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Recent Progress in Perturbative Gauge Theory

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I Introduction

(a) Goal and motivation

(b) Definition of the amplitudes

II Tree-Level Amplitudes of Gluons

(a) What was known before 2004

(b) New techniques and results

III One-Loop Amplitudes of Gluons

(a) What was known before 2004

(b) New techniques and results in $\mathcal{N} = 4, 1, 0$ SYM

IV Conclusions

Introduction

The goal of this talk is to review the exciting developments in perturbative gauge theory that were triggered by the introduction of twistor string theory (Witten 12/2003).

Surprises: Previously thought impossible calculations are now simple exercises!

Motivations I (String Theory)

- Twistor string theory (Witten 12/2003): a string theory (topological B-model) with target twistor space (Penrose 1967).
- Twistor string theory describes $\mathcal{N} = 4$ SYM at weak coupling and leads to new insights into its perturbation theory. Complementary to the AdS/CFT correspondence (Maldacena 1997) which describes the strong coupling regime.

Motivation II (Field Theory)

- Why do we compute perturbative QCD amplitudes? Background in Hadron colliders like Tevatron and LHC.
- In principle perturbation theory is under control: Feynman diagrams!

  But not in practice or conceptually.

  In practice: $\#$ of FD grows very rapidly with $\#$ gluons and $\#$ of loops.

  Conceptually: After simplifying a huge $\#$ of FD the final answer is often simple and elegant.

In this talk I will present many new techniques that lead directly to the simple and elegant expressions.

Why does perturbation theory exhibit such an amazing simplicity?

Definition of the Amplitudes

(Reviews: Z.Bern TASI 92, L.Dixon TASI 95, Sterman TASI 04.)

We want to compute scattering amplitudes of n gluons. Each gluon carries the following information: $g_i = \{p_i^\mu, \epsilon_i^\mu, a_i\}$.

Color Decomposition

(Berends, Giele, Mangano, Parke, Xu 80's)

$$\mathcal{A}_n^{\rm tree}(\{p_i^\mu, \epsilon_i^\mu, a_i\}) = i g^{n-2} \delta^{(4)}(p_1 + \ldots + p_n) \times$$

$$\sum_{\sigma \in S_n / Z_n} {\rm Tr}(T^{a_{\sigma(1)}} \ldots T^{a_{\sigma(n)}}) A_n^{\rm tree}(\sigma(p_1^\mu, \epsilon_1^\mu), \ldots, \sigma(p_n^\mu, \epsilon_n^\mu))$$

Color Ordered Partial Amplitudes. (At one-loop the same decomposition can be done but it also includes double trace terms.)

Spinor-Helicity Formalism

$\{p_i^\mu, \epsilon_i^\mu\} \Longrightarrow$ Large number of redunbant Lorentz invariant combinations.

In four dimensions: Complexify the Lorentz group.

$$SO(3,1,C) \cong SL(2,C) \times SL(2,C).$$

Spinors of $\pm$ chirality: λ in $(1/2,0)$ and $\tilde{\lambda}$ in $(0,1/2)$.

Vector: $(1/2,1/2)$. Bispinor:

$$P_{a\dot{a}} = \sigma^\mu_{a\dot{a}} P_\mu = \lambda_a \tilde{\lambda}_{\dot{a}} + \lambda'_a \tilde{\lambda}'_{\dot{a}}$$

Null vector:

$$p_{a\dot{a}} = \lambda_a \tilde{\lambda}_{\dot{a}}$$

Lorentz invariant inner products:

$$\langle \lambda,\ \lambda' \rangle = \epsilon_{ab} \lambda^a \lambda'^b \qquad [\tilde{\lambda},\ \tilde{\lambda}'] = \epsilon_{\dot{a}\dot{b}} \tilde{\lambda}^{\dot{a}} \tilde{\lambda}'^{\dot{b}} \qquad 2p \cdot q = \langle p,\ q \rangle [p,\ q]$$

Example:

$$2p_1 \cdot p_2 = \langle 1,\ 2 \rangle [1,\ 2] = \langle 1\ 2 \rangle [1\ 2]$$

The main simplification comes from:

$$(-)-\text{helicity}: \quad \epsilon^{(i)}_{a\dot a} = \frac{\lambda^{(i)}_a \tilde\mu_{\dot a}}{[\tilde\lambda^{(i)},\ \tilde\mu]}$$

$$(+)-\text{helicity}: \quad \epsilon^{(i)}_{a\dot a} = \frac{\mu_a \tilde\lambda^{(i)}_{\dot a}}{\langle\mu,\ \lambda^{(i)}\rangle}$$

Example:

$$A_5^{\text{tree}}(1^-,2^+,3^-,4^+,5^+) = \frac{\langle 1\ 3\rangle^4}{\langle 1\ 2\rangle\langle 2\ 3\rangle\langle 3\ 4\rangle\langle 4\ 5\rangle\langle 5\ 1\rangle}$$

- Berends, Kleiss, De Causmaecker, Gastmans, Wu (1981)
- De Causmaecker, Gastmans, Troost, Wu (1982)
- Kleiss, Stirling (1985)
- Xu, Zhang, Chang (1987)
- Gunion, Kunszt (1985)

Outline

I Introduction

II Tree-Level Amplitudes of Gluons

(a) What was known before 2004

(b) Twistor string theory: Connected, disconnected instantons and localization

(c) MHV diagrams, extensions and applications

(d) BCFW construction

(e) BCF recursion relations

III One-Loop Amplitudes of Gluons

IV Conclusions

What was known before 2004:

Parke-Taylor Amplitudes: (Parke, Taylor 1986; Berends, Giele 1989)

$$A(1^+, 2^+, 3^+, \ldots, n^+) = 0, \qquad A(1^-, 2^+, 3^+, \ldots, n^+) = 0$$

Maximal helicity violating or MHV amplitudes:

$$A(1^+, 2^+, \ldots, r^-, \ldots, s^-, \ldots, n^+) = \frac{\langle r\, s\rangle^4}{\langle 1\, 2\rangle\langle 2\, 3\rangle \ldots \langle n-1\, n\rangle\langle n\, 1\rangle}$$

Some Next-to-MHV amplitudes:

$$A(1^-, 2^-, 3^-, 4^+, 5^+, 6^+) = \frac{\langle 2\, 3\rangle^2 [5\, 6]^2 (\langle 1\, 2\rangle [2\, 4] + \langle 1\, 3\rangle [3\, 4])^2}{(p_2 + p_3 + p_4)^2 (p_2 + p_3)^2 s_{34} s_{56} s_{61}} + \ldots$$

All other six-gluon helicity configurations: (Berends, Giele 1987; Mangano, Parke, Xu 1988).

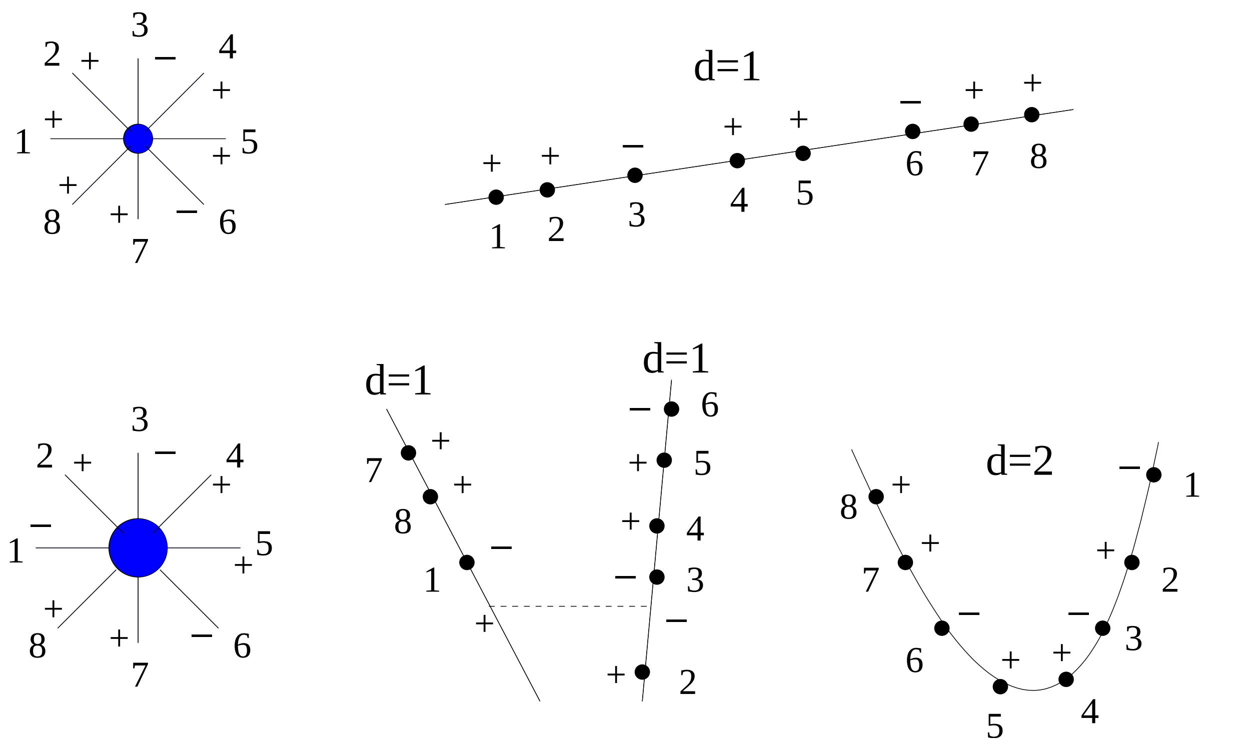
All seven-gluon NMHV amplitudes (Berends, Giele, Kuijf 1990)

Infinite series of NMHV: $A(1^-, 2^-, 3^-, 4^+, \ldots, n^+)$ (Kosower 1990).

Twistor String Theory: (Witten 12/2003)

Twistor space (Penrose 1967): $(Z_1, Z_2, Z_3, Z_4) = (\lambda^1, \lambda^2, \mu^{\dot{1}}, \mu^{\dot{2}})$, with $\mu_{\dot{a}} = -i\partial/\partial\tilde{\lambda}^{\dot{a}}$.

Twistor string theory: Topological B-model on $\mathrm{CP}^{3|4}$ in the presence of N D5-branes and D-instantons. (*Alternative open string formulation*: Berkovits 02/2004).



Connected Instantons: (Roiban, Spradlin, Volovich 02/,03/2004)

$Z^I = P^I(u,v), \quad \psi^A = \chi^A(u,v)$ where (u,v): Coordinates of a CP^1.

$$A_n = \int d\mathcal{M}_{d=m-1} \prod_{i=1}^{n} \int_C \frac{\langle u_i, du_i \rangle}{\prod_k \langle u_k, u_{k+1} \rangle} \phi_i(\lambda_i(u_i), \mu_i(u_i), \psi_i)$$

Obs: Not really an integral $\Rightarrow$ Solving polynomial equations!

Disconnected Instantons: (F.C., Svrček, Witten 03/2004)

$$A_n = \sum_{\mathcal{D}} \prod_{k=1}^{d=m-1} A_k^{\mathrm{MHV}} \prod_{\{ij\} \in \mathrm{Links}} \frac{1}{P_{ij}^2}$$

Obs: This is simple and systematic!

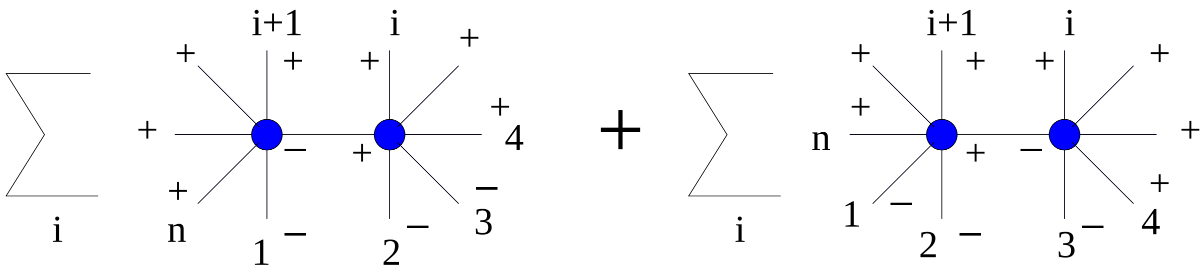
Surprise: Both formulas reproduce the full amplitude!

Localization Argument: The integral over the moduli space localizes to singular configurations. (Gukov, Motl, Neitzke 04/2004)

MHV Diagrams: (F.C., Svrček, Witten 03/2004)

Claim: All tree-level amplitudes can be computed by sewing MHV amplitudes (continued off-shell) with Feynman propagators.

$$A_n(1^-, 2^-, 3^-, 4^+, \ldots, n^+) =$$



$$\sum_{i=3}^{n-1} \left(\frac{\langle 1,\ P_i \rangle^3}{\langle P_i, i+1 \rangle \langle i+1, i+2 \rangle \ldots \langle n, 1 \rangle} \right) \frac{1}{P_i^2} \left(\frac{\langle 2, 3 \rangle^3}{\langle P, 2 \rangle \langle 3, 4 \rangle \ldots \langle i, P \rangle} \right)$$

$$+ \sum_{i=4}^{n} \left(\frac{\langle 1, 2 \rangle^3}{\langle 2, P_i \rangle \langle P_i, i+1 \rangle \ldots \langle n, 1 \rangle} \right) \frac{1}{P_i^2} \left(\frac{\langle P_i, 3 \rangle^3}{\langle 3, 4 \rangle \ldots \langle i-1, i \rangle \langle i, P_i \rangle} \right)$$

where $\langle k, P_i \rangle = \epsilon_{ab} \lambda_k^a P_i^{b\dot{b}} \eta_{\dot{b}}$ and η is a fixed reference spinor.

Can this simple construction be equivalent to the sum of a huge number of Feynman diagrams? Yes, it is!

Proof: MHV diagrams possess the following properties: (F.C., Svrček, Witten 03/2004)

- All correct collinear ($(p_1 + p_2)^2 \to 0$) and multi-particle ($(p_1 + p_2 + \ldots + p_i)^2 \to 0$) factorization limits.
- η independent. Lorentz invariant. Unphysical poles $1/\langle i\ P\rangle$ are spurious.

Therefore, $A_n^{(1)}$ computed from MHV diagrams has the same poles and residues as $A_n^{(2)}$ computed from Feynman diagrams. At tree level this is enough to conclude that $A_n^{(1)} = A_n^{(2)}$.

Q: Is there a systematic way of constructing an amplitude from its singularities?

Partial List of Extensions and Applications:

- Amplitudes of gluons with fermions and scalars. (Georgiou, Khoze 04/, Wu, Zhu 06/2004)
- Amplitudes with quarks, etc. (Georgiou, Glover, Khoze 07/; Su, Wu 07/2004).
- Application to Higgs plus partons (Dixon, Glover, Khoze 11/; Badger, Glover, Khoze 12/2004) .
- Multicollinear limits in QCD: Calculation of universal split functions. (Birthwright, Glover, Khoze, Marquard 03; 05/2005)
- Electroweak vector boson currents (Bern, Forde, Kosower, Mastrolia 12/2004)

One-loop applications: Wait until part III of the talk!

BCFW Construction

(Britto, F.C., Feng, Witten 01/2005)

Q: Is there a systematic way of constructing an amplitude from its singularities?

Consider any amplitude of gluons: $A(p_1, \ldots, p_n)$

Define the following function of a complex variable z:

$$A(z) = A(p_1, \ldots, p_{k-1}, p_k(z), p_{k+1}, \ldots, p_{n-1}, p_n(z))$$

where

$$\tilde{\lambda}_k \to \tilde{\lambda}_k - z\tilde{\lambda}_n, \qquad \lambda_n \to \lambda_n + z\lambda_k.$$

In other words,

$$p_k(z) = p_k - z\lambda_k\tilde{\lambda}_n, \qquad p_n(z) = p_n + z\lambda_k\tilde{\lambda}_n.$$

Note: $A(z)$ is a physical amplitude for all z:

$$p_i^2 = 0 \;\; \forall i \quad \text{and} \quad \sum_{j=1}^{n} p_j = 0$$

All we need is good control of the analytic structure of $A(z)$ which comes from physical singularities.

Application: BCF Recursion Relations

Consider the BCFW construction at tree-level: $A^{\rm tree}(z)$.

Claims:

- $A(z)$ is a rational function. Therefore, its only singularities are poles.
- $A(z)$ only has simple poles. (Propagators that depend on z)
- $A(z)$ vanishes as $z \to \infty$. (Easy to prove from Feynman diagrams)

$$A(z) = \sum_{i,j} \sum_{h=\pm} A_L^h(z_{ij}) \frac{1}{P_{ij}(z)^2} A_R^{-h}(z_{ij})$$

A_L and A_R are physical on-shell amplitudes!

Set $z = 0$ and get recursion relations for physical amplitudes of gluons.

BCF recursion relations: originally conjectured for $k = n - 1$ (Britto, F.C., Feng 12/2004):

Some Applications

Alternating 8-gluon amplitude: (Britto, F.C., Feng 12/2004).

$$A(1^-, 2^+, 3^-, 4^+, 5^-, 6^+, 7^-, 8^+) = [T] + [U] + [V].$$

$$T = \frac{[1\ 3]^4 [5\ 7]^4 \langle 4\ 8\rangle^4}{[1\ 2][2\ 3][5\ 6][6\ 7] t_{123} t_{567} \langle 4|2+3|1] \langle 4|5+6|7] \langle 8|1+2|3] \langle 8|6+7|5]}$$

Split Helicity Amplitudes: (Britto, Feng, Roiban, Spradlin, Volovich 03/2005)

$$A(1^-, \ldots, q^-, (q+1)^+, \ldots, n^+) = \sum_{k=0}^{\min(q-3, n-q-2)} \sum_{A_k, B_{k+1}} \frac{N_1 N_2 N_3}{D_1 D_2 D_3}$$

where $A_k \subset \{2, \ldots q-2\}$, $B_{k+1} \subset \{q+1, \ldots, n-1\}$,

$N_1 = \langle b_1 + 1, b_1 \rangle \langle b_2 + 1, b_2 \rangle \ldots \langle b_{k+1}, b_{k+1} \rangle$,

$D_1 = (p_2 + \ldots + p_{b_1})^2 (p_{b_1+1} + \ldots + p_{a_1})^2 \ldots (p_{b_{k+1}+1} + \ldots + p_{q-1})^2$,

etc.

Impossible $\Rightarrow$ Possible!

Scientific discoveries do not usually follow the most natural order!

A striking example:

- IR behavior of one-loop amplitudes in $\mathcal{N} = 4$ SYM is well understood. It depends on the tree-level amplitude. (Catani 1998)
- This leads to new formulas for tree amplitudes of gluons. (Bern, Dixon, Kosower 2004)
- One-Loop $\mathcal{N} = 4$ amplitudes can be computed from tree-level amplitudes of gluons, scalars and fermions. (Britto, F.C., Feng 12/2004)
- New formulas for tree amplitudes of gluons as products of amplitudes of gluons, fermions and scalars! Trees from loops! (Roiban, Spradlin, Volovich 12/2004)
- BCF recursion relations: Amplitudes of gluons as products of amplitudes of gluons!
- BCFW construction restores the natural order!

More Extensions and Applications: A Partial List

- RR for amplitudes of gluons and fermions. (Luo, Wen 01,02/2005)
- RR for amplitudes of gravitons. (Bedford, Brandhuber, Spence, Travaglini 02/, F.C., Svrcek 02/2005)
- Relation of RR to twistor diagrams. (Hodges 03/2005)
- RR for gauge theory amplitudes with massive particles. (Badger, Glover, Khoze, Svrcek 04/2005)
- RR for one-loop amplitudes: Finite QCD (Bern, Dixon, Kosower 01/,05/2005)
- More on one-loop applications later!

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(a) What was known before 2004

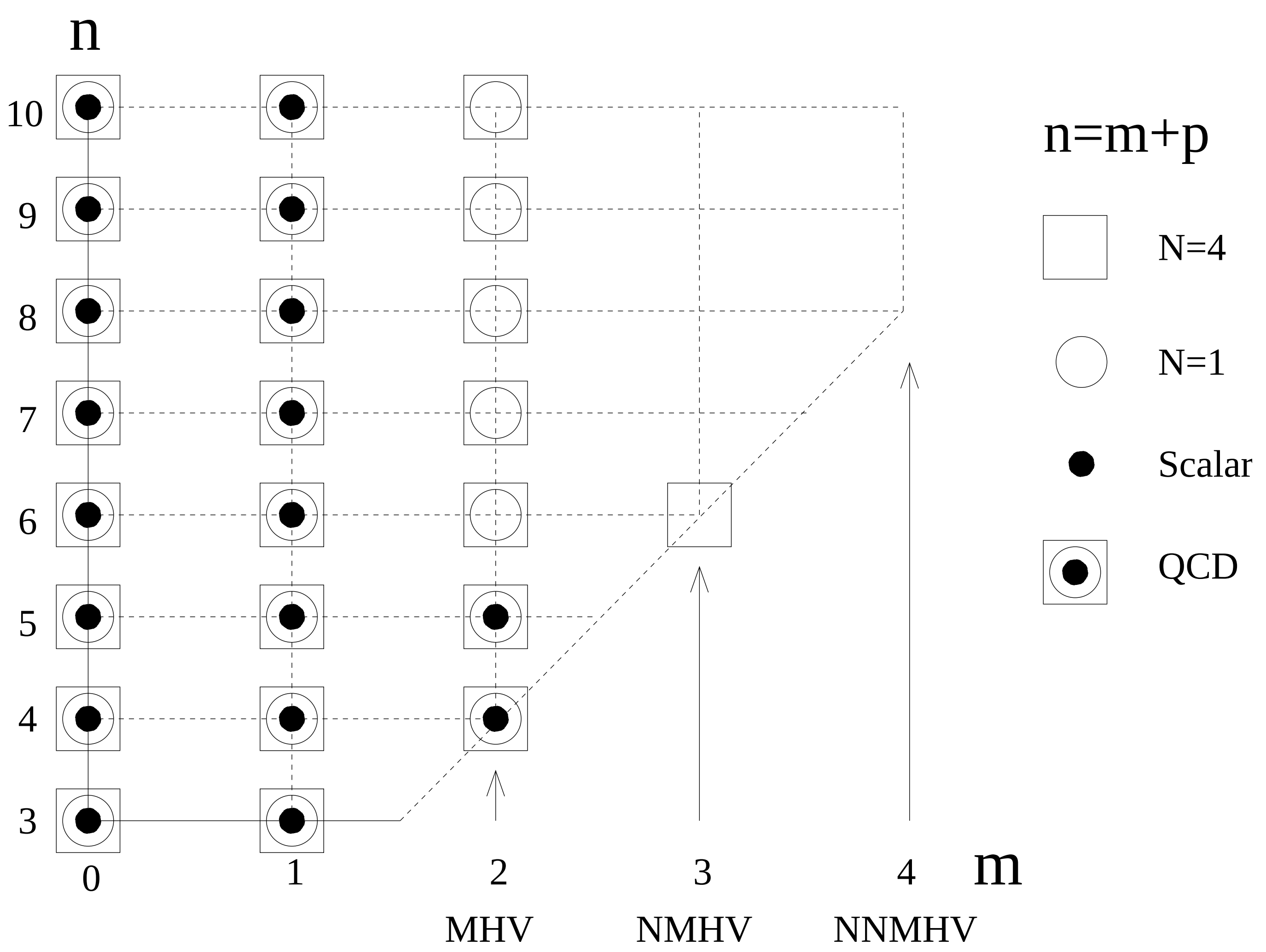
(b) New techniques and results in $\mathcal{N} = 4, 1, 0$ SYM

$$A^{\rm QCD} = A^{\mathcal{N}=4} - 4A^{\mathcal{N}=1}_{\rm chiral} + A^{scalar}$$

$$g = (g + 4f + 3s) - 4(f + s) + s$$

What was known before 2004

One-loop amplitudes of gluons: $A^{\mathrm{QCD}} = A^{\mathcal{N}=4} - 4A^{\mathcal{N}=1}_{\mathrm{chiral}} + A^{scalar}$

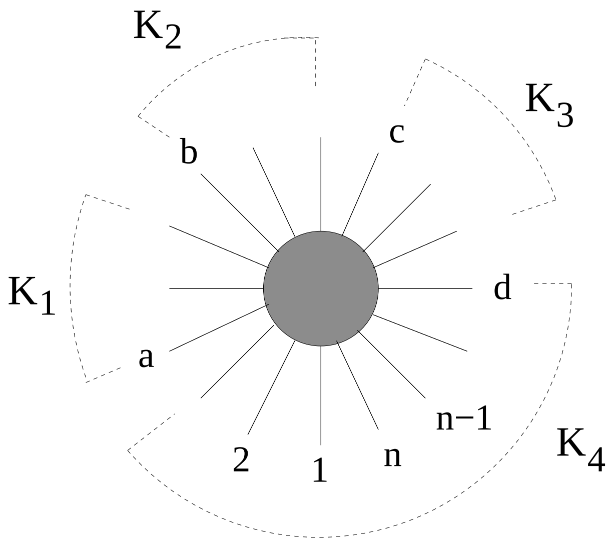


(Taken from: Dixon, TASI 95)

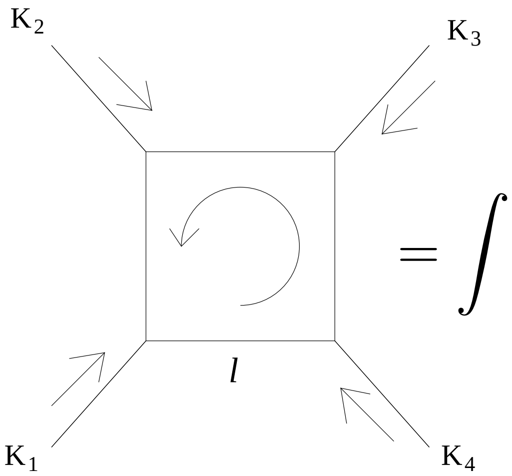
One-Loop Amplitudes in $\mathcal{N} = 4$ SYM

- Supersymmetric amplitudes of gluons are four-dimensional cut-constructible. This means that the amplitude is completely determined by its finite branch cuts and discontinuities. (Bern, Dixon, Dunbar, Kosower 1994)

- All tensor integrals in a Feynman graph calculation of the amplitudes can be reduced to a set of scalar box integrals. (Passarino-Veltman reduction. In Dim. Reg: Bern, Dixon, Kosower 1993)



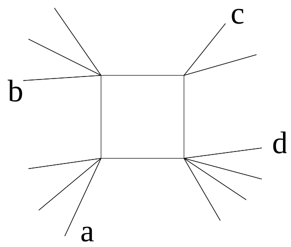
Scalar Box Integrals



$$= \int d^4\ell \frac{1}{\ell^2(\ell-K_1)^2(\ell-K_1-K_2)^2(\ell+K_4)^2}$$

Any n-gluon one-loop amplitude can be written as: (Bern, Dixon, Kosower 1993 & with Dunbar 1994)

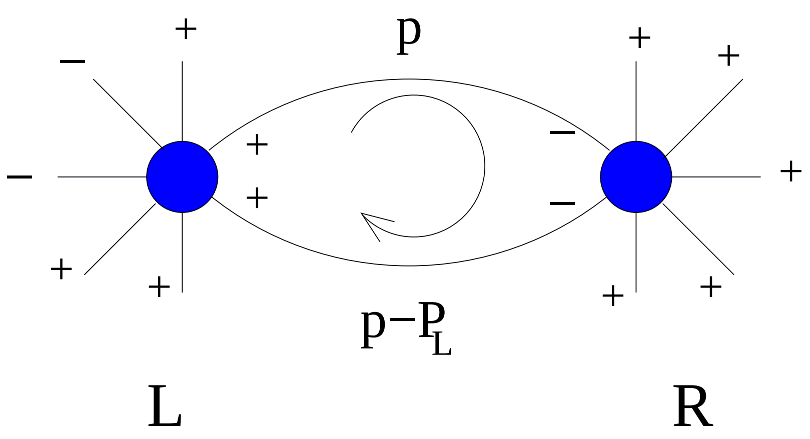
$$A_n^{1-\text{loop}} = \sum_{1<a<b<c<d<n} B_{abcd} \times$$



Observation: The problem of computing $\mathcal{N}=4$ one-loop amplitudes is reduced to that of computing the coefficients B_{abcd}, which are rational functions of $\langle i\ j\rangle$ and $[i\ j]$.

MHV Diagrams: One-Loop

MHV amplitudes: (Brandhuber, Spence, Travaglini 2004)



$$A_{\rm MHV}^{1-{\rm loop}} = \sum_{\mathcal{D},h} \int d^{4-2\epsilon}p A_L(\lambda_p, \lambda_{p-P_L}) \frac{1}{p^2(p-P_L)^2} A_R(\lambda_p, \lambda_{p-P_L})$$

where $\lambda_p^a = p^{a\dot{a}}\eta_{\dot{a}}$ and $\lambda_{p-P_L}^a = (p-P_L)^{a\dot{a}}\eta_{\dot{a}}$.

Answer: (Bern, Dixon, Dunbar, Kosower 1994) $B_{abcd} = \{A_n^{\rm tree\ MHV} \ \text{ or } \ 0\}$

Open problem: What about next-to-MHV?

Twistor Space Localization

There are differential operators: F and K that determine collinearity and coplanarity in twistor space. (Witten 12/2003)

At one-loop, amplitudes failed to be annihilated by the operators. The reason is a holomorphic anomaly. (F.C., Svrček, Witten 2004)

The failure to give zero resulted in a rational function. This led to a purely algebraic approach for the computation of B_{abcd}. (F.C. 10/2004)

Application: The first new result at one-loop!

$A_7(1^-,2^-,3^-,4^+,5^+,6^+,7^+)$ 35 coefficients (Britto, F.C., Feng 10/2004)

$$-\frac{\langle 1\,2\rangle^3\langle 2\,3\rangle^3[5\,6]^3}{\langle 7\,1\rangle\langle 3\,4\rangle\langle 2|3+4|5]\langle 2|7+1|6](\langle 7\,1\rangle\langle 2|3+4|1]-t_{234}\langle 7\,2\rangle)(s_{71}\langle 2\,4\rangle-\langle 3\,4\rangle\langle 2|7+1|3])}$$

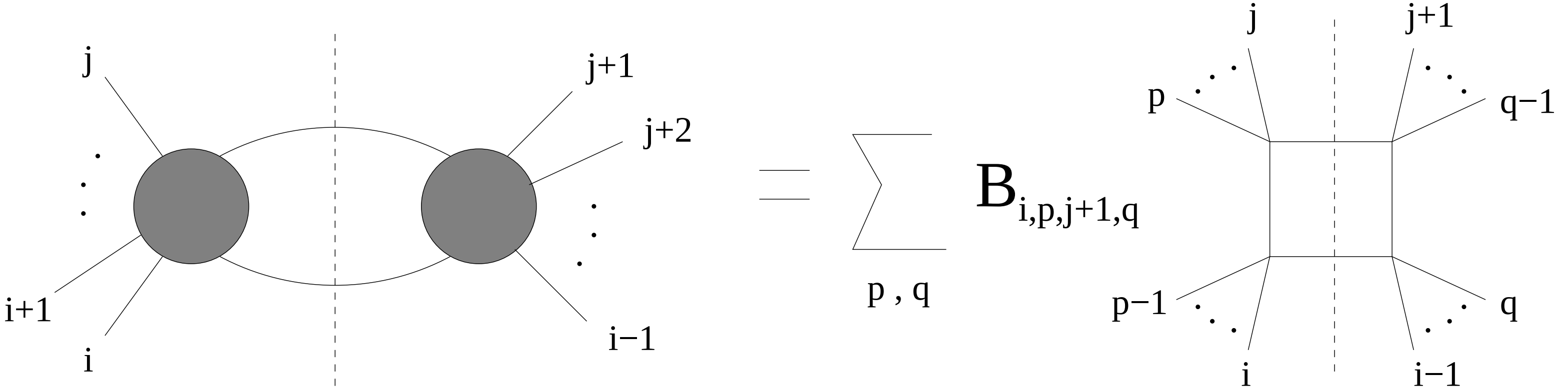
with $t_{234}=(p_2+p_3+p_4)^2$ and $s_{71}=(p_7+p_1)^2$.

This result, along with all other helicity configurations, was reproduced using a purely field theoretic method, the unitarity-based method. (Bern, Del Duca, Dixon, Kosower 10/2004)

Unitarity Cuts

Recall that SUSY amplitudes are four-dimensional cut constructible. Then it is natural to use unitarity cuts to compute them. This goes under the name of the unitarity-based method. (Bern, Dixon, Dunbar, Kosower 1994)

Observation: Methods based on unitarity cuts have the disadvantage that always several unknown coefficients show up at once in a given cut.



Reduction techniques or the algebraic approach have to be used in order to disentangle the information about the different coefficients.

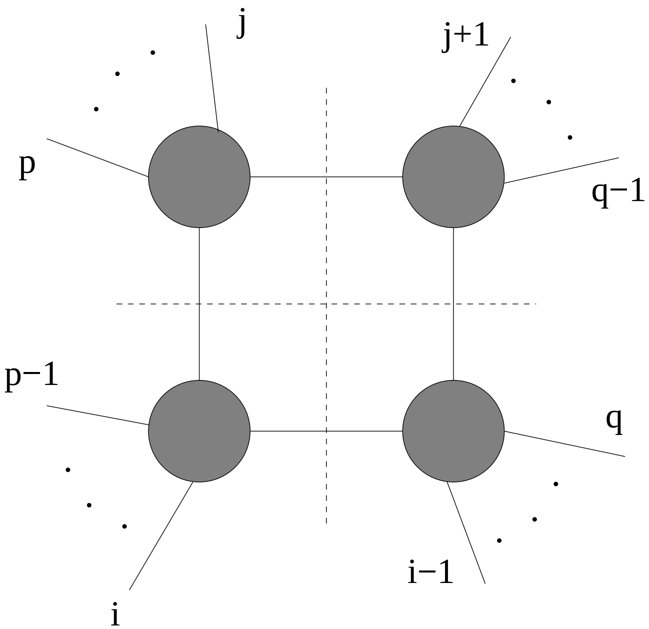
Blobs denote complete tree-level amplitudes! (The idea of combining FD that share the same cut can be found e.g. in "*The Analytic S-Matrix*" by Eden et al 1966)

Quadruple Cuts

(Britto, F.C., Feng, 12/2004)

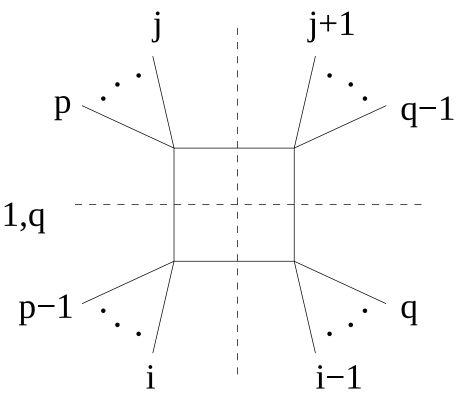
Just as unitarity cuts compute the discontinuity across a branch cut, higher cuts compute discontinuity across other singularities.

It turns out that each scalar box integral has a unique singularity! The discontinuity across it is computed by a quadruple cut!



$$=$$

$$\mathrm{B}_{\mathrm{i,p,j+1,q}}$$



$$\int d\mu A_1^{\mathrm{tree}} A_2^{\mathrm{tree}} A_3^{\mathrm{tree}} A_4^{\mathrm{tree}} = B \int d^4\ell \delta^{(+)}(\ell_1^2)\delta^{(+)}(\ell_2^2)\delta^{(+)}(\ell_3^2)\delta^{(+)}(\ell_4^2)$$

But the use of multiple cuts in gauge theory is not a new idea! (Feynman 1963). Modern applications: (Bern, Dixon, Kosower 1997,2000,2004)

A Major Problem and a Simple Solution

Problem: The unique singularity does not exist for real momenta in Minkowski space. A related problem: $A_3^{\rm tree}(p_1,p_2,p_3)=0$.

Observation: In the twistor string theory construction it is natural to consider $(++--)$ signature: $A_3^{\rm tree}(p_1,p_2,p_3)\neq 0$.

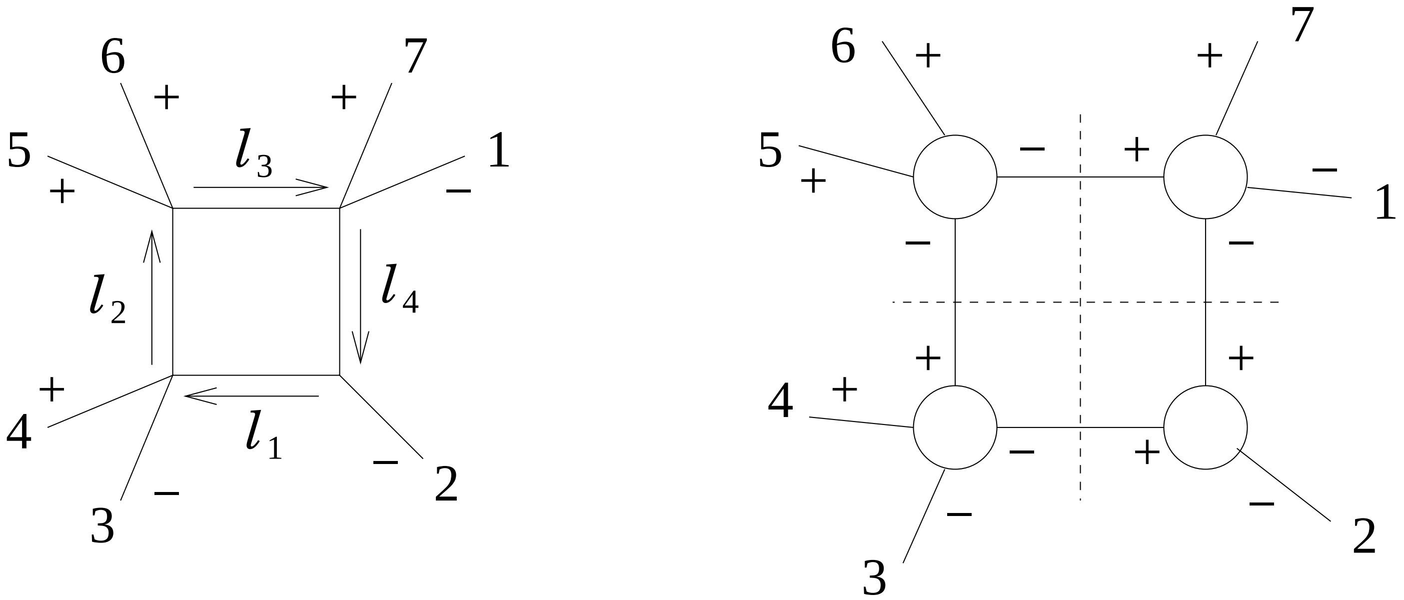
Solution: Use $(++--)$ signature with real momenta.

Final Formula:

$$B_{abcd}=\frac{1}{2}\sum_{\mathcal{S}}\sum_{h}A_1^{\rm tree}A_2^{\rm tree}A_3^{\rm tree}A_4^{\rm tree}$$

$$\mathcal{S}=\{\,\ell\,|\,\ell^2=0,\ (\ell-K_1)^2=0,\ (\ell-K_1-K_2)^2=0,\ (\ell+K_4)^2=0\}$$

An Example: $A(1^-, 2^-, 3^-, 4^+, 5^+, 6^+, 7^+)$

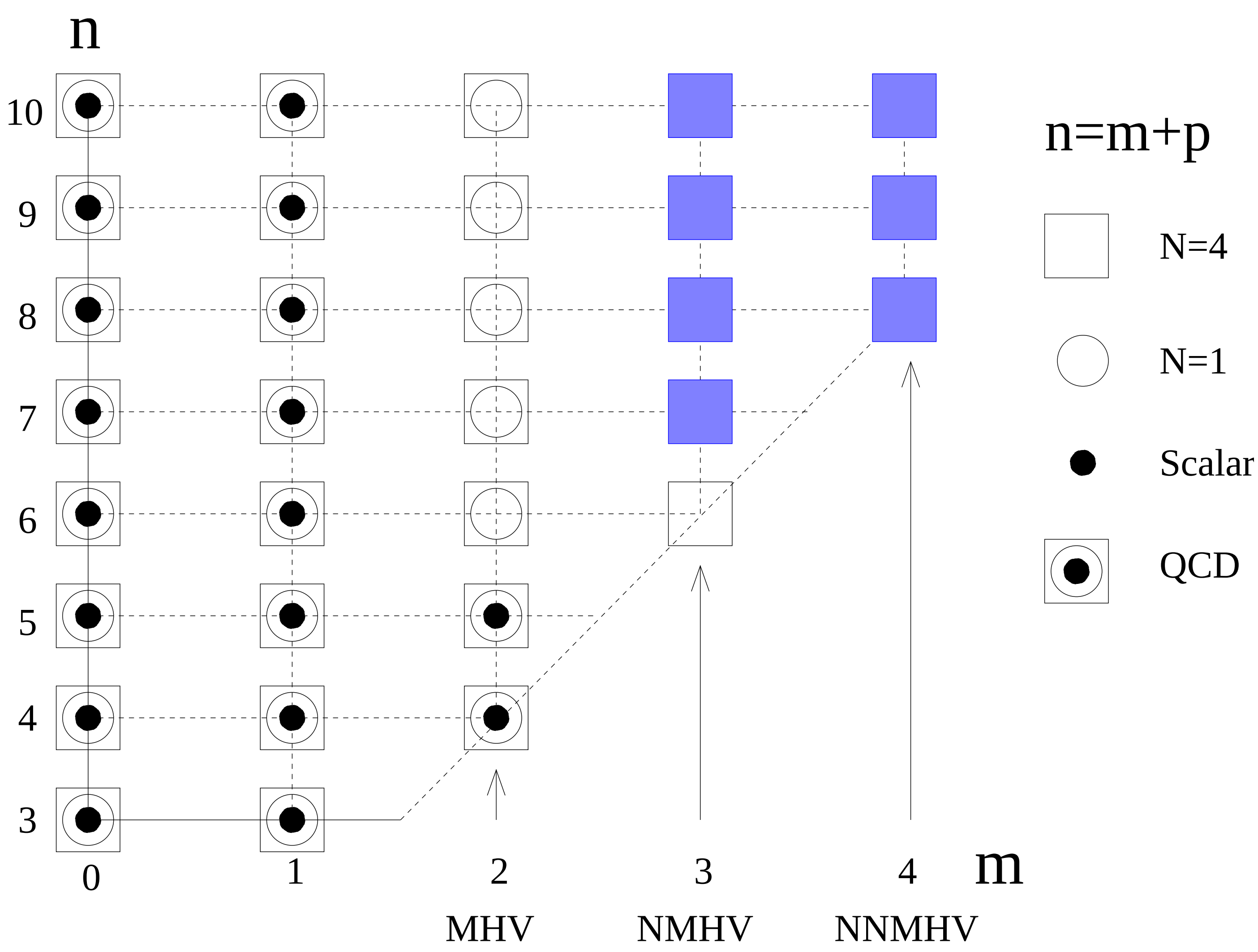


(a) (b)

$$B_{3572} = \frac{1}{2}\frac{[\ell_1\,\ell_4]^3}{[\ell_1\,2][2\,\ell_4]}\frac{[4\,\ell_2]^3}{[\ell_2\,\ell_1][\ell_1\,3][3\,4]}\frac{[5\,6]^3}{[6\,\ell_3][\ell_3\,\ell_2][\ell_2\,5]}\frac{[\ell_3\,7]^3}{[7\,1][1\,\ell_4][\ell_4\,\ell_3]}$$

$$= -\frac{\langle 1\,2\rangle^3\langle 2\,3\rangle^3[5\,6]^3}{\langle 7\,1\rangle\langle 3\,4\rangle\langle 2|3+4|5]\langle 2|7+1|6](\langle 7\,1\rangle\langle 2|3+4|1] - t_2^{[3]}\langle 7\,2\rangle)(s_{71}\langle 2\,4\rangle - \langle 3\,4\rangle\langle 2|7+1|3])}$$

Conclusion: All $\mathcal{N}=4$ one-loop amplitudes of gluons are under control!

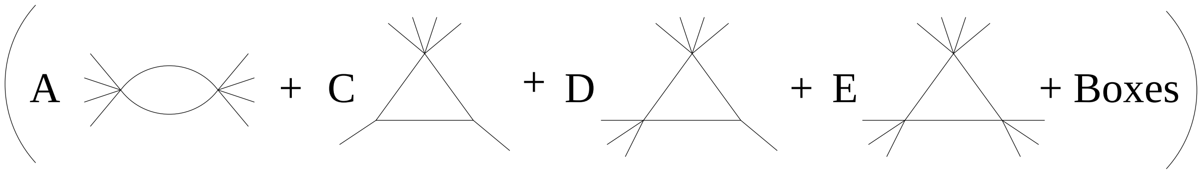


One-Loop $\mathcal{N}=1$ SYM: Chiral Φ

Recall: $A^{\rm QCD} = A^{\mathcal{N}=4} - 4A^{\mathcal{N}=1}_{\rm chiral} + A^{scalar}$

Chiral multiplet: $\Phi = \{f, s\}$.

$A^{\mathcal{N}=1}_{\rm chiral} =$



<u>MHV Diagrams:</u>

All $\mathcal{N}=1$ MHV amplitudes have been reproduced. (Quigley, Rozali; Bedford, Brandhuber, Spence, Travaglini 10/2004)

Open problem: What about NMHV?

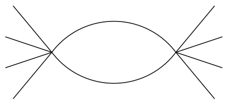
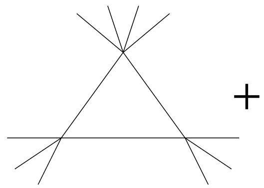
<u>Twistor Space Localization:</u>

New Amplitude: $A(1^-, 2^-, 3^-, 4^+, 5^+, 6^+)$ (Bidder, Bjerrum-Bohr,Dixon, Dunbar 10/2004)

Generalized Cuts and Cut Integration

$A(1^-, 2^-, 3^-, 4^+, ..., n^+)$: Quadruple cuts for boxes and triple cuts for 1m and 2m triangles. (Bidder, Bjerrum-Bohr, Dunbar, Perkins 02/2005)

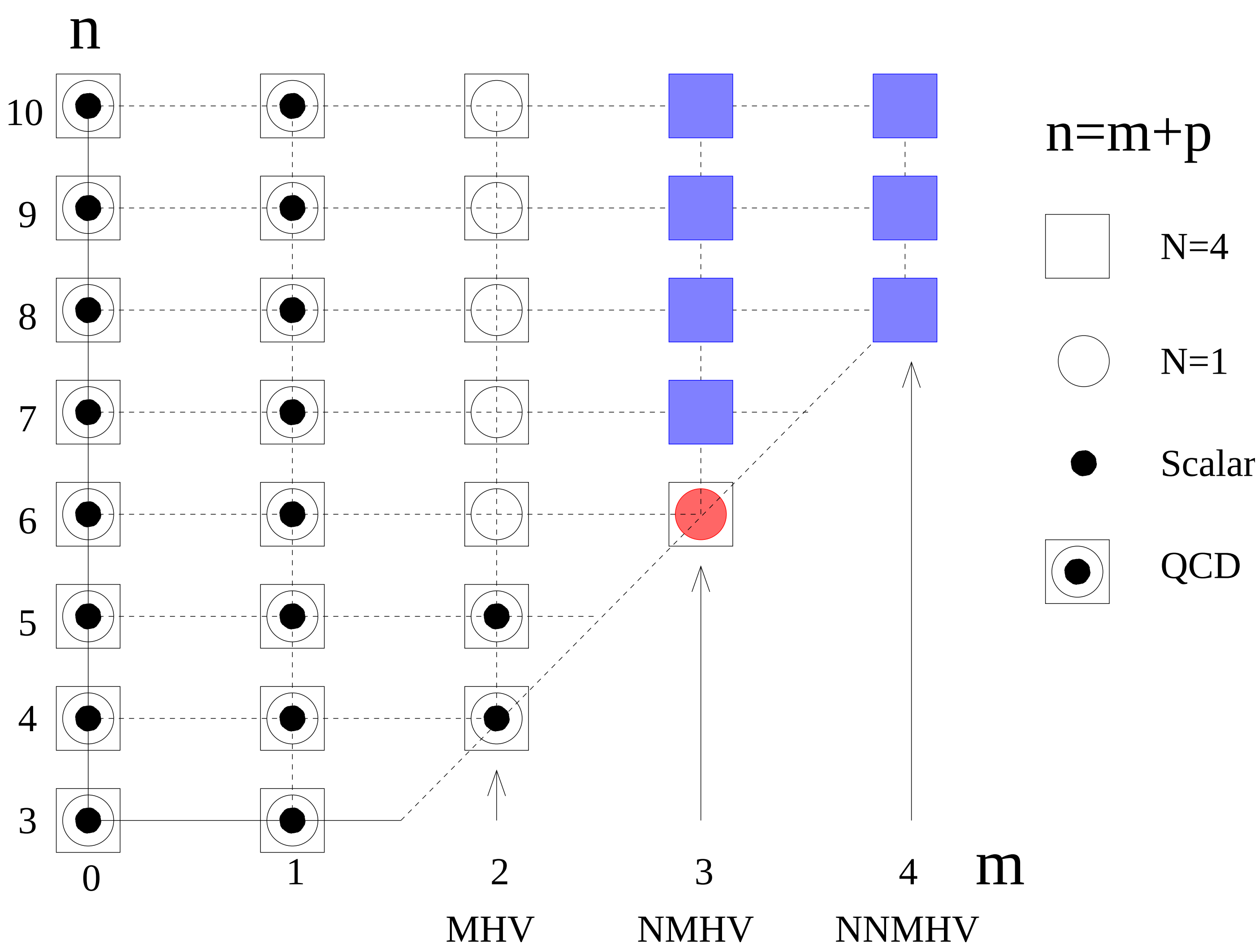
New Basis: (Britto, Buchbinder, F.C., Feng 03/2005)

$A^{\mathcal{N}=1}_{\text{chiral}} = \Bigg($ A  + E  + Finite Boxes $\Bigg)$

Cut Integration: Use a measure for unitarity cuts given as a contour integral over $R^+ \times CP^1 \times CP^1$ with contour the diagonal CP^1. This is again coming from the twistor string theory description of MHV diagrams (F.C., Svrcek, Witten 03/2004)

New Amplitudes: $A(1^-, 2^-, 3^+, 4^-, 5^+, 6^+)$ and $A(1^-, 2^+, 3^-, 4^+, 5^-, 6^+)$. This completes the six-gluon one-loop $\mathcal{N} = 1$ amplitudes!

Conclusion: New Efficient Techniques for $\mathcal{N} = 1$ One-Loop Amplitudes of Gluons



One-Loop Scalar: Last piece of A^{QCD}!

$$A^{\mathrm{QCD}} = A^{\mathcal{N}=4} - 4A^{\mathcal{N}=1}_{\mathrm{chiral}} + A^{scalar}.$$

Cut Constructible and Non-cut Constructible Pieces

$A^{scalar} = C + R$ where C is some function that reproduces the unitarity cuts of A^{scalar} while R is a rational function. This splitting is not unique.

Purely R Amplitudes:

$$A^{\mathrm{QCD}}(\pm, +, \ldots, +) = A^{\mathrm{scalar}}(\pm, \ldots, +) = R.$$

Use the BCFW construction: Consider $A(z)$ and study its singularities. (New ingredient: double poles!) Recursion relations lead to very compact formulas! (Bern, Dixon, Kosower 01/,05/2005)

C piece of scalar “MHV” Amplitudes:

$A(1^-, 2^-, 3^+, \ldots, n^+)$ (Bern, Dixon, Dunbar, Kosower 1994.)

$A(1^-, 2^+, \ldots, i^-, \ldots, n^+)$ First new result of MHV diagrams at one-loop! (Bedford, Brandhuber, Spence, Travaglini 12/2004)

Very Recently: A Week Ago! (Bern, Dixon, Kosower)

- Use the BCFW construction for MHV scalar amplitudes.
- New ingredient: Branch cuts.
- In cases when $A(z) \longrightarrow 0$ for $z \longrightarrow \infty$ one has

$$0 = \oint_C \frac{dz}{z} A(z) = A(0) + \sum \mathrm{Poles} + \sum \int \mathrm{Disc} A(z)$$

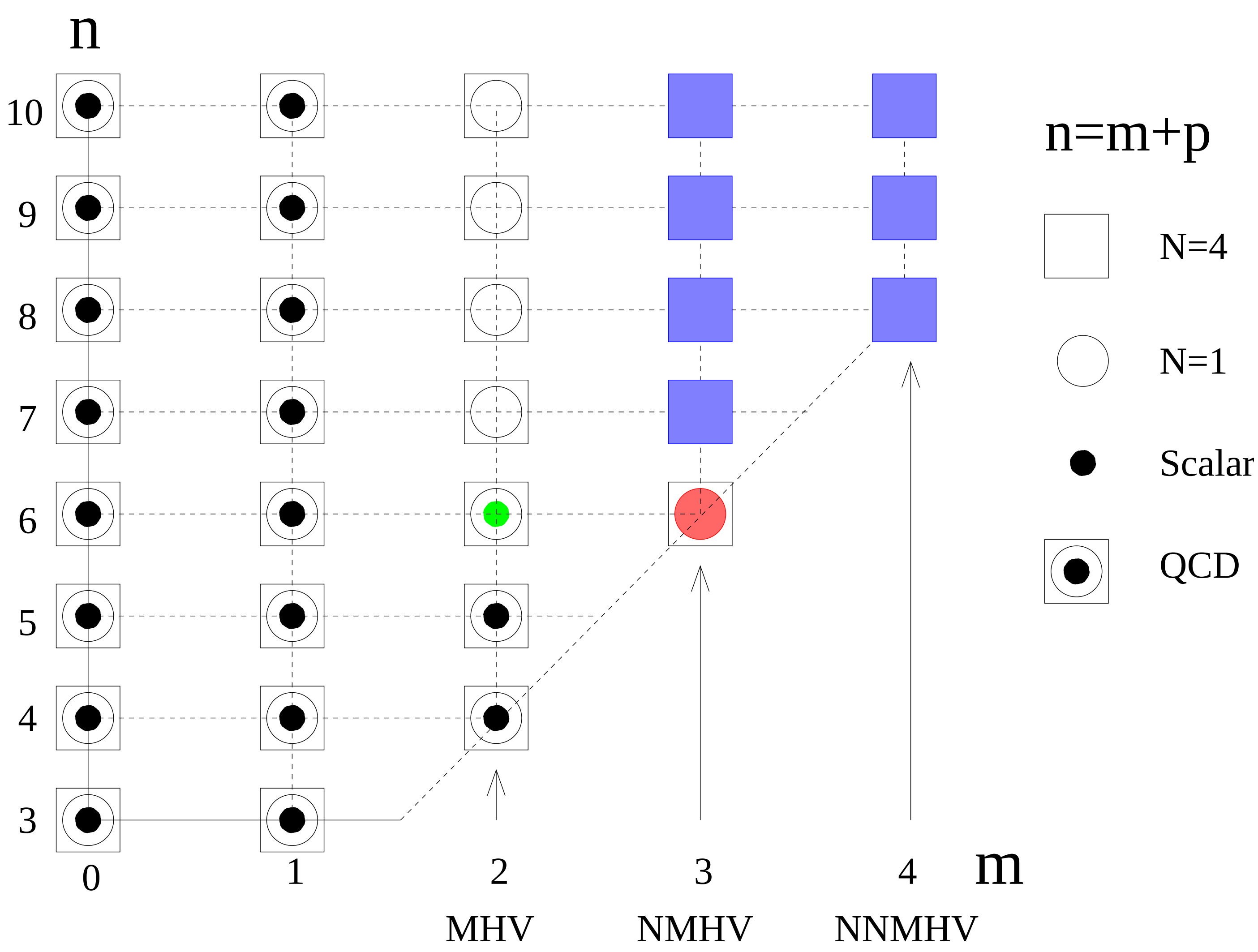
- This leads to a clean separation of C and R.
- It allows to get recursion relations for R.
- New results: $A_6^{\mathrm{scalar}}(1^-, 2^-, 3^+, 4^+, 5^+, 6^+)$

  Clear path towards A_7^{scalar}.

Striking example of: Impossible $\longrightarrow$ Possible (Simple).

Summary: One-Loop

$$A^{\mathrm{QCD}} = A^{\mathcal{N}=4} - 4A^{\mathcal{N}=1}_{\mathrm{chiral}} + A^{scalar}$$



Conclusion:

Perturbative gauge theory has some hidden beauty that is not apparent from Feynman diagrams!

Future Directions

- Twistor string theory (TST) also contains conformal supergravity (CSG) in the spectrum. At one-loop CSG mixes with the $\mathcal{N} = 4$ SYM contribution. (Berkovits, Witten 2004) Q: Is there a TST that only contains $\mathcal{N} = 4$ SYM?

- There is some evidence that MHV diagrams work well at one-loop. Q: Could MHV diagrams provide a completely new full perturbative expansion of $\mathcal{N} = 4$ SYM or even QCD?

- The BCFW construction only uses complex analysis in a single complex variable z and the physical singularities of the amplitudes. Q: Could this be done systematically to all-loop orders?

- Higher loops in $\mathcal{N}=4$ SYM. ABDK conjecture (Anastasiou, Bern, Dixon, Kosower 1998)

$$M_n^{(2)}(\epsilon) = \frac{1}{2}\left(M_n^{(1)}(\epsilon)\right)^2 + f(\epsilon) M_n^{(1)}(2\epsilon) - \frac{5}{4}\zeta_4 + \mathcal{O}(\epsilon^0)$$

Two weeks ago: Three-loop formula for MHV amplitudes (Bern, Dixon, Smirnov).

New conjecture: General all loop formula for MHV amplitudes:

$\sum_{L=0}^{\infty} \lambda^L F_n^{(L)} = B(\lambda) \mathrm{Exp}\left(\gamma(\lambda) F_n^{(1)}\right)$

Q: Could any of the techniques presented here be used to provide a proof of the conjecture?

Q: What is the relation to the AdS/CFT correspondence?

Q: What is the relation to integrability?