Progress on Quantum Network Coding

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$:[CRC, CFI, OIT, NSERC, CIAR]

Attitude/personality test:

Q1. If you're to give a talk titled "quantum network coding" , how would you motivate it?

(a) It's our future !

You're a cheerful optimist. You may have some problem coping with reality.

(b) To control something (q-sys), you get to understand it !

You're strategic & cautious in your pursuit, though some may call you a control freak.

(c) Who cares ? It's fun !

You know how to enjoy life, good luck with grant apps.

(d) None of the above
This can't be motivated !

Have a good nap.

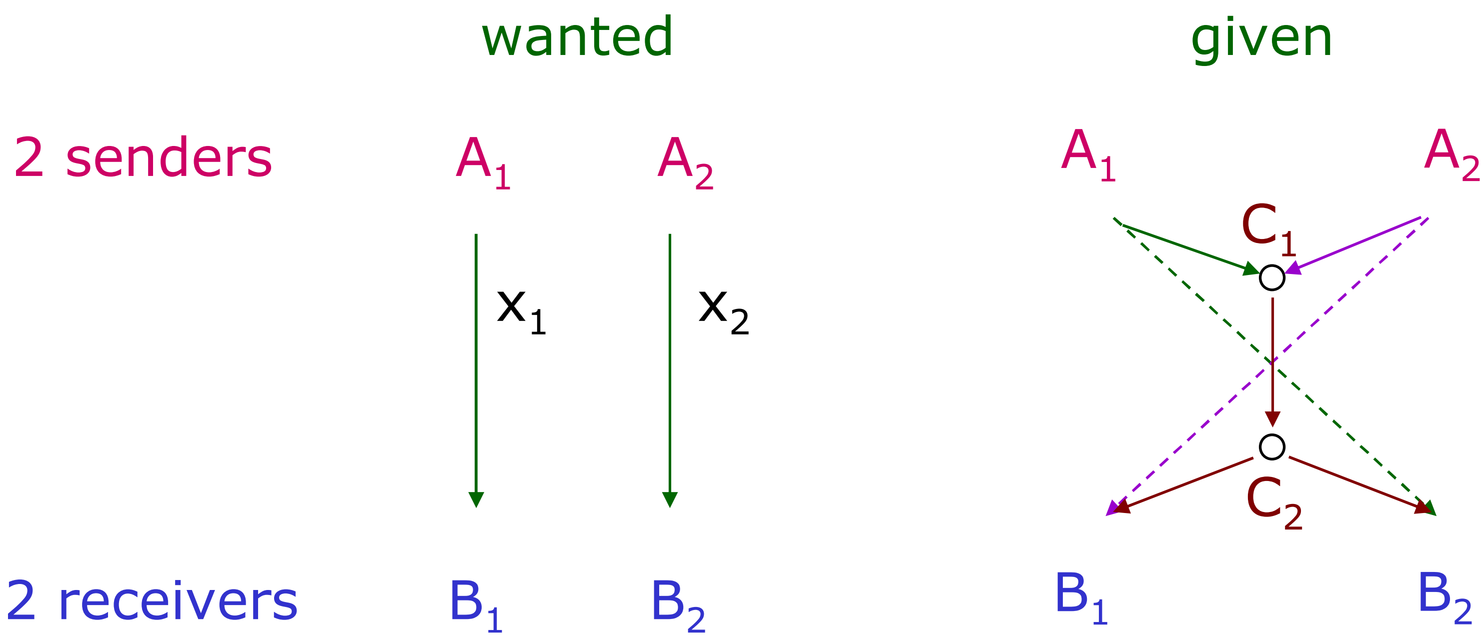
Plan:

- Motivating example (the butterfly network)
  - The Problem & Solution (classical)
  - Prior work in quantum setting
  - Define our problems & solutions

- More general networks
  - Partial results and conjectures

Motivating example : the butterfly network

The problem (classical):



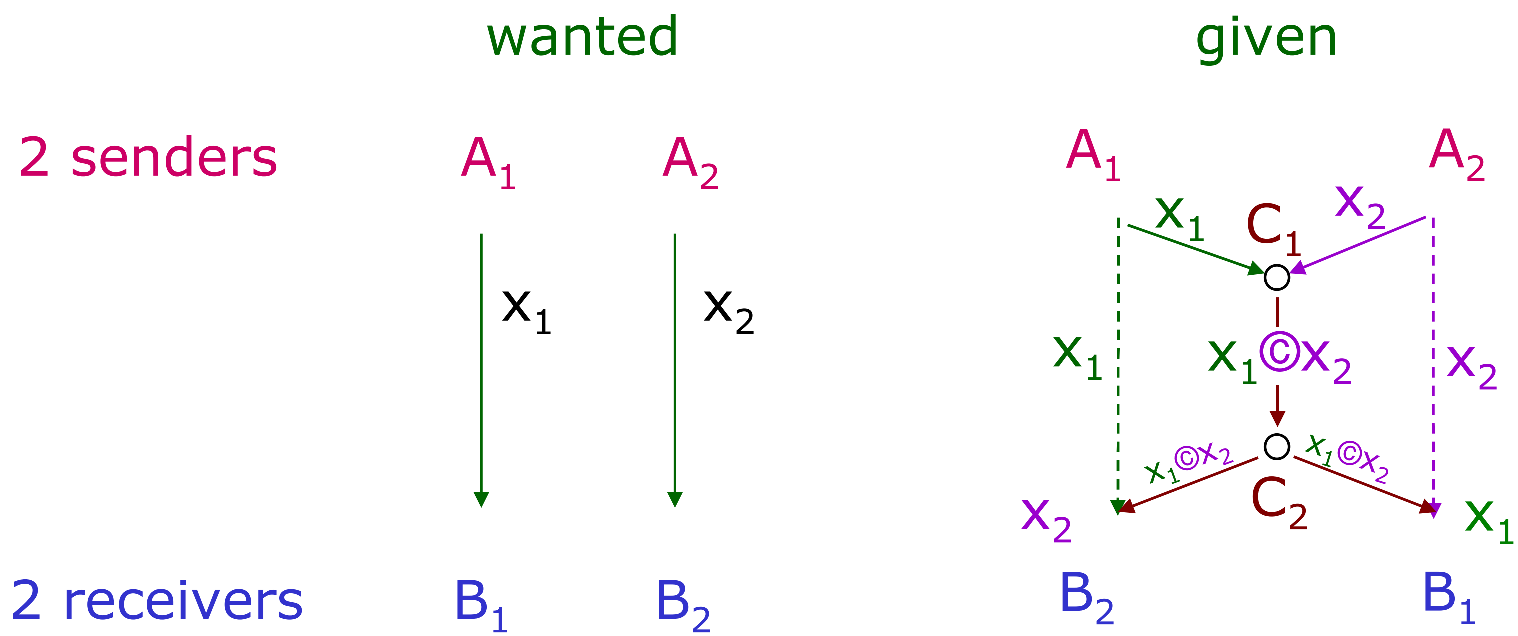
Assumption: "network" (all 7 channels) called as a package
Qn: how to "best" communicate from A_1 to B_1 and A_2 to B_2?

Moving from attitude test to IQ test   Online slides: © should be ⊕

Motivating example : the butterfly network

For independent bits x_1, x_2, a "best" defines itself

B_1, B_2 both get both x_1, x_2



Assumption: "network" (all 7 channels) called as a package

Qn: how to "best" communicate from A_1 to B_1 and A_2 to B_2?

"Best" -- exact, 1-shot, & individual-rate optimal

Any question about the classical result?

Now, let's go quantum.

Attitude test (ctd):

Q2. How would you give the punchline of your result "quantum info sent in network is like waterflow in pipes, that simple rerouting is optimal unlike the classical case" ?

(a) Due to monogamy of entanglement, quantum information sent in a network is incompressible, thus admitting simple, elegant, optimal solution that has no classical counterpart.

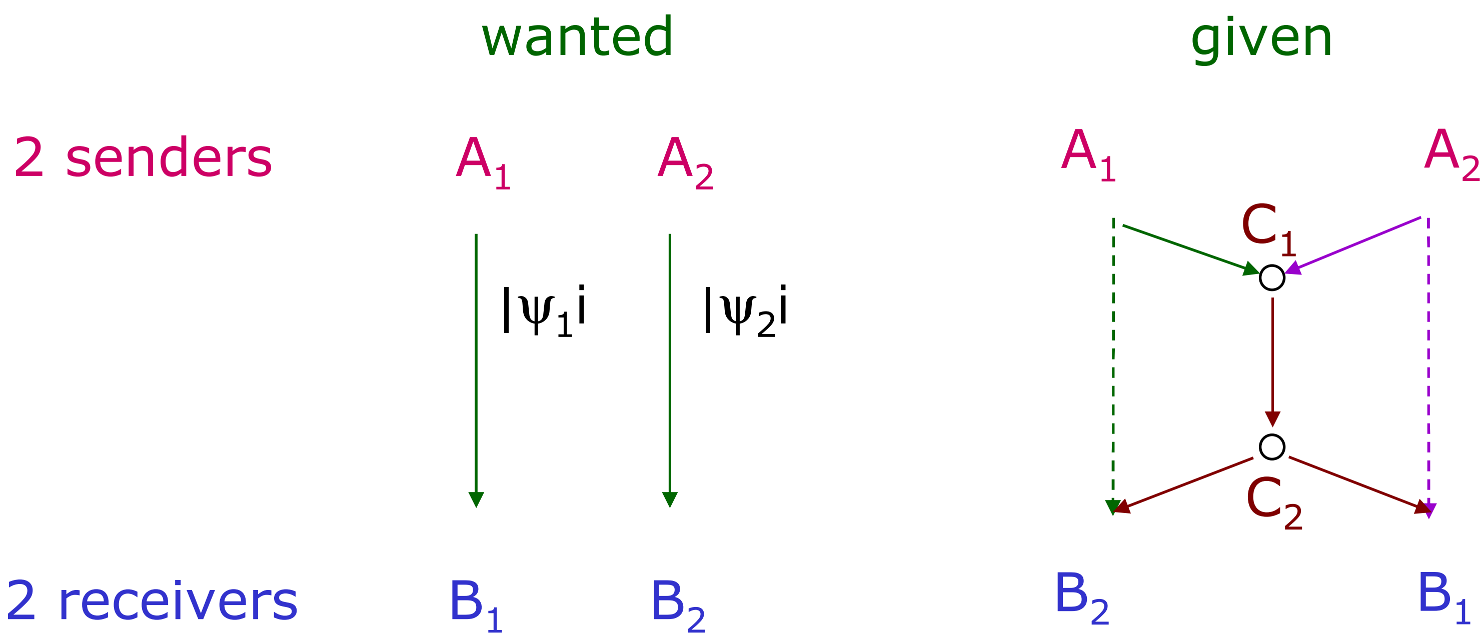
you're appreciative

(b) The surprise is the lack of surprise in quantum network coding compared to its classical counterpart.

you're honest and harsh

Motivating example : the butterfly network

Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$



Assumption: "network" (all 7 channels) called as a package

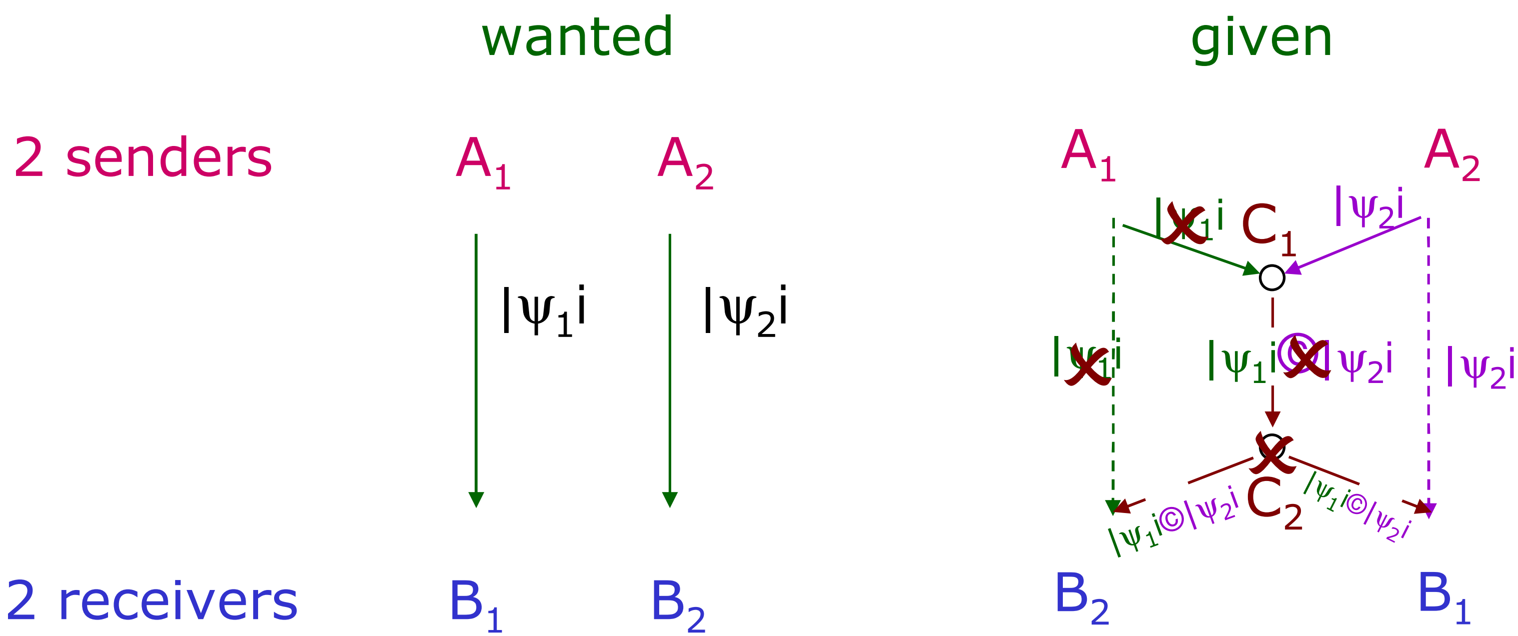
Qn: how to "best" communicate from A_1 to B_1 and A_2 to B_2?

Online slides: $|\psi_1 i$ should be $|\psi_1\rangle$

Motivating example : the butterfly network

Quantum: for independent $|\psi_1 i$, $|\psi_2 i$

drunkness-test



Assumption: "network" (all 7 channels) called as a package

Qn: how to "best" communicate from A_1 to B_1 and A_2 to B_2?

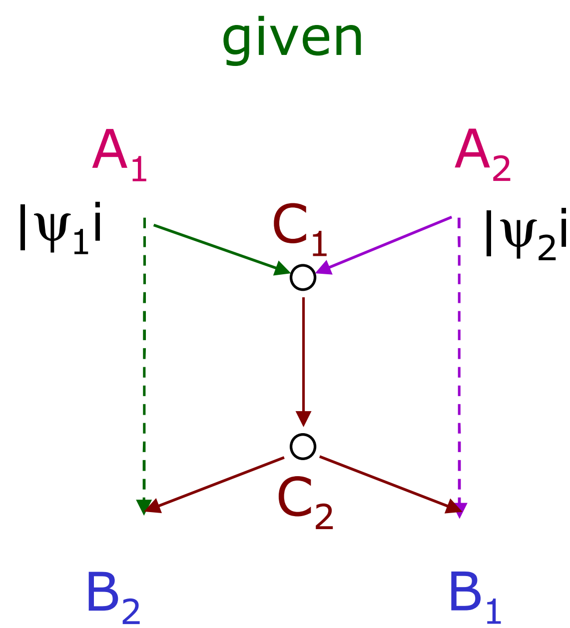
No all-optimal solution -- need to define notion of optimality

Motivating example : the butterfly network

Quantum: for independent $|\psi_1 i$, $|\psi_2 i$

Prior work: [Hayashi, Iwama, Nishimura, Raymond, Yamashita 0601088] 1-use of network, fixed rate, min distortion (fidelity ¼ 1/2-2/3).

Here:
asymptotic: n uses, n_i-qubit $|\psi_i i$
max #qubits sent per network use
impose near-perfect transmission
Study optimal trade-off



Assumption: "network" (all 7 channels) called as a package
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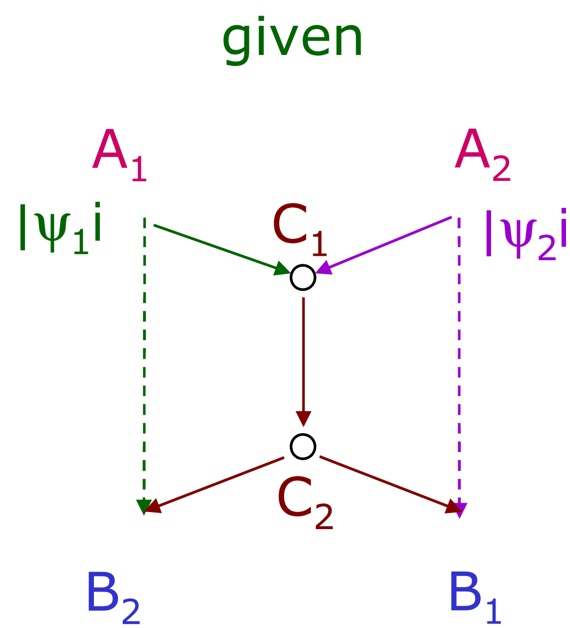
Motivating example : the butterfly network

Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$

Def [achievable rate region]:
If
  n-uses of the network enables
  n_i qubits to be faithfully sent
  from A_i to B_i ,
then,
 rate pair $(r_1,r_2) = (n_1/n, n_2/n)$
 is "achievable."



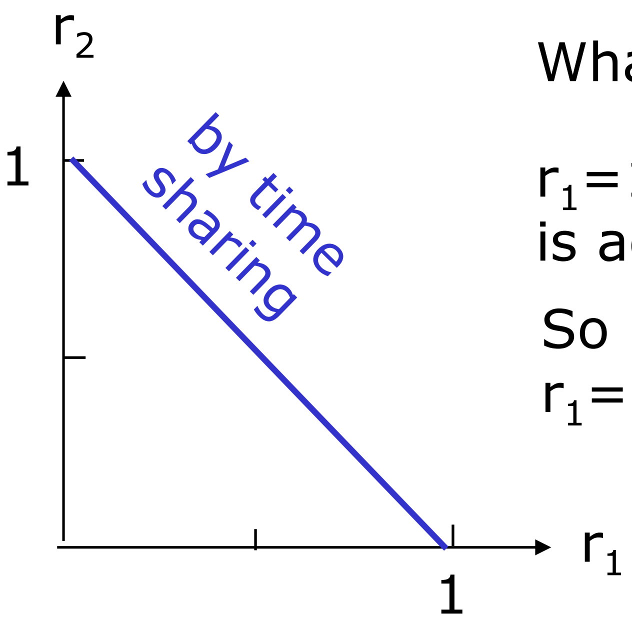
Achievable rate region is the set
of all achievable rate pairs.

Goal: Find entire achievable rate region

Will consider various "assistance" (i.e., free auxiliary resources)

Motivating example : the butterfly network

Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ no assistance



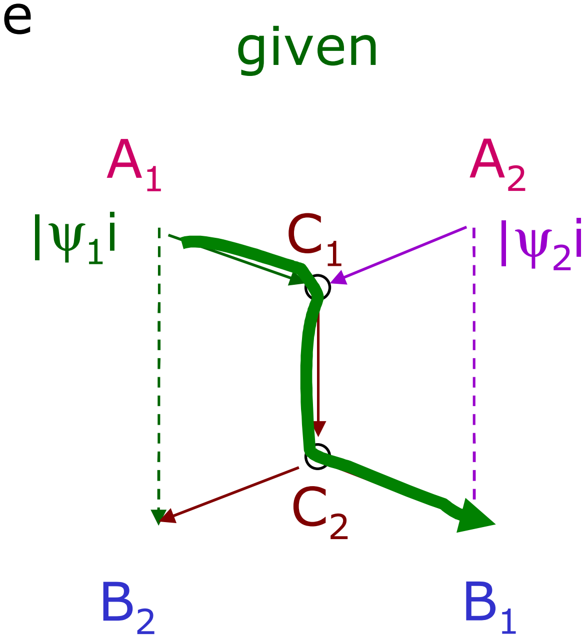
achievable

no-go beyond

What can be done

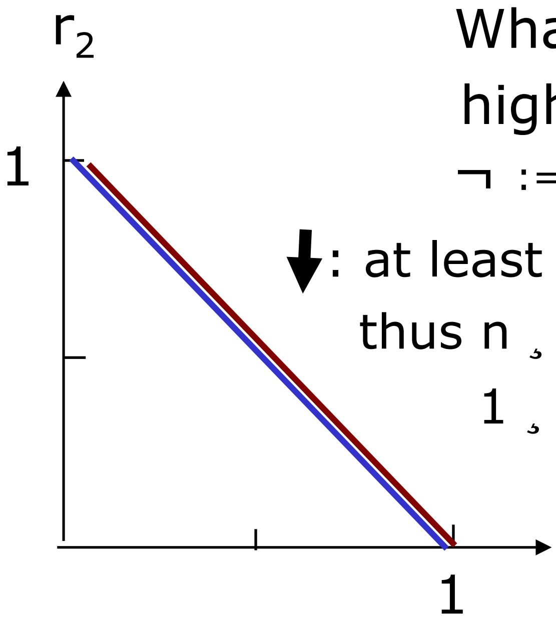
$r_1=1$, $r_2=0$ is achievable

So is $r_1=0$, $r_2=1$



Motivating example : the butterfly network

Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ no assistance



What cannot be done

high fidelity

¬ := uncorrelated with

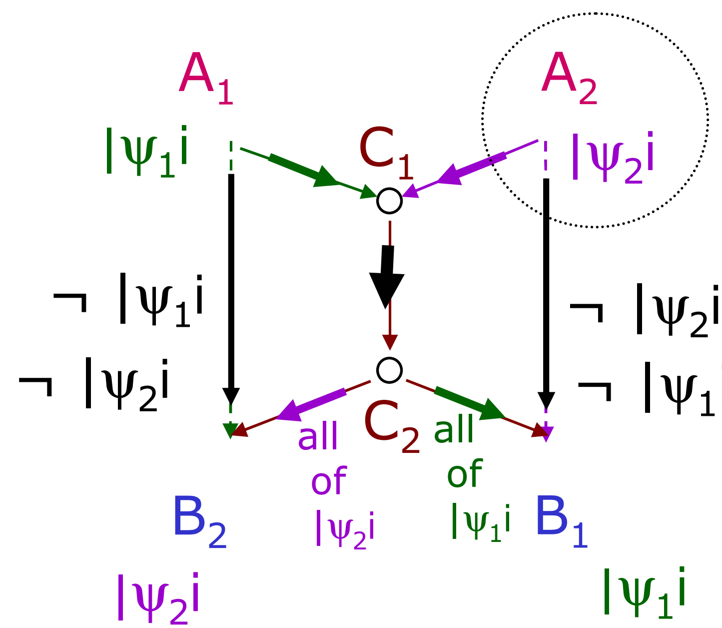
⬇ : at least n_1+n_2 qubits

thus $n \geq n_1+n_2$

$1 \geq r_1+r_2$

each line: n-qubit thick

given



achievable

no-go beyond

Optimal protocol: time sharing
between 2 trivial 1-shot solutions

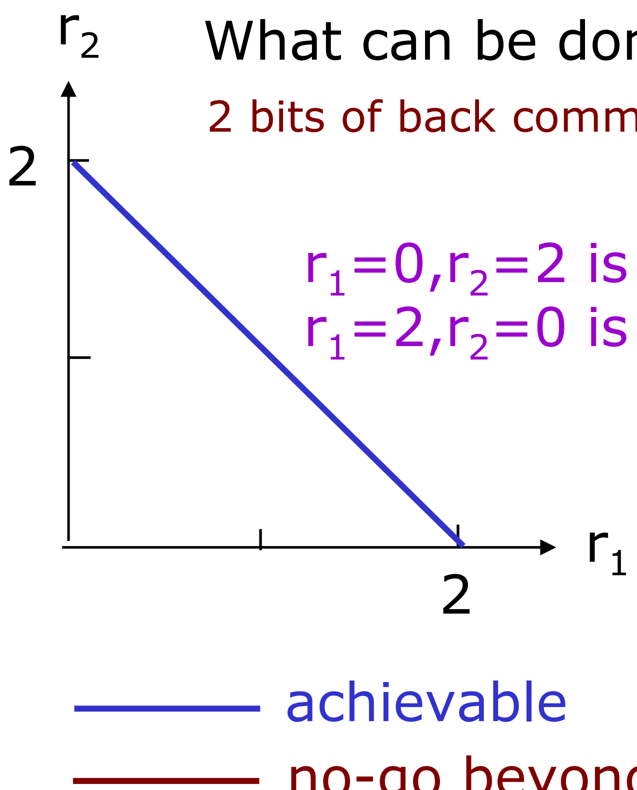
Any question about the quantum case with no assistance?

Motivating example : the butterfly network

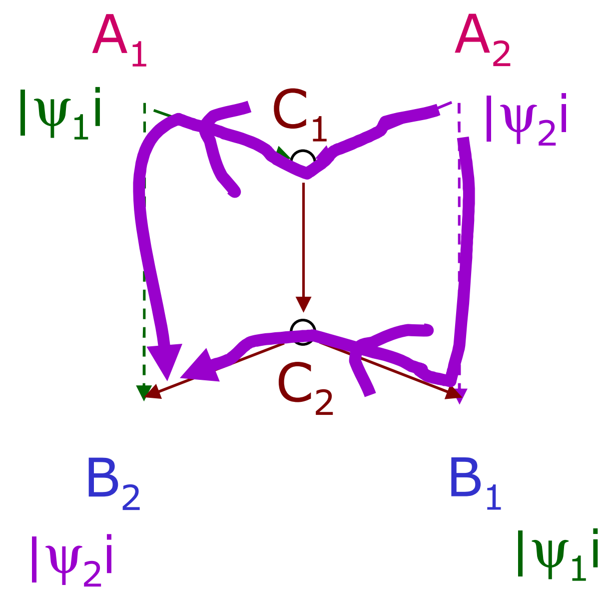
Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ free 2-way CC

What can be done

2 bits of back comm reverts a quantum channel



$r_1=0, r_2=2$ is achievable
$r_1=2, r_2=0$ is achievable



Motivating example : the butterfly network

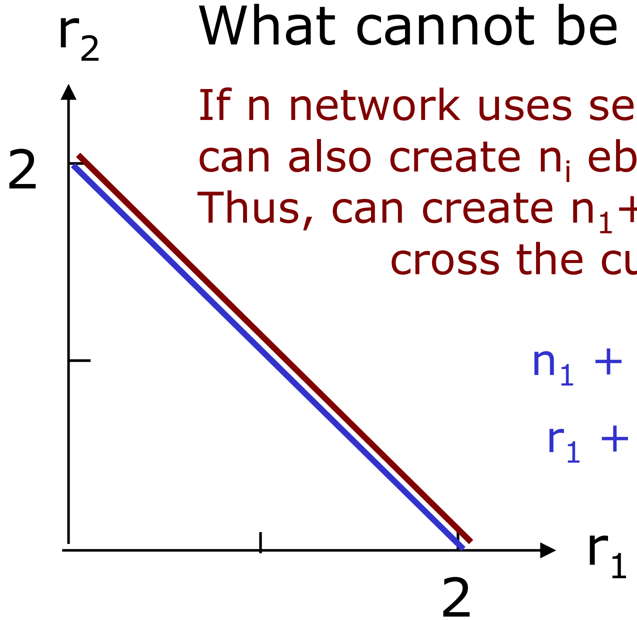
Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ free 2-way CC

What cannot be done asymptotically

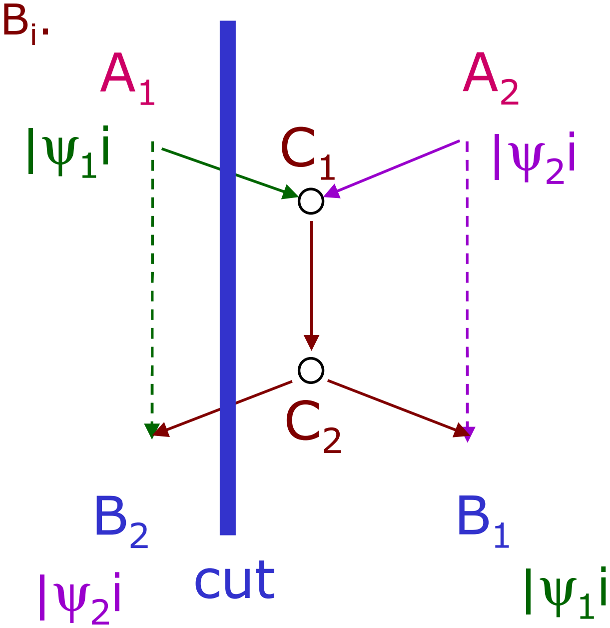
If n network uses send n_i qubits from A_i to B_i ,
can also create n_i ebits between A_i & B_i.
Thus, can create n_1+n_2 ebits
cross the cut

$n_1 + n_2 \le 2n$

$r_1 + r_2 \le 2$



achievable
no-go beyond

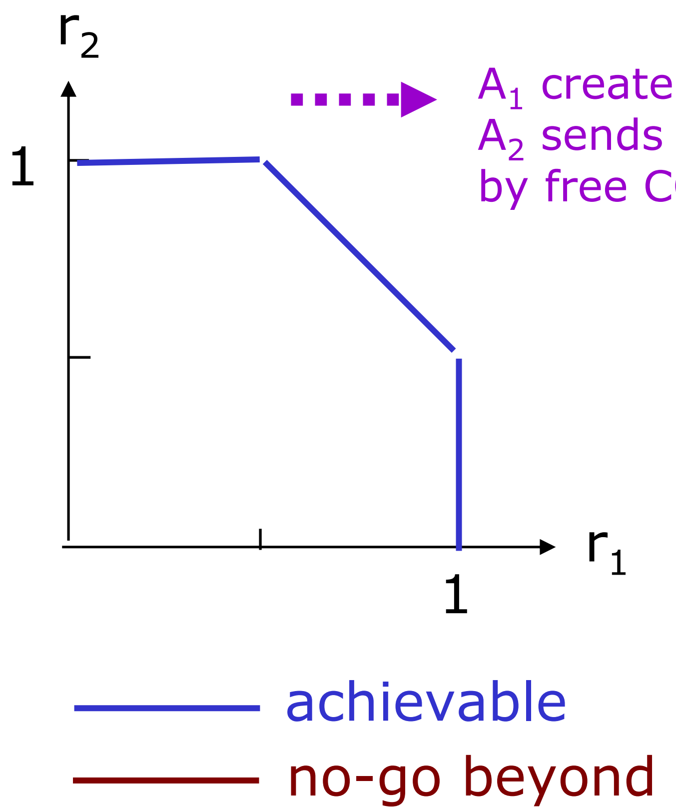


Optimal solution : time sharing of two 1-shot protocols
Note: free two-way CC no better than 4 bits of back comm

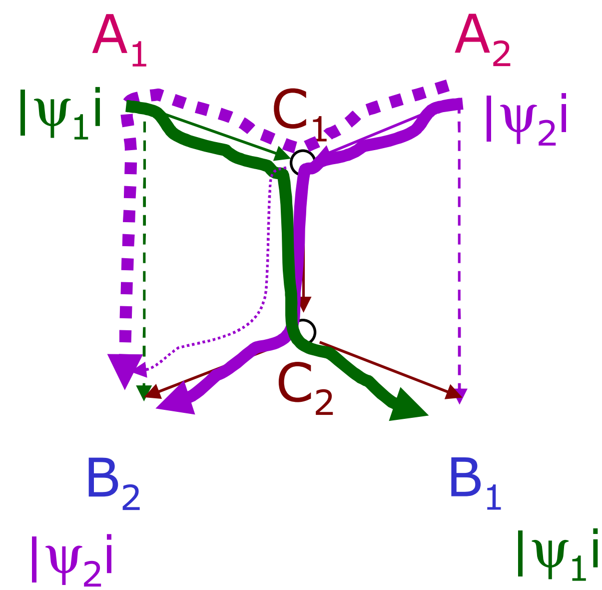
Any question about the quantum case
with free classical communication assistance?

Motivating example : the butterfly network

Quantum: for independent $|\psi_1\rangle$, $|\psi_2\rangle$ free FORWARD CC



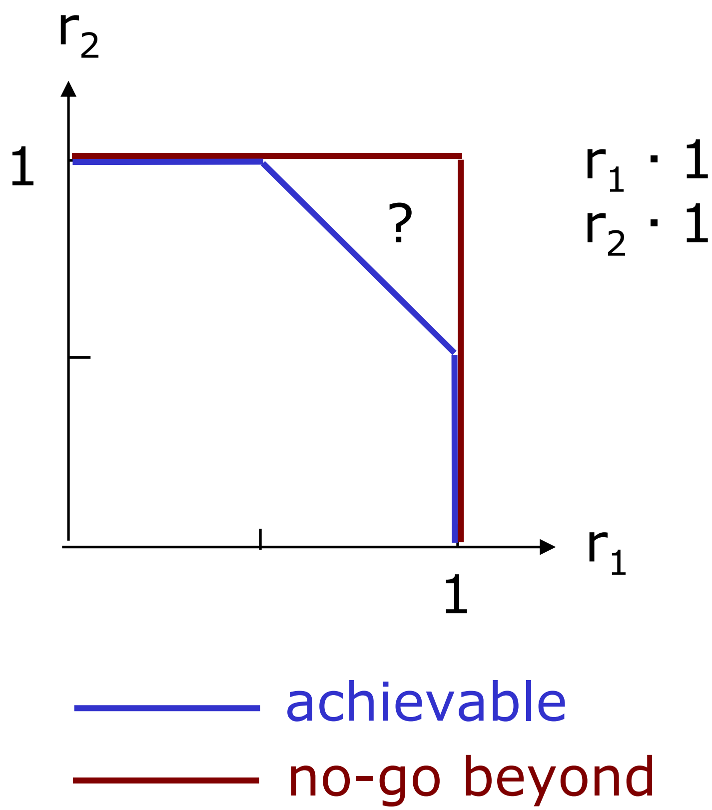
A_1 creates an ebit between C_1 and B_2
A_2 sends $|\psi_2\rangle$ to C_1 who teleports it to B_2
by free CC to C_2 to B_2

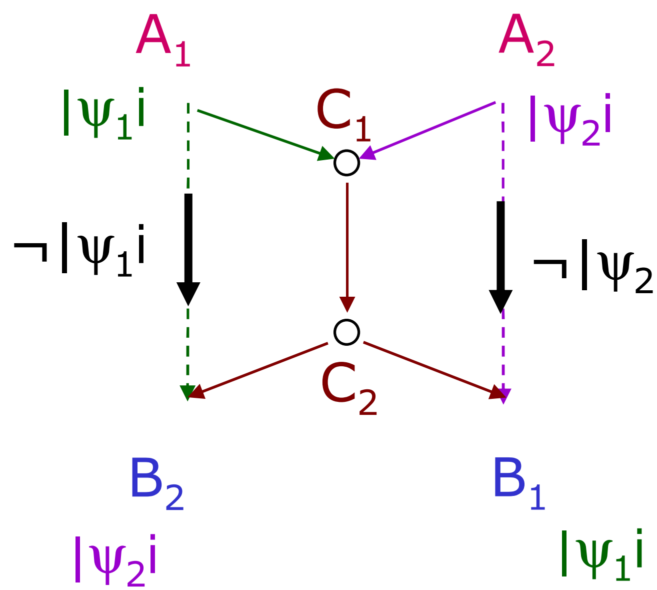


$n_1=1$, $n_2=2$ for $n=2$, hence $(r_1, r_2) = (0.5,1)$ achievable
So is $(1,0.5)$ by symmetry. Time sharing the two.

Motivating example : the butterfly network

Quantum: for independent $|\psi_1 i$, $|\psi_2 i$ free FORWARD CC





Motivating example : the butterfly network

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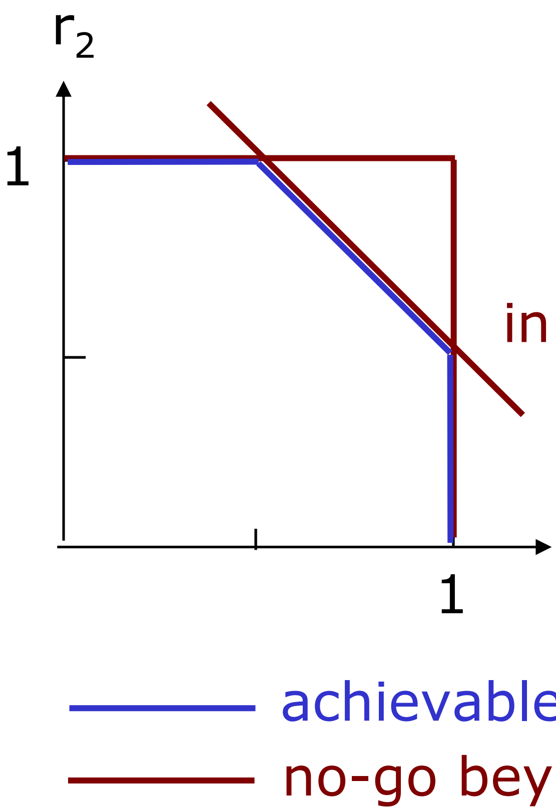
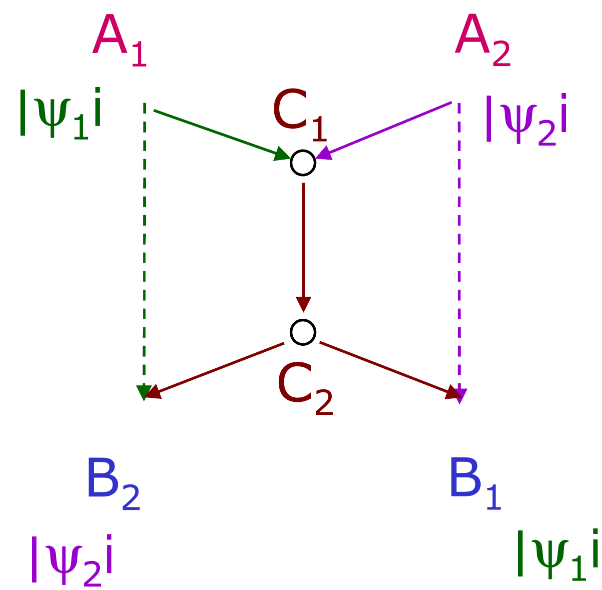


$r_1 \le 1$
$r_2 \le 1$

in fact

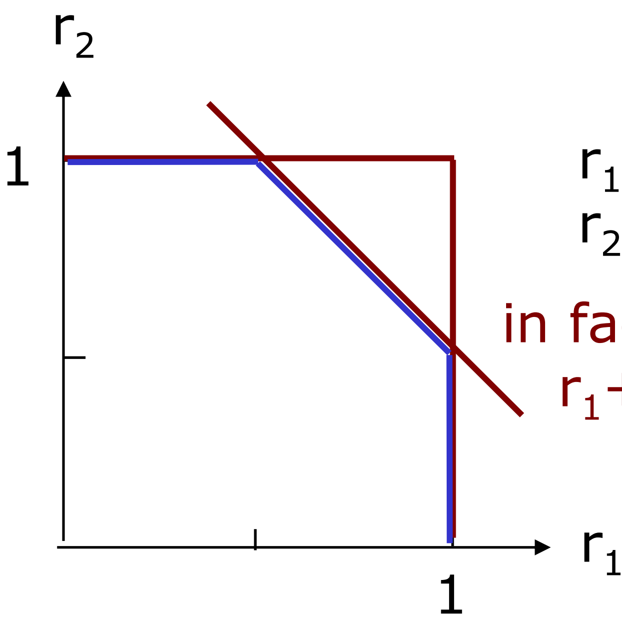
$r_1+r_2 \le 3/2$

How many ways can A_2 send to B_2?



Motivating example : the butterfly network

Quantum: for independent $|\psi_1 i$, $|\psi_2 i$ free FORWARD CC

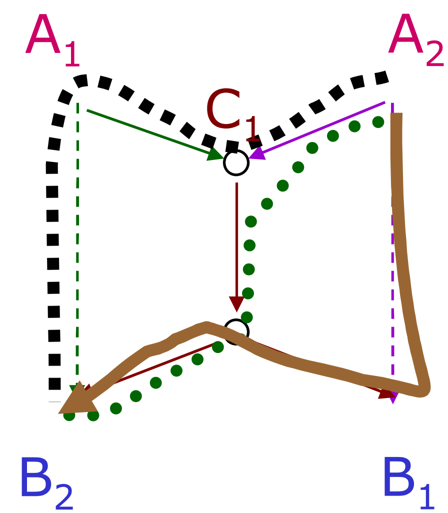


$r_1 \cdot 1$
$r_2 \cdot 1$

in fact
$r_1+r_2 \cdot 3/2$

achievable
no-go beyond

How many ways can A_2 send to B_2?

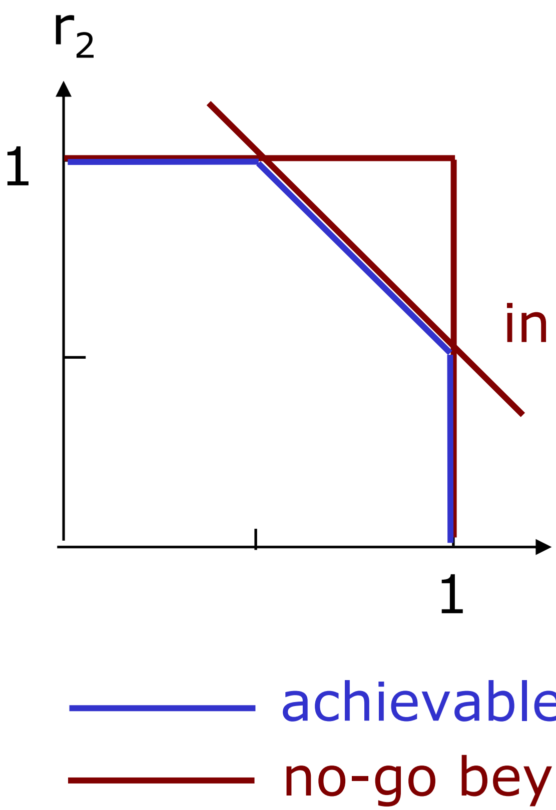


Path via B_1 (a sink) is invalid!

i.e. all comm from A_2 to B_2 goes through C_1. Similarly for A_1 to B_1

Motivating example : the butterfly network

Quantum: for independent $|\psi_1$i, $|\psi_2$i free FORWARD CC



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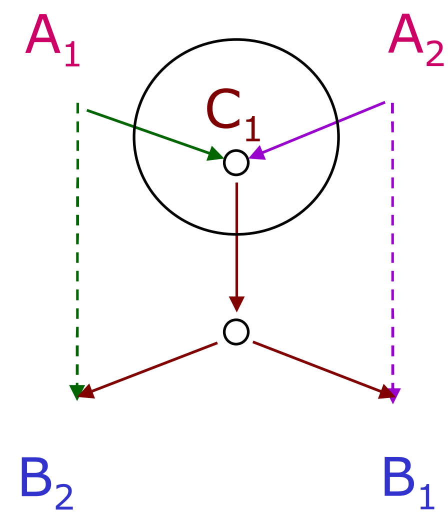
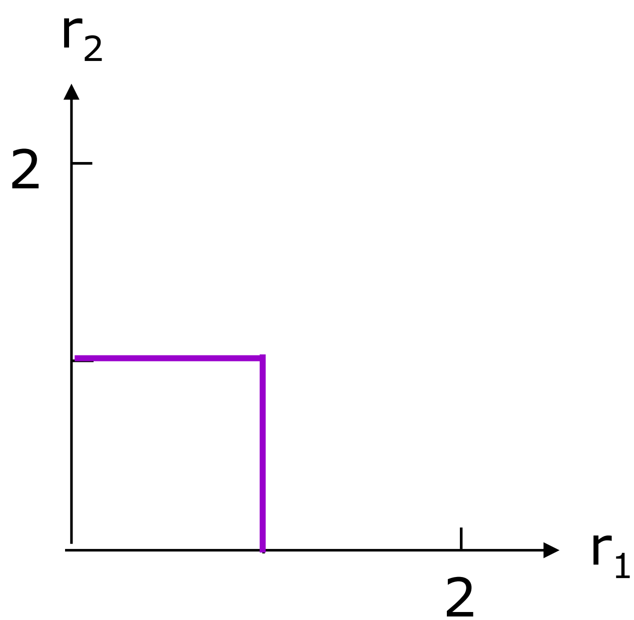


To send a qubit from A_i to B_i require 2 edges going in/out of C_1. Thus, if n_1+n_2 qubits are sent using n networks, $n_1+n_2 \cdot 3n/2$

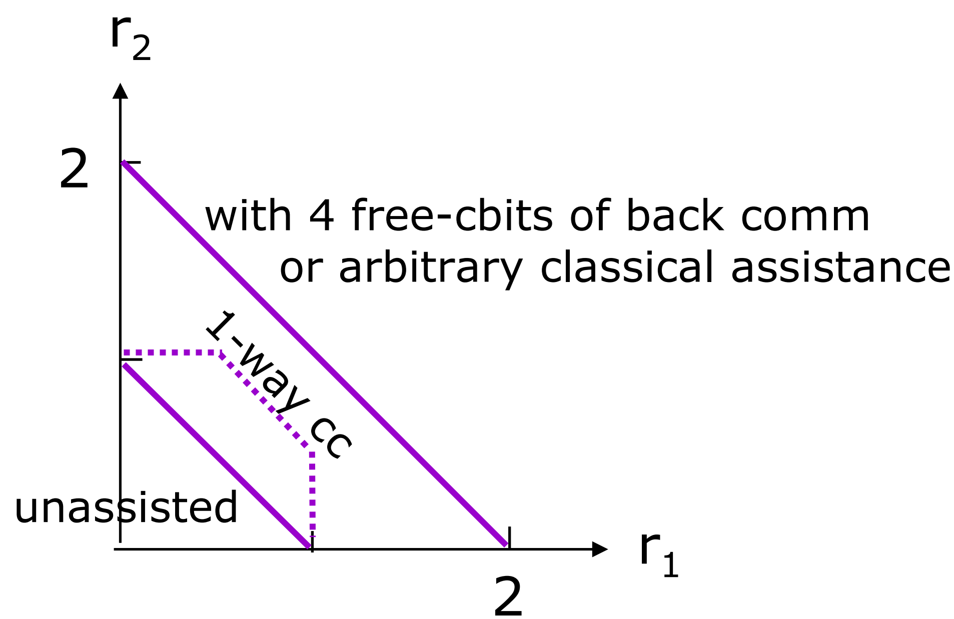
Any question about the quantum case with free forward classical communication assistance?

Summary for the butterfly network:

Uniquely optimal
classical rate region



Rate optimal - high fidelity
quantum rate regions



Summary for the butterfly network:

Quantum case:

The surprising is the simpicity (boringness) of the optimal solutions : each "leg" is either used for A_1 ! B_1 comm or A_2 ! B_2 comm

Contrast to the classical case, quantum info runs down a network like incompressible water running down pipes.

Does this simplifying feature hold in more general networks?

Generalization:

- arbitrary number of receivers (B_i's)
  (focusing on the case when they are sinks)
- arbitrary number of "helpers" (C_i's)
- arbitrary number of senders (A_i's) who may also be helpers
- arbitrary directed graph
  - vertices represent players
  - weighted edges represent noiseless channels of weighted capacity available in 1 network call

Task: communication of independent messages between specific sender-receiver pairs.

Goal: find asymptotic achievable rate region

Generalization:

Progress report:

- All cases of classical communication assistance : optimality of "rerouting" holds for "shallow" networks (low depth treating the network as a circuit) 98%

- with back communication, 2 senders, 2 receivers, the mincut method give tight bound on the rate sum 98%

- for broadcasting (sharing a cat state between reference and all receivers, the classical solution holds) 97%

- entanglement-assisted case : trivial "superdense coding protocol" occurs to be optimal, but rigorous proof of optimality needed 93%

Conjecture : rerouting is optimal in general graph in all scenarios 5%

Any other question?

Reference: 0608666
???

The analysis of the attitude test does NOT reflect the opinion of the speaker

Thank you ...