

Marginal deformations of field theories and their gravity duals

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Outline

- Marginal deformations of $\mathcal{N} = 4$ SYM.
- Deforming geometries with $U(1)^2$ symmetry.
- Applications to IIB SUGRA:
 - deformations of $AdS_5 \times S^5$
 - more general theories
- Applications to M theory
- Summary.

Marginal deformations of $\mathcal{N} = 4$ SYM

- Superpotential of $\mathcal{N} = 4$ theory:

$$\text{Tr}(\Phi_1\Phi_2\Phi_3 - \Phi_1\Phi_3\Phi_2)$$

- Superpotential of $\mathcal{N} = 1$ conformal theory:

$$A\text{Tr}(q\Phi_1\Phi_2\Phi_3 - q^{-1}\Phi_1\Phi_3\Phi_2) + h\text{Tr}(\Phi_1^3 + \Phi_2^3 + \Phi_3^3)$$

Parkes, West '84; ... ; Leigh, Strassler '95

- Symmetries for $h = 0$:

	Φ_1	Φ_2	Φ_3
$U(1)_R$	1	1	1
$U(1)_1$	0	1	-1
$U(1)_2$	-1	1	0

- β deformation and noncommutativity ($q = e^{i\pi\beta}$):

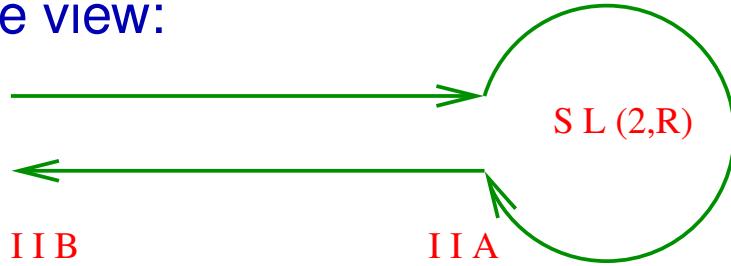
$$f \star g = \exp \left[i\pi\beta \left(Q_f^1 Q_g^2 - Q_g^1 Q_f^2 \right) \right] f g$$

Gravity solutions containing T^2

- String theory on T^2 : $SL(2, \mathbb{Z})$ symmetry.
- SUGRA: $SL(2, \mathbb{R})$ transformations:

$$\tau = B_{12} + i\sqrt{g} \rightarrow \tau' = \frac{a\tau + b}{c\tau + d}$$

- Alternative view:



- Constant non-commutativity of T^2 :

$$\theta^{12} = \left(\frac{1}{g + B} \right)_A^{12} = -\frac{B_{12}}{\det g + B_{12}^2} = -\text{Re} \frac{1}{\tau}$$

Seiberg, Witten '99

- Rigid T^2 : ordinary non-commutativity

Hashimoto, Itzhaki; Maldacena, Russo '99

- Contractible T^2 : potential singularities.

Deforming $AdS_5 \times S^5$

- Conformal symmetry: warped AdS_5
- Small deformation: perturbative solution

Aharony, Kol, Yankielowicz '02

- $\mathcal{N} = 1$ SUSY \rightarrow Killing spinors on S^5

$$\varepsilon \sim \exp\left[\pm \frac{i}{2}(\phi_1 + \phi_2 + \phi_3)\right]$$

- Selection of T^2

- metric on S^5

$$ds_S^2 = \sum_1^3 (d\mu_i^2 + \mu_i^2 d\phi_i^2), \quad \sum \mu_i^2 = 1$$

- $U(1)_R \times U(1)_1 \times U(1)_2$ symmetry:

$$\phi_1 = \psi - \varphi_2, \quad \phi_2 = \psi + \varphi_1 + \varphi_2, \quad \phi_3 = \psi - \varphi_1$$

- Avoiding singularities

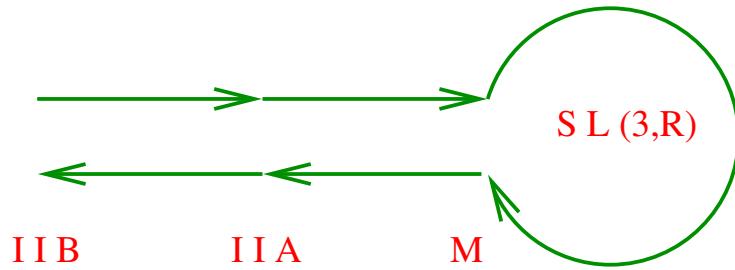
$$\tau_2 \rightarrow 0 \quad \Rightarrow \quad \tau \rightarrow 0, \quad \tau' = \tau + O(\tau^2)$$

$$SL(2, R) \rightarrow U(1) : \quad \tau' = \frac{\tau}{1 + \gamma\tau}$$

Deformations with two parameters

- Motivation: β is complex–valued
- Enhancing the symmetry to $SL(3, R)$:

IIA on $T^2 \rightarrow$ M theory on T^3



- Regularity of the solution: 2 + 2 parameters
- Metric of the deformed solution

$$ds^2 = G^{-1/4} \left[ds_{AdS}^2 + \sum d\mu_i^2 \right] \\ + G^{-1/4} \left[G \sum \mu_i^2 d\phi_i^2 + |\beta|^2 G \mu_1^2 \mu_2^2 \mu_3^2 (\sum d\phi_i)^2 \right], \\ G^{-1} \equiv 1 + |\beta|^2 (\mu_1^2 \mu_2^2 + \mu_1^2 \mu_3^2 + \mu_2^2 \mu_3^2)$$

- $SL(3, Z)$ symmetry of string theory:

$$\beta \sim \beta + 1, \quad \beta \sim \beta + i$$

Dorey, Hollowood, Kumar, '02

Properties of the solution

- Chiral primaries

- poinlike strings: $(J, J, J), (J, 0, 0), \dots$
- pp wave limits: solvable theory
- rational $\beta = m/n$: new states

$$(J_1, J_2, J_3) = (k_1, k_2, k_3), \quad k_1 = k_2 = k_3 \bmod n$$

Berenstein, Jejjala, Leigh, '99

- gravity side: wrapped strings for real β :

$$\varphi'_1 = n_1, \quad \varphi'_2 = n_2 \rightarrow J_3 = J_2 + \frac{n_1}{\beta}, \quad J_2 = J_1 + \frac{n_2}{\beta}$$

- Coulomb branch for $\beta = m/n$

- field theory — NC torus from F-term eqn

$$\Phi_1 = xU^m, \quad \Phi_2 = yV, \quad \Phi_3 = zV^{-1}U^{-m}$$

$$UV = e^{2\pi i/n} VU \quad \text{Berenstein, Leigh '99}$$

- string side: DBI action for m D5 branes on T^2 has a flat direction for $\beta = m/n$.

Further developments

- Probe strings and dual operators
 - fast string with $(J, J, 0)$ and XXZ spin chain:
agreement for 1-loop anomalous dimensions
Frolov, Roiban, Tseytlin
 - worldsheet counterpart of the transformation
leads to the Lax pair for the string
Frolov
 - twist in Bethe ansatz
Beisert, Roiban
- Chiral primaries in field theory
 - new chiral primary $\text{Tr}(\Phi_1 \Phi_2)$ is discovered
Freedman, Gursoy
 - zero anomalous dimension in 2-loops
Penati, Santambrogio, Zanon

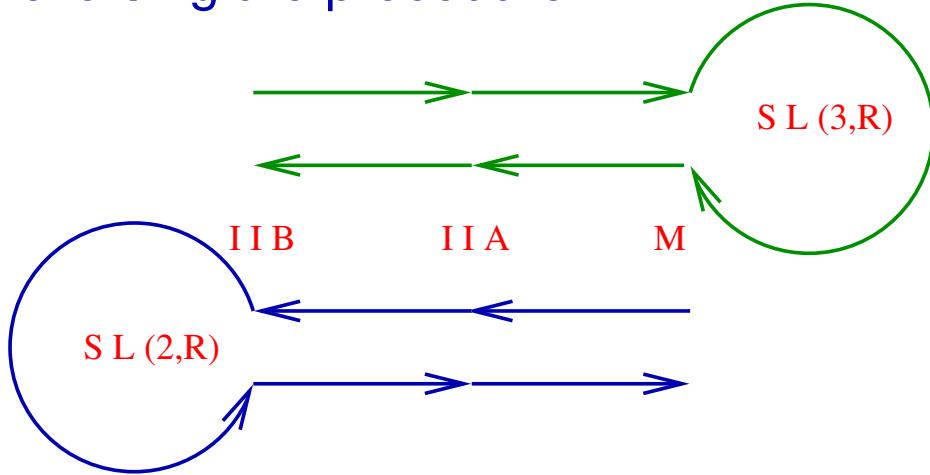
Application to other solutions

- Duals of superconformal field theories
 - product of AdS_5 and Sasaki–Einstein manifold
 - quiver gauge theories and $Y^{p,q}$
 - Gauntlett, Martelli, Sparks, Waldram;
 - Benvenuti, Franco, Hanany, Kennaway, Vegh, Wecht, ...
 - our procedure: gauge group doesn't change,
product becomes non-commutative
 - combination: β –deformation of quivers
- Applications to non-conformal theories
 - we only need two $U(1)$ symmetries
 - example: Klebanov–Strassler solution
 - new feature: deformation grows in the UV
 - another application: Maldacena–Nunez

Gursoy, Nunez

Deforming M theory solutions

- Assumption: $U(1)^3 \times U(1)_R$ symmetry
- Reversing the procedure:



- Regularity: only one-parameter family
- Action on the modulus:

$$\tau = C_{123} + i\sqrt{G} : \quad \frac{1}{\tau'} = \frac{1}{\tau} + \gamma$$

- Example: starting from M2 brane

$$ds^2 = ds_4^2 + \sum_{i=1}^4 (d\mu_i^2 + \mu_i^2 d\phi_i^2)$$

- Other applications: $AdS_4 \times$ 7D Sasaki–Einstein
Ahn, Vazquez-Poritz; Gauntlett, Lee, Mateos, Waldram

Summary

- Regular geometries dual to β deformations
 - relation to non-commutativity
 - dualities and twists on T^2
 - two-parameter family of solutions
- Examples
 - Leigh–Strassler deformation of $\mathcal{N} = 4$ SYM
 - applications to Sasaki–Einstein manifolds
 - deformation of Klebanov–Strassler solution
- Chiral primaries and Coulomb branch
- Generating solutions of M theory with $U(1)^3$
- Open problem: dual of the deformation

$$\text{Tr}(\Phi_1^3 + \Phi_2^3 + \Phi_3^3)$$