Robust Airline Fleet Assignment

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Agenda

- Fleet Assignment Model
- Robustness
- Spoke Purity
- Station Decomposition (SDM)
- Convex combinations of duals
  - A dual ascent version of D-W decomp
  - Second stage master
- Integer Answers
- The (more real) bottom line

Fleet Assignment Model (FAM)

- FAM assigns aircraft types to an airline timetable in order to maximize profit
- Widely used in the airline industry
  - AA and DL reported 1% profit from FAM
  - FAM was originally deployed in the 1980s
- Environment for fleet assignment has evolved
  - Hub and spoke schedule structure
  - Impact on planning, marketing and operations
    - Crew and maintenance planning and ops

Airline Planning Integration

- **Enterprise Planning**
  - **Labor planning**
  - **Markets**
  - **Fleets**
- **Product Planning**
  - **Sched & price**
- **Tactics and Operations**
  - **Crew and plane planning**
  - **Capacity, price, availability optimization**

Time Horizon	• 18 Months +	• 18 Months – 1 Month	• 3 months – Departure
Objective	• Maximize NPV of Future Profits	• Maximize NPV of Future Profits	• Maximize NPV of Future Profits
Decisions	• Route Structure • Fleet • Maintenance Bases • Crew Bases • Facilities	• Schedule • Pricing Policies	• Price • Restrictions • Availability
Constraints	• Financial Resources • Regulation	• Route Structure • Fleet • Maintenance • Crew Bases • Facilities	• Schedule • Pricing Policies

FAM Background

- **Given a flight schedule and available fleet of aircraft, FAM maximizes operating profit subject to operational constraints**
- **The basic FAM constraints include:**
  - **Cover: Each flight in the schedule must be assigned exactly one aircraft type**
  - **Plane Count: The total number of aircraft assigned cannot exceed the number available in the fleet**
  - **Balance: Aircraft cannot appear or disappear from the network**

FAM Notation

2.1.1 Sets

A: set of airports indexed by a.
L: set of flight legs, indexed by i.
F: set of fleet types, indexed by f.
T: set of all departure and arrival events, indexed by t.
N: set of timeline nodes, indexed by $\{f,a,t\}$.
$CL(f)$: set of flight legs crossing the counting line flow by fleet f.
$I(f,a,t)$: set of flight legs inbound to $\{f,a,t\}$.
$O(f,a,t)$: set of flight legs outbound from $\{f,a,t\}$.

2.1.2 Decision Variables

$$x_{f,i} = \begin{cases} 1 \text{ if fleet } f \in F \text{ is assigned to flight leg } i \in L \\ 0 \text{ otherwise} \end{cases}$$

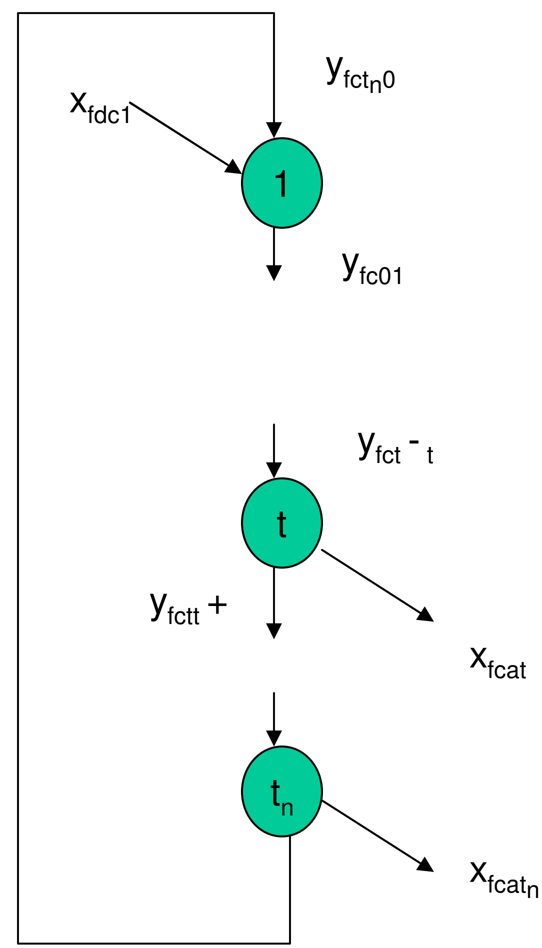
y_{f,a,t_j^+}: the number of aircraft on the ground for fleet type $f \in F$, at airport $a \in A$, on the ground arc just following time $t_j \in T$.

y_{f,a,t_j^-}: the number of aircraft on the ground for fleet type $f \in F$, at airport $a \in A$, on the ground arc just prior to time $t_j \in T$.

2.1.3 Parameters

N_f: the number of aircraft available of fleet type $f \in F$.
Cap_f: the seating capacity of fleet type $f \in F$.
$C_{f,i}$: operating cost of assigning fleet $f \in F$ to flight leg $i \in L$.
$R_{f,i}$: revenue produced by assigning fleet $f \in F$ to flight leg $i \in L$.

Timeline city c, fleet f



FAM Formulation

Maximize:

$$\sum_{f \in F} \sum_{i \in L} (R_{f,i} - C_{f,i}) x_{f,i}$$

Subject to:

$$\sum_{f \in F} x_{f,i} = 1, \forall i \in L$$

$$y_{f,a,t^-} + \sum_{i \in I(f,a,t)} x_{f,i} - y_{f,a,t^+} - \sum_{i \in O(f,a,t)} x_{f,i} = 0, \forall f, a, t$$

$$\sum_{a \in A} y_{f,a,t_m} + \sum_{i \in CL(f)} x_{f,i} \le N_f, \forall f \in F$$

$$x_{f,i} \in \{0,1\}, \forall f \in F, \forall i \in L$$

$$y_{f,a,t} \ge 0, \forall f, a, t$$

Modeling Robustness in FAM

- The goal of robustness is to produce FAM solutions that anticipate subsequent planning and operational recovery
  - Planning: reduce crew costs by avoiding low frequency service to a spoke by a given fleet
  - Reduce maintenance costs by reducing the total number of fleet/station combinations
  - Operations: limit diversity of aircraft types at stations in order to create more possibilities for crew swaps and aircraft swaps

Spoke Purity in FAM

- We impose station purity to make FAM solutions more favorable to crew planning, maintenance planning, and operations
  - Limit the number of fleet types at spokes
  - Multiple levels of purity based on station size
  - Relax fleet purity to family purity
- We also penalize the number of station/family pairs
  - Leads to more clustered use of fleet families

Station Purity Formulation

$$w_{f,s} \geq x_{f,i} \forall f \in F, s \in A, i \in L$$

$$\sum_{f \in F} w_{f,s} \leq SP_s \forall s \in A$$

$$w_{f,s} \in \{0,1\} \forall f \in F, s \in A$$

FAM Scenarios

- Star7: Mid-size international weekly schedule, one hub, 7 fleet types
- Int7: Mid-size international weekly schedule, multiple hubs, 7 fleet types
- US7: US domestic daily schedule, multiple hubs, 7 fleet types
- US19: US domestic daily schedule, multiple hubs, 19 fleet types
- Test environment: Pentium 4 processor (2.0 gHz, 1.5 g RAM), ILOG CPLEX 8.0, ILOG Concert 1.2

FAM Scenarios

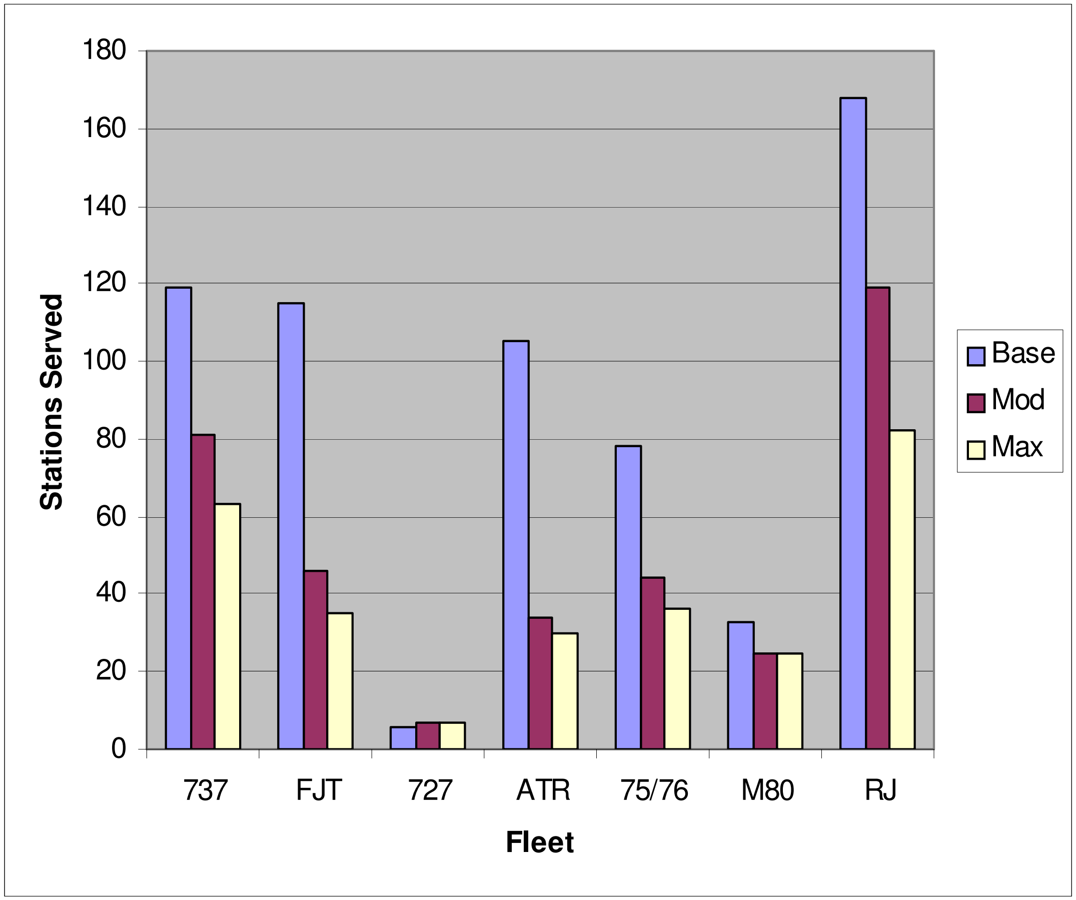
Models and computational results – no purity

Case	Cities	Flights	Fleet Types	Legal Assignments	Rows	Cols	Profit*	Time**	
								LP	IP
Star7	44	1568	7	4991	4747	8163	65.38	6.30	2.31
Int7	50	2358	7	6537	6900	11072	82.54	12.64	7.24
US7	210	4182	7	27698	16547	40056	17.52	11.83	0.52
US19	210	4182	19	71096	35899	102794	19.36	253.00	8.53

* $ x 1,000,000

** CPU seconds

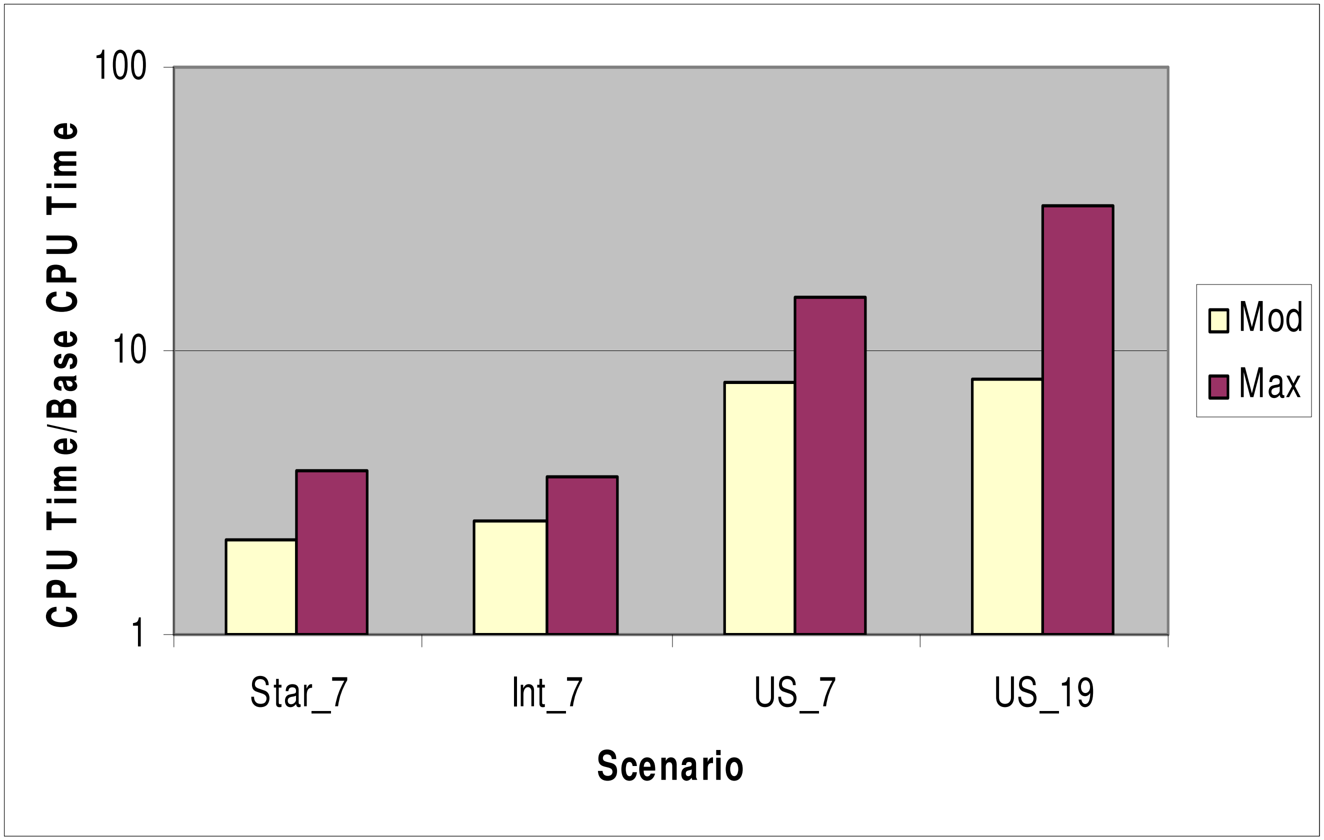
Station Purity Limits Dispersion of Fleets US7



- Base: no purity
- Mod: 1 fleet type at small stations, 2 at larger stations, no restrictions at hubs
- Max: 1 fleet type at all non-hub stations

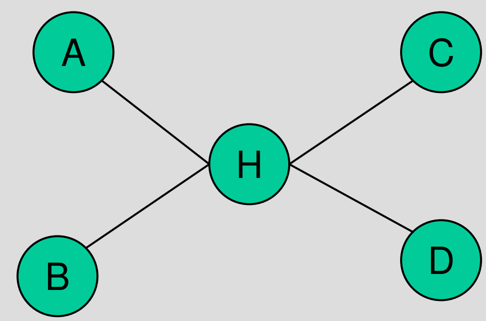
Purity	Singletons	Total Combinations
Base	164	630
Moderate	33	357
Maximum	22	280

Purity Increases FAM Solution Times



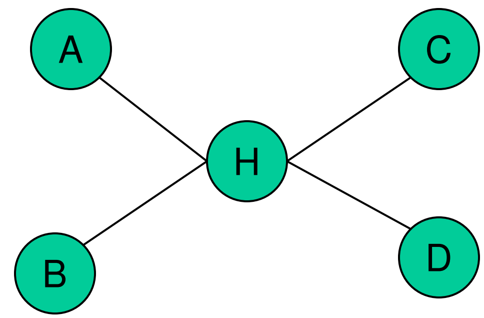
Station Decomposition Model

- **Master Problem – determine fleeting based on plans between hub and spokes**
  - **Reduce number of rows – spoke balance moved to subproblems**
  - **Reduce number of decision variables**
  - **Remove/reduce impact of station-level constraints**
- **Subproblem – generate plan(s) for a spoke**
  - **Relatively small, simple IP**



A plan defines fleeting for all flights between the hub and a spoke

Decomposition for Star Network



Master
Maximize Op Profit
Balance at H
Plane Count
Cover for Plans

Subproblem A
Max Reduced Cost
Balance at A
Cover for Flights H-A

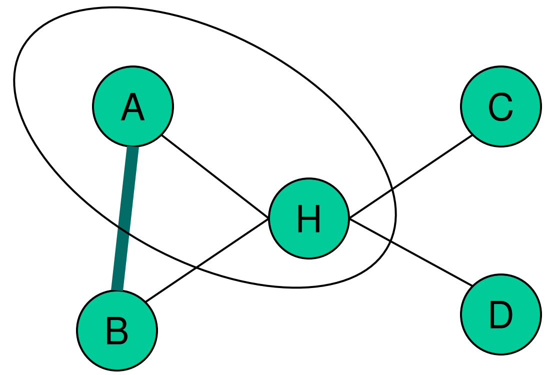
Subproblem B
Max Reduced Cost
Balance at B
Cover for Flights H-B

Subproblem C
Max Reduced Cost
Balance at C
Cover for Flights H-C

Subproblem D
Max Reduced Cost
Balance at D
Cover for Flights H-D

General Network

Formulation assumes that flights operate between hubs and spokes, no S to S flights

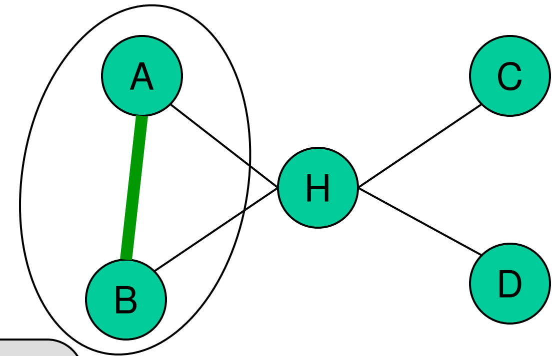


A-B Flight
Make A a Hub

	Master ***Balance at H & A*** ***Plane Count*** ***Cover for Plans and AH***	
Subproblem B ***Balance at B*** ***Cover for Flights H-B, A-B***	***Subproblem C*** ***Balance at C*** ***Cover for Flights H-C***	***Subproblem D*** ***Balance at D*** ***Cover for Flights H-D***

General Network

Need to pick hubs/groups to strike a balance between:
- Small master size
  (Low number of hubs)
- Small max subproblem size
  (More hubs)



A-B Flight
Make A&B a Group

Master
Balance at H
Plane Count
Cover for Plans

Subproblem A&B
Balance at A
Balance at B
Cover for Flights:
H-A, H-B, A-B

Subproblem C
Balance at C
Cover for Flights H-C

Subproblem D
Balance at D
Cover for Flights H-D

SDM Notation

3.1.1 SDM Sets

H: set of hub airports, indexed by h. $H \subset A$.
S : set of spoke airports, indexed by s. $S \subset A$, $H \cap S = \varnothing$.
G : set of station groups, indexed by g.

A^g : set of airports in station group $g \in G$, indexed by a.

P : set of assignment plans, indexed by p.

P^s : set of plans for spoke s, indexed by p.

P^g :set of plans for station group $g \in G$, indexed by p.

L^g : set of flight legs within group $g \in G$, indexed by i. $L^g \subset L$

L^h: set of hub-to-hub flight legs $h \in H$, indexed by i.
$L^h \subset L$, $L^h \cap L^g = \varnothing$.

3.1.2 Decision Variables

$$x_p = \begin{cases} 1, & \text{if plan p is in the SDM solution} \\ 0, & \text{otherwise} \end{cases}$$

$$x_{f,i}^p = \begin{cases} 1, & \text{if leg } i \in L \text{ is assigned fleet } f \in F \text{ in plan } p \in P \\ 0, & \text{otherwise} \end{cases}$$

SDM Notation

3.1.1 Parameters and Data

R_p: revenue for plan p, $R_p = \sum_{f \in F} \sum_{i \in L} R_{f,i} x_{f,i}^p$.

C_p: cost for plan p, $C_p = \sum_{f \in F} \sum_{i \in L} C_{f,i} x_{f,i}^p$.

PC_f^p: the number of aircraft from fleet $f \in F$ on the ground or in the air at the counting time in plan $p \in P$.

$$q_{f,h,t,p} = \begin{cases} \text{1, if plan p includes an arrival of aircraft type f at hub h, time t} \\ \text{-1, if plan p includes a departure of aircraft type a at hub h, time t} \\ \text{0, otherwise} \end{cases}$$

SDM Master

Maximize:

$$\sum_{f\in F}\sum_{i\in L}(R_{f,i}-C_{f,i})x_{f,i}+\sum_{p\in P}(R_p-C_p)x_p$$

Subject to:

$$\sum_{f\in F}x_{f,i}=1,\forall i\in L^h,\forall h\in H$$

$$y_{f,h,t^-}+\sum_{i\in I(f,h,t),i\in L^h}x_{f,i}-y_{f,h,t^+}-\sum_{i\in O(f,h,t),i\in L^h}x_{f,i}+\sum_{p\in P}q_{f,h,t,p}x_p=0,\forall f,h,t$$

$$\sum_{h\in H}y_{f,h,t_m}+\sum_{i\in CL(f),i\in L^h}x_{f,i}+\sum_{p\in P}PC_f^p x_p\le N_f,\forall f\in F$$

$$\sum_{p\in P^g}x_p=1,\forall g\in G$$

$$x_{f,i}\in\{0,1\},\forall f\in F,\forall i\in L^h$$

$$x_p\in\{0,1\},\forall p\in P$$

$$y_{f,h,t}\ge 0,\forall f,h,t$$

π_g^{conv} : dual variable on the convexity constraint for group $g\in G$.

π_f^{pc} : dual variable on the plane count constraint for fleet $f\in F$.

$\pi_{f,h,t}^{bal}$: dual variable on the balance constraint for fleet $f\in F$, hub $h\in H$, node $t\in T$.

SDM SubProblem

Maximize:

$$z^g = \sum_{f\in F}\sum_{i\in L^g}(R_{f,i} - C_{f,i})x_{f,i} - \sum_{f\in F}\sum_{a\in A^g}\pi_f^{pc} y_{f,a,t_0}$$
$$+\sum_f\sum_h\sum_{i\in I(f,h,t)}\pi_{f,h,t}^{bal}x_{f,i} - \sum_f\sum_h\sum_{i\in O(f,h,t)}\pi_{f,h,t}^{bal}x_{f,i}\forall f,h,t$$

Subject to:

$$\sum_{f\in F} x_{f,i} = 1, \forall i\in L^g$$

$$y_{f,a,t^-} + \sum_{i\in I(f,a,t),i\in L^g} x_{f,i} - y_{f,a,t^+} - \sum_{i\in O(f,a,t),i\in L^g} x_{f,i} = 0, \forall f, a\in A^g, t$$

$$w_{f,a} \geq x_{f,i} \forall f\in F, a\in A^g, i\in L^g$$

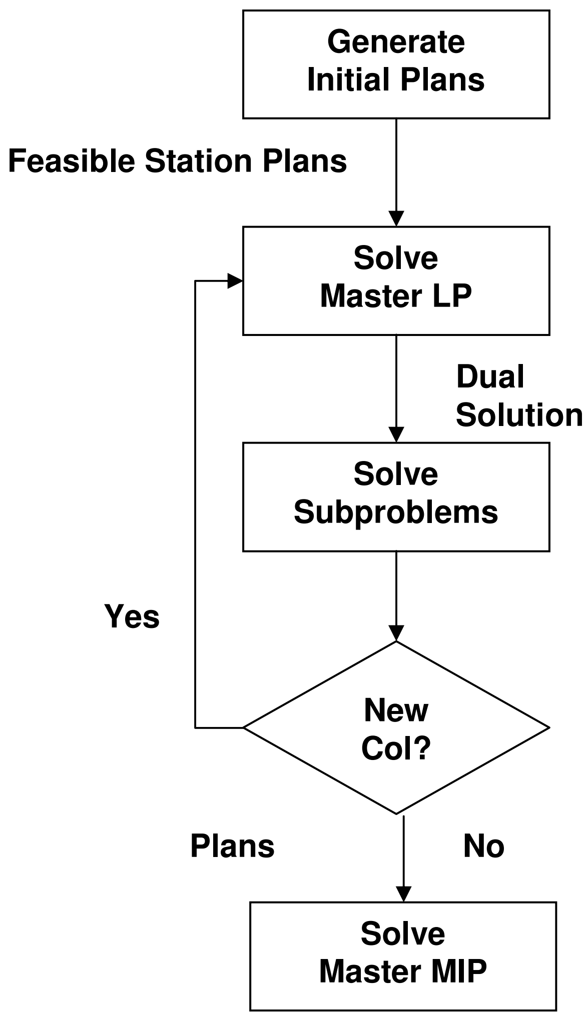
$$\sum_{f\in F} w_{f,a} \leq SP_a \forall a\in A^g$$

$$x_{f,i}\in\{0,1\}, \forall f\in F, \forall i\in L^g$$

$$y_{f,a,t}\geq 0, \forall f, a\in A^g, t$$

$$w_{f,a}\in\{0,1\}\forall f\in F, a\in A^g$$

Station Decomposition Flow

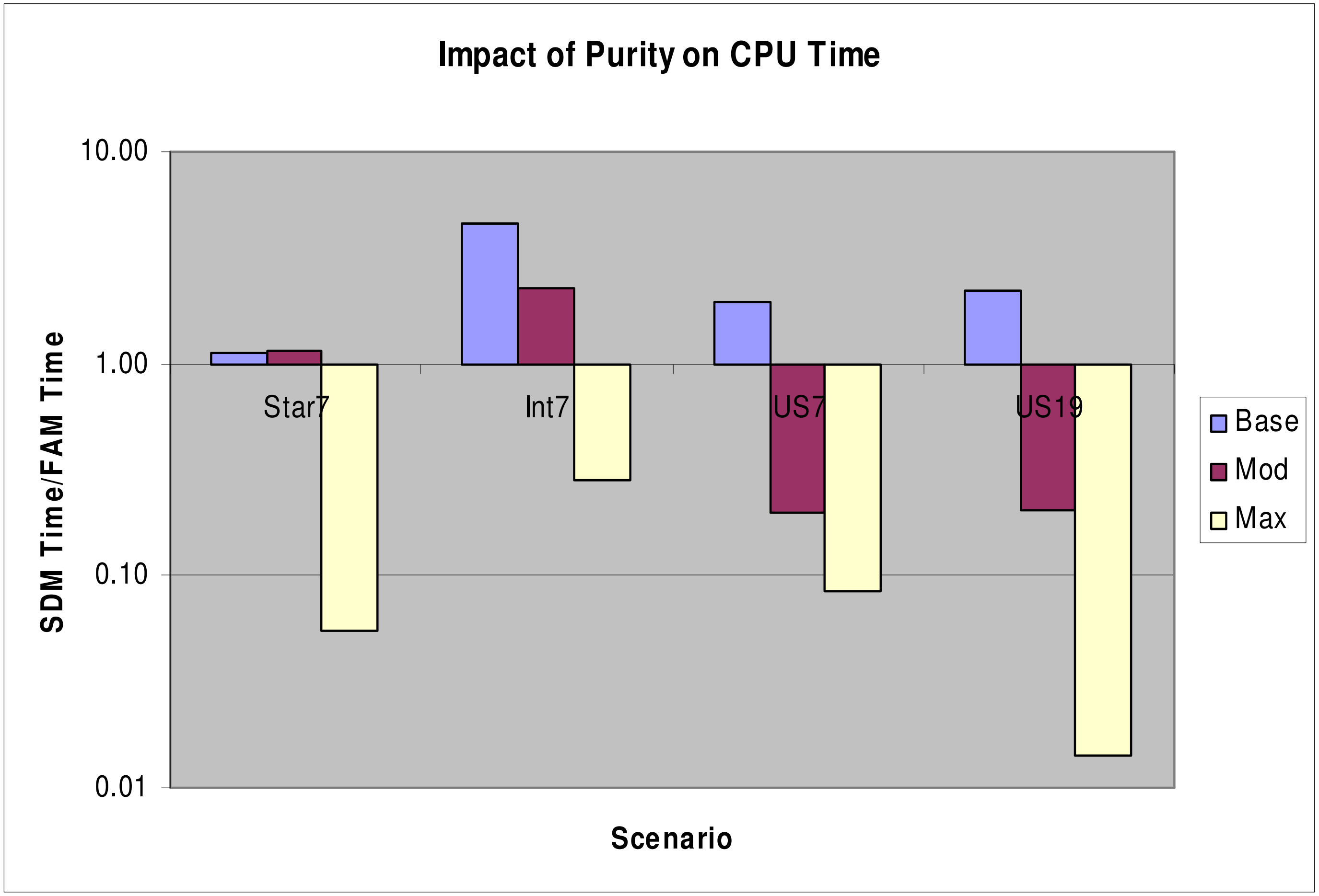


Stopping criterion:

If $\max \{ RC^s : s \varepsilon S\} \leq kz$

Where $RC^s = z^s - \pi^s$
and $k = .0001$

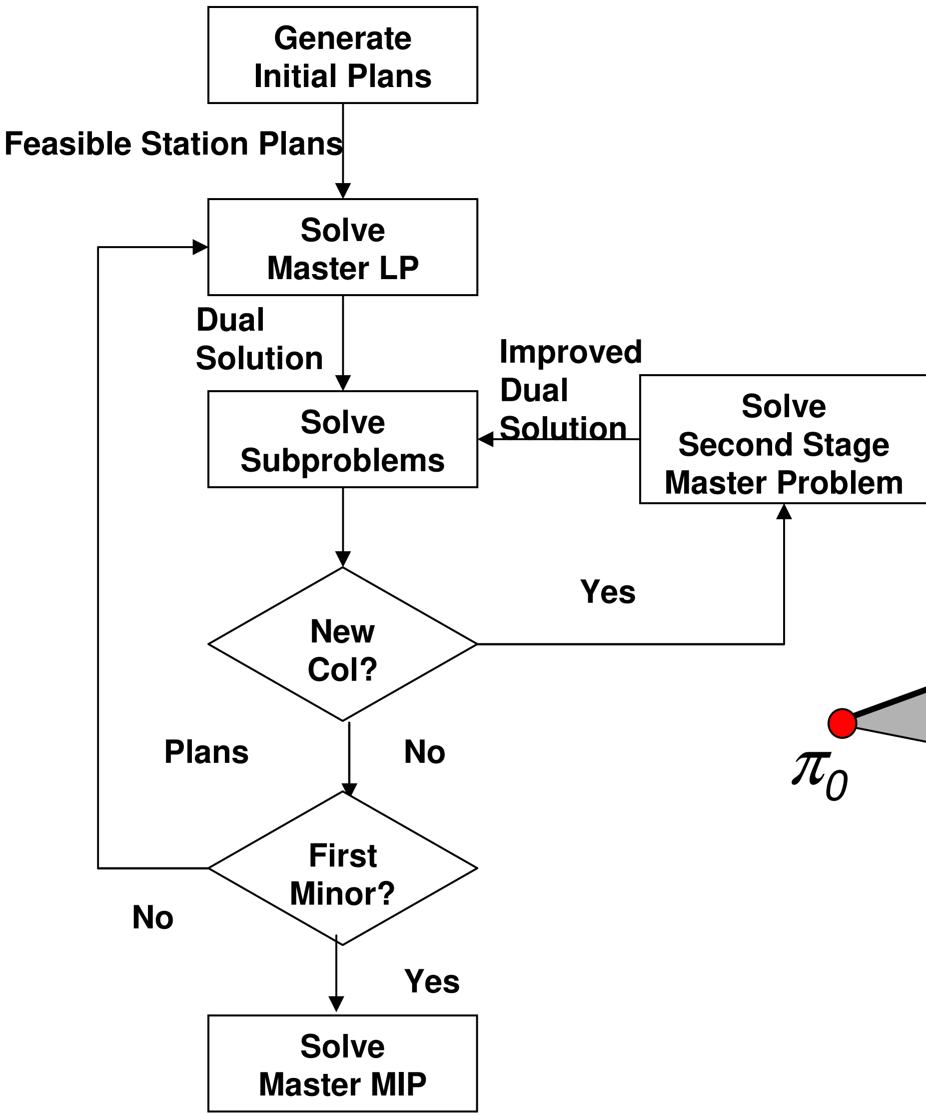
SDM Performance vs FAM

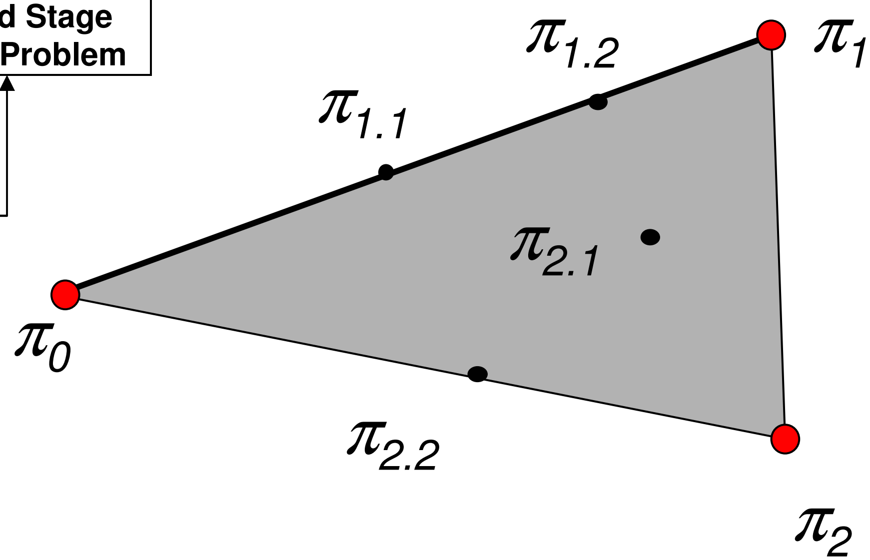


SDM Dual Improvement Approach

- The quality of plans generated is limited by the quality of the current dual solution
  - On successive master iterations, the dual may not improve so plans may not be optimum
  - May require many iterations
- Second Stage Master Problem (SSMP) finds the optimal dual solution in a convex region defined by previous dual solutions
  - Improves dual solution at each iteration
  - Generates plans without resolving the master
  - Reduce the number of major iterations
  - Reduce CPU time

Station Decomposition with SSMP





SDM Master and Dual

SDM master problem:

Maximize: $C_H X_H + \sum_{g \in G} \sum_{p \in P^G} P_p x_p$

Subject to:

$$\sum_{p \in P^G} x_p = 1, \forall g \in G$$

$$A_H X_H + \sum_{g \in G} \sum_{p \in P^g} A^p x_p = b$$

$$X_H \geq 0, x_p \geq 0, \forall p \in P$$

Dual of the SDMmp is:

Minimize: $\pi b + \sum_{g \in G} \pi_g^{conv}$

Subject to:

$$\pi A_H \geq C_H$$

$$\pi_g^{conv} + \pi A^p \geq P_p, \forall g \in G, p \in P^g$$

C_H : vector of objective function coefficients for H2H flight legs (revenue minus cost).

X_H : vector of decision variables associated with H2H flight assignments and hub ground flows.

$P_p = R_p - C_p$: profit associated with plan p.

A_H : matrix of coefficients for H2H cover, hub balance and plane count constraints.

A^p : vector of coefficients for plane count and hub incidence for plan *p*.

b : vector of right hand side values associated with plane count and hub balance constraints.

Restrict L(π) to Convex Region

- Let λ be a vector of weights on these solutions, and let π^{λ} be the convex combination of the previous solutions based on these weights. The restricted lagrangian minimization becomes:

$$\min_{\{\pi^1,\ldots,\pi^t\}} L(\pi^{\lambda}) = \min\left\{\pi^{\lambda} b + \sum_{s} S^{s}(\pi^{\lambda}) \mid \pi^{\lambda} = \sum_{j} \lambda_j \pi^{j}, \sum_{j} \lambda_j = 1, \lambda_j > 0\right\}$$

- This is equivalent to:

$$\min_{\{\pi^1,\ldots,\pi^t\}} L(\pi^{\lambda}) = \min\left\{\begin{array}{l} \pi^{\lambda} b + \sum_{g\in G} \pi_g^{\ conv} \mid \\ \pi_g^{\ conv} \geq P_p - \pi^{\lambda} A^{p} \forall p \in P^{g} \\ \pi^{\lambda} = \sum_{j=1..t} \lambda_j \pi^{j}, \sum_{j=1..t} \lambda_j = 1, \lambda_{1..t} > 0 \end{array}\right\}$$

or:

$$\min_{\{\pi^1,\ldots,\pi^t\}} L(\pi^{\lambda}) = \min\left\{\begin{array}{l} \sum_{j=1..t} \lambda_j \pi^{j} b + \sum_{g\in G} \pi_g^{\ conv} \mid \\ \sum_{j=1..t} \lambda_j \pi^{j} A^{p} + \pi_g^{\ conv} \geq P_p, \forall p \in P^{g} \\ \sum_{j=1..t} \lambda_j = 1, \lambda_{1..t} \geq 0 \end{array}\right\}$$

SSMP: Second Stage Master

Maximize: $z_0 + \sum_{p \in P} P_p x_p$

Subject to:

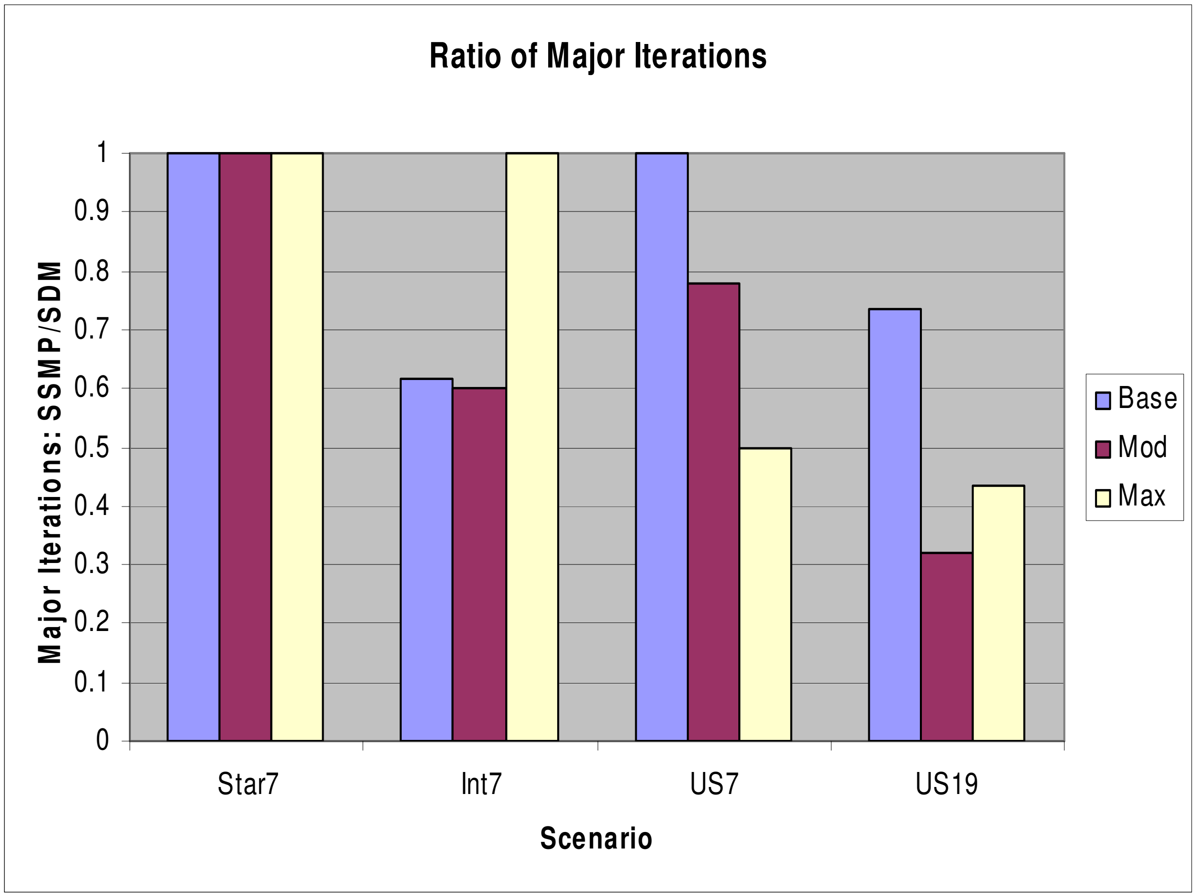
$$\sum_{p \in P^g} x_p = 1, \forall g \in G$$

$$z_0 + \sum_{p \in P} \pi^j A^p x_p = \pi^j b, \forall j = \{1..t\}$$

$$x_p \geq 0$$

- z_0 is the dual of the convexity constraint on λ
- λ_j is the dual on the j^th constraint
- Given an optimal solution to the SSMP, the improved dual π^λ is found as: $\pi^\lambda = \sum_{j=1..t} \lambda_j \pi^j$

SSMP Performance: Major Iterations



Solving SDM MIP

- The SDM LP relaxation is tighter than the FAM relaxation
  - SDM solutions are convex combinations of integer subproblem solutions
  - Should give better results if solved to optimality
- Unfortunately there are problems:
  - SDM solution is highly fractional
  - Good LP columns may not produce a feasible MIP solution
- Used typical strategy to solve MIP: variable fixing, branching on subproblem variables using SOS1
- This results in higher MIP gap and longer MIP times
- Developed a “fix and price” heuristic to fix and later unfix variables while generating new columns
  - This results in faster runtimes and lower MIP gaps

Fix and Price Heuristic for Solving SDM MIP

1. Solve LP relaxation to specified stopping criteria
2. Fix plan variables close to 1
3. Solve master
4. If profit drops below a specified threshold:
   - Unfix low reduced cost variables
   - Add cut to avoid this solution in future iterations
5. Generate new plans for unfixed stations
6. If some stations are unfixed Go to Step 2
7. Solve MIP

SDM MIP Results with Fix and Price

Schedule	Purity	FAM			SDM			SDM w/ F&P		
		Time	Nodes	Gap	Time	Nodes	Gap	Time	Nodes	Gap
Star7	None	**8.61**	**1**	**0.00%**	**8.47**	**1**	**0.00%**	**9.34**	**1**	**0.00%**
	Max	**88.74**	**9**	**0.60%**	**1.90**	**1**	**0.00%**	**2.00**	**1**	**0.00%**
	Mod	**21.27**	**1**	**0.01%**	**26.37**	**1**	**0.00%**	**27.87**	**1**	**0.00%**
Int7	None	**19.88**	**1**	**0.01%**	**65.34**	**8**	**0.05%**	**39.93**	**1**	**0.00%**
	Max	**45.55**	**1**	**0.00%**	**12.32**	**1**	**0.00%**	**12.34**	**1**	**0.00%**
	Mod	**55.33**	**8**	**0.03%**	**74.27**	**15**	**0.10%**	**48.29**	**1**	**0.00%**
US7	None	**12.35**	**7**	**0.00%**	**26.93**	**7**	**0.00%**	**26.08**	**1**	**0.00%**
	Max	**188.77**	**3**	**0.01%**	**20.71**	**39**	**0.11%**	**15.18**	**1**	**0.00%**
	Mod	**92.74**	**6**	**0.01%**	**25.62**	**36**	**0.10%**	**16.12**	**1**	**0.00%**
US19	None	**261.53**	**20**	**0.01%**	**621.44**	**163**	**0.18%**	**858.59**	**2**	**0.01%**
	Max	**8686.23**	**30**	**0.20%**	**175.17**	**90**	**0.29%**	**115.86**	**15**	**0.33%**
	Mod	**2167.55**	**32**	**0.04%**	**663.47**	**318**	**0.55%**	**260.74**	**1**	**0.00%**

ODFAMplr and Family Purity

- **Fleet-level purity has a significant negative impact on profit**
- **We can achieve many of the benefits associated with purity by imposing it at a crew-compatible family level**
- **ODFAMplr results in more reasonable profit impact at family level**
- **Family purity results in significant runtime impacts**

Schedule	Purity	Level	ODFAMplr		
			Profit	B&B Nodes	Cplex Time
Star7	Base		65.80	1	18.75
	Mod	Family	65.79	1	12.36
	Max	Family	64.87	1986	4775.73
Int7	Base		83.93	3	42.56
	Mod	Family	83.93	1	45.01
	Max	Family	83.86	12	65.09
US19	Base		19.34	1	1718.77
	R Mod	Family	19.31	1	4870.06
	R Max	Family	19.13	1	13721.50

Family/Station Penalties

- **AA Maintenance stated that each fleet/sta combination increases annual operating costs by $500k**
- **Added a penalty to the FAM formulation to estimate potential benefits of reducing combinations**
- **Three cases were run for each scenario:**
  - **Base: No purity**
  - **$0: Moderate family purity no penalty**
  - **$X0,000: Moderate family purity with daily or weekly penalty on family/station**
- **Annual benefits for ODFAM and purity are significant**

Schedule	Penalty	Profit (mm)	FS	ΔFS	Net Profit
Star7	Base	$65.79	64		65.15
	$0	$65.79	62	2	65.17
	$30,000	$65.75	56	8	65.19
	$50,000	$65.59	54	10	65.05
	$500,000	$65.00	51	13	64.49
Int7	Base	$83.91	79		83.12
	$0	$83.91	79	0	83.12
	$30,000	$83.86	72	7	83.14
	$50,000	$83.82	71	8	83.11
	$500,000	$83.56	70	9	82.86
US19	Base	$19.32	508		18.36
	$0	$19.30	488	20	18.63
	$1,500	$19.24	370	138	18.73
	$3,000	$19.21	370	138	18.70

Crew Pairing Cost

- For the US7 RJ fleet and three FAM solutions, we solved the planning problem for crew pairings
  - Base FAM solution - no purity constraints
  - Mod and max purity FAM solutions for
- Consistent rules for all three
  - Rules came from Sabre for client airline
- Measured excess pay-and-credit percentages
  - Base: 26.2%, Mod 5.6%, Max 4.2%

The more real bottom line

- Just from down-line planning
- Base solution daily operating profit:

  19.36 – 505x1370 = 18.67 (maintenance)

  18.67 - .42 = 18.25 (excess crew cost)
- Mod solution daily operating profit:

  19.26 - 374x1370 = 18.75 (maintenance)

  18.75 - .14 = 18.61 (excess crew cost)
- Operational recovery savings should be comparable

The more real bottom line

- Why is clustered fleets more robust?
- Leads to clustered crews
  – Can swap pilots if e.g. a flight is late getting in and cuts crew rest too much
- More planes are same fleet family
  – Can have fewer overnight maintenance stations because more planes of a given fleet are on the ground
  – Can swap planes as well

Conclusions

- **Robustness - potential benefits and computational issues associated with fleet and family purity**
- **Station Decomposition Model (SDM) – developed a workable formulation for a general airline network**
- **Dual Improvement Algorithm makes column generation more efficient by using convex combinations of previous duals**
- **Fix and price is an approach to solving the SDM MIP that provides significant improvements in solution quality and efficiency compared to the column generation->MIP approach**