Direct Characterization of Quantum Dynamics

Masoud Mohseni[1,2]

and Daniel Lidar[2]

[1] Department of Physics,
University of Toronto

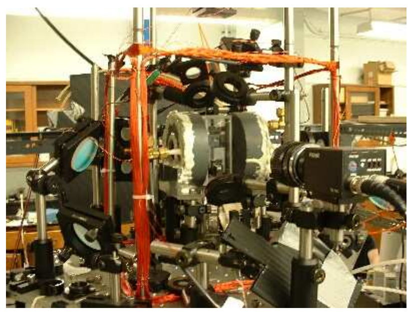
[2]Department of Chemistry and
Electrical Engineering-Systems,
University of Southern California

Conference on Quantum Information & Quantum Control, 2006

Motivation

"Black Box"

$$\rho \longrightarrow$$



$$\longrightarrow \varepsilon(\rho)$$

Aephraim Steinberg's laboratory at
University of Toronto

The characterization of dynamics of open quantum systems

- among the central modern challenges of quantum physics and chemistry
- a fundamental problem in quantum information science and quantum control

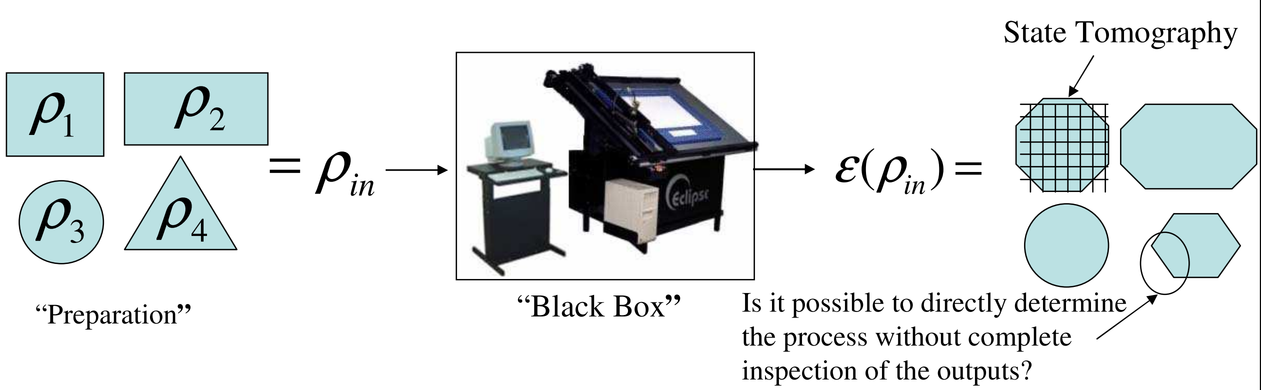
> Ø Verifying/Monitoring the performance of a quantum device
> Ø Design of decoherence prevention/correction methods

Quantum Process Tomography

- Subjecting an ensemble of identical quantum systems (prepared in a member of a set of quantum states) to the same quantum dynamics, and measuring the output via quantum state tomography

Problems with QPT:

- The number of required measurements grows exponentially with the number of degrees of freedom of the system.
- The dynamics is characterized indirectly, using state tomography.



Motivation

Ø **Is it possible to directly characterize the "real-world" quantum dynamics, including decoherence and losses? (i.e., without state tomography)**

Key observation: using quantum error detection schemes one can directly obtain some partial information about the nature of errors acting on the state of a quantum system

What is the relationship between quantum error detection (QED) and quantum process tomography (QPT)?

Is it possible to completely characterize quantum dynamics for arbitrary open quantum systems using QED?

- How the physical resources scale-up with the system's degrees of freedom?
- Which kind of applications such a theory may have?
- Does entanglement play a fundamental role?
- Does it lead to new ways of understanding/controlling quantum dynamical systems?

UofT

We present:

An optimal algorithm for complete and direct characterization of quantum dynamics

- **Optimal in the sense of overall required experimental configurations and quantum operations over all known quantum process tomography schemes in a given Hilbert space.**

- **A Quadratic reduction in the number of experimental configuration** over all separable quantum process tomography schemes.

- Quantum dynamics is characterized **directly**, **without any state tomography**.

- **Entanglement** is a required physical resource in both preparation and measurement.

- **Quantum error detection** is utilized for complete characterization of dynamics

- Applications for **partial characterization** of quantum dynamics.

  E.g., Hamiltonian Identification, simultaneous determination of T1 and T2 in a single measurement.

Outline

Introduction

- Quantum dynamical maps
- Standard quantum process tomography
- Ancilla-assisted process tomography
- Comparing the required physical resources

Direct Characterization of Dynamics for two-level systems

- Error detection schemes
- Characterization of quantum dynamical population
- Characterization of quantum dynamical coherence
- Physical realization

Generalizations and applications

Conclusion

Quantum Dynamical Maps

$$\varepsilon(\rho) = \sum_{m,n=0}^{d^2} \chi_{mn} E_m \rho E_n^{\dagger}$$

χ is positive-Hermitian matrix

Superoperator, χ, has

4x4 = 16 elements

$$\rho = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{01}^{*} & \rho_{11} \end{pmatrix} \longrightarrow \begin{pmatrix} \rho'_{00} & \rho'_{01} \\ \rho'^{*}_{01} & \rho'_{11} \end{pmatrix} = \varepsilon(\rho)$$

Quantum dynamical population

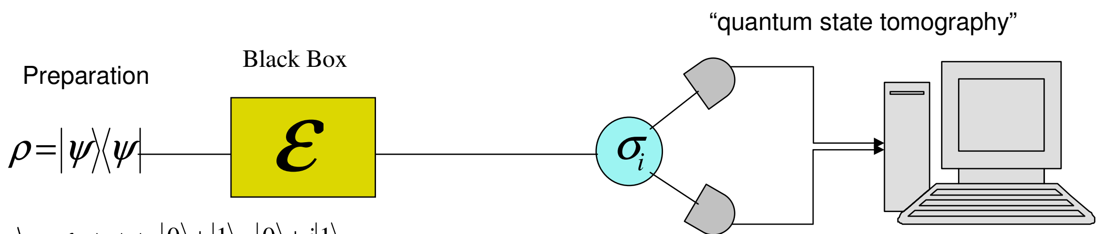
$$\chi = \begin{pmatrix} \chi_{00} & \chi_{01} & \chi_{02} & \chi_{03} \\ \chi_{01}^{*} & \chi_{11} & \chi_{12} & \chi_{13} \\ \chi_{02}^{*} & \chi_{12}^{*} & \chi_{22} & \chi_{23} \\ \chi_{03}^{*} & \chi_{13}^{*} & \chi_{23}^{*} & \chi_{33} \end{pmatrix}$$

Quantum dynamical coherence

The number of independent real parameters of superoperator for a qudit: d^4

(For a trace-preserving map becomes $d^4 - d^2$)

Standard Quantum Process Tomography



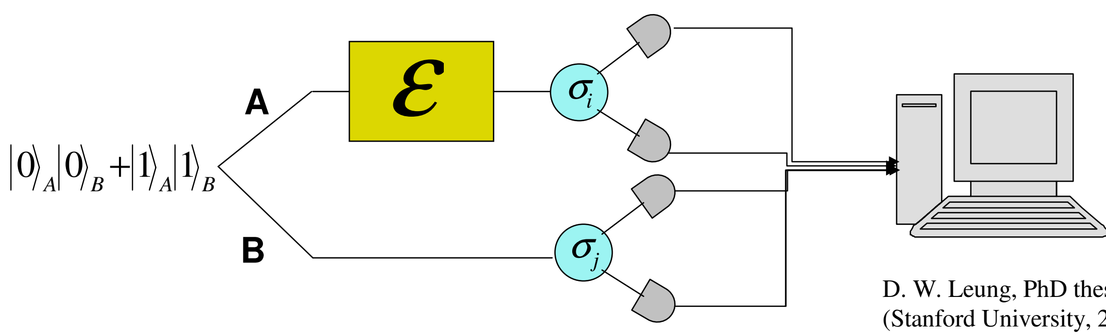
$|\psi\rangle \in \{|0\rangle, |1\rangle, \frac{|0\rangle+|1\rangle}{\sqrt{2}}, \frac{|0\rangle+i|1\rangle}{\sqrt{2}}\}$

$\sigma_i \in \{I, \sigma_x, \sigma_y, \sigma_z\}$

Overall number of measurements = 4 **x** 4 = 16

J. F. Poyatos, J. I. Cirac, and P. Zoller, PRL (97); I.L. Chuang and M.A. Nielsen, J. Mod. Opt. (97)

Ancilla-Assisted Process Tomography



D. W. Leung, PhD thesis (Stanford University, 2000); D'Ariano, G. M. & Lo Presti, P. PRL. **86**, (01).

Overall number of measurements = 1 **x** 16 = 16

Comparing the Required Physical Resources for n Qubits

The number of required non-commuting measurements at each output state

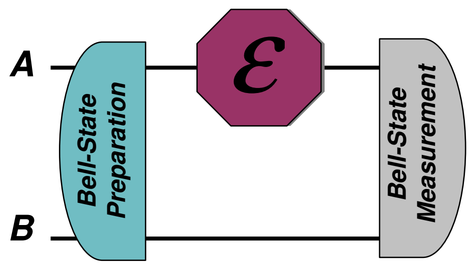
Not available experimentally

	n Ancilla qubits	Entanglement	# Input states	# Measurements correspond to each input state	# Overall Measurements	Type pf measurements	Required Physical Interactions
SQPT	Not required	N/A	4^n	4^n	16^n	**Single-qubit Measurements**	**N/A**
Separable AAPT	Required	Not Required	1	16^n	16^n	**Joint Single-qubit Measurements**	**N/A**
Non-separable AAPT	Required	Not Required	1	4^n+1	4^n+1	**Non-separable Measurement (e.g. MUB)**	**Many-body Interactions**
DCQD	Required	**Required**	4^n	**1**	$\mathbf{4^n}$	**Bell–State Measurements**	**Two-body Interactions**

A single measurement of commuting observables at each output state

quadratic reduction in the number of measurements over separable process tomography schemes

Direct Characterization of Superoperator for a Single Qubit



Input state	Measurement: Stabilizer	Measurement: Normalizer	Output
$(\|0\rangle\|0\rangle + \|1\rangle\|1\rangle)/\sqrt{2}$	ZZ, XX	N/A	$\chi_{00}, \chi_{11}, \chi_{22}, \chi_{33}$
$\alpha\|0\rangle\|0\rangle + \beta\|1\rangle\|1\rangle$	ZZ	XX	χ_{03}, χ_{12}
$\alpha\|+\rangle_x\|+\rangle_x + \beta\|-\rangle_x\|-\rangle_x$	XX	ZZ	χ_{01}, χ_{23}
$\alpha\|+\rangle_y\|+\rangle_y + \beta\|-\rangle_y\|-\rangle_y$	YY	ZZ	χ_{02}, χ_{13}

$|\alpha| \neq |\beta| \neq 0$

Four independent real parameters of superoperator are determined in each measurement

Quantum Error Detection

Ø Encoding in a larger Hilbert space (with redundancy) such that any arbitrary error that may occur on the data can be detected.

Stabilizer code: $S_i|\psi_C\rangle = |\psi_C\rangle$; $|\psi_C\rangle \in V_C$; $S_i \in S \subset G_n$

Error detection:

E.g., $|\phi^+\rangle = (|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)/\sqrt{2}$

$$\sigma_x^A\sigma_x^B|\phi^+\rangle = |\phi^+\rangle$$

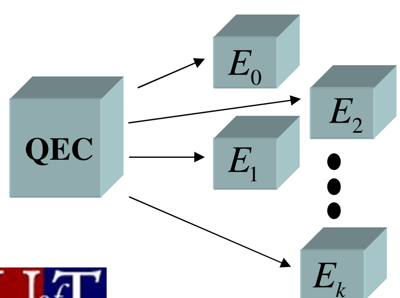
$$\sigma_z^A\sigma_z^B|\phi^+\rangle = |\phi^+\rangle$$

Error operator $E_m^A \in \{I^A, \sigma_x^A, \sigma_y^A, \sigma_z^A\}$

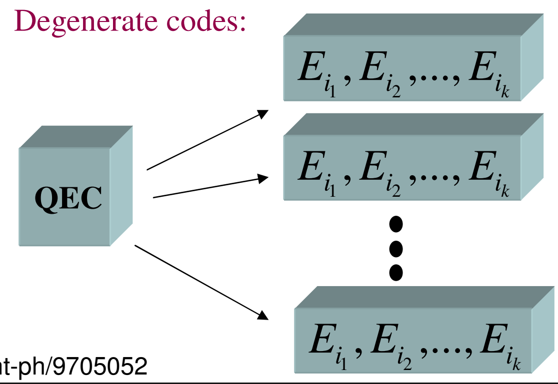
$$\sigma_z^A\sigma_z^B E_m^A|\phi^+\rangle = \pm E_m^A|\phi^+\rangle$$

$$\sigma_x^A\sigma_x^B E_m^A|\phi^+\rangle = \pm E_m^A|\phi^+\rangle$$

Non-degenerate codes:



Degenerate codes:



D. Gottesman, quant-ph/9705052

Characterization of Quantum Dynamical Population

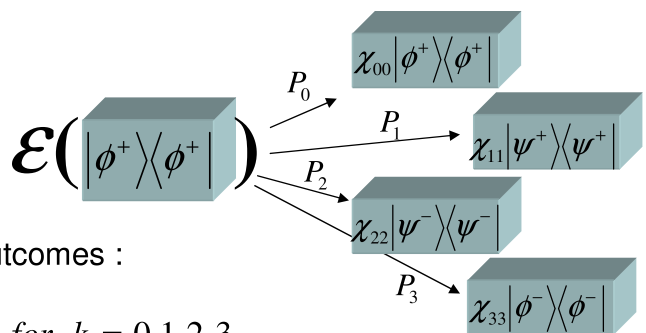
1- Prepare the Input state: $|\psi_C\rangle = (|0\rangle_A |0\rangle_B + |1\rangle_A |1\rangle_B)/\sqrt{2}$

2- Apply the unknown quantum dynamics to qubit A: $\varepsilon(\rho) = \sum_{m,n} \chi_{mn} \sigma_m^A \rho \sigma_n^A$

3- Perform the Bell-state measurement $P_k \in \{P_0, P_1, P_2, P_3\}$ as:

$$\varepsilon(\rho) \longrightarrow P_k \varepsilon(\rho) P_k$$

$$P_0 = |\phi^+\rangle\langle\phi^+| \quad P_2 = |\psi^-\rangle\langle\psi^-|$$
$$P_1 = |\psi^+\rangle\langle\psi^+| \quad P_3 = |\phi^-\rangle\langle\phi^-|$$



4-Calculate the probabilities of these outcomes :

$$Tr[P_k \varepsilon(\rho)] = \chi_{kk} \qquad for \;\; k = 0,1,2,3$$

Outputs: $\chi_{00}, \chi_{11}, \chi_{22}, \chi_{33}$

We obtain four independent parameters of superoperator in a single measurement

Characterization of Quantum Dynamical Coherence

1- Prepare the Input state: $|\psi_C\rangle = \alpha|0\rangle_A|0\rangle_B + \beta|1\rangle_A|1\rangle_B \qquad |\alpha| \neq |\beta| \neq 0$

Sole stabilizer generator: $\sigma_z^A\sigma_z^B|\psi_C\rangle = |\psi_C\rangle$ ← Degenerate stabilizer code $(I^A, \sigma_z^A);(\sigma_x^A, \sigma_y^A)$

2- Apply the unknown dynamics to qubit A: $\varepsilon(\rho) = \sum_{m,n}\chi_{mn}\sigma_m^A\rho\sigma_n^A$

3- Measure stabilizer generator $\sigma_z\sigma_z$:

$\mathcal{E}\left(\rho = |\psi_C\rangle\langle\psi_C|\right)$

+1 → $\chi_{00}\rho + \chi_{33}\sigma_z^A\rho\sigma_z^A + \chi_{03}\rho\sigma_z^A + \chi_{03}^*\sigma_z^A\rho$

-1 → $\chi_{11}\sigma_x^A\rho\sigma_x^A + \chi_{22}\sigma_y^A\rho\sigma_y^A + \chi_{12}\sigma_x^A\rho\sigma_y^A + \chi_{12}^*\sigma_y^A\rho\sigma_x^A$

4-Calculate the probability of these outcomes:

$$Tr[P_+\varepsilon(\rho)] = \chi_{00} + \chi_{33} + 2\,\mathrm{Re}(\chi_{03})Tr(\sigma_z\rho)$$

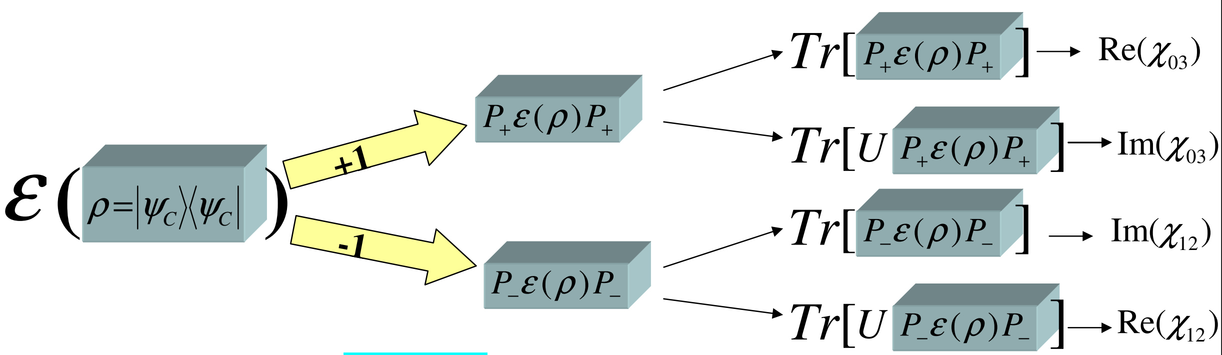
$$Tr[P_-\varepsilon(\rho)] = \chi_{11} + \chi_{22} + 2\,\mathrm{Im}(\chi_{12})Tr(\sigma_z\rho)$$

5- Calculate the expectation values of a normalizer U (e.g., $\sigma_x^A \sigma_x^B$):

$$Tr[UP_+\varepsilon(\rho)] \rightarrow \mathrm{Im}(\chi_{03})$$
$$Tr[UP_-\varepsilon(\rho)] \rightarrow \mathrm{Re}(\chi_{12})$$

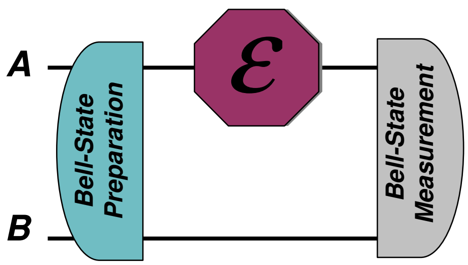
The combined stabilizer and normalizer measurement is equivalent to measuring Hermitian operators:

$$\text{Stabilizer} \begin{cases} P_+ = |\phi^+\rangle\langle\phi^+| + |\phi^-\rangle\langle\phi^-| \\ P_- = |\psi^+\rangle\langle\psi^+| + |\psi^-\rangle\langle\psi^-| \end{cases} \qquad \text{Normalizer} \begin{cases} |\phi^+\rangle\langle\phi^+| - |\phi^-\rangle\langle\phi^-| \\ |\psi^+\rangle\langle\psi^+| - |\psi^-\rangle\langle\psi^-| \end{cases}$$



Outputs: χ_{03}, χ_{12}

Direct Characterization of Superoperator for a Single Qubit



Input state	Measurement: Stabilizer	Measurement: Normalizer	Output
$(\vert 0\rangle\vert 0\rangle + \vert 1\rangle\vert 1\rangle)/\sqrt{2}$	ZZ , XX	N/A	χ_{00} , χ_{11} , χ_{22} , χ_{33}
$\alpha\vert 0\rangle\vert 0\rangle + \beta\vert 1\rangle\vert 1\rangle$	ZZ	XX	χ_{03} , χ_{12}
$\alpha\vert +\rangle_x\vert +\rangle_x + \beta\vert -\rangle_x\vert -\rangle_x$	XX	ZZ	χ_{01} , χ_{23}
$\alpha\vert +\rangle_y\vert +\rangle_y + \beta\vert -\rangle_y\vert -\rangle_y$	YY	ZZ	χ_{02} , χ_{13}

$$|\alpha| \neq |\beta| \neq 0$$

Four independent real parameters of superoperator are determined in each measurement

Physical Realization

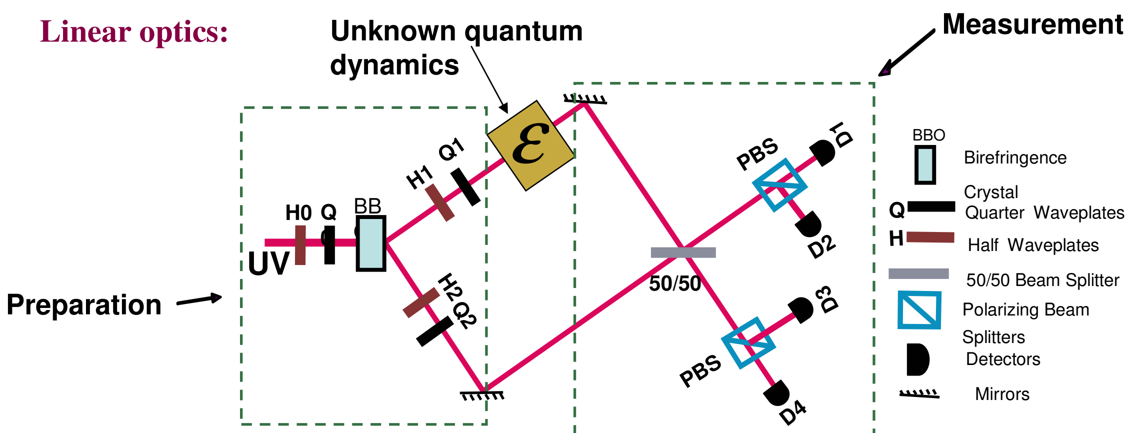
Required physical resources are:

1) **Bell-state generation**
2) **Single-qubit rotations**
3) **Bell-state measurement**

E.g.,

Trapped ions and liquid-state NMR: possible with current state of technology

Linear optics:



Conclusions

- **Direct** and **Complete** characterization of quantum dynamics, without state tomography.
- **Optimal in the sense of overall required experimental configurations and quantum operations over all known quantum process tomography schemes in a given Hilbert space.**
- **A Quadratic reduction in the number of experimental configuration** over all separable quantum process tomography schemes.
- Introduce error detection schemes to fully measure the coherence in a quantum dynamical process.

UofT

Future Works

- Direct Hamiltonian Identification
- Simultaneous determination of T1 and T2
- Generalization to Higher dimensional systems
- Optimization of the required non-maximally entangled input states
- Continuous characterization of quantum dynamics

References:

M. Mohseni and D. A. Lidar, quant-ph/0601033 and quant-ph/0601034.