdot s^{zero}(t) + I(t, T) \cdot u(t) + J(t, T) \cdot \omega(t)]$$

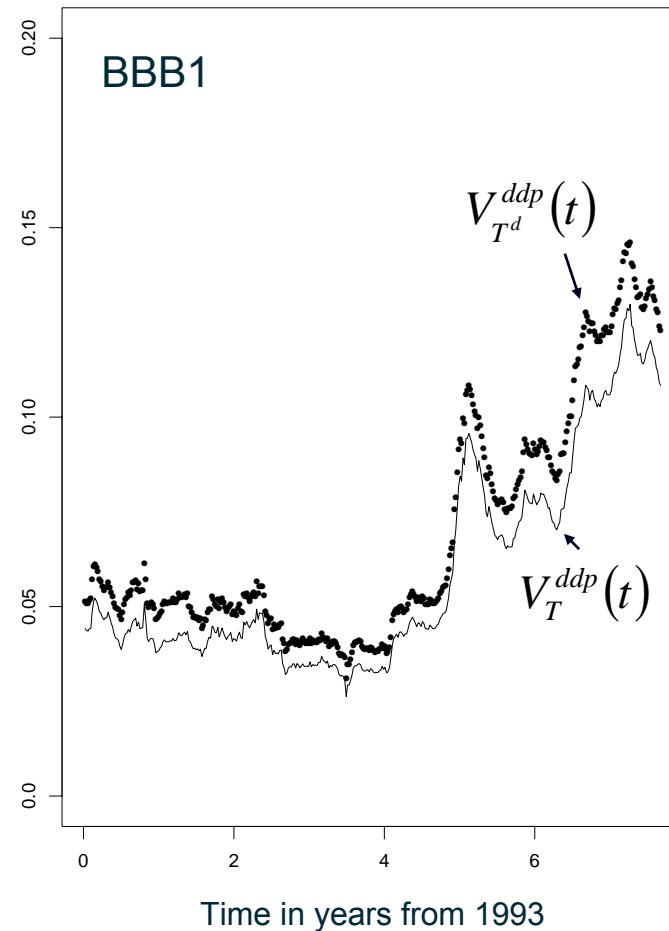
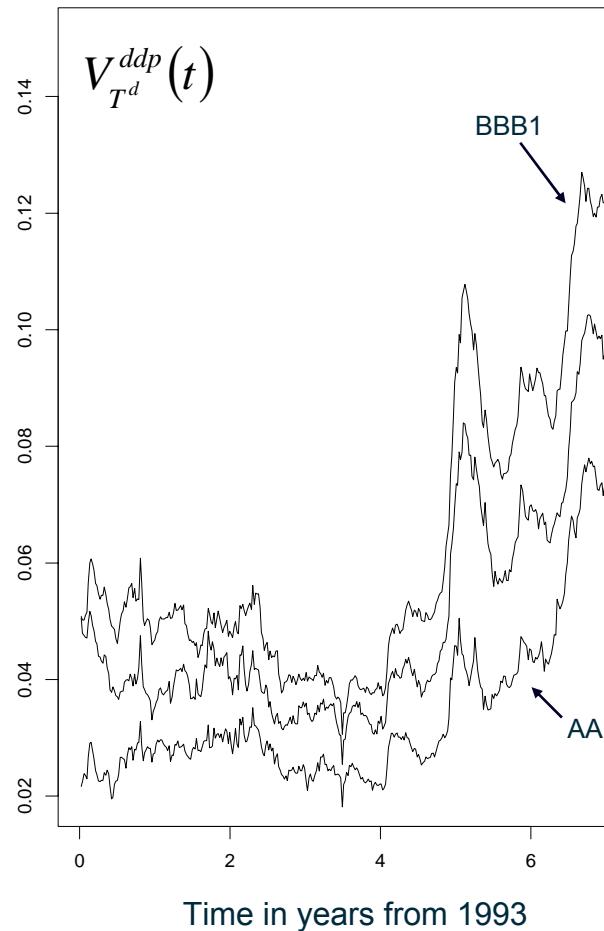
where

$$H(t, T) = e^{-a_s \cdot (T-t)}, \quad I(t, T) = -b_{su}^{zero} \cdot \frac{e^{-a_s \cdot (T-t)} - e^{-a_u \cdot (T-t)}}{a_s - a_u}, \quad J(t, T) = b_{s\omega}^{zero} \cdot \frac{e^{-a_s \cdot (T-t)} - e^{-a_\omega \cdot (T-t)}}{a_s - a_\omega},$$

$$\begin{aligned} G(t, T) &= -\frac{1}{2} \cdot \left[(\sigma_s^{zero} \cdot C(t, T))^2 + (\sigma_u \cdot D^{zero}(t, T))^2 \right] + \theta_u \cdot D^{zero}(t, T) + \theta_s^{zero} \cdot C(t, T) \\ &\quad + \theta_\omega \cdot (E^{d,zero}(t, T) - E(t, T)) - \int_t^T \sigma_\omega^2 \cdot E^{d,zero}(\tau, T) \cdot J(\tau, T) d\tau. \end{aligned}$$

Pricing Credit Derivatives

Default Digital Options for $z=0.5$



Pricing Credit Derivatives

Pricing of Default Options

- A **default option** with maturity T replaces the difference to par on a defaultable zero-coupon bond with maturity $T^* \geq T$ at default and 0 else.

Theorem 8.

The price at time $t < \min\{T, T^d\}$ of a default option on a defaultable zero-coupon bond with maturity T^* is given by

$$V^{dp}(t) = V_{T^d}^{ddp}(t) - P^d(t, T^*) + P^{d,*}(t, T, T^*) .$$

with

$$P^{d,*}(t, T, T^*) = E_Q \left[e^{-\int_t^T r(l) dl} P^d(T, T^*) \cdot 1_{\{T < T^d\}} \mid \mathcal{I}_t \right] = E_Q \left[e^{-\int_t^T [r(l) + s^{zero}(l)] dl} P^d(T, T^*) \mid \mathcal{I}_t \right] .$$

Pricing Credit Derivatives

Pricing of Default Options

Corollary 9.

Let $T=T^*$. The price at time $t < \min\{T, T^d\}$ of a default option on a defaultable zero-coupon bond with maturity T^* is given by

$$V^{dp}(t) = V_{T^d}^{ddp}(t) - P^d(t, T) + P^{d, \text{zero}}(t, T).$$

Pricing Credit Derivatives

Pricing of Default Options

Theorem 10.

The price $P^{d,*}(t, T, T^*) = P^{d,*}(r, s, u, \omega, t, T, T^*)$ at time $t < \min\{T, T^*\}$ is given by

$$P^{d,*}(t, T, T^*) = e^{A^{d,*}(t, T, T^*) - B(t, T^*) \cdot r - C^*(t, T, T^*) \cdot s - D^*(t, T, T^*) \cdot u - E^{d,*}(t, T, T^*) \cdot \omega}$$

where

$$C^*(t, T, T^*) = \frac{C(t, T^*) - z \cdot e^{-a_s \cdot (T-t)} C(T, T^*)}{1-z}$$

$$D^*(t, T, T^*) = e^{-a_u \cdot (T-t)} \cdot D(T, T^*) + \frac{1}{1-z} \cdot D(t, T) - b_{su} \cdot C(T, T^*) \cdot \left(\frac{e^{-a_u \cdot (T-t)} - e^{-a_s \cdot (T-t)}}{a_u - a_s} \right),$$

$$E^{d,*}(t, T, T^*) = E(t, T^*) - e^{-a_\omega \cdot (T-t)} \cdot E^*(T, T^*) - \frac{1}{1-z} \cdot E^*(t, T) + b_{s\omega} \cdot C(T, T^*) \cdot \left(\frac{e^{-a_\omega \cdot (T-t)} - e^{-a_s \cdot (T-t)}}{a_\omega - a_s} \right),$$

$$\begin{aligned} A^{d,*}(t, T, T^*) &= A^d(T, T^*) + \frac{1}{2} \int_t^T \sigma_r^2 \cdot B^2(\tau, T^*) + \sigma_s^2 \cdot C^{*2}(\tau, T, T^*) d\tau \\ &\quad + \frac{1}{2} \int_t^T \sigma_u^2 \cdot D^{*2}(\tau, T, T^*) + \sigma_\omega^2 \cdot E^{d,*2}(\tau, T, T^*) d\tau - \int_t^T \theta_r(\tau) \cdot B(\tau, T^*) d\tau \\ &\quad - \int_t^T \theta_s \cdot C^*(\tau, T, T^*) + \theta_u \cdot D^*(\tau, T, T^*) + \theta_\omega \cdot E^{d,*}(\tau, T, T^*) d\tau \end{aligned}$$

Pricing Credit Derivatives

Pricing of Default Swaps

- In a **default swap** with maturity T one party pays the difference to par on a defaultable zero-coupon bond with maturity $T^* \geq T$ (reference asset) at default and 0 else in exchange to a regular fee S (default swap rate) at previously fixed payment dates t $t \leq T_1 \leq \dots \leq T_m = T$ which is paid only as long as no default occurred.

Theorem 11.

The price at time $t < \min\{T, T^d\}$ of a default swap is given by

$$V^{ds}(t) = V^{dp}(t) - \sum_{i=1}^m S \cdot P^{d,zero}(t, T_i)$$

The corresponding par swap rate $S(t)$ is given by

$$V^{ds}(t) = 0 \Leftrightarrow S(t) = \frac{V^{dp}(t)}{\sum_{i=1}^m P^{d,zero}(t, T_i)}$$

Pricing Credit Derivatives

Pricing of Default Swaps

- In the case that the reference asset is a defaultable coupon bond with maturity $T^* \geq T$ and coupon $C(\tau_i)$ at the coupon payment dates $t \leq \tau_1 \leq \dots \leq \tau_n = T^*$, we set

$$P_C^d(t, T^*) = \sum_{i=1}^n C(\tau_i) \cdot P^d(t, \tau_i)$$

and

$$P_C^{d,*}(t, T, T^*) = \sum_{i=1}^n C(\tau_i) \cdot P^{d,*}(t, T, \tau_i) \text{ with } P^{d,*}(t, T, \tau_i) = P^{d,zero}(t, \tau_i) \text{ if } \tau_i \leq T.$$

- The corresponding default options and default swaps with fixed payment dates $t \leq T_1 \leq \dots \leq T_m = T$ are therefore given by

$$\begin{aligned} V_C^{dp}(t) &= V_{T^d}^{ddp}(t) - P_C^d(t, T^*) + P_C^{d,*}(t, T, T^*), \\ S_C(t) &= \frac{V_C^{dp}(t)}{\sum_{i=1}^m P^{d,zero}(t, T_i)} \end{aligned}$$

Pricing Credit Derivatives

Pricing of Default Swaps (5-Year CDS, Payment at Default)

- **Kimberley Clark:**

Underlying bond: Rating AA, Coupon: 6.875% s.a., Maturity: 15.02.2014,
Issue date: 18.02.1994, 1st coupon date: 15.08.1994
Estimation: $z=87.72181\%$, Equivalent price just before default: 45.59869

- **Caterpillar:**

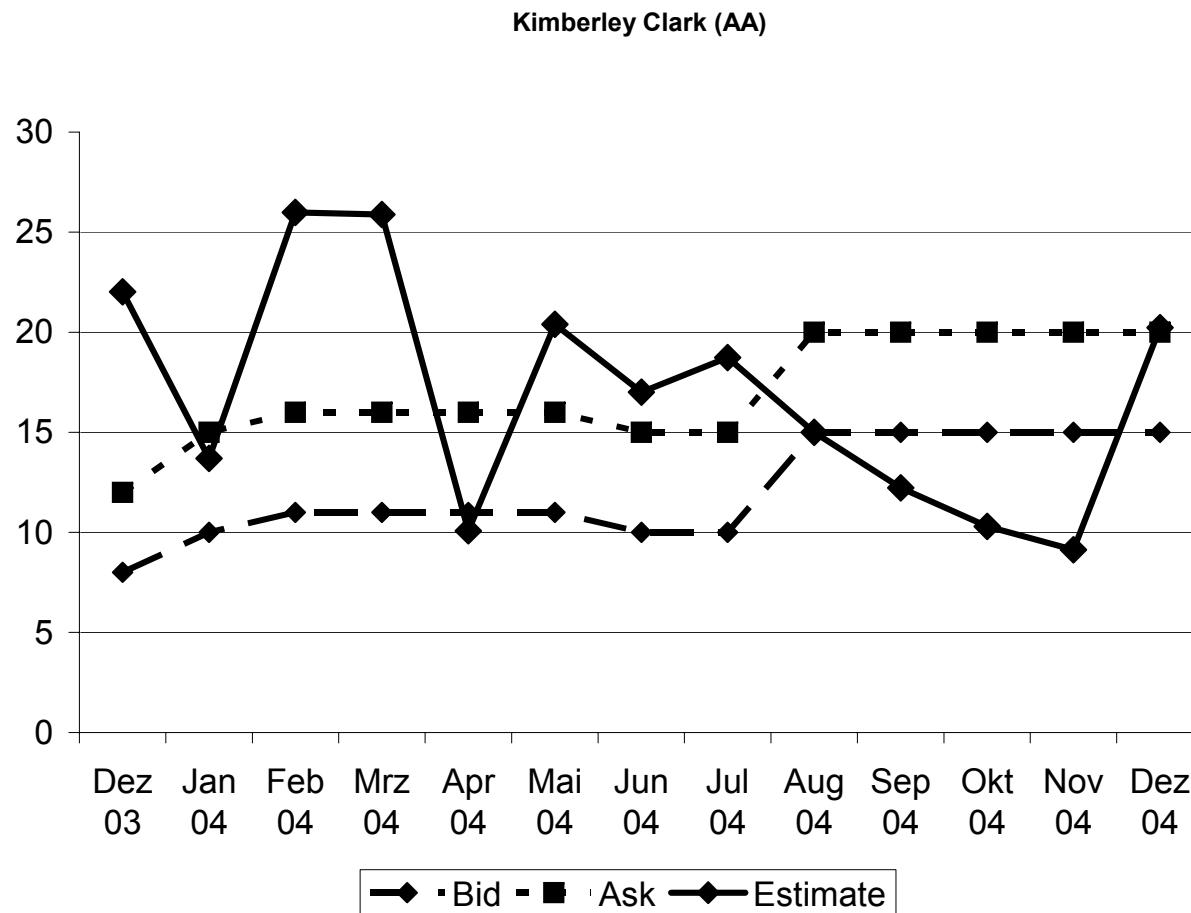
Underlying bond: Rating A2, Coupon: 6.55% s.a., Maturity: 01.05.2011,
Issue date: 08.05.2001, 1st coupon date: 01.11.2001
Estimation: $z=88.73718\%$, Equivalent price just before default: 45.07693

- **Masco:**

Underlying bond: Rating BBB1, Coupon: 5.875% s.a., Maturity: 15.07.2012,
Issue date: 24.06.2002, 1st coupon date: 15.01.2003
Estimation: $z=89.55659\%$, Equivalent price just before default: 44.66450

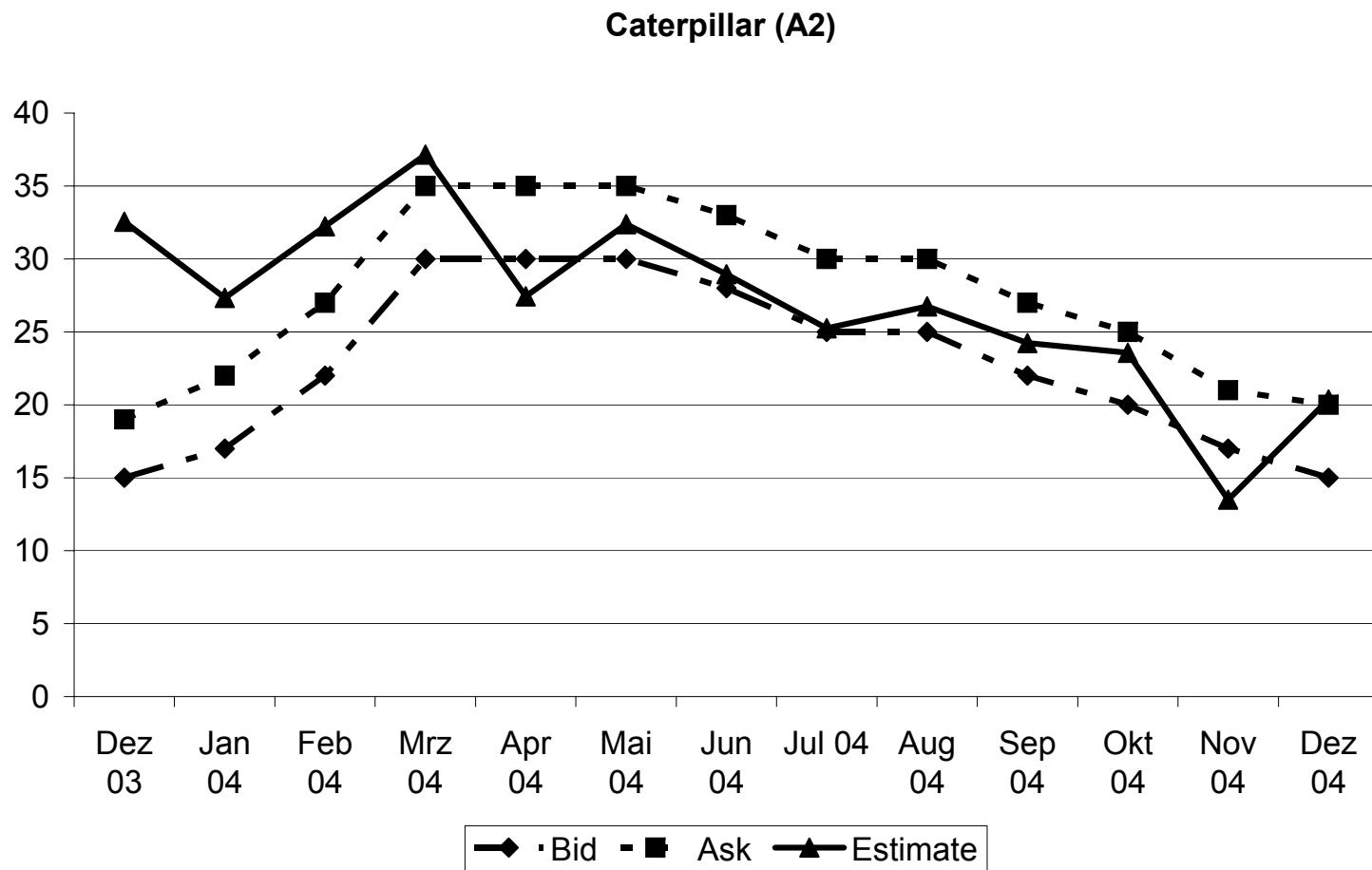
Pricing Credit Derivatives

Pricing of Default Swaps (in basis points)



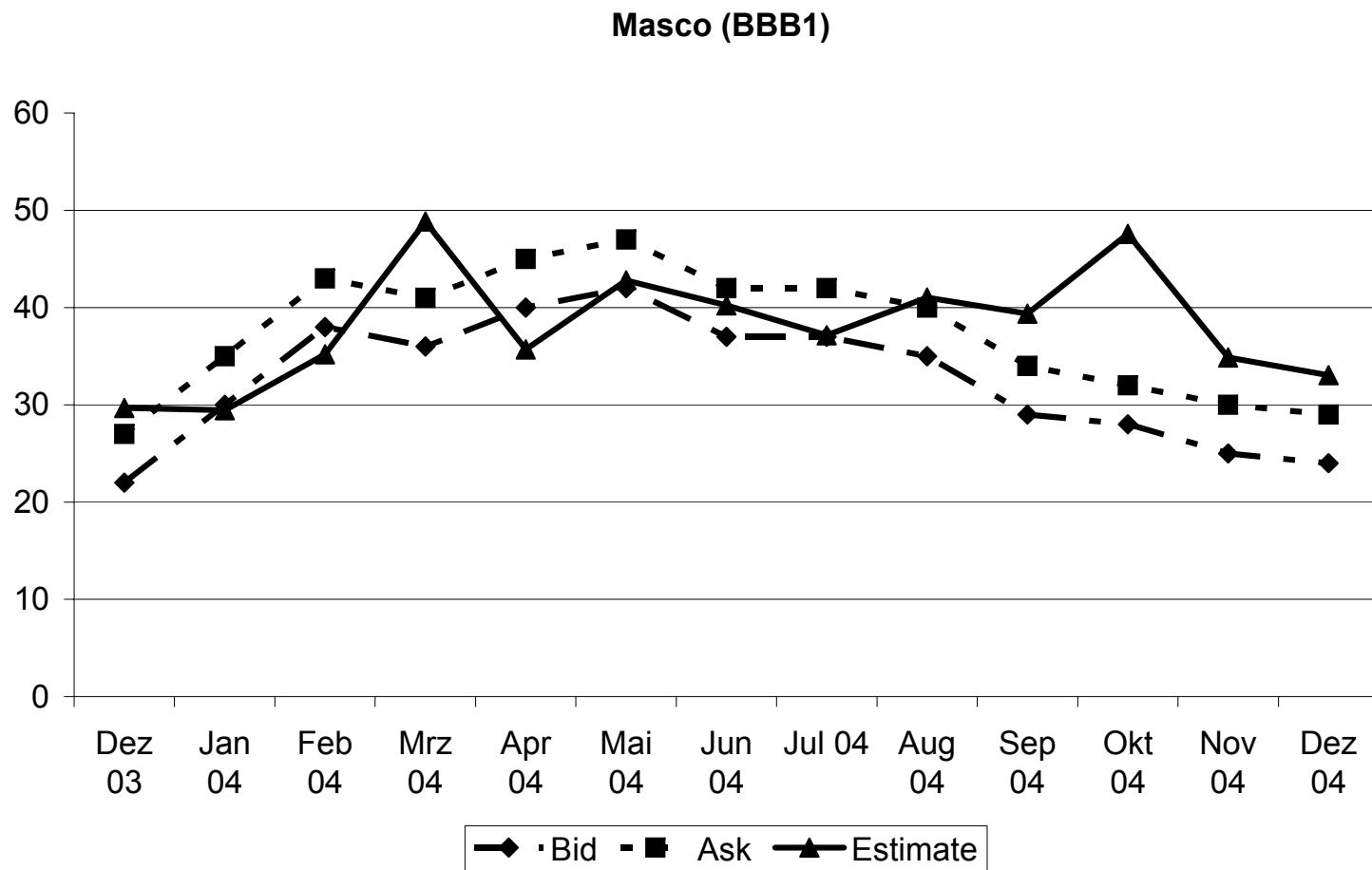
Pricing Credit Derivatives

Pricing of Default Swaps (in basis points)



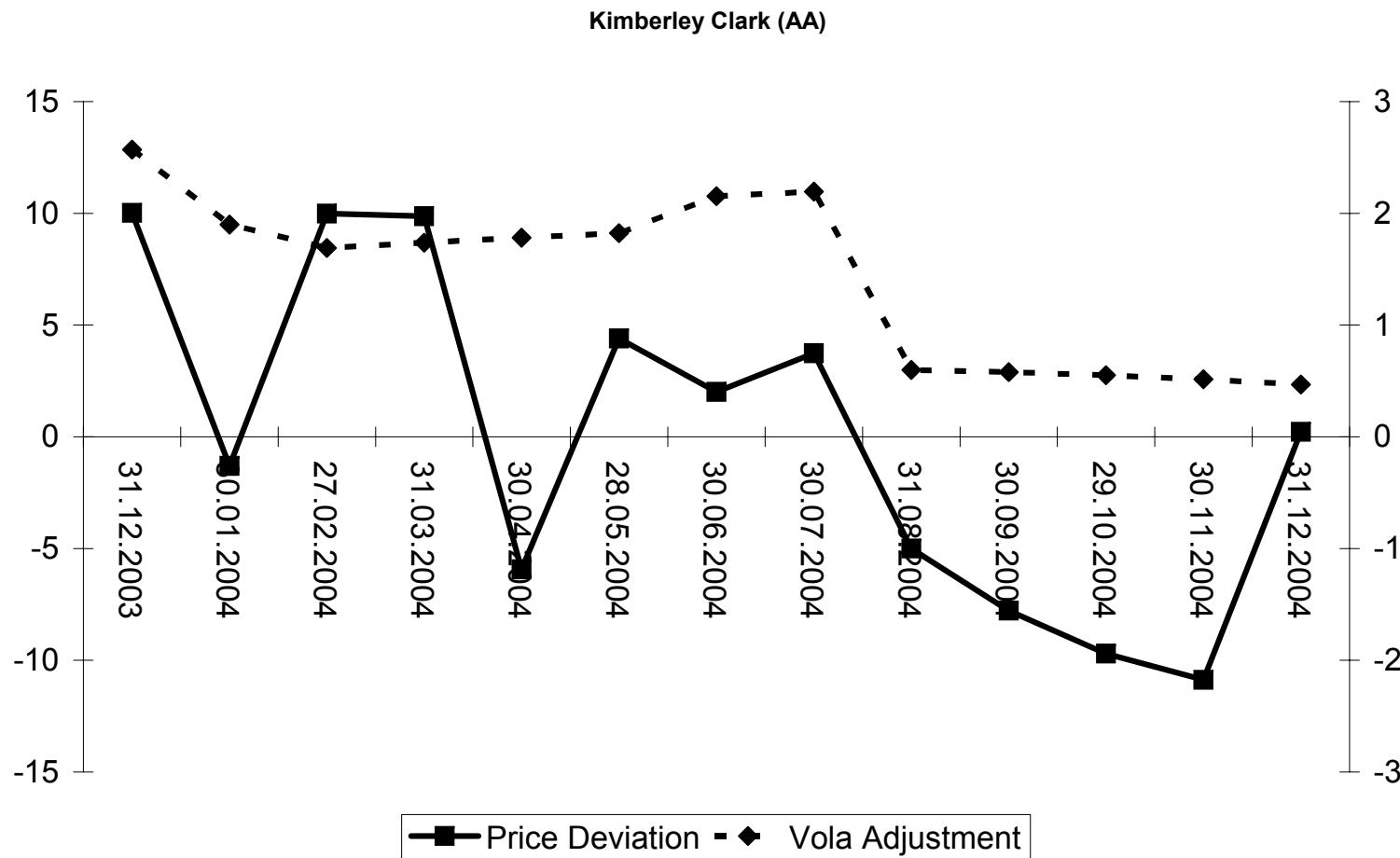
Pricing Credit Derivatives

Pricing of Default Swaps (in basis points)



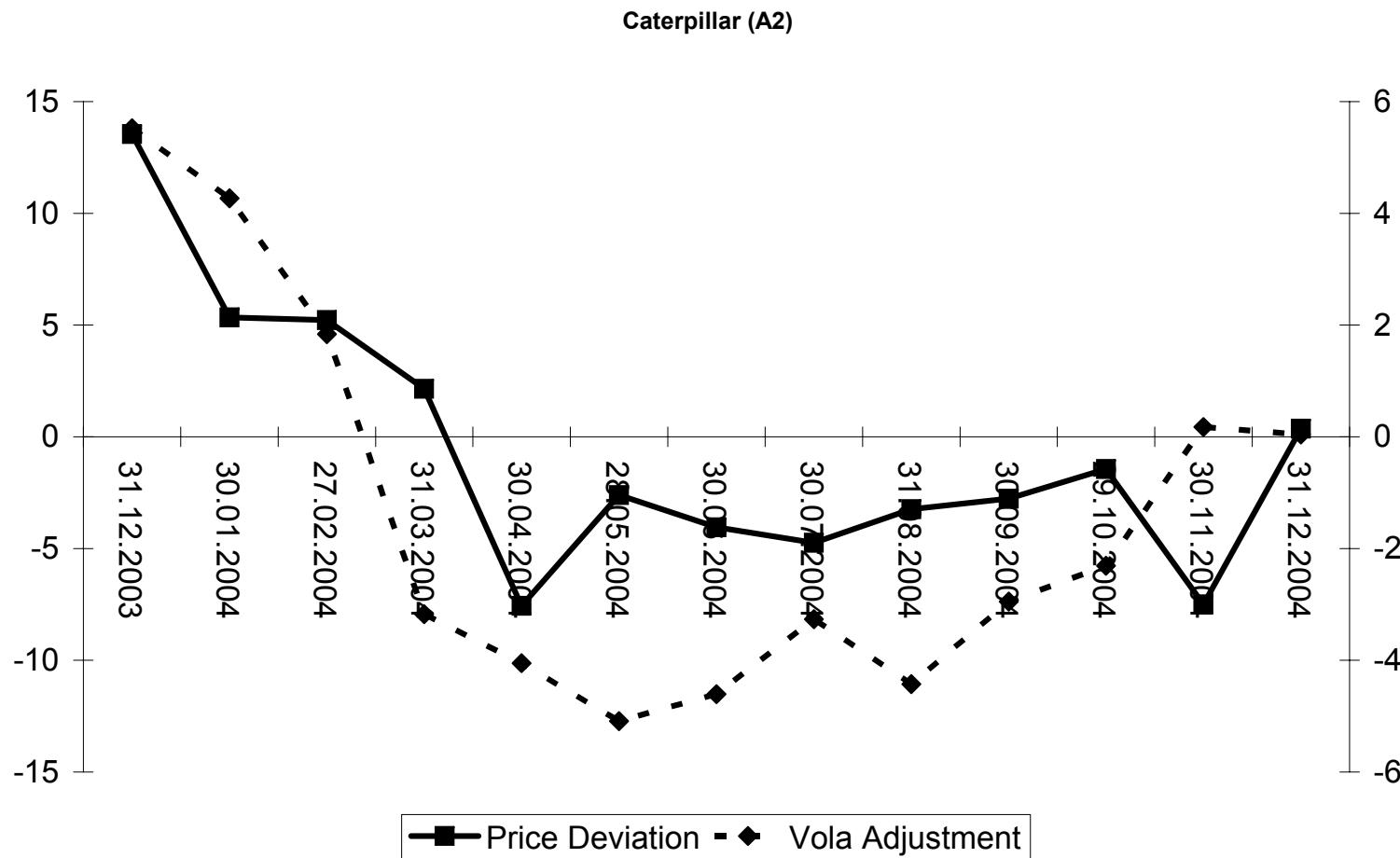
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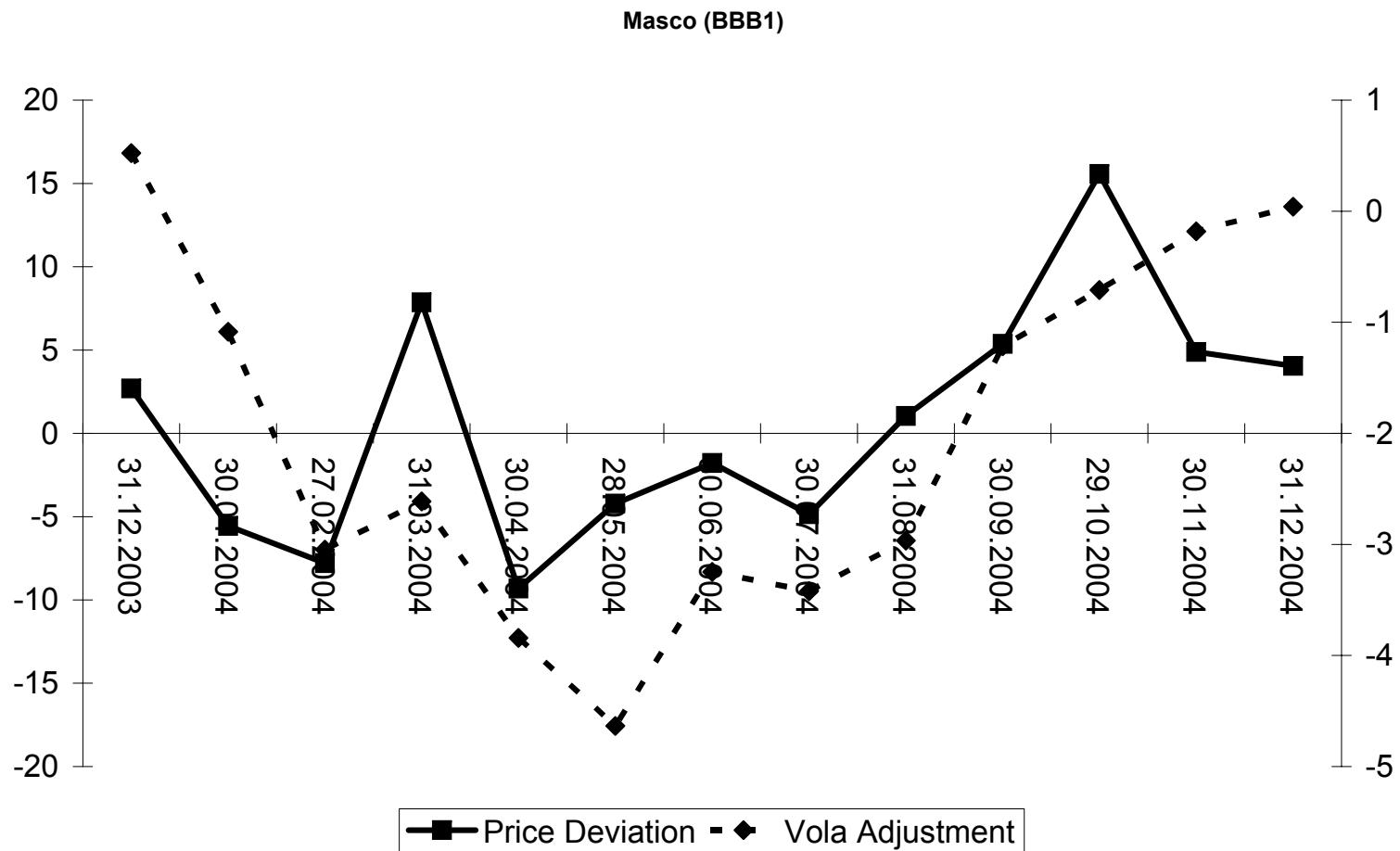
Pricing Credit Derivatives

Pricing of Default Swaps (in basis points)



Pricing Credit Derivatives

Pricing of Default Swaps (in basis points)



Pricing of Credit Derivatives

Summary

- **We introduced a new model which**
 - is simple and leads to closed-form solutions
 - consistently models long-term economic and market behaviour
 - can be extended to a complete ALM model
- **We derived the prices for**
 - Defaultable Zero-Coupon Bonds with and without recovery
 - Default Digital Options and Default Options
 - Default Swaps
- **and found**
 - an implicit price just before default of around 45%
 - a difference between the empirical and implied volatility between $\pm 5\text{bp}$.