State-of-the-Art in the Solution of Control-Related Nonlinear Optimization Problems

Hans D Mittelmann

Department of Mathematics and Statistics
Arizona State University

Industrial Optimization Seminar, Fields Institute
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Outline

1. Our Optimization-Related Services
   - "Decision Tree for Optimization Software"
   - "Benchmarks for Optimization Software"
   - Our NEOS Solvers
2. PDE Constrained Optimization
   - Our related work
   - Others solving our problems
3. System Identification Problems
   - Crest Factor Minimization and spatial Distribution
   - How to improve the Distribution?

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Where to find Links to Software?
An advanced Google search

An advanced Google search for "optimization software" yields in order:

- Decision Tree for Optimization Software
- Benchmarks for Optimization Software
- NEOS Guide: Optimization Software
- NEOS Guide

Decision Tree for Optimization Software

The entry Page

Problems/ Software | Benchmarks | Testcases | Books/ Tutorials | Tools | Websub-mission | Other Sources

Decision Tree for Optimization Software

Search the Decision Tree
Web statistics for server Plato

Welcome! This site aims at helping you identify ready to use solutions for your optimization problem, or at least to find some way to build such a solution using work done by others. If you know of useful sources not listed here, please let us know. If something is found to be erroneous, please let us know, too. Where possible, public domain software is listed here.

Problems/Software

Links to these Subpages

- Global Optimization
- LP/NLP - Linear and Nonlinear Optimization
  - Unconstrained
  - Constrained
  - Least squares (other norms -> Approximation)
- ZERO
- MCP - Complementarity Problem
- Multi-objective Optimization
- Discrete Optimization
- Approximation

Problems/Software

Number of Links on each Subpage

- Global Optimization 40
- LP/NLP - Linear and Nonlinear Optimization
  - Unconstrained 53
  - Constrained 246
  - Least squares (other norms -> Approximation) 70
- ZERO 64
- MCP - Complementarity Problem 14
- Multi-objective Optimization 14
- Discrete Optimization 65
- Approximation 32

The Constrained NLO Problem

Major Parts of this Page

- The LP-problem, also mixed integer and stochastic
- The QP-problem, also mixed integer
- Semidefinite and second-order cone programming
- Geometric programming
- The general nonlinear problem (dense, sparse, nonsmooth, SIP)
- Mixed integer nonlinear programming
- Network constraints
- Special/constraint solvers
- Control problems
- other collections/problems

The general nonlinear problem

Further Substructuring

- Using function values only
- f general, bound constraints only
- General linear constraints
- General nonlinear constraints, convex problems
- General nonlinear constraints, dense linear algebra
- General nonlinear constraints, sparse linear algebra
- Minimization of Nonsmooth Functions
- Semi-infinite Programming

The other Main Pages

With Number of Links per Page

- Benchmarks 41
- Testcases 99
- Tutorials, Books 65
- Web-Submission 15
- Other Sources 19

Is that of Interest to Industry?

Accesses from Industry

From webstatistics (on entry page) of September 05
Reverse Domain Lookup for .com .co.jp

bankofamerica	deutsche-boerse	learjet	unilever
bmwgroup	dupont.eurx	mathworks	unisys
boeing	fidelity	microsoft	wellsfargo
capitalone	francetelecom.rd	motorola	jp.co.aventis
chase	halliburton	nintendo	jp.co.hitachi
commerzbank	honeywell	oracle	jp.co.mizuho-ri
cray	ibm.almaden	raytheon	jp.co.sumitomo
daimlerchrysler	intel.sc	shell	jp.co.toyota
delphiauto	kodak	sun	

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Our Benchmarks

Current Benchmarks

- COMBINATORIAL OPTIMIZATION
  - Concorde-TSP with different LP solvers (1-25-2006)
- LINEAR PROGRAMMING
  - Benchmark of commercial LP solvers (1-19-2006)
  - Benchmark of free LP solvers (2-6-2006)
- SEMIDEFINITE/SQL PROGRAMMING
  - Several SDP codes on problems from SDPLIB (10-11-2005)
  - Newer SDP/SOCP-codes on the 7th DIMACS Challenge problems(10-11-2005)
  - Several SDP codes on sparse and other SDP problems (2-13-2006)
  - SOCP (second-order cone programming) Benchmark (6-23-2005)

Our Benchmarks

Current Benchmarks (cont.)

- NONLINEAR PROGRAMMING
  - AMPL-NLP Benchmark, IPOPT, KNITRO, LOQO, PENNON & SNOPT (1-31-2006)
- MIXED INTEGER LINEAR PROGRAMMING
  - MILP Benchmark - free codes (1-23-2006)
- MIXED INTEGER NONLINEAR PROGRAMMING
  - MIQP Benchmark (1-16-2006)
- PROBLEMS WITH EQUILIBRIUM CONSTRAINTS
  - MPEC Benchmark (1-28-2006)

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Which solvers are "ours"?

including input formats

- Combinatorial Optimization
  - concorde [TSP Input] (only one in category)
- Linear Programming
  - bpmpd [AMPL Input][LP Input][MPS Input][QPS Input]
- Mixed Integer Linear Programming
  - feaspump [AMPL Input][CPLEX Input][LP Input][MPS Input]
  - scip [AMPL Input][CPLEX Input][LP Input][MPS Input][ZIMPL Input]
- Nondifferentiable Optimization
  - condor [AMPL Input] (only one in category)
- Semi-infinite Optimization
  - nsips [AMPL Input] (only one in category)

Which solvers are "ours"?
continued

- Stochastic Linear Programming
  - bnbs [SMPS Input]
  - ddsip [MPS input] [LP input]
- Semidefinite Programming (all but one in category)
  - csdp [MATLAB BINARY Input][SPARSE SDPA Input]
  - penbmi [MATLAB Input][MATLAB BINARY Input]
  - pensdp [MATLAB BINARY Input][SPARSE SDPA Input]
  - sdpa [MATLAB BINARY Input][SPARSE SDPA Input]
  - sdpa-c [MATLAB BINARY Input][SPARSE SDPA Input]
  - sdplr [MATLAB BINARY Input][SDPLR Input][SPARSE SDPA Input]
  - sdpt3 [MATLAB BINARY Input][SPARSE SDPA Input]
  - sedumi [MATLAB BINARY Input][SPARSE SDPA Input]

Input formats are important!

count of NEOS solvers

Including the number of input formats our solvers make up

36 of 90 solvers or **40** percent

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Half a dozen papers

reports starting in 1998

H. Maurer and H. D. Mittelmann,
Optimization Techniques for Solving Elliptic Control Problems with Control and State Constraints.
Part 1: Boundary Control
Comp. Opt. Applic. 16, 29-55 (2000)
Part 2: Distributed Control
Comp. Opt. Applic. 18, 141-160 (2001)

H. D. Mittelmann and H. Maurer,
Interior Point Methods for Solving Elliptic Control Problems with Control and State Constraints. Boundary and Distributed Control,
J. Comp. Math. Applic. 120, 175-190 (2001)

Our related work (cont.)

H. D. Mittelmann,
Verification of Second-Order Sufficient Optimality Conditions for Semilinear Elliptic and Parabolic Control Problems.
Comp. Opt. Applic. 20, 93-110 (2001)

H. D. Mittelmann,
Sufficient Optimality for Discretized Parabolic and Elliptic Control Problems,
in Fast solution of discretized optimization problems, K.-H. Hoffmann, R.H.W. Hoppe, and V. Schulz (eds.), ISNM 138, Birkhäuser, Basel, 2001

Our related work (cont.)

H. D. Mittelmann and F. Tröltzsch, *Sufficient Optimality in a Parabolic Control Problem.* in Proceedings of the first International Conference on Industrial and Applied Mathematics in Indian Subcontinent, P. Manchanda, A.H. Siddiqi, and M. Kocvara (eds), Kluwer, Dordrecht, The Netherlands, 2001

all papers accessible (PS/PDF/HTML) through:

http://plato.asu.edu/papers.html

Subsequent Activities in US
we were not invited

first sandia workshop on large-scale pde-constrained optimization
april 4-6, 2001 the bishop's lodge santa fe, nm

Large-Scale PDE-Constrained Optimization Series: Lecture Notes in Computational Science and Engineering, Vol. 30 Biegler, L.T.; Ghattas, O.; Heinkenschloss, M.; Bloemen Waanders, B.v. (Eds.) 2003

second sandia workshop on large-scale pde-constrained optimization
may 19-21, 2004 the bishop's lodge santa fe, nm

Boundary Control Problems

$$F(y,u) = \int_{\Omega} f(x,y)\,dx + \int_{\Gamma_1} g(x,y,u)\,dx + \int_{\Gamma_2} k(x,u)\,dx$$

subject to the elliptic state equation

$$-\Delta y(x) + d(x,y(x)) = 0\,, \quad \text{for} \quad x \in \Omega\,,$$

boundary conditions of Neumann or Dirichlet type

$$\begin{aligned} \partial_\nu y(x) &= b(x,y(x),u(x))\,, && \text{for} \quad x \in \Gamma_1 \\ y(x) &= a(x,u(x))\,, && \text{for} \quad x \in \Gamma_2\,, \end{aligned}$$

and control and state inequality constraints

$$\begin{aligned} C(x,u(x)) &\le 0\,, && \text{for} \quad x \in \Gamma \\ S(x,y(x)) &\le 0\,, && \text{for} \quad x \in \Omega\,. \end{aligned}$$

Necessary Optimality Conditions

adjoint equation and boundary conditions:

$$\begin{aligned} -\Delta\bar{q} + \bar{q}(x)\, d_y(x,\bar{y}) + f_y(x,\bar{y}) + S_y(x,\bar{y})\,\bar{\mu} \quad &= 0 \quad \text{on} \;\; \Omega \\ \partial_\nu \bar{q} - \bar{q}(x) b_y(x,\bar{y},\bar{u}) + g_y(x,\bar{y},\bar{u}) \quad &= 0 \quad \text{on} \;\; \Gamma_1 \\ \bar{q} \quad &= 0 \quad \text{on} \;\; \Gamma_2 \end{aligned}$$

minimum condition for $x \in \Gamma_1$:

$$g_u(x,\bar{y},\bar{u}) - \bar{q}\, b_u(x,\bar{y},\bar{u}) + \bar{\lambda}\, C_u(x,\bar{u}) = 0$$

minimum condition for $x \in \Gamma_2$:

$$k_u(x,\bar{u}) + \partial_\nu \bar{q}\, a_u(x,\bar{u}) + \bar{\lambda}\, C_u(x,\bar{u}) = 0$$

complementarity conditions (J active sets):

$$\begin{array}{llllll} \bar{\lambda}(x) \geq 0 & \text{on} & J(C) & \bar{\lambda}(x) = 0 & \text{on} & \Gamma \setminus J(C)\,, \\ d\bar{\mu} \geq 0 & \text{in} & J(S) & d\bar{\mu} = 0 & \text{in} & \Omega \setminus J(S) \end{array}$$

Typical Applications

In many applications, the cost functional is of *tracking type*

$$F(y,u) = \frac{1}{2}\int_{\Omega} (y(x) - y_d(x))^2\,dx + \frac{\alpha}{2}\int_{\Gamma} (u(x) - u_d(x))^2\,dx\,,$$

with given functions $y_d \in C(\bar{\Omega})$, $u_d \in L^\infty(\Gamma)$, and nonnegative weight $\alpha \geq 0$. The control and state constraints are taken to be box constraints of the simple type

$$y(x) \leq \psi(x) \quad \text{in} \;\; \Omega \qquad u_1(x) \leq u(x) \leq u_2(x) \quad \text{on} \;\; \Gamma\,,$$

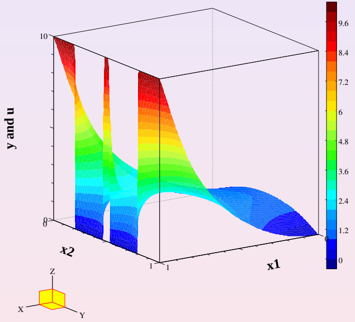
with functions $\psi \in C(\bar{\Omega})$ and $u_1, u_2 \in L^\infty(\Gamma)$

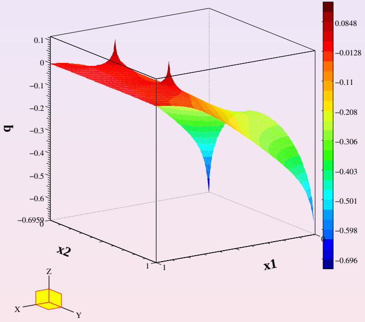
Letting $\Omega = [0,1]^2$, $\Gamma_2 = \{\, (x_1, 1) \mid 0 \le x_1 \le 1 \,\}$ and $\Omega_0 = [0.25, 0.75]^2$, the control problem is to determine a function $u \in L^\infty(\Gamma_2)$ which minimizes

$$F(y,u) = \frac{1}{2} \int_{\Omega_0} (y(x) - 1)^2 \, dx \; + \; \frac{\alpha}{2} \int_{\Gamma_2} u(x)^2 \, dx \qquad (1)$$

subject to the state equation, Neumann and Dirichlet boundary conditions and control and state inequality constraints,

$$\begin{array}{rcllll}
-\Delta y(x) & = & 0 & \text{in} & \Omega \,, & \\
\partial_\nu y(x) & = & 0 & \text{for} & x_2 = 0, & 0 \le x_1 \le 1 \,, \\
\partial_\nu y(x) & = & y(x) - 5 & \text{for} & x_1 \in \{0, 1\}, & 0 \le x_2 \le 1 \,, \\
y(x) & = & u(x) & \text{for} & x_2 = 1, & 0 \le x_1 \le 1 \,, \\
y(x) & \le & 3.15 & \text{in} & \Omega_0 \,, & \\
y(x) & \le & 10 & \text{in} & \Omega \setminus \Omega_0 \,, & \\
0 \le u(x) & \le & 10 & \text{for} & x_2 = 1, & 0 \le x_1 \le 1 \,.
\end{array}$$





Case $\alpha = 0$ (bang-bang control):

```
param z{i in 1..n, j in 1..n} := 1;
var x{0..n-1, 0..n-1};

minimize f:
.5*h2*sum{i in n1/4..3*n1/4,j in n1/4..3*n1/4}
(x[i,j]-z[i,j])^2+.5*h*a*sum{i in 1..n} x[i,n1]^2;

s.t. pde{i in 1..n, j in 1..n}:
4*x[i,j]-sum{k in {-1,1}}(x[i+k,j]+x[i,j+k])=g*h2;

s.t. bc1{i in 1..n}: x[i,0]=x[i,1];
s.t. bc2{i in 1..n}: x[0,i]-x[1,i]=h*(x[0,i]-t);
s.t. bc3{i in 1..n}: x[n1,i]-x[n,i]=h*(x[n1,i]-t);
s.t. sc{i in 0..n1,j in 0..n}: 0<=x[i,j]<= if n1/4
<=i<=3*n1/4 && n1/4<=j<=3*n1/4 then 3.15 else 10;
s.t. cc{i in 1..n}: 0<=x[i,n1]<=10;
```

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Benchmarks we provide

imitation is the sincerest form of flattery

Bonettini, Silvia,*A nonmonotone inexact Newton method.*
Optim. Methods Softw. 20 (2005), 475–491

Bonettini, S., Galligani, E., and Ruggiero, V.,
An inexact Newton method combined with Hestenes multipliers' scheme for the solution of indefinite KKT systems.
Appl. Math. Comput. 168 (2005), 651–676

Schenk, O., Waechter, A., and Hagemann, M.,
Combinatorial Approaches to the Solution of Saddle-Point Problems in Large-Scale Parallel Interior-Point Methods,
Research Report RC23824, Dec 2005, IBM Research,
T. J. Watson Research Center, Yorktown Heights, NY

Largest Cases Solved

three of twelve cases chosen

For the above **boundary** control problem Waechter et al push N to 2500 (6.25 million variables&constraints) and solve it in 4 hrs (1 proc)

For a **distributed** control problem (same N, 12.5 million variables & 6.25 million constraints) they need 4 hrs (2 proc)

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The Problem and its Realization

$$CF(y) = \frac{||y||_\infty}{||y||_2}$$

```
minimize obj: t;
s.t. bound31{j in N..2*N-1}: -t<=y[j,1];
s.t. bound32{j in N..2*N-1}: y[j,1]<= t;
s.t. bound41{j in N..2*N-1}: -t<=y[j,2];
s.t. bound42{j in N..2*N-1}: y[j,2]<= t;
```

or

```
-t*sqrt((1/N)*sum{i in N..2*N-1}y[i,1]^2)<=y[j,1];
y[j,1]<= t*sqrt((1/N)*sum{i in N..2*N-1}y[i,1]^2);
-t*sqrt((1/N)*sum{i in N..2*N-1}y[i,2]^2)<=y[j,2];
y[j,2]<= t*sqrt((1/N)*sum{i in N..2*N-1}y[i,2]^2);
```

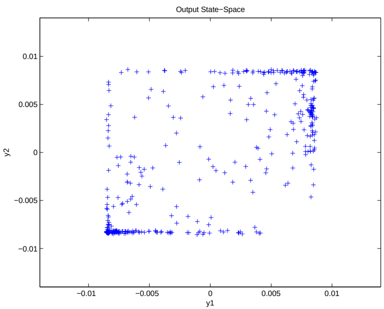
The Computational Effort

Using output crest factor minimization

```
=====================================================
problem       nv/nc      IPOPT KNITRO  LOQO SNOPT
=====================================================
NARX_CFy   43973/46744   685    240    fail     t
=====================================================
nv/nc: no. of var./cons. t: time exceeded (2hr)
=====================================================
```

The Distribution

Using minCFy



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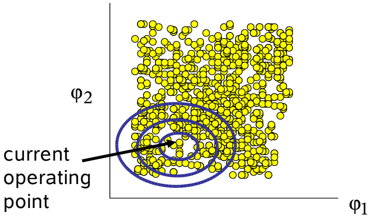
Why improve the Distribution?

Model-on-Demand Estimation

(Stenman, 1999)

- A modern data-centric approach developed at Linkoping University
- Identification signals geared for MoD estimation should consider the geometrical distribution of data over the state-space.



Utilize historical result

Theorem (H. Weyl, 1916)

A sequence $\{y_1(n), y_2(n)\}$ *is equidistributed in* $[0,1)^2$ *if and only if*

$$\lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} e^{2\pi i (l_1 y_1(n) + l_2 y_2(n))} = 0$$

$\forall$ *sets of integers* l_1, l_2 *not both zero.*

The AMPL Realization

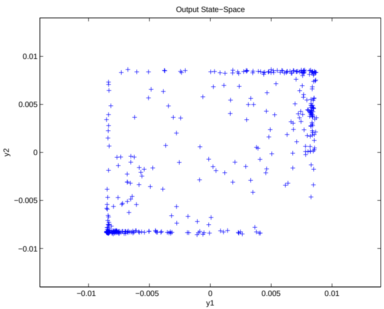
plus gradual increase of lim

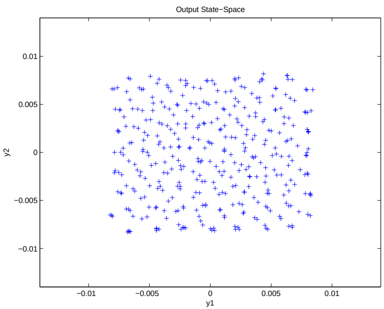
```
param t := 1e-2;
param lim := 7;
set S := -lim..0 union 1..lim;
set W := {i in S,j in S:not(i==0 and j==0)};

s.t. weyl1{(m1,m2) in W}:
       sum{n in N+1..2*N-1}
           cos(2*pi*(m1*(y[n,1]+bdy)/(2*bdy)
           + m2*(y[n,2]+bdy)/(2*bdy))) <= t;
s.t. weyl2{(m1,m2) in W}:
       sum{n in N+1..2*N-1}
           sin(2*pi*(m1*(y[n,1]+bdy)/(2*bdy)
           + m2*(y[n,2]+bdy)/(2*bdy))) <= t;
```

The Distribution

Using both Methods

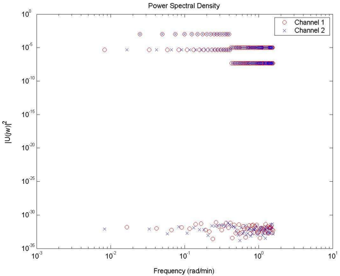
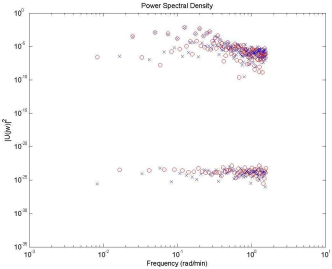




The Spectra

Using both Methods

min Crest Factor vs Weyl-based Signals - Power Spectra

Modified Zippered, min CF (y) Signal	Modified Zippered, Weyl-based signals
	

All harmonic coefficients are selected by the optimizer in the Weyl-based problem formulation

The Computational Effort

Using both Methods

problem	nv/nc	IPOPT	KNITRO	LOQO	SNOPT
NARX_CFy	43973/46744	685	240	fail	t
NARX_Weyl	44244/45568	t	3835	fail	t

nv/nc: no. of var./cons. t: time exceeded (2hr)

Does that allow improved MoD Control?

Yes, see this paper:

D. E. Rivera, H. Lee, H. D. Mittelmann, and G. Pendse,
Optimization-based Design of Plant-Friendly Input Signals for Data-Centric Estimation and Control,
Annual AIChE Meeting, paper 242k, Cincinnati, OH, October 31 - November 4, 2005

THE END

Thank you for your attention

Questions or Remarks?

PDF of talk at: http://plato.asu.edu/talks/

Other links on http://plato.asu.edu/