New Phenomena in 2d String Theory

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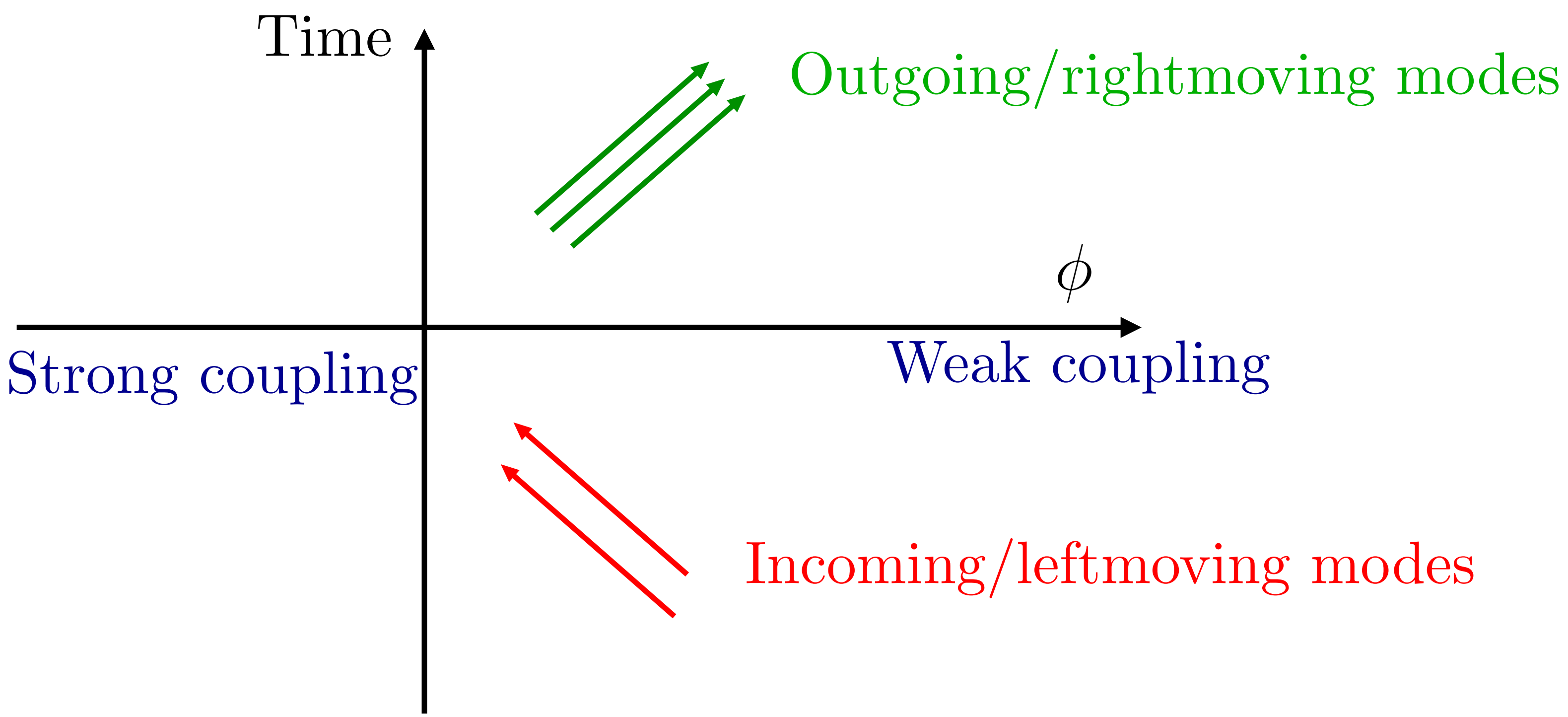
Low Dimensional String Theories

Matrix models give complete nonperturbative definitions of some string theories – only known well defined string theories which are exactly solvable.
They can be used as laboratories for new stringy effects.

Minimal String Theories ($c < 1$) describe strings in one Euclidean dimension (many examples).
They exhibit: D-branes, holography, RR-flux, connections to integrable systems, topological strings...

The two dimensional theories have time and hence are richer.

Two Dimensional Theories



Scattering is from and to null infinity at the weak coupling end (the strong coupling region is effectively compact).

The bosonic, 0A and 0B string theories have known formulations in terms of matrix models which allow us to explore their strong coupling region.

We will discuss other theories: IIA, IIB, HO and HE (no known matrix model). We will focus on the simple physics in the weak coupling region.

They raise many issues including:

- The excitations visible in the worldsheet cannot have a unitary S-matrix – need massless solitons.
- New stringy phase transitions – peculiar thermodynamics.

Spectrum of the Simplest Theories

Bosonic: massless “tachyon” $T(p)$

0A: massless “tachyon” $T(p)$

0B: massless “tachyon” $T(p)$
massless RR scalar $C(p)$
(nonperturbative massless solitons of C)

Type II

Orbifold the type 0 theories by leftmoving worldsheet fermion number. $T(p)$ and $C_-(p)$ are projected out. The twisted sectors have spacetime fermions.

IIA: Majorana fermion	$\Psi_-(p \leq 0)$	$\longleftarrow$
	$\Psi_+(p \geq 0)$	$\longrightarrow$
IIB: Weyl fermion	$\Psi_-(p \leq 0)$	$\longleftarrow$
	$\widetilde{\Psi}_-(p \leq 0)$	$\longleftarrow$
Chiral scalar	$C_+(p \geq 0)$	$\longrightarrow$

Comments

The projection in the twisted sectors is opposite to 10d.

IIA is worldsheet chiral; IIB is spacetime chiral.

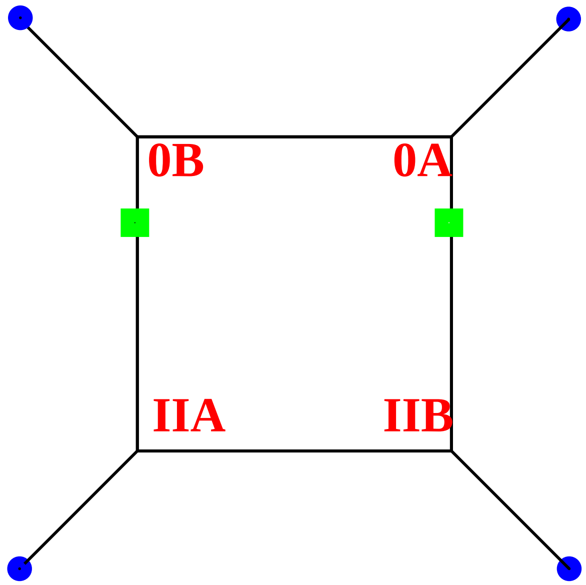
Finite number of particles – no Hagedorn density of states.

No unitary S-matrix of these excitations!
Expect: additional massless excitations – solitons made out of the chiral scalar C.

Compactifications

Compactify Euclidean time $x \sim x + 2\pi R$

Each of these theories has a $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry which we can twist by. The moduli space



Comments

All four theories are connected.

T duality relates different theories or a theory to itself.

The circles are selfdual points with continuous symmetry.

The squares were introduced in [Kutasov and NS].

Physical vertex operators – states in 1d – have either momentum ($w = 0$) or winding ($p = 0$). All of them except $p = w = 0$ are massive.

Torus Amplitudes

$$\Gamma = aR + \frac{b}{R}$$

$a =$ vacuum energy density – independent of the compactification. It is infinite in field theory but finite in string theory.

b is calculable in field theory as $\sum |p|$. For thermal circles ($R \sim \frac{1}{T}$) b measures the number of degrees of freedom.

T duality relates a and b of different compactifications. Therefore, a can be calculated as $\sum |w|$.

2d Heterotic String

First discussed in [McGuigan, Nappi and Yost].

HO: has $Spin(24)$ symmetry,

24	massless "tachyons"	$T^I(p)$	$\leftrightarrows$

HE: has $Spin(8) \times E_8$ symmetry,

$\mathbf{8_v}$	massless "tachyons"	$T^i(p)$	$\leftrightarrows$
$\mathbf{8_s}$	leftmoving fermions	$\Psi^\alpha(p \geq 0)$	$\longleftarrow$
$\mathbf{8_c}$	rightmoving fermions	$\widetilde{\Psi}^{\dot{\alpha}}(p \leq 0)$	$\longrightarrow$

Again, no Hagedorn density of states.

Compactifications

Depending on the radius and Wilson lines

$$\mathcal{M} = SO(13,1,\mathbb{Z})\backslash SO(13,1)/SO(13)$$

Infinite number of states with both p and w (not only pure p or w)!
No tachyons.

The thermal circles are selfdual

$$Spin(24) \times U(1) \to Spin(26)$$

$$Spin(8) \times E_8 \times U(1) \to Spin(10) \times E_8$$

Torus Amplitude

The torus amplitude depends on the moduli in $\mathcal{M}$.

Consider, for example, the HE theory on a thermal circle

$$\Gamma = \begin{cases} \dfrac{1}{R} & R \geq 1 \\ R & R \leq 1 \end{cases}$$

Note, the vacuum energy density (a) vanishes.
As expected, it is T-dual ($R \to \frac{1}{R}$), but it not smooth at the selfdual point $R = 1$!

Thermodynamics

The one loop approximation of $\mathrm{Tr}\, e^{-H/T}$ is smooth as a function of $T \sim 1/R$ (no Hagedorn), but it is not T-dual!

The standard proof that $\mathrm{Tr}\, e^{-H/T}$ = Euclidean circle amplitude is valid only for sufficiently small T (after Poisson resummation $[\int, \sum] \neq 0$ beyond some T).

The Euclidean time torus amplitude and $\mathrm{Tr}\, e^{-H/T}$ differ for small R!

$$\Gamma = \begin{cases} \dfrac{1}{R} & R \geq 1 \\ R & R \leq 1 \end{cases}$$

$$-\frac{F}{T} = \frac{1}{R} \qquad 0 < R < \infty$$

Physics of the Transition

The transition is driven by the $p = w/2 = \pm 1$ modes with $m(R) = \frac{1}{2}|R - \frac{1}{R}|$.
They extend the **8** of $Spin(8)$ tachyons to **10** of $Spin(10)$ at the selfdual point.
Their effective action

$$\mathcal{L}_\Phi = \frac{1}{2}|\partial_\phi \Phi|^2 + \frac{1}{2} m(R)^2 |\Phi|^2$$

leads to

$$Z_\Phi = -\int \frac{dp}{2\pi} \log(p^2 + m(R)^2) = -|m(R)| + \text{const}$$

which is not analytic in R!

Euclidean Circle $\stackrel{?}{=}$ Temperature

If yes:

First order transition with negative latent heat

Lower entropy for higher T [Atick and Witten]

Standard thermodynamics inequalities are not satisfied

(is the system unstable? to what?)

If no:

Is T meaningful above the transition point T_c?

Perturbation theory is bad at T_c: $[T \to T_c, g_s \to 0] \neq 0$

Conclusions

There are many interesting 2d string theories.

New phenomena: chiral, massless nonperturbative states are needed for unitarity, new phase transitions, peculiar thermodynamics.

It will be nice to have nonperturbative formulations (e.g. matrix models) of these theories.