IBM TJ Watson Research Center

Stochastic Programming Applications in Deregulated Electricity Markets

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Outline

The goal of this talk is two fold: To introduce the audience to the interesting field of electric power and to present some results that relate to managing short-term electricity contracts.

- The ABCs of Electric Power
- The Deregulation of the US Electric Power Industry
- Opportunities for Mathematical Modeling
- Robust Modeling of Short-Term Electricity Contracts
  - Two-Stage Stochastic Linear Programs
  - Robust Two-Stage Stochastic Linear Programs
    - Issues with Using an Arbitrary Variability Measure
    - A Special Class of Variability Measures
    - Numerical Results
  - Two-Stage Robust Stochastic Program with Binary First-Stage Variables
    - A Branch-and-Cut Approach
    - Numerical Results

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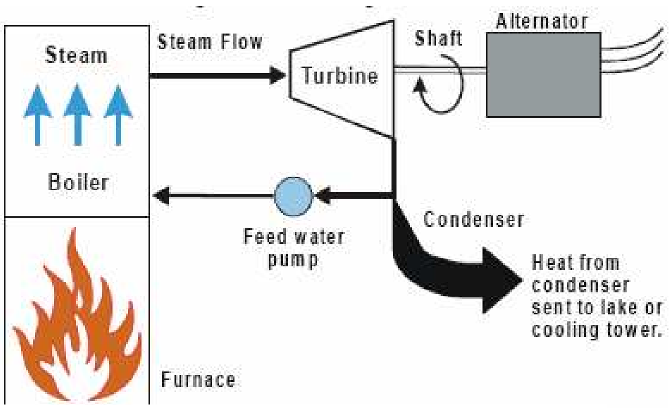
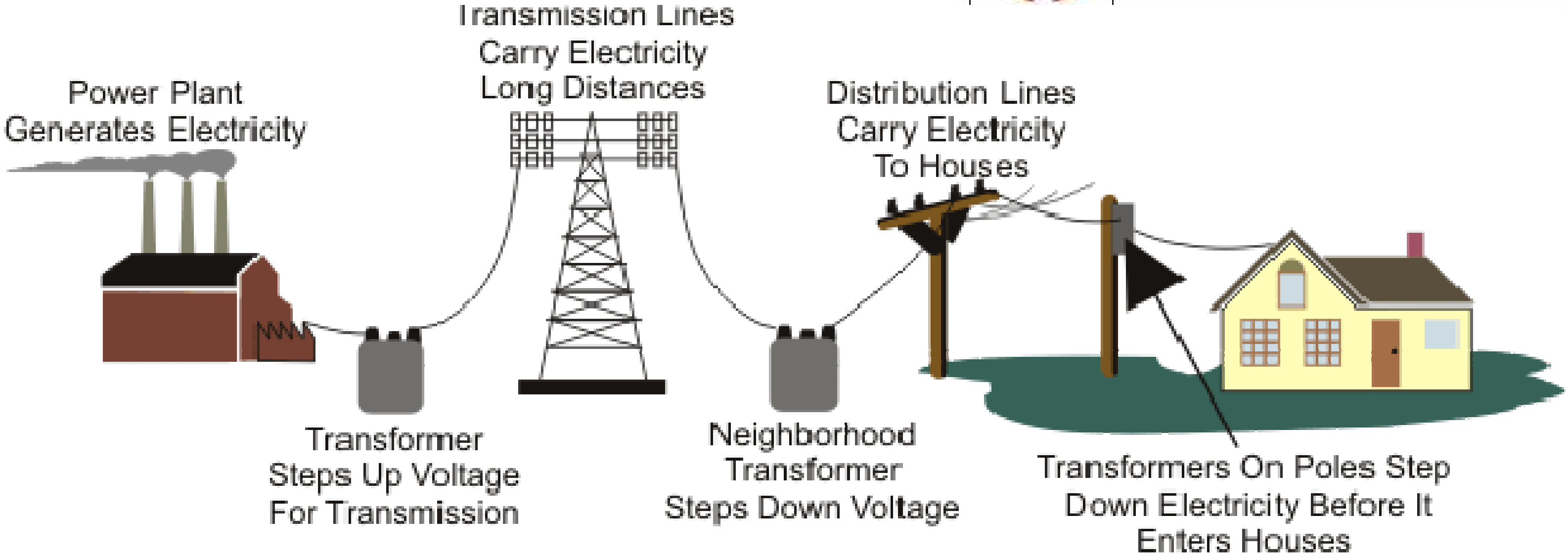
The Supply Chain of Electric Power

Electricity is a secondary energy source as it is generated by converting other sources of energy, like coal, natural gas, oil, and nuclear power.





Source: Energy Information Administration http://www.eia.doe.gov

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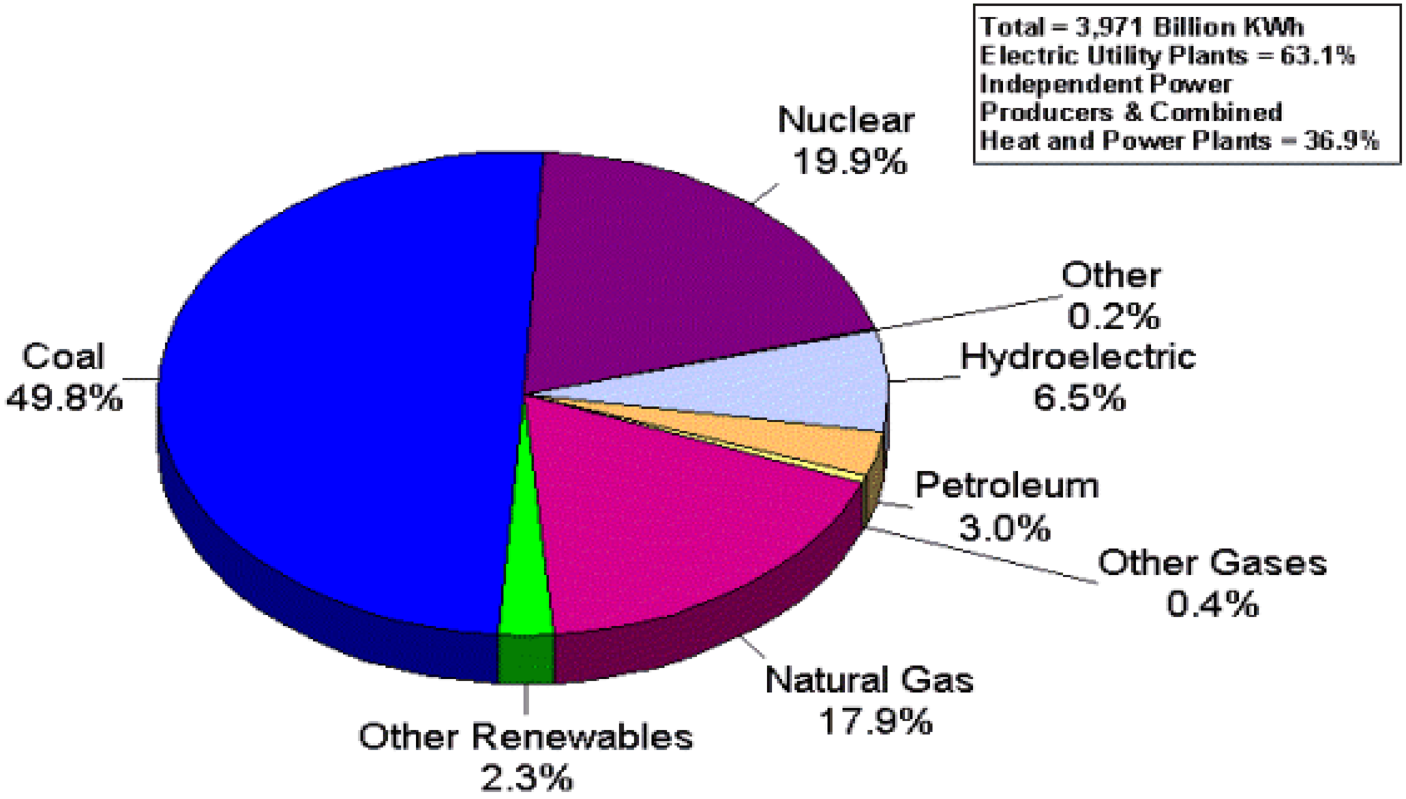
The US Electricity Market at a Glance

In 2002, the US consumed 3,971 BKWh of the world's 14,275 BKWh of electric power. For the same year, Canada produced 549 BKWh, of that 316 BKWh was generated from hydro and 29 BKWh was exported to the US.



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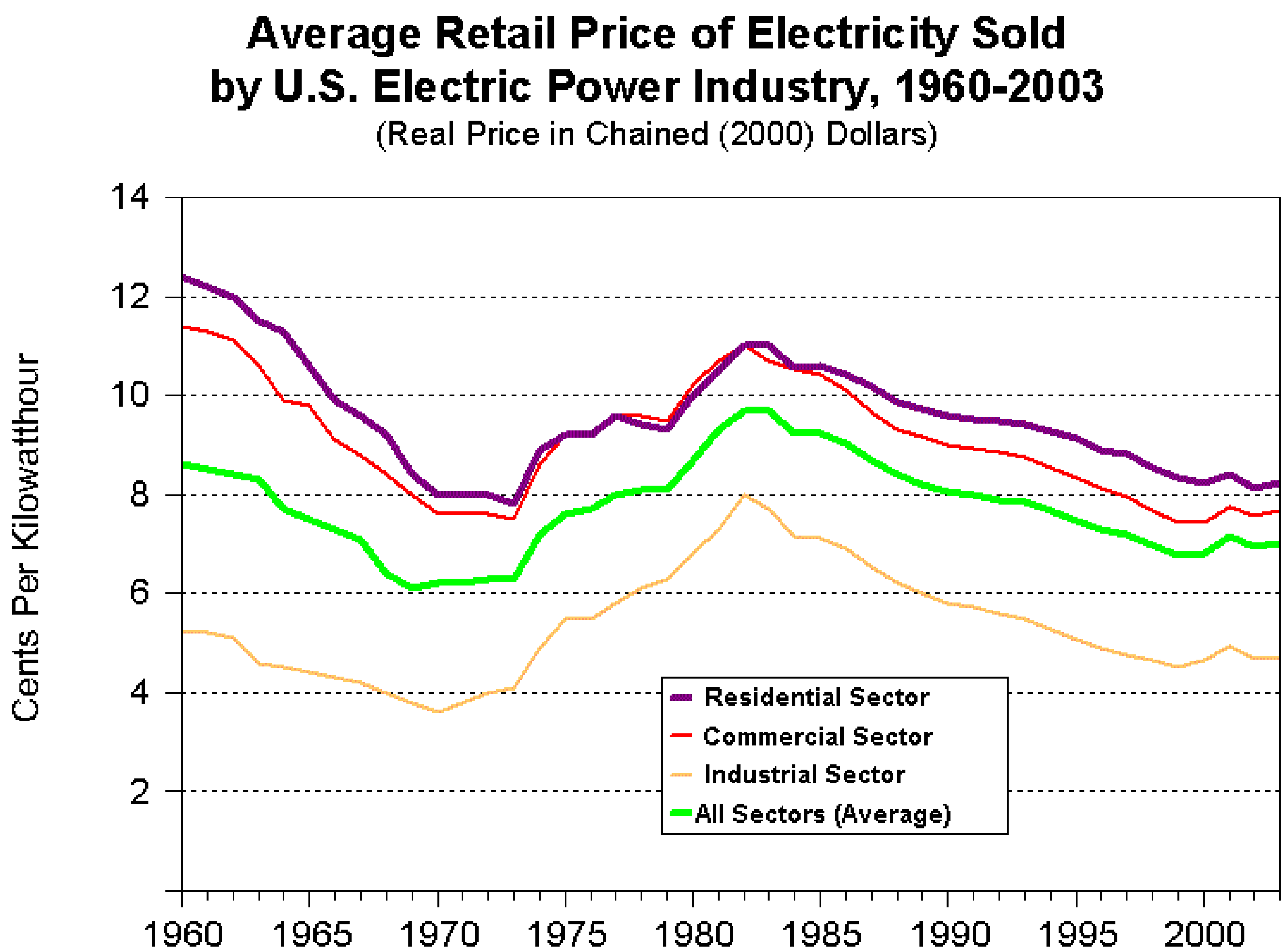
The Price of Electricity Depends on Who and Where You Are

The average price varies from 4.10 Cents per KWh in the State of Washington to 10.40 Cents in New York. The price reached $6 per KWh in 1998 in the Midwest US.



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Electric Power and the Environment

Electric power plants are the 2nd largest user of freshwater in the United States after agriculture. Each day the electric power industry withdraws 190 billion gallons of water, 39 percent of all the freshwater withdrawals in the nation.

U.S. Electric Power Industry Emissions

	1993	1998	2003
	(Thousand Metric Tons)		
Carbon Dioxide	2,034,206	2,313,013	2,408,961
Sulfur Dioxide	14,968	12,509	10,594
Nitrogen Oxides	7,997	6,235	4,396

Source: Energy Information Administration http://www.eia.doe.gov and Office of Fossil Energy http://www.fossil.energy.gov

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A Brief History of Electricity Deregulation

The competitive parts of the newly restructured systems are generation and retailing, while the regulated parts are distribution and transmission.

- Public Utility Regulatory Policies Act of 1978 opened the wholesale market to non-utility generators
  - "*Section 210 of PURPA requires electric utilities to interconnect with and buy whatever amount of capacity and energy is offered from any facility meeting the criteria for a qualifying facility*"
- Energy Policy Act of 1992 authorized FERC to open the transmission systems to wholesale suppliers
- The act of 1994 unbundled generation and transmission
  - Comparable access refers to the belief that owners of the transmission grid should offer third parties access to the grid on the same or comparable basis and under the same or comparable terms and conditions as the transmission owner's use of the system.
- FERC order 889 in 1996 established Open Access Same-Time Information System
- In December 1999, FERC asked all transmission-owning utilities to place their transmission systems under the control of Regional Transmission Organization

Source: Energy Information Administration http://www.eia.doe.gov

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Power Trading Requires Tightly Interconnected Systems

In order to maintain the reliability of bulk power operations, electric systems are already Interconnected and their operations are governed by NERC

Source: North American Electric Reliability Council http://www.nerc.com

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Opportunities for Mathematical Modeling

Relevant areas include game theory, mathematical programming, statistics, …

- Long-Term Modeling
  - Demand forecasting based on population and economic growth
  - Electricity price trends based on economic and competitive forecasts
  - Market design using game theory and simulation tools
  - Contract evaluation and investment assessment – new power lines and plants – in light of technological advancements
  - Modeling fuel contracts and operations, such as reservoirs and natural gas trends
- Short-Term Modeling
  - Demand forecasting based on weather
  - Electricity price forecasting as a function of supply and demand while considering the strategic behavior of different players in the market place
  - Managing the portfolio of power plants and contracts to maximize profit under price uncertainty
- Frequently Overlooked Problems
  - Assigning repair personnel and vehicles to maintenance calls
  - Determining optimal inventory
  - Workforce issues such as hiring, training, aging, …
  - Maintenance scheduling
  - Emission trading and its relationship to generation
  - Optimal operation of power plants and early problem detection – statistical quality control

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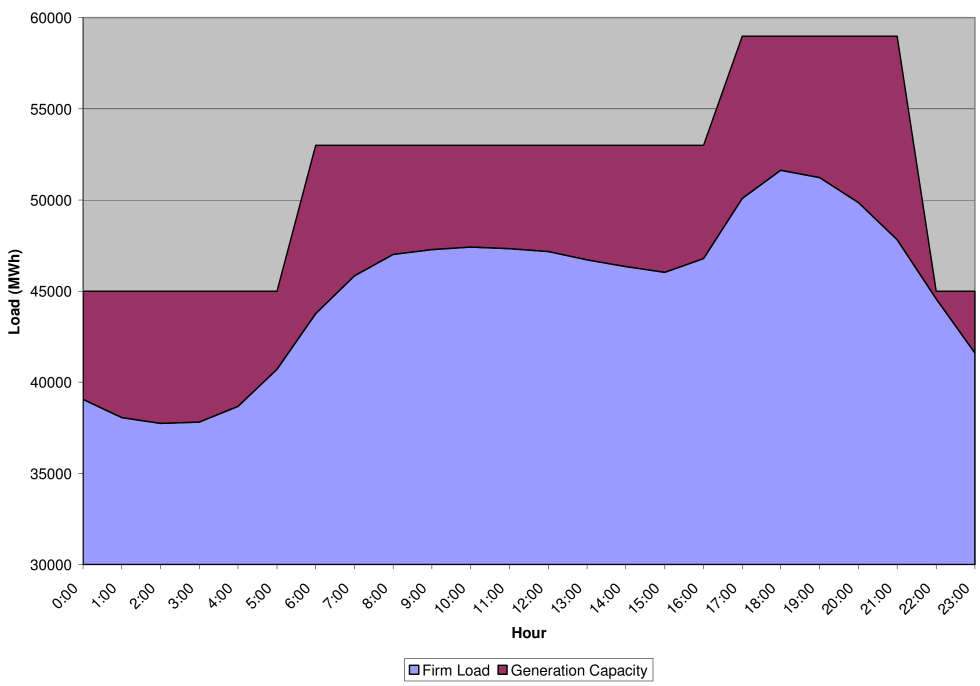
The Structure of a Short-Term Electricity Contract

A typical contract has two components: capacity and energy. The price is governed by the type of Power.

- Determine optimal schedules for the generating units
- Refine the load forecast which may result in residual capacity
- Auction the capacity through a bidding process where profit is maximized
  - Cost is a function of amount of power produced to fulfill the contractual obligations
  - Revenue from a power contract has two components
    - Capacity which is known at bid time
    - Energy which is a function of the true consumption
- Provide a robust solution



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A Two-Stage Robust Formulation

- The first stage selects the bids – binary decisions
- The second stage is a multi-period linear model for determining optimal generation. It contains all of the generation-related constraints
- The objective is to maximize expected profit while controlling second-stage variability
- Mathematically, the goal is to

$$\min \left\{ \sum_{i=1}^{I} c_i x_i + \sum_{k=1}^{K} p_k Q_k(x) + \lambda V(Q_1(x), \ldots, Q_K(x)) \mid x_i \in \{0,1\} \right\}$$

where

$$Q_k(x) := \min \left\{ \sum_{t=1}^{T} q_{tk}(y_{tk}) \mid \sum_{i=1}^{n} d_{tk}^i x_i \le y_{tk} \le C_{tk}, t = 1, \ldots, T \right\}.$$

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Two-Stage Stochastic Linear Programs

A two-stage stochastic linear program has the form

$$\min_x \left\{ cx + \sum_{k=1}^{K} p_k Q_k(x) \mid Ax = b,\ x \geq 0 \right\},$$

where

$$Q_k(x) := \min_y \left\{ q_k y \mid D_k y = h_k - T_k x,\ y \geq 0 \right\}.$$

This is equivalent to

$$\begin{array}{rl} \min_{x,y_k} & cx + \sum_{k=1}^{K} p_k q_k y_k \\ \text{s.t.} & Ax = b,\ x \geq 0, \\ & Dy_k = h_k - T_k x,\ y_k \geq 0. \end{array}$$

Reference: John R. Birge and Francois Louveaux. Introduction to Stochastic Programming. Springer Verlag, 1997

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Solving Two-Stage Stochastic Linear Programs

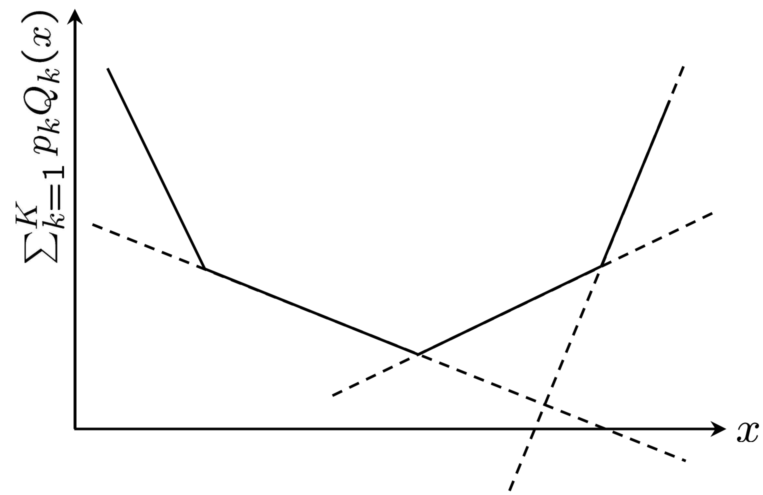
Observe that the second-stage objective is piecewise linear function of the first-stage decision. Therefore, cutting-plane methods are often used to solve the problem efficiently.

Re-write

$$\min_x \left\{ cx + \sum_{k=1}^{K} p_k Q_k(x) \mid Ax = b,\ x \geq 0 \right\}$$

as

$$\min_x \left\{ cx + \Psi \mid Ax = b,\ x \geq 0,\ \psi \geq \alpha x + \beta \right\}.$$



The problem can then be solved iteratively

- Solve the first-stage problem
- Solve the (separable) second-stage problems
- Add a cut to the first-stage problem
- Repeat

Reference: John R. Birge and Francois Louveaux. Introduction to Stochastic Programming. Springer Verlag, 1997

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Adding a Variability Measure to Control the Second-Stage Objective May Result in Loss of Optimality

$$\begin{array}{ll} \text{min} & cx + \sum_{k=1}^{K} p_k Q_k(x) \\ \text{s.t.} & Ax = b, x \geq 0 \end{array} \quad \Leftrightarrow \quad \begin{array}{ll} \text{min} & cx + \sum_{k=1}^{K} p_k q_k y_k \\ \text{s.t.} & Ax = b, x \geq 0 \\ & Dy_k = h_k - T_k x, \;\; y_k \geq 0 \end{array}$$

$$\downarrow \qquad\qquad\qquad \downarrow$$

$$\begin{array}{ll} \text{min} & cx + \sum_{k=1}^{K} p_k Q_k(x) \\ & +\lambda V(Q_1(x), \ldots, Q_K(x)) \\ \text{s.t.} & Ax = b, x \geq 0 \end{array} \quad \not\Leftrightarrow \quad \begin{array}{ll} \text{min} & cx + \sum_{k=1}^{K} p_k q_k y_k \\ & +\lambda V(q_1 y_1, \ldots, q_K y_K) \\ \text{s.t.} & Ax = b, x \geq 0 \\ & Dy_k = h_k - T_k x, \;\; y_k \geq 0 \end{array}$$

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Adding a Variability Measure to Control the Second-Stage Objective May Result in Loss of Optimality

Consider a 2-scenario problem

$$\begin{array}{ll} \min & 2x + 0.5y_1 + 0.5y_2 + \lambda V(y_1, y_2) \\ \text{s.t.} & x + y_1 \geq 3, \\ & x + y_2 \geq 2, \\ & x \geq 0,\ y_1 \geq 0,\ y_2 \geq 0, \end{array}$$

where $V(y_1, y_2) = (y_1 - y_2)^2/4$.

The optimal solution is

- If $\lambda \leq 1$, then $x = 0,\ y_1 = 3,\ y_2 = 2$
- If $\lambda > 1$, then $x = 0,\ y_1 = 3,\ y_2 = 3 - 1/\lambda$

Reference: S. Takriti and S. Ahmed. "On Robust Optimization of Two-Stage Systems," Mathematical Programming, vol.99, pp.109-126, 2004

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Why Do We Lose Optimality?

Let y_k, $k = 1, \ldots, K$, *be a feasible solution for*

$$\begin{array}{ll} \min & \sum_{k=1}^{K} p_k q_k y_k + \lambda V(q_1 y_1, \ldots, q_K y_K) \\ \text{s.t.} & D y_k = h_k - T_k x, \;\; y_k \geq 0. \end{array}$$

Then, y_k *satisfies the KKT optimality conditions if and only if there exists a subgradient* $g = (g_1, \ldots, g_K) \in \partial V$ *such that:*

- *If* $p_k + \lambda g_k > 0$ *then* y_k *solves*

$$\min_y \{ q_k y \mid D y = h_k - T_k x, \;\; y \geq 0 \}$$

- *If* $p_k + \lambda g_k < 0$, *then* y_k *solves*

$$\max_y \{ q_k y \mid D y = h_k - T_k x, \;\; y \geq 0 \}.$$

Reference: S. Takriti and S. Ahmed. "On Robust Optimization of Two-Stage Systems," Mathematical Programming, vol.99, pp.109-126, 2004

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A Sufficient Condition for Maintaining Second-Stage Optimality is to Choose a Non-Decreasing Variability Measure

- When the variability measure is non-decreasing, we can show

$$\begin{aligned} \min \quad & cx + \textstyle\sum_{k=1}^K p_k Q_k(x) \\ & +\lambda V(Q_1(x), \ldots, Q_K(x)) \\ \text{s.t.} \quad & Ax = b, x \geq 0 \end{aligned} \quad \Leftrightarrow \quad \begin{aligned} \min \quad & cx + \textstyle\sum_{k=1}^K p_k q_k y_k \\ & +\lambda V(q_1 y_1, \ldots, q_K y_K) \\ \text{s.t.} \quad & Ax = b, x \geq 0 \\ & Dy_k = h_k - T_k x, \;\; y_k \geq 0 \end{aligned}$$

- Extend the L-shaped method to solve the problem efficiently
  - Solve the first-stage problem
  - Solve the (separable) second-stage problems
  - *Scale the dual multipliers*
  - Add a cut to the first-stage problem
  - Repeat
- An example of such a variability measure is squared cost above target

$$V(q_1 y_1, \ldots, q_K y_K) = \sum_{k=1}^{K} [q_k y_k - R^*]_+^2$$

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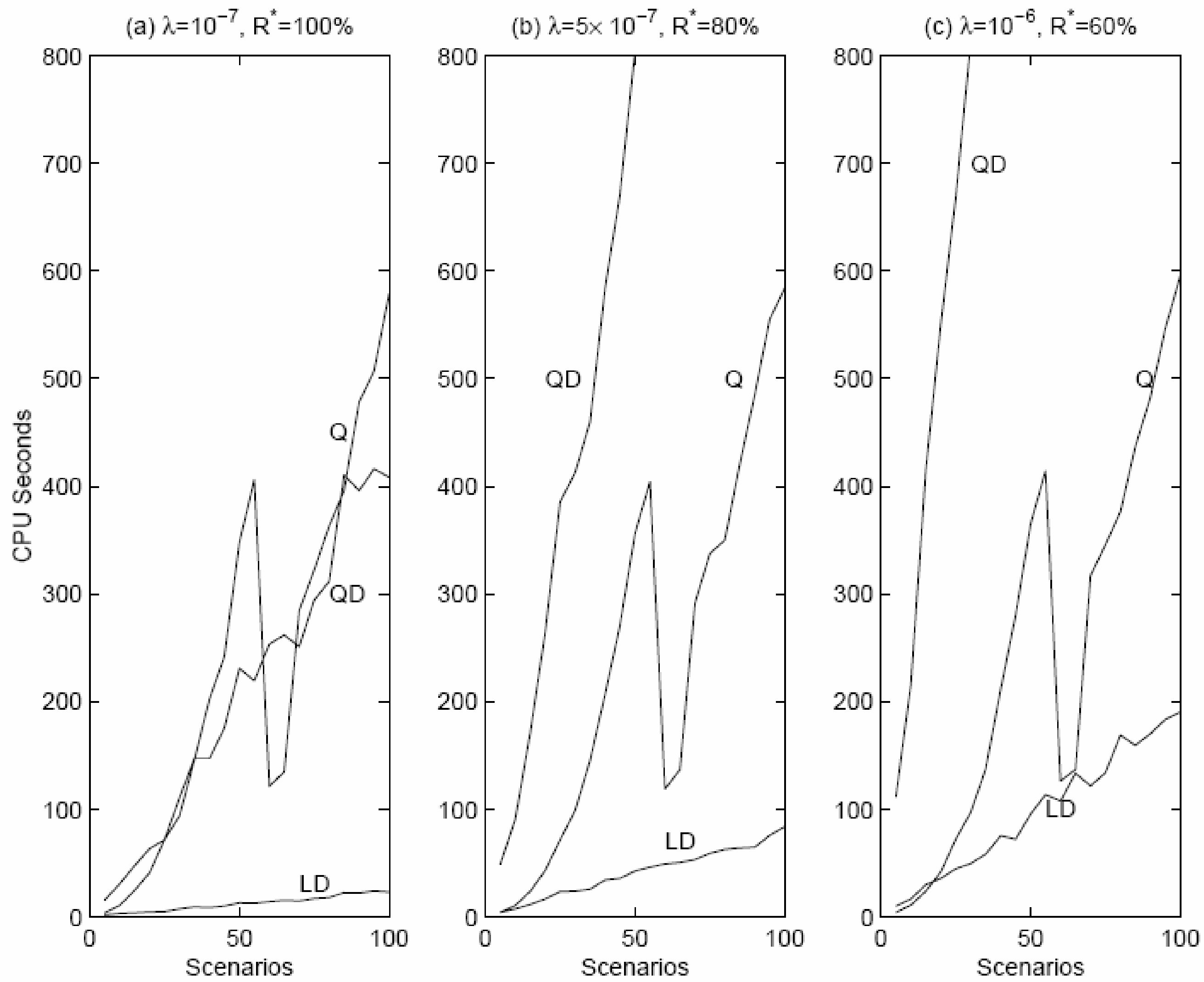
An Example with Quadratic Variability Measure – STORM



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An Example with Quadratic Variability Measure – STORM

R^*	λ	Cuts	cx^*	$EQ(x^*)$	Cost	$V(.)$	CPU
–	0	50	5818723	9679627	15498350	–	26.70
100%	1	44	5853398	9645328	15498726	4990	23.40
	5	57	5899601	9601937	15501537	4104	29.46
	10	52	5941350	9564835	15506186	3454	25.21
90%	1	47	5942561	9563800	15506361	82936	24.39
	5	121	6487955	9159346	15647301	29997	59.99
	10	120	6880558	8890016	15770574	11749	59.69
80%	1	89	6123922	9417984	15541905	290568	44.54
	5	162	7823787	8251453	16075240	35771	83.61
	10	150	8257502	7967127	16224629	13913	77.42
70%	1	115	6626946	9063971	15690917	533799	59.24
	5	146	9249246	7319257	16568503	39551	77.91
	10	191	9715286	7020317	16735603	15029	109.45
60%	1	134	7775395	8283304	16058699	622996	67.56
	5	219	10599947	6472183	17072130	53542	137.30
	10	291	11140819	6155373	17296192	20731	188.72

The value of λ is in multiples of 10^{-7} and V is in multiples of 10^7

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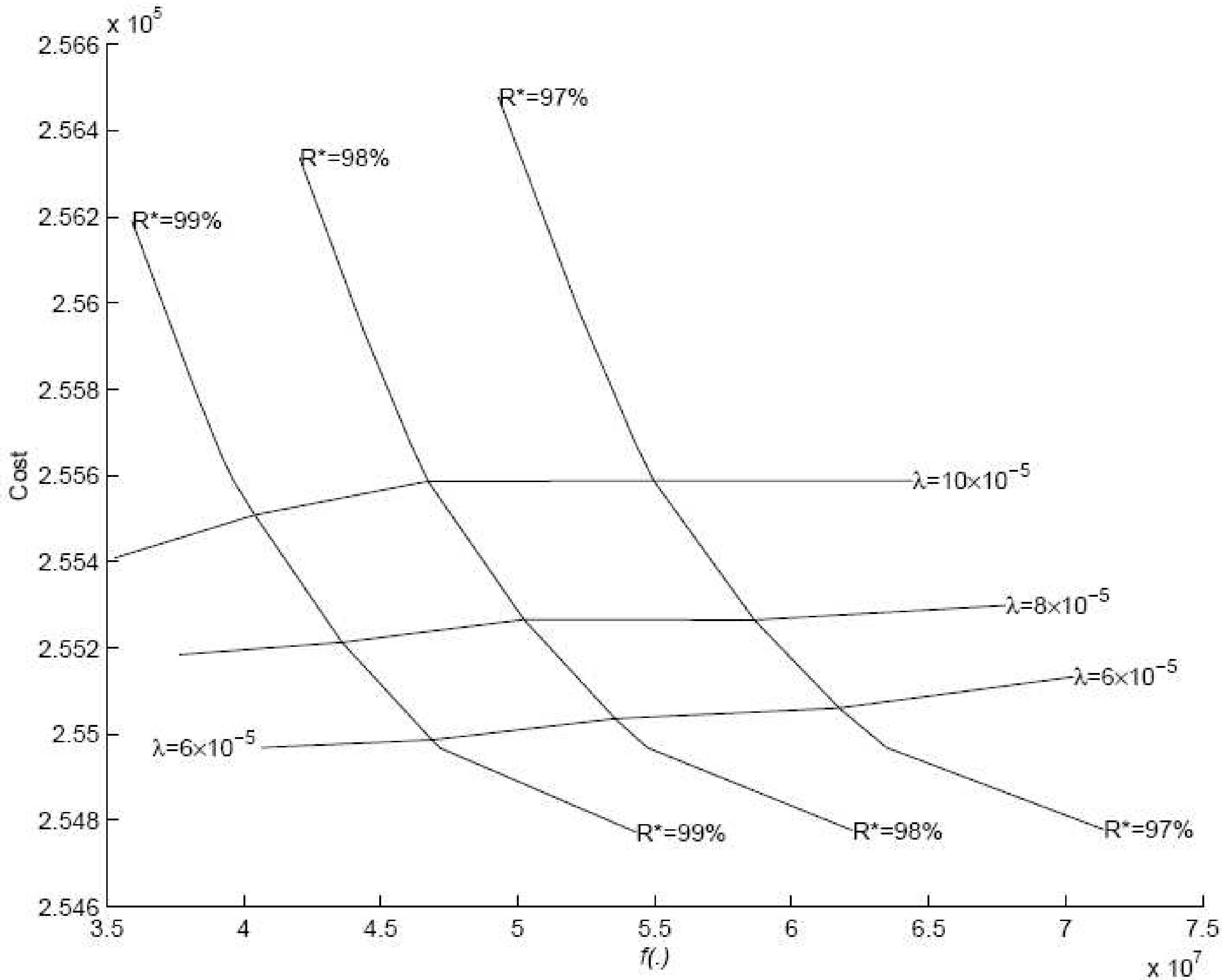
An Example with Quadratic Variability Measure – 20TERM



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How to Handle the Original Case with Integer First Stage?

Use a branch-and-cut approach where the cuts are generated as discussed before. Additional cuts are added at integer nodes.

- Solve the linear relaxation by adding cuts as described before
- Start the branch-and-bound process
  - If a node is fractional, continue the branch-and-bound process
  - If a node is integer, add the necessary cut as described before
    - If the solution remains integer, then fathom the node
    - Otherwise, continue branching

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A Contract Selection Problem with 100 Bids and 200 Scenarios

		Root		Branch & Bound			Objective	
R^*	λ	Cuts	CPU	Nodes	Inc.	CPU	Cost	$f(.)$
–	0	368	22.51	14899	179	107.84	-663305	–
100%	1	361	25.99	17249	212	128.87	-663305	630
	5	368	27.78	27435	342	218.2	-661306	6
	10	372	27.53	23272	307	182.06	-661306	6
90%	1	341	24.85	17300	310	136.82	-658629	2630
	5	382	30.07	12262	217	113.92	-652026	724
	10	452	37.78	10415	164	116.85	-651215	639
80%	1	359	26.07	5204	201	69.38	-652026	24091
	5	168	9.54	63	2	10.49	-613865	8501
	10	113	6.06	3980	97	18.64	-563850	2129
70%	1	255	16.02	3258	106	34.54	-641675	71495
	5	33	1.72	20	4	2.12	-511100	18130
	10	34	1.8	57	7	2.32	-419576	4020
60%	1	177	10.11	682	28	13.89	-628611	145902
	5	27	1.41	240	17	2.63	-344328	21001
	10	27	1.43	755	14	2.89	-246979	6347

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Thank You – Any Questions?

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