The Wavefunction of the Universe

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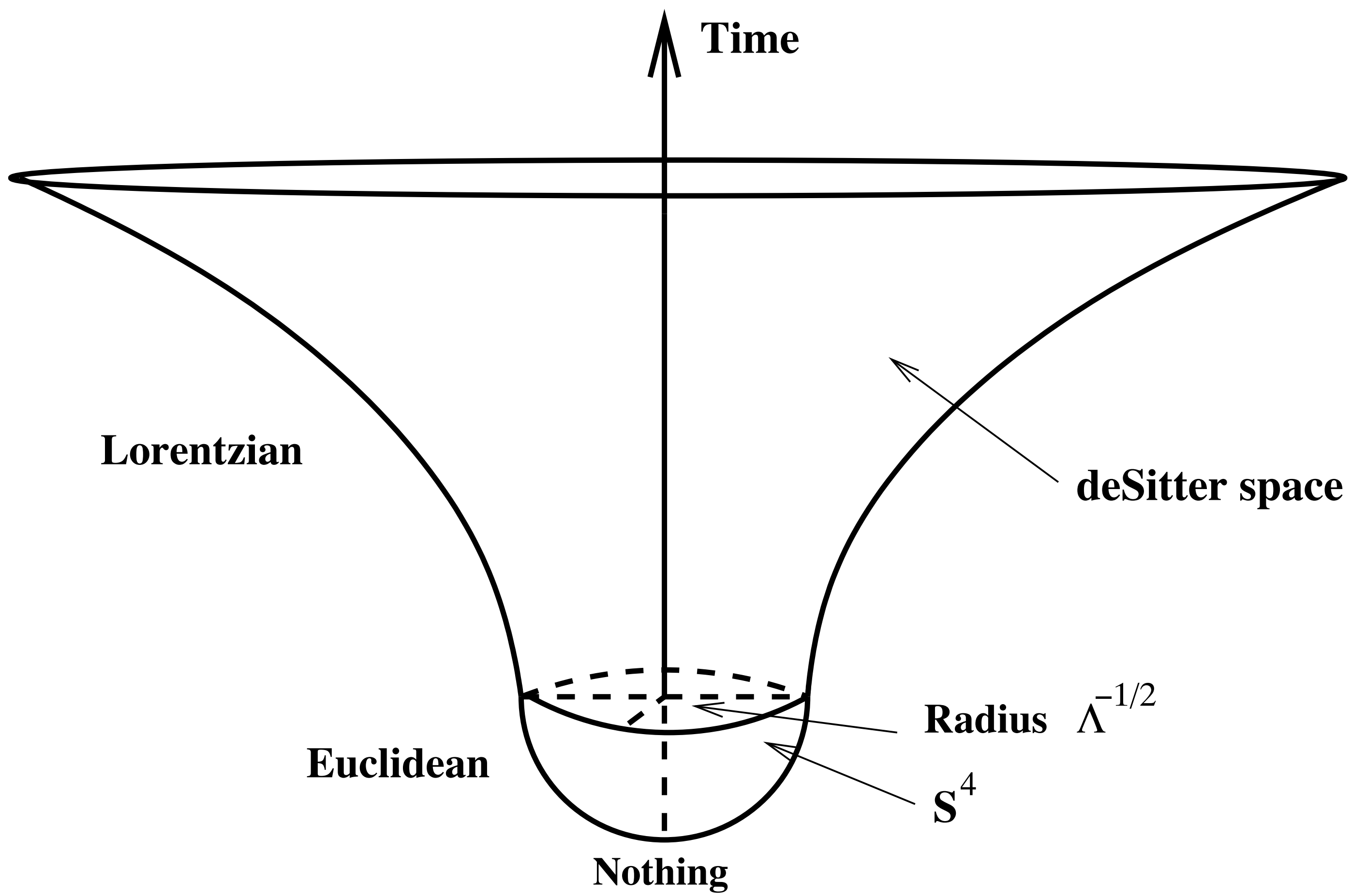
Cosmic Landscape

- Why we end up at the particular vacuum in the cosmic landscape ?
- How we end up here ? Start with the original universe which evolves cosmologically to today's vacuum.
- We like to have an alternative to the anthropic principle.
- A speculative approach : Tunneling from Nothing.
- We need to improve the Hartle-Hawking wavefunction.
- Application of the improved wavefunction to SOUP - Selection of the Original Universe Principle

Tunneling from Nothing



Vilenkin, 1982, 1983
Hartle and Hawking, 1983

Review

- The $4-D$ Euclidean Action:

$$S_E = -\frac{1}{16\pi G}\int d^4x\sqrt{|g|}\,(R-2\Lambda)$$

- Metric ansatz (minisuperspace):

$$ds^2 = d\tau^2 + a^2(\tau)d\Omega_3^2$$

- Euclidean Einstein's Equations (closed universe):

$$-2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} + \frac{1}{a^2} = \Lambda/3 = H^2$$

$$-\dot{a}^2 + 1 = H^2a^2$$

which gives the S^4 instanton solution:

$$a(\tau) = \frac{1}{H}\cos(H\tau)$$

- continued to Lorentzian signature :

$$a(t) = \frac{1}{H}\cosh(Ht)$$

Hartle-Hawking Wavefunction

- $\Psi_{HH} = \int_O^{h_{ij}} D[g] e^{-S_E[g]}$ $\qquad$ $P = |\Psi_{HH}|^2$

- The Euclidean action in minisuperspace :

$$S_{E,0} = \frac{1}{2} \int d\tau \left(-a\dot{a}^2 - a + \lambda a^3 \right)$$

- $R = 12H^2 = 4\Lambda \qquad V_4 = 8\pi^2/3H^4$

  so the value of $S_{E,0}$ of the S^4 instanton :

$$S_{E,0} = -3\pi/G\Lambda$$

- Entropy $= 3\pi/G\Lambda$

  Gibbons, Hawking, Perry

- Tunneling probability :

$$P \simeq e^{-S_{E,0}} = e^{3\pi/G\Lambda}$$

Problems

In models where Λ is dynamical, e.g., models with four-form flux,

(Brown-Teitelbaum, Bousso-Polchinski)

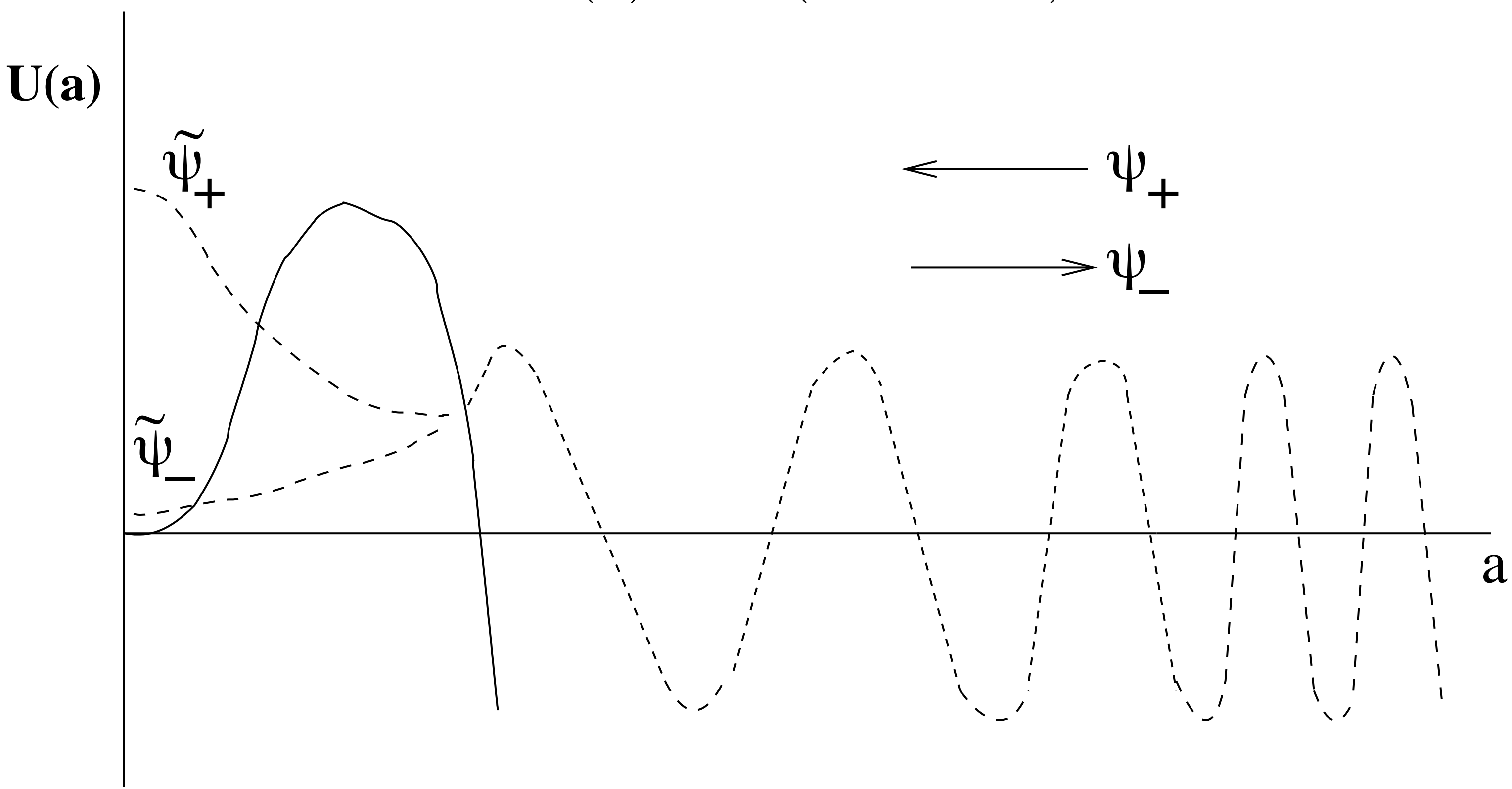
- $\Psi_{HH} \sim e^{3\pi/2G\Lambda} \quad \rightarrow \quad$ Such a universe prefers $\Lambda \rightarrow 0$, $\Psi \rightarrow \infty$ and Size $\rightarrow \infty$ ($a \sim \frac{1}{\sqrt{\Lambda}}$)
- This means the Euclidean action does not have a minimum : $S_E \rightarrow -\infty$.
- This renders Ψ_{HH} unnormalizable.
- This infrared divergence is related to the lack of a lower bound to the Euclidean action in theories with dynamical Λ.
- Loop and/or string corrections not helpful.
- Problem with other topologies:

  Coleman, Weinberg, Hawking, Preskill, Strominger, Giddings, Klebanov, Susskind, Banks, Fischler, Morgan, Polchinski etc

Upon quantization, $p = \dot{a}a \rightarrow id/da$, and the Wheeler-DeWitt equation becomes

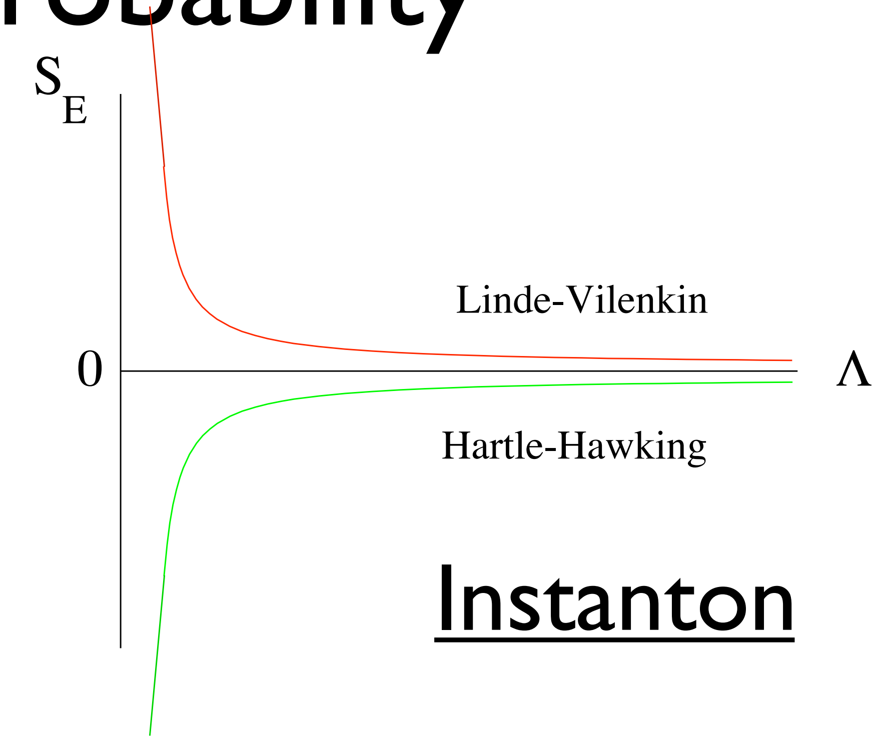
$$\left(\frac{-d^2}{da^2} + U(a) \right) \Psi(a) = 0$$

$$U(a) = a^2(1 - H^2 a^2)$$



Tunneling Probability

A 20 year old puzzle



WKB

Instanton

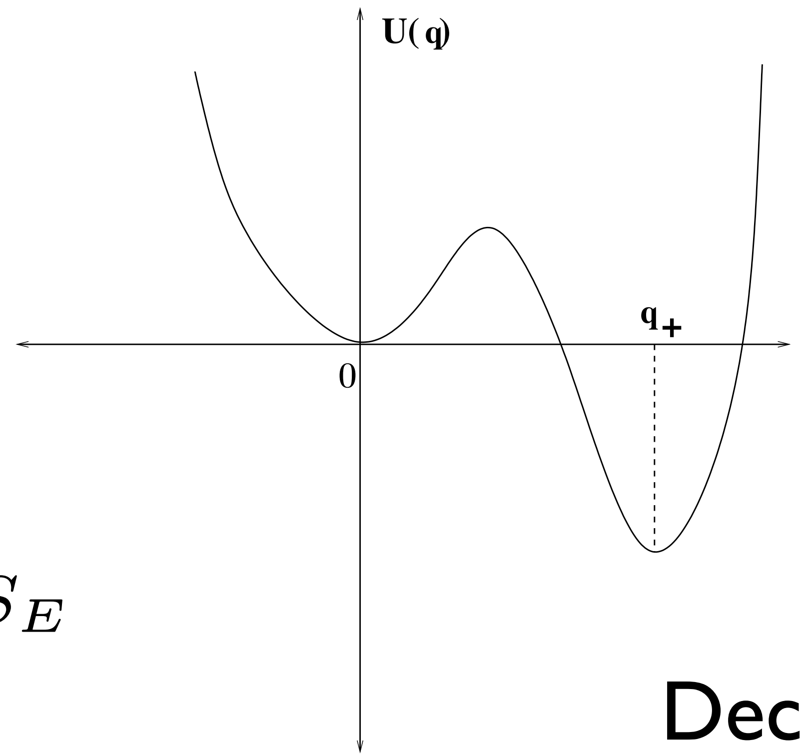
QFT : $$e^{-2\int\sqrt{2V}} \quad = \quad e^{-S_E}$$

↙ ↘

Gravity : $e^{-2\int\sqrt{2V}} = e^{+S_E}$ $\qquad$ $e^{+2\int\sqrt{2V}} = e^{-S_E}$

Linde-Vilenkin $\qquad$? $\qquad$ Hartle-Hawking

Tunneling in QM



$$P = e^{-S_E}$$

Decoherence

In the presence of other degrees of freedom (the environment) interacting with the system q, tunneling of q is suppressed.

1983 A. Caldeira and A. Leggett, J. Sethna, ...

Comments

- Increase in S_E is due to the longer path length in the many dimensional (q, x_α) space.

$$S_E = \int \sqrt{2MU(q, x_\alpha)} ds$$

$$ds^2 = dq^2 + \sum \frac{m_\alpha}{M} dx_\alpha$$

- Interaction of q with x_α interferes with its attempt to tunnel. This interaction can be seen as attempts to observe q. Repeated measurements of q suppresses the tunneling rate.

- Interaction of q with the environment introduces decoherence, which makes the system to behave less quantum and more like classical.

- For bounded case, this is merely a correction.

- For gravity, decoherence shall provide a bound to Euclidean action. So the change is qualitative.

Decoherence in Euclidean Gravity

- In mini-superspace, only the cosmic scale factor a(t) is kept.
- Metric perturbations around a(t) as well as matter fields should be included.
- Since these other modes are not measured, they constitute the environment and so should be integrated out. That should generate a decoherence term which should suppress tunneling.
- However, integrating them out in the S^4 background does not yield a correction term we are looking for.
- Instead, integrating out the environment with arbitrary a(t) does generate a new term, resulting in a modified action.
- Solving the modified action generates a decoherence term.
- That is, the back-reaction is crucial.

Calculations

Including the metric perturbations, i.e., tensor modes, and light scalar fields :

$$S_E = \frac{1}{2}\int d\tau a^3 \left(-\frac{\dot{a}^2}{a^2} + \Lambda - \frac{1}{a^2} + \frac{\nu}{a^4}\sum n\right)$$

$$\sum n = \alpha a^4 + \beta a^2 + \frac{c}{\Lambda^2}$$

$$c \geq 0$$

$$a_0(\tau) \to a(\tau) = a_0(\tau) + \delta a$$

$$\sum n \sim a^4 = (a_0(\tau) + \delta a)^4$$

The new terms becomes a radiation term after tunneling.

The Modified Bounce

- Tracing out the environment leads to

$$S_{E,dC} = S_{E,0}[a] + D[a]$$

$$= \frac{1}{2}\int d\tau \left(-a\dot{a}^2 - a + \Lambda a^3 + \frac{\nu}{\Lambda^2 a}\right)$$

- $\nu \sim M_s^4$ is related to the large wavelength (Hubble) and small wavelength (*string*) cutoffs.

- Equation of Motion

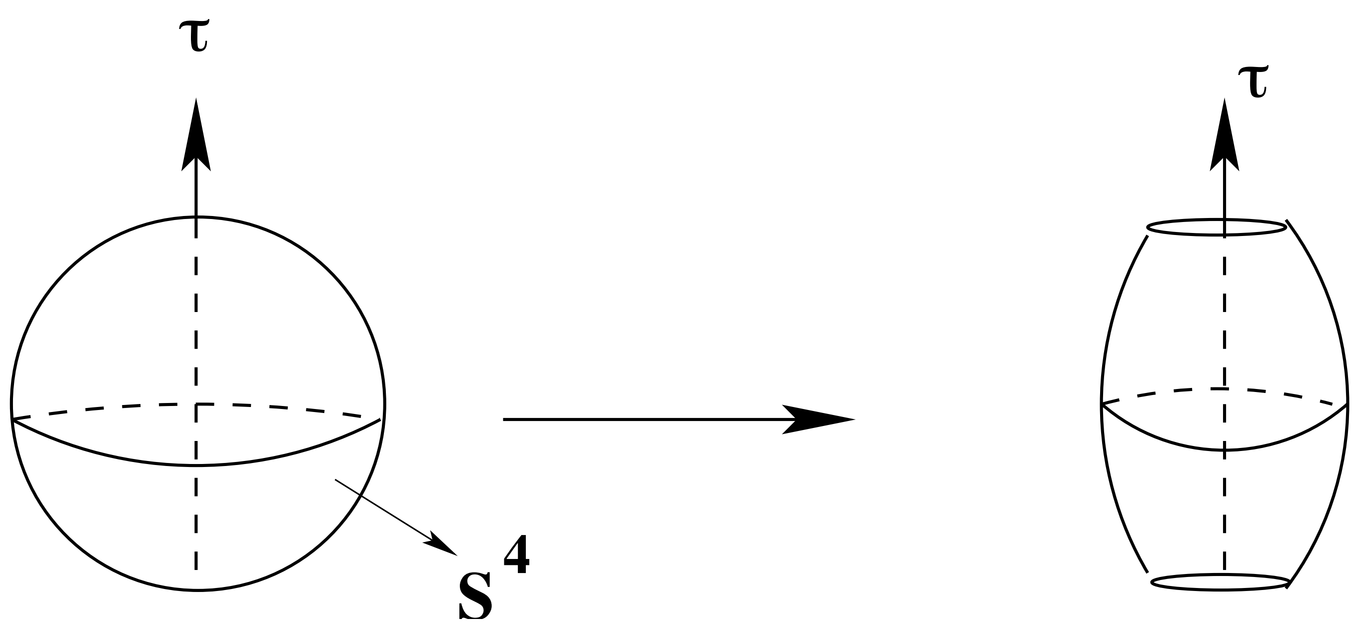
$$-2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} + \frac{1}{a^2} = \frac{\Lambda}{3} - \frac{\nu}{\Lambda^2 a^4}$$

- Modified Bounce Solution

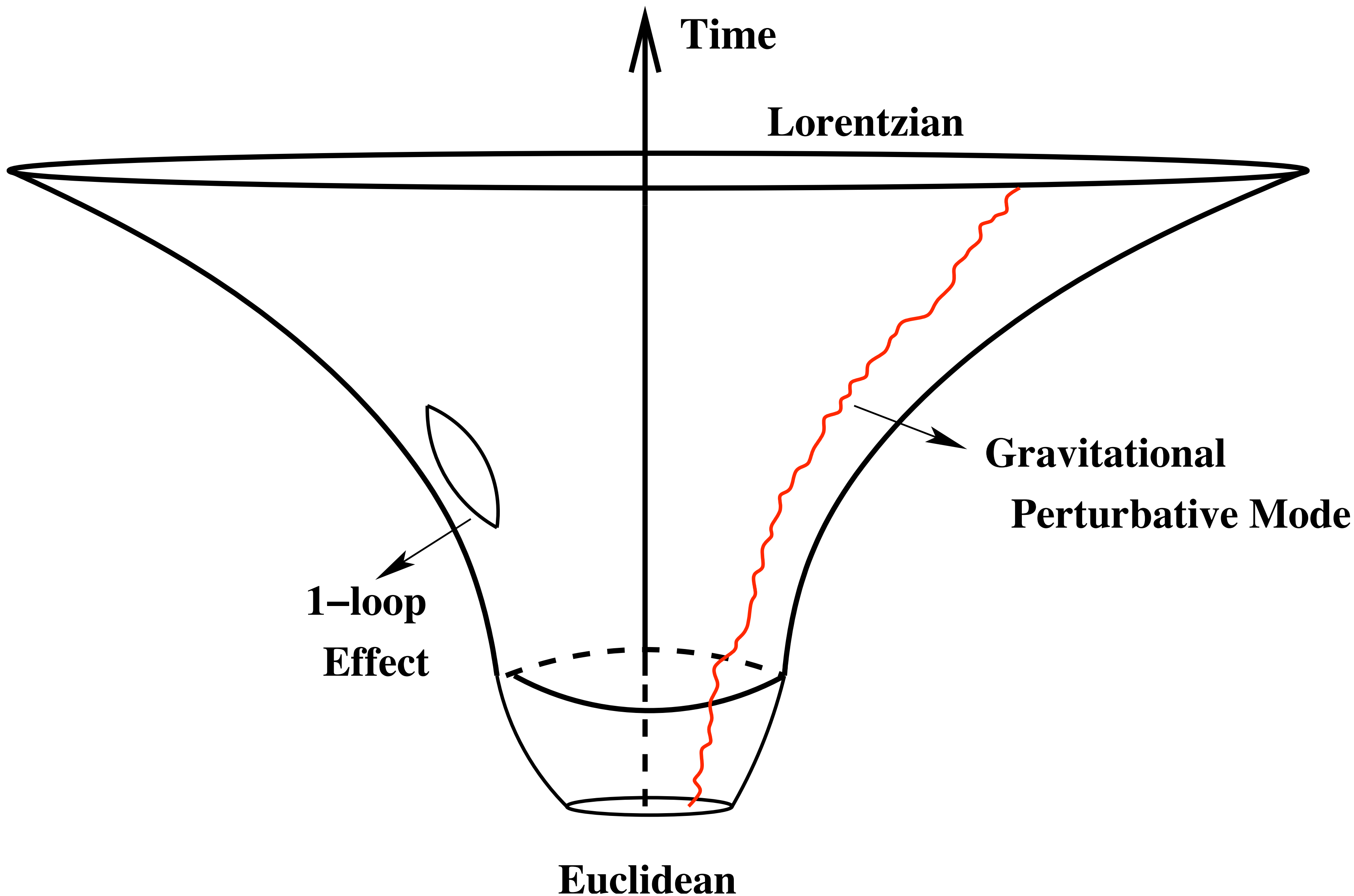
$$a(\tau) = \frac{1}{\sqrt{2}H}\sqrt{1 + \sqrt{\left(1 - \frac{4\nu}{\Lambda}\right)}\cos(2H\tau)}$$

- S^4 recovered when $\nu = 0$.

Decoherence effect



Tunneling from Nothing

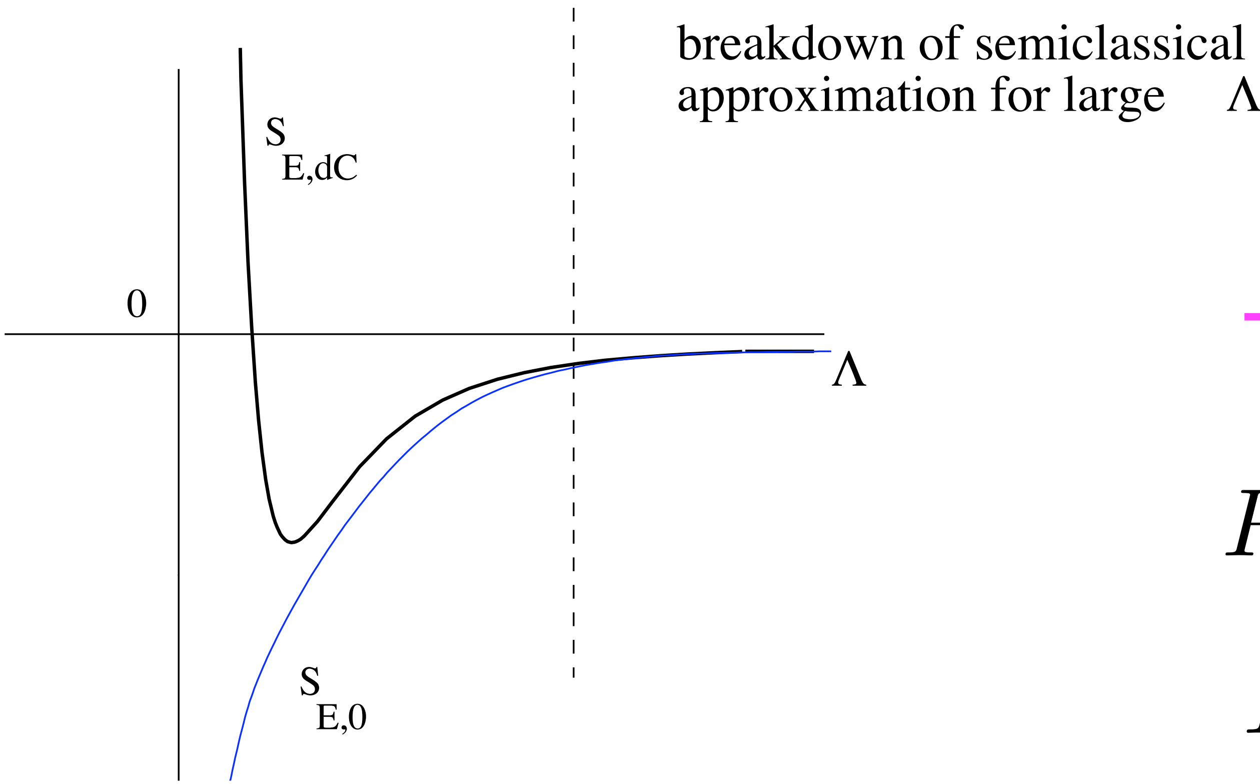


Nothing

The physical picture

- The new term is simply radiation.
- The S4 Euclidean action leads to pure deSitter space, while the barrel-shaped instanton leads to a universe with a cosmological constant and some radiation.
- With a S4 spherical instanton, radiation (environment) cannot be present, so there is no decoherence. So back-reaction is crucial.
- For small universe (large Λ), decoherence is negligible. For large universes, decoherence is very important.
- Since string scale is below the Planck scale, the semi-classical approximation is valid for intermediate values of Λ.
- Hartle-Hawking wavefunction is improved.

The improved Euclidean action $S_{E,dC}$ vs $S_{E,0}$



Tunneling probability

$$P \simeq e^{-S_{E,dC}} = e^{F}$$

$$F = \frac{3\pi}{G\Lambda} - \frac{12 n_d}{\Lambda^2 l_s^4}$$

$S_{E,0}$ is unbounded from below, but the interaction with the environment has made $S_{E,dC} = -F$ bounded from below.

Inclusion of environment will enhance tunneling in the Linde-Vilenkin scenario

and

suppress tunneling in the Hartle-Hawking scenario

Inclusion of environment also provides a lower bound to the Eclidean action, so the improved wavefunction is normalizable.

Improved Wavefunction in 10 Dimensions

In 10-D,

$$S_{E,dC} \simeq S_{E,10} + c\left(\frac{V_{10}}{l_s^{10}}\right)$$

where $S_{E,10}$ is the 10-D Euclidean action determined in mini-superspace and V_{10} is the 10-dimensional volume of the instanton. In effective 4-D theory, $S_{E,10}$ reduces to $-3\pi/G\Lambda$, since

$$8\pi G = M_{Pl}^{-2} = \frac{g_s^2 l_s^8}{4\pi V_6}$$

where V_6 is the 6-D compactification volume.

$$c \simeq \frac{n_d}{\pi}\frac{1}{M_{Pl}^2 l_s^2 g_s^2}$$

Tunneling Probabilities

$$P \simeq e^{F} = e^{-S_{E,dC}}$$

- Tunneling to an inflationary universe (KKLMMT model with fluxes M and K fixed to maximize F)
  $F \sim 10^{14}$.

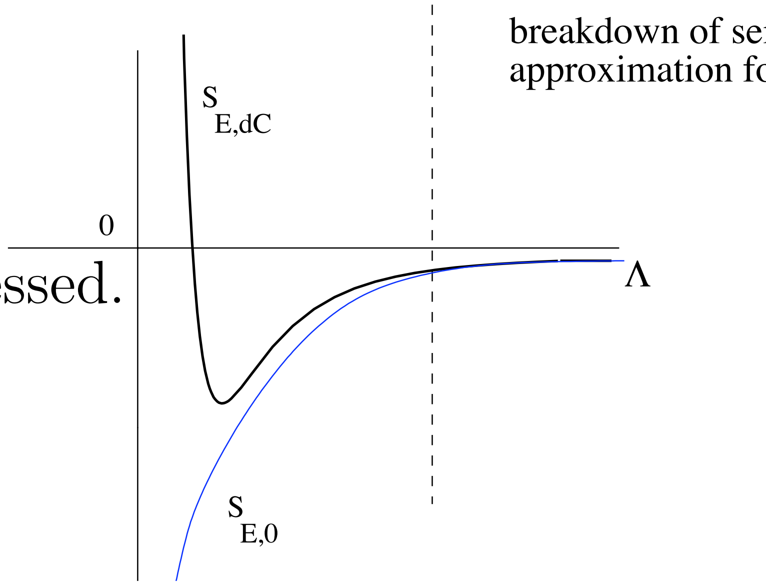
- To 10-D deSitter space S^{10}
  $F \sim 10^{4}$.
  Similarly, for $S^4 \times S^6$, $S^5 \times S^5$, etc.

- Supersymmetric vacua are totally suppressed.

- Quantum foam is very suppressed.

- To a KKLT-like vacuum
  $F < 0$ or zero tunneling probability.

- To a vacuum with today's cosmological constant
  $F < -10^{100}$ or zero tunneling probability.



SOUP

Selection of the Original Universe Principle

- Decoherence term must be included in the improved wavefunction. This is the simplest extension beyond the minimal minisuperspace.
- The improved wavefunction gives the probability of tunneling from nothing to any point in the landscape.
- Comparing the probabilities will tell which vacua are preferred.
- Once the preferred vacuum (likely to be an inflationary universe) is created, it can follow a path that leads to today's vacuum with a low cosmological constant.

Summary and Conclusion

- The modified wavefunction with the inclusion of the environmental effect can be used as a selection principle on the cosmic landscape.
- Decoherence effect provides a lower bound to the Euclidean gravitational action.
- A 4-D inflationary universe seems to be favored over supersymmetric vacua, KKLT-like vacua and vacua such as S^{10}, $S^4 \times S^6$ etc.
- The final universe must lie on the path that the preferred vacuum evolves along. We call this SOUP - Selection of the Original Universe Principle.
- Find other vacua, especially vacua with other large spatial dimensions; and determine the tunneling probability from nothing to any one of them. Find out whether 4D is selected by SOUP or not.