Optimal Control Strategies for Real Energy Generation Assets

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Thanks

Financial support provided by

- MITACS
- NSERC
- Ontario Power Generation (OPG)
- Dydex Research and Capital Ltd

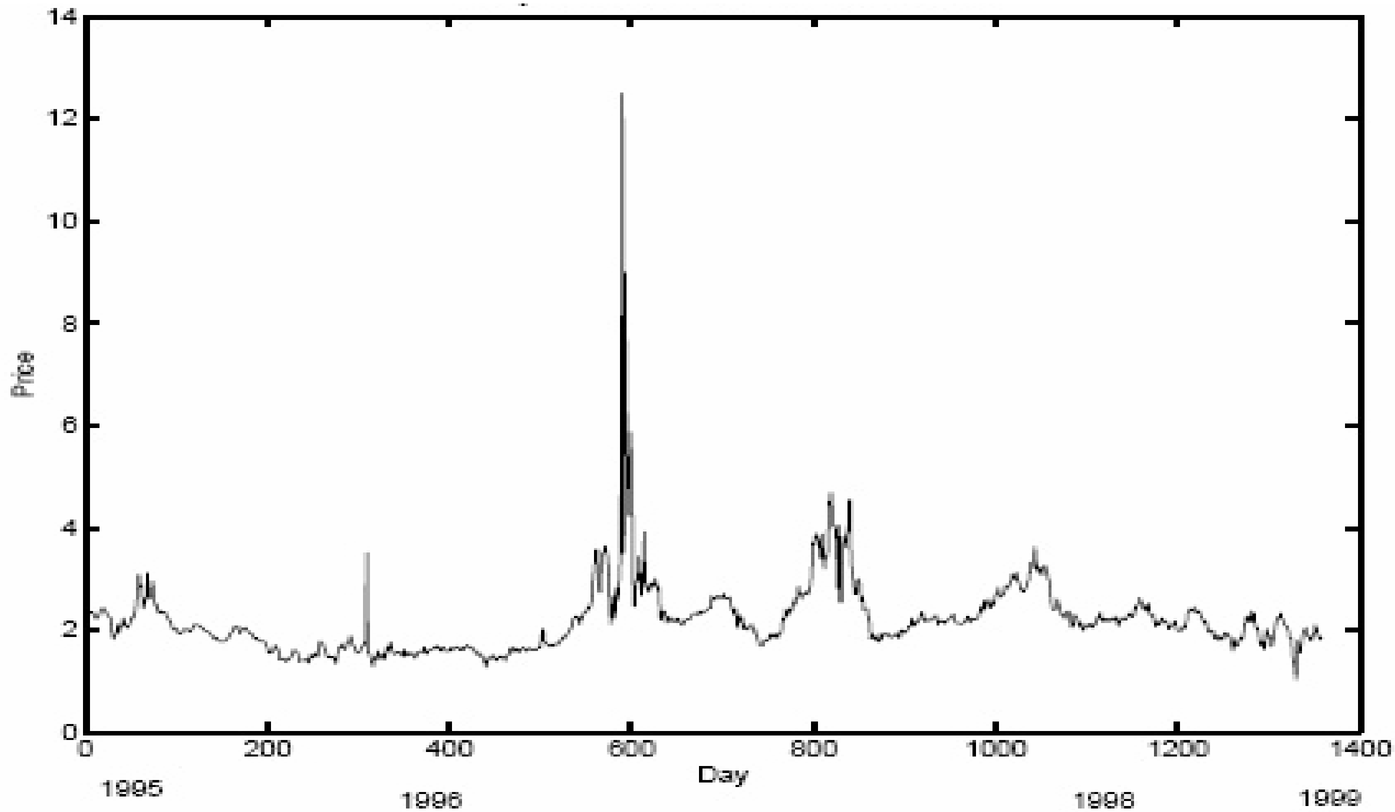
Outline

- Natural gas storage
- Pump storage facilities
- Thermal power plant
- Future work

Natural Gas Price

Buy low and sell high

Time-series of Henry Hub natural gas prices 1995-1999



1. Natural Gas

Procedure

- Using Merton's application of Bellman's principle to finance
- Incorporate engineering details

Natural Gas Storage Facilities

Natural gas can be stored underground in

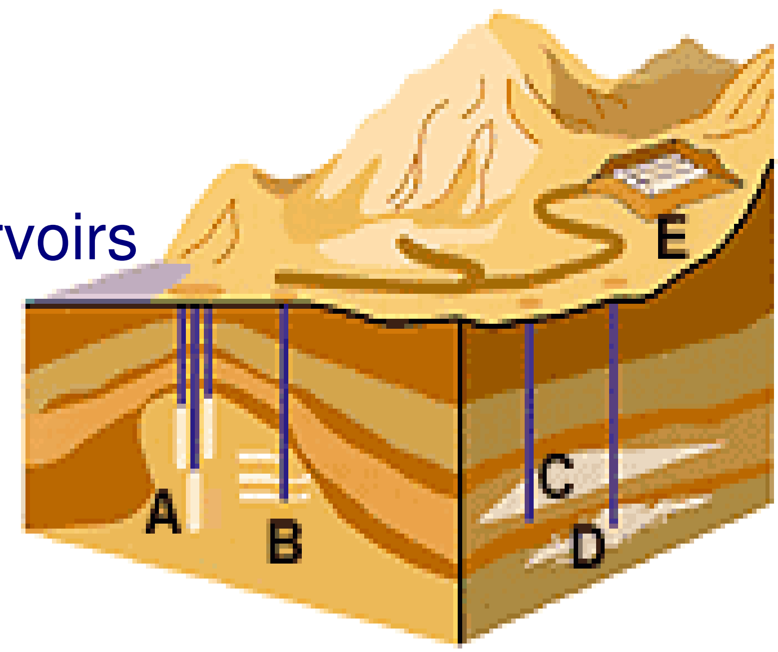
(A) salt caverns

(B) mines

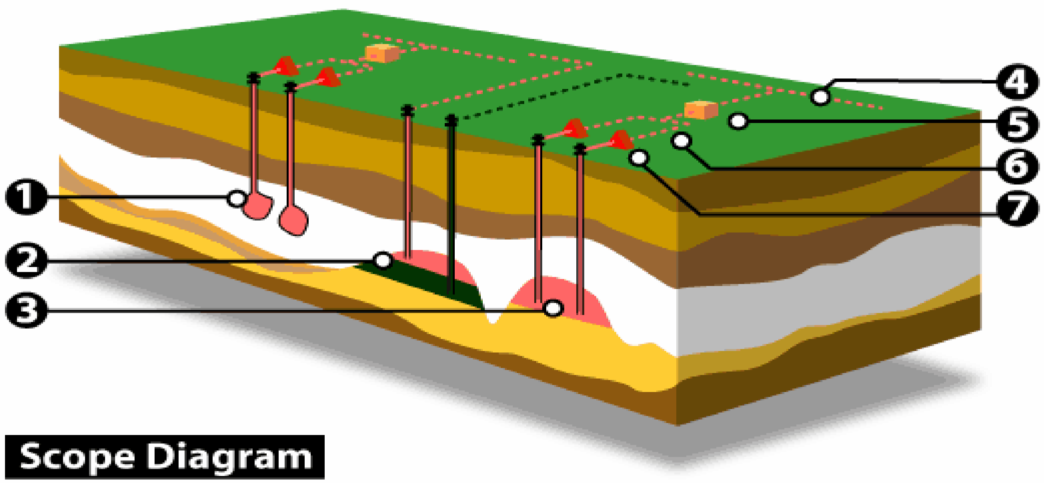
(C) aquifers

(D) depleted oil/gas reservoirs

(E) hard rock mines



Resources in Ontario



Scope Diagram

1. Salt cavern hydrocarbon storage
2. Oil & gas reservoir
3. Natural gas reservoir storage
4. Transmission pipeline
5. Compressor
6. Gathering pipeline
7. Emergency shut down valve

1. Natural Gas

Physics/Engineering pV=nRT

- Base gas capacity
  - Required for reservoir pressure
  - Never removed
- Working gas capacity
  - Amount of gas available to produce and sell
- Deliverability
  - Rate at which gas can be released
  - Depends on gas level
- Injection capacity
  - Rate at which natural gas can be added
  - Depends on gas level
- Cycling
  - Salt caverns are HDMC
- Reservoir seepage
- Cost to pump gas

Variables in General Gas Storage Equations

P – price per unit of natural gas;

I – current amount of working natural gas inventory;

c – control variable gas injected (c > 0) / stored (c < 0);

I_{max} – max storage capacity of facility;

I_{min} -- base gas capacity;

$\mathbf{c_{max}(I)}$ – max deliverability rate as function of storage level;

$\mathbf{c_{min}(I)}$ – min injection rate as function of storage level;

$a(I,c)$ – amount of gas lost given c units of gas released/injected;

Optimization Framework I

The objective function

$$\max_{c(P,I,t)} E[\int_0^T e^{-\rho\tau}(c - a(I,c))Pd\tau]$$

Subject to

$$c_{min}(I) \leq c \leq c_{max}(I)$$

Change in $\mathbf{I}$ obeys ODE

$$dI = -[c + a(I,c)]dt$$

Change in **P** obeys Markov process

$$dP = \mu_1(P,t)dt + \sigma_1(P,t)dX_1 + \sum_{k=1}^{N} \circ_k(P,t,J_k)dq_k$$

Optimization Framework II

To simultaneously determine optimal strategy c(P, I, t) and corresponding optimal value V(p, I, t), let

$$V(P,W,f,t;c) = \max_c E\left[\int_t^T e^{-\rho(\tau-t)}(c-a(I,c))Pd\tau\right]$$

Split integral to get

$$V = \max_c E\left[\int_t^{t+dt} e^{-\rho(\tau-t)}(c-a(I,c))Pd\tau + e^{-\rho dt}V(P+dP,I+dI,t+dt)\right]$$

Moving towards Bellman's equation

Standard Taylor Series arguments

Employ Ito's lemma to obtain Taylor series

$$V = \max_c E\left[(c - a(I,c))Pdt + (1-\rho dt)V + (1-\rho dt)\left(V_t + \frac{1}{2}\sigma_1^2 V_{PP} + \mu_1 V_P - (c + a(I,c))V_I\right)dt \right. \\ \left. +(1-\rho dt)\sum_{k=1}^{N}(V_k^+ - V)dq_k + (1-\rho dt)(\sigma_1 V_P dX_1)\right]$$

Eliminate all higher order terms and simplify

$$\max_c E\left[\left(V_t + \frac{1}{2}\sigma_1^2 V_{PP} + \mu_1 V_P - (c + a(I,c))V_I + (c - a(I,c))P - \rho V\right)dt + \sum_{k=1}^{N}(V_k^+ - V)dq_k + \sigma_1 V_P dX_1\right] = 0$$

Take expectations and divide by dt

$$\max_c \left[V_t + \frac{1}{2}\sigma_1^2 V_{PP} + \mu_1 V_P - (c + a(I,c))V_I - \rho V + (c - a(I,c))P + \sum_{k=1}^{N}\epsilon_k E[V_k^+ - V]\right] = 0$$

The PDE

- The optimal value for c maximizes

$$\max_c[-(c+a(I,c))V_I+(c-a(I,c))P]$$

Subject to

$$c_{min}(I)\leq c\leq c_{max}(I)$$

- The PDE

$$V_t+\frac{1}{2}\sigma_1^2V_{PP}+\mu_1V_P-(c+a(I,c))V_I-\rho V+(c-a(I,c))P+\sum_{k=1}^{N}\epsilon_kE[V_k^+-V]=0$$

Initial condition:

$$V(P,I,T;c)=0$$

Boundary conditions:

$$V_{PP}\to 0 \text{ for } P \text{ large}$$

$$V_{PP}\to 0 \text{ as } P\to 0$$

The Numerical Difficulties

- Hyperbolic in I
  - direction of information flow
  - upwind finite differencing
- Total variation diminishing schemes
  - Slope limiting method works best
- Method of lines approach (Mukadam)

A Sample Problem

The Stratton Ridge facility

- Working gas capacity of 2000 MMcf
- Base gas requirement 50 MMcf
- Minimum capacity infectivity 80 MMcf/day
- Injection pump requirement 1.7MMcf /day
- No seepage from reservoir
- Ideal gas law and Bernoulli's law apply

$$dP = 0.25 * (2.5 - P)dt + 0.2PdX + (J - P)dq$$

$$dq = \begin{cases} 0 & \text{with probability } 1 - 2dt \\ 1 & \text{with probability } 2dt, \end{cases} \qquad J \in N(6, 4).$$

- Prices in MMbtus
- Time measured in years
- Discount rate 10%

The PDE

The function $\mathbf{a}$

$$a(I,c) = a(c) = \begin{cases} 0 & \text{for } c \geq 0 \\ 1.7 * 365 & \text{for } c < 0 \end{cases}$$

The PDE

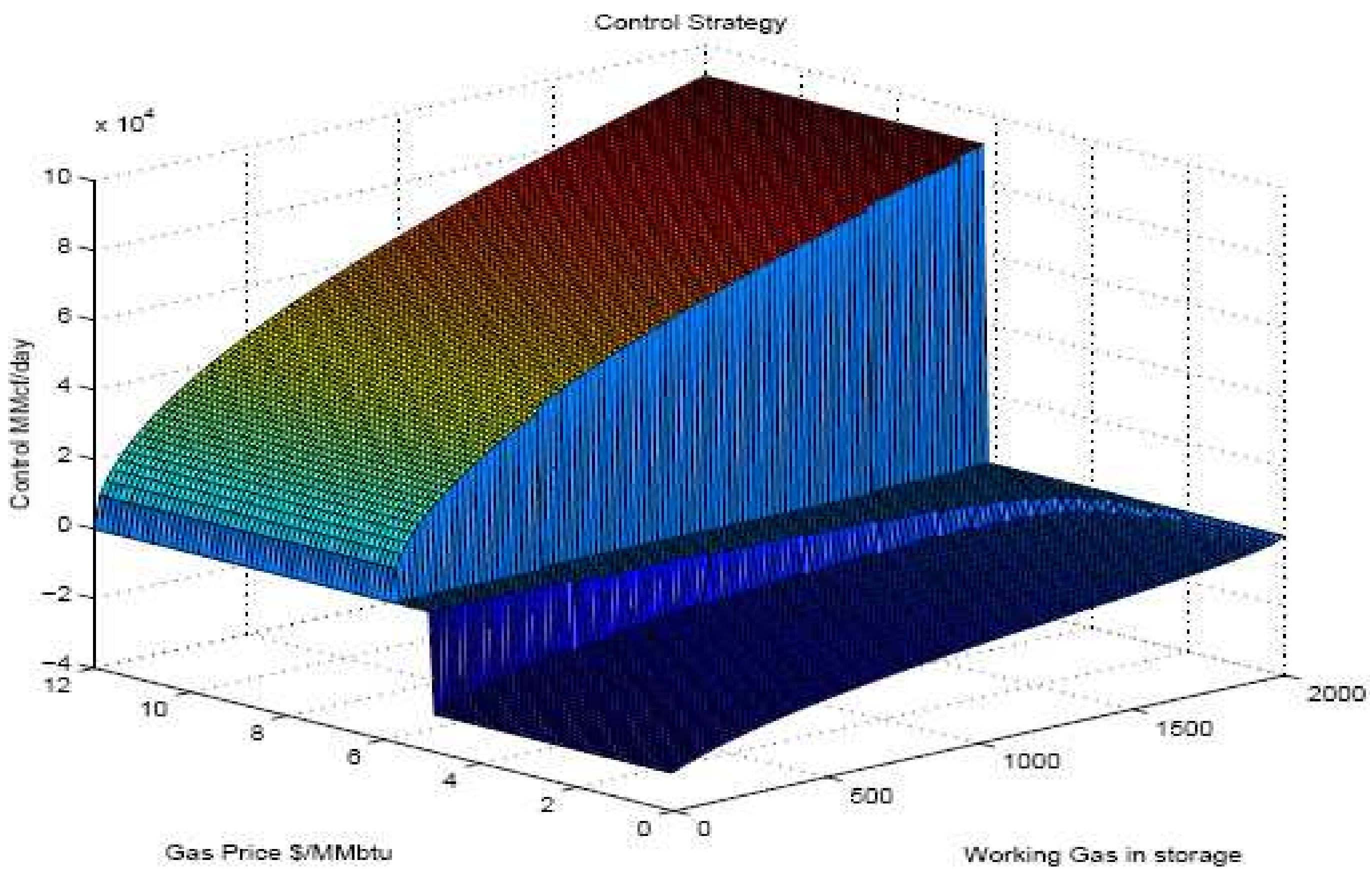
$$V_t + \frac{1}{2}(0.04P^2 V_{PP}) + 0.25(2.5 - P)V_P - (c + a(c))V_I - 0.1V + 1000(c - a(c))P$$
$$+2\int_0^\infty (V(J,I,t) - V(P,I,t))\frac{1}{\sqrt{8\pi}}e^{-(J-6)^2/(8)}dJ = 0$$

Then

$$c_{max}(I) \approx 2040.41\sqrt{I}$$

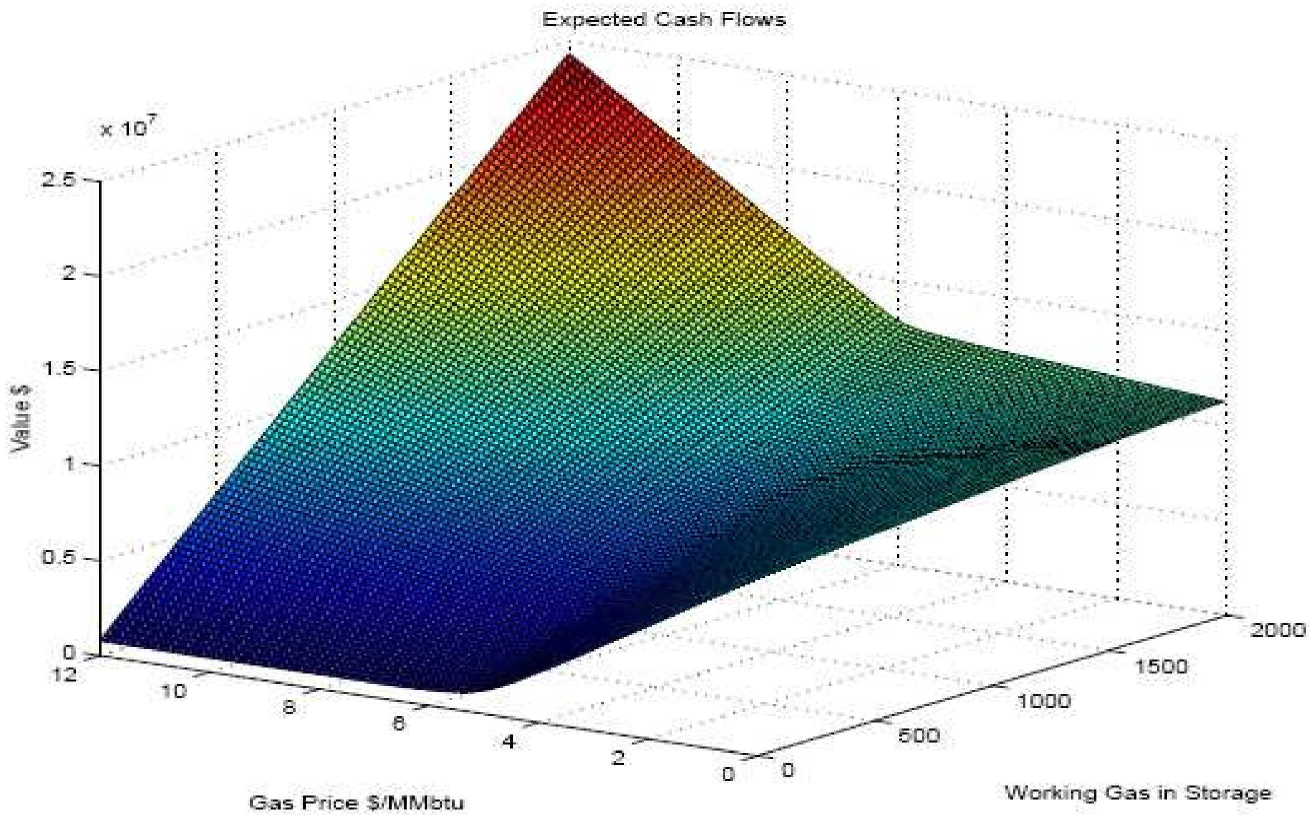
$$c_{min}(I) = -K_1\sqrt{\frac{1}{I + I_b} + K_2}$$

Natural Gas Control Surface



1. Natural Gas

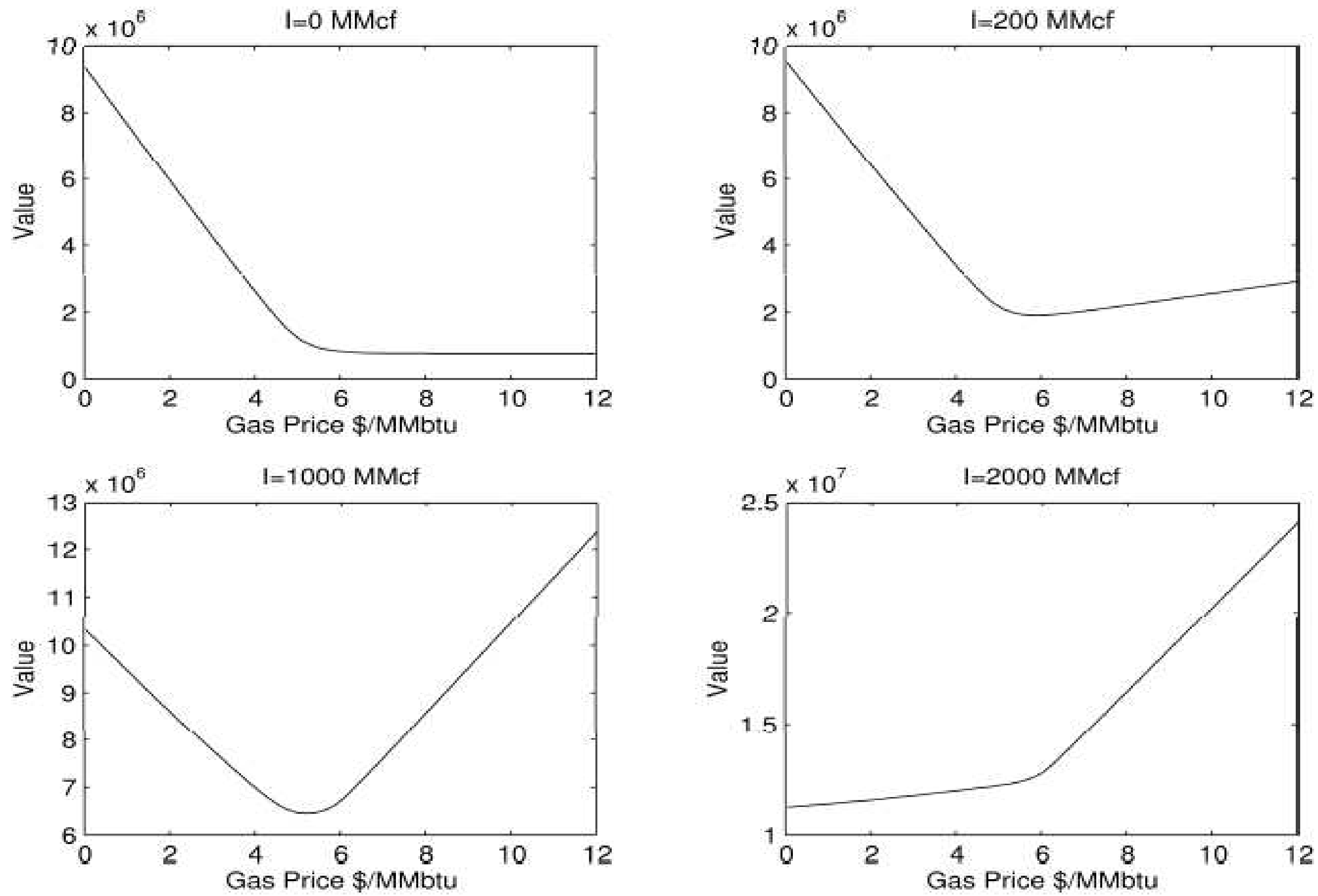
Natural Gas Value Surface



1. Natural Gas

Put and Call

Put: Give right to **sell** at fixed price.
Call: Give right to **buy** at fixed price.



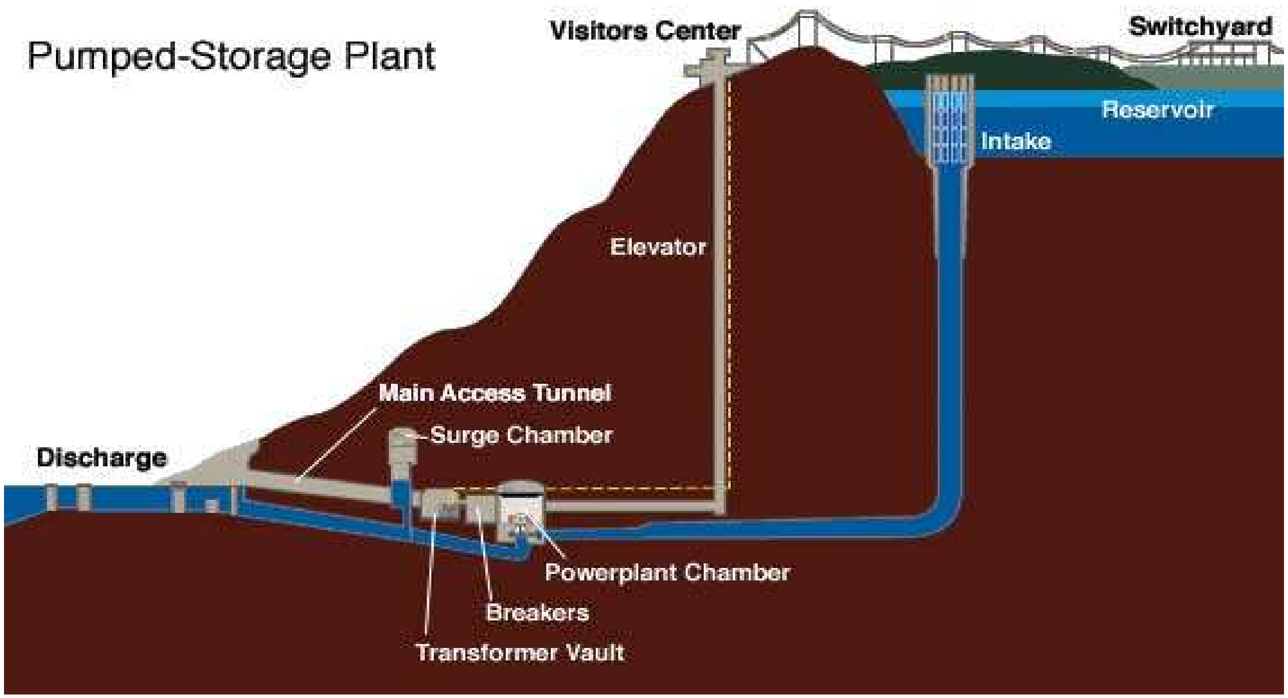
1. **Natural Gas**

Pump Storage Facilities

- Conversion of mechanical to electrical energy is efficient
- Get 80% round trip efficiency from
  electricity à running water à electricity
- Pump water when power price is low
- Use water to run turbine when power price is high
- What is the best way of doing this?

Pump Storage II

- Pump storage plant



Stochastic Optimal Control

- Valuation and Optimal Operation of electric power plants in competitive markets
- Continuous time model for power prices including Poisson jumps
- Price dynamics

$$dP = \mu_1(P,t)dt + \sigma_1(P,t)dX_1 + \sum_{k=1}^{N} \gamma_k(P,t,J_k)dq_k,$$

where μ, σ and the γ_k can be any arbitrary functions of price and/or time.

For detailed discussion, see M. Thompson, MD & H. Rasmussen (2004), *Operations Research* **52**, 546-562.

The PIDE

- Merton-style portfolio optimization problem
- Plus engineering fluid mechanics
- Leads to PIDE with initial and boundary conditions:

$$V_t + \frac{1}{2}\sigma(P)V_{pp} + \mu(P,t)V_p - \frac{3600c}{20000}V_h - (r + \lambda_{up}(P) + \lambda_{down}(P))V + H(c,h)P$$

$$+\lambda_{up}(P)\int_{-\infty}^{\infty} V(J_1,h,t)\frac{1}{100\sqrt{2\pi}}\exp\left(\frac{-(S-700)^2}{2(10)^2}\right)dJ_1$$

$$+\lambda_{down}(P)\int_{-\infty}^{\infty} V(J_2,h,t)\frac{1}{10\sqrt{2\pi}}\exp\left(\frac{-(S-100)^2}{2(10)^2}\right)dJ_2 = 0,$$

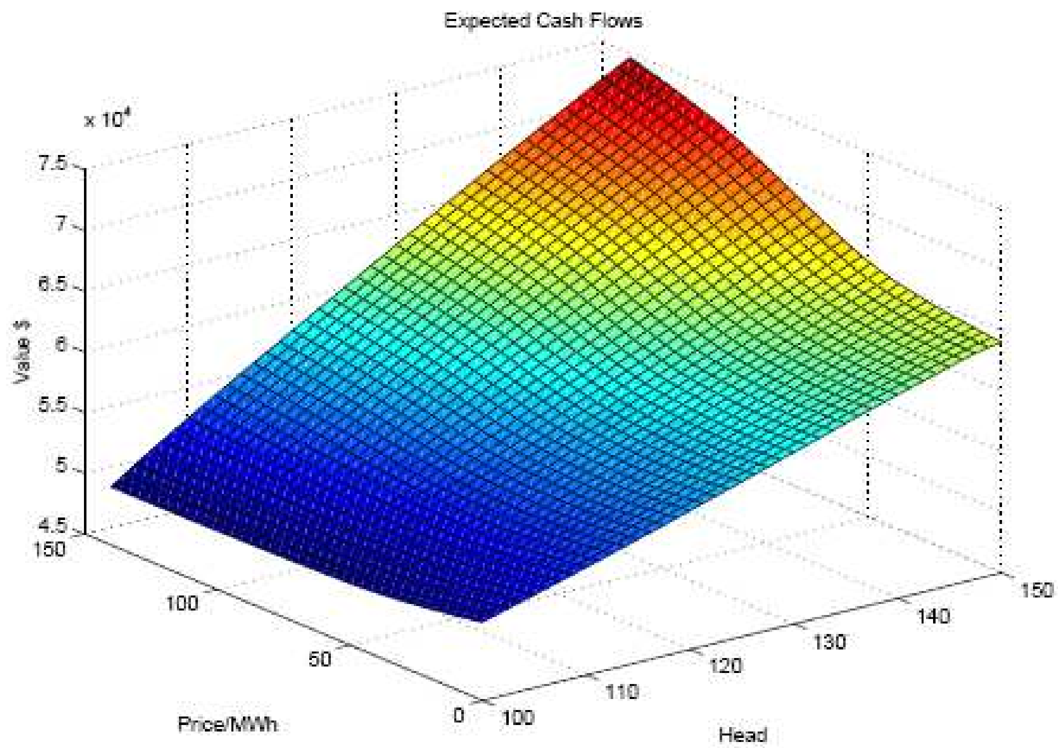
$$\text{Initial condition: } V(P,h,T) = 0,$$

$$\text{Boundary conditions: } V_{PP} \to 0 \text{ (for } P \text{ large)},$$

$$V_{PP} \to 0 \ (as\ P \to 0).$$

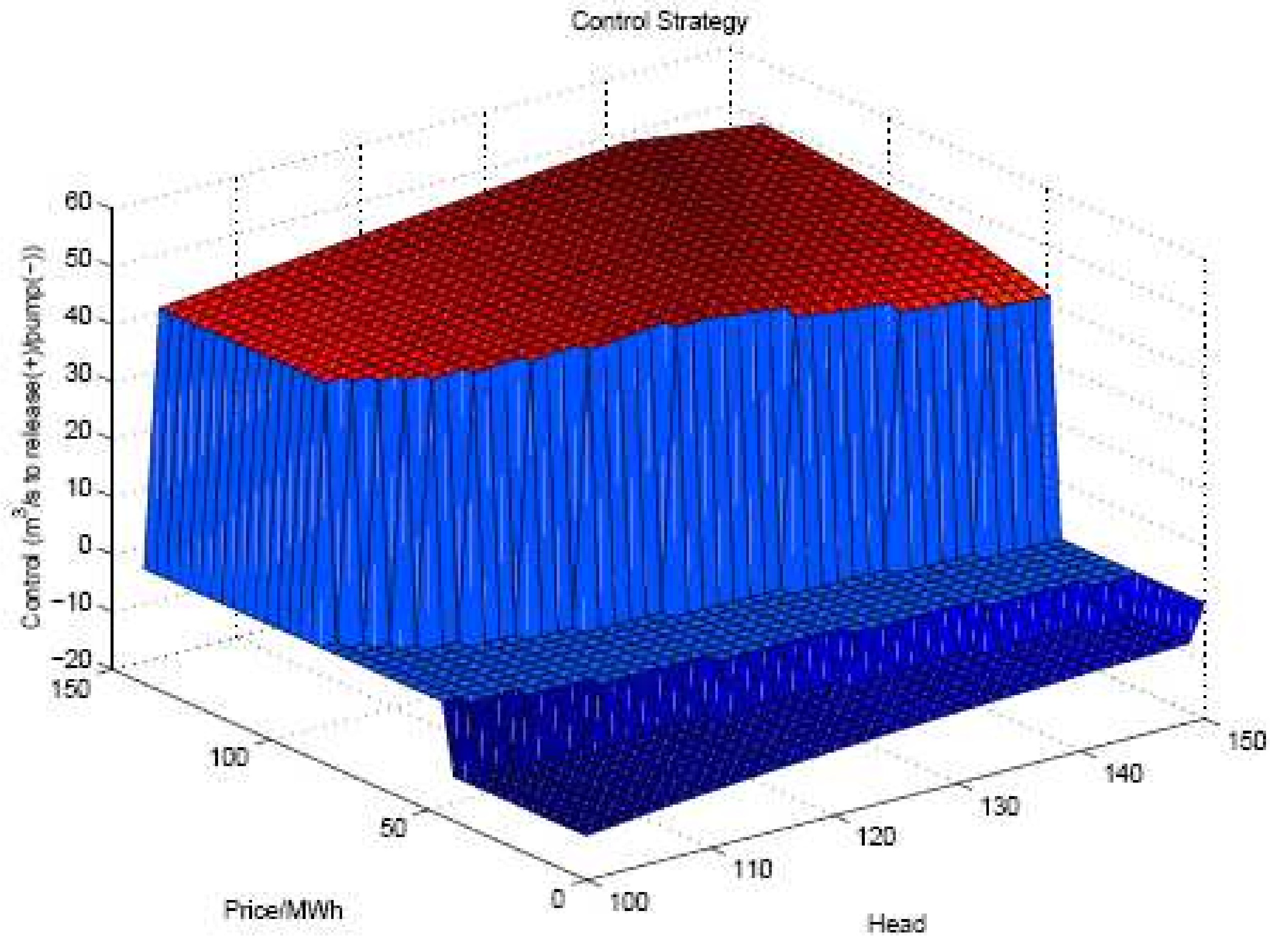
Pump Storage Value Surface

Solve the equation numerically using flux limiters to get:



2. Managing Load Shape

Pump Storage Control Surface



2. Managing Load Shape

Thermal Power Plant Optimization

- Random price electricity
- Random cost of fuel
- Variable start-up time
- Control response time lags
- Variable start-up and product costs
- Ramp rate limits
- Minimum generation levels
- Variable output rates
- Minimum up/down time requirement

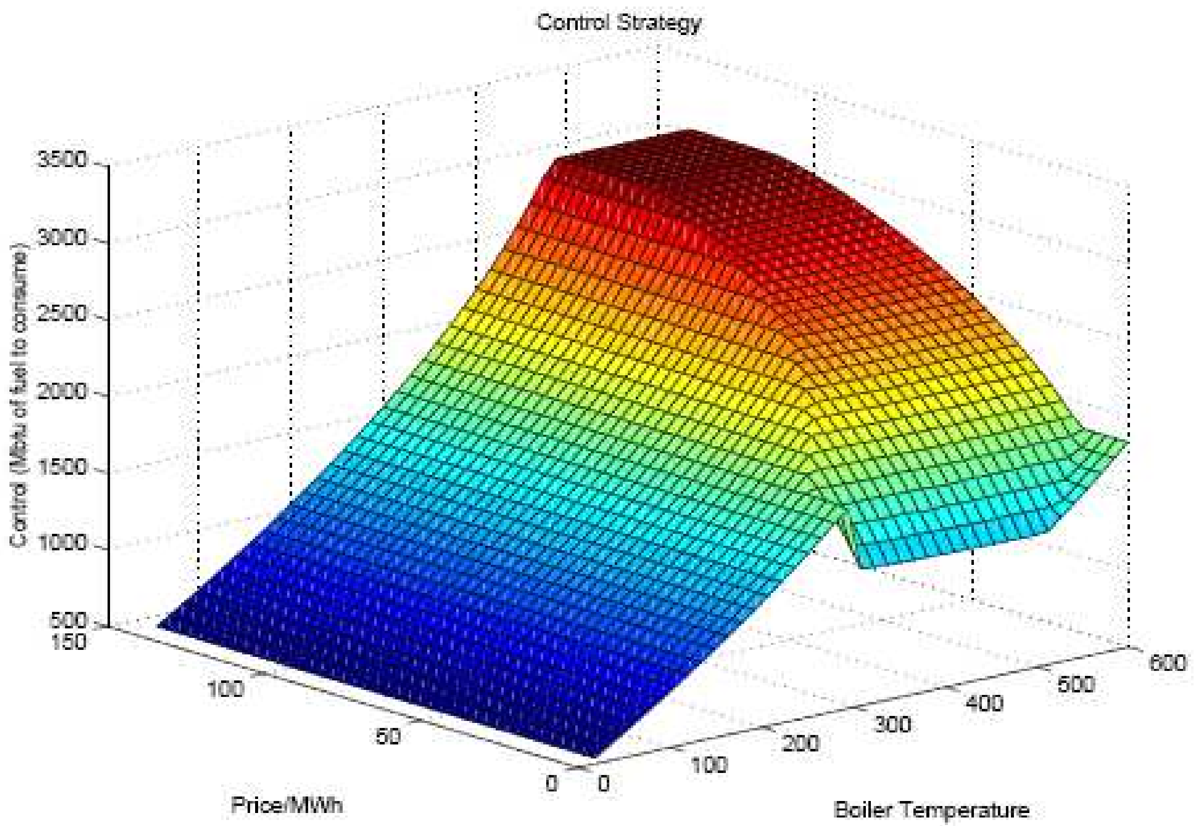
The Equation

$$V_t + \frac{1}{2}\sigma(P)V_{pp} + \mu(P,t)V_p + 0.1(\overline{L}(c) - L)V_L$$
$$-(r + \lambda_{down}(P)) + H(L)P - 3.5c$$
$$+\lambda_{up}(P)\int_{-\infty}^{\infty} V(j,h,t)\frac{1}{100\sqrt{2\pi}} \bullet \exp\left(\frac{-(s-700)^2}{2(100)^2}\right) dJ_1$$
$$+\lambda_{down}(P)\int_{-\infty}^{\infty} V(J_2,h,t)\frac{1}{10\sqrt{2\pi}} \exp\left(\frac{-(S-100)^2}{2(10)^2}\right) dJ_2 = 0$$

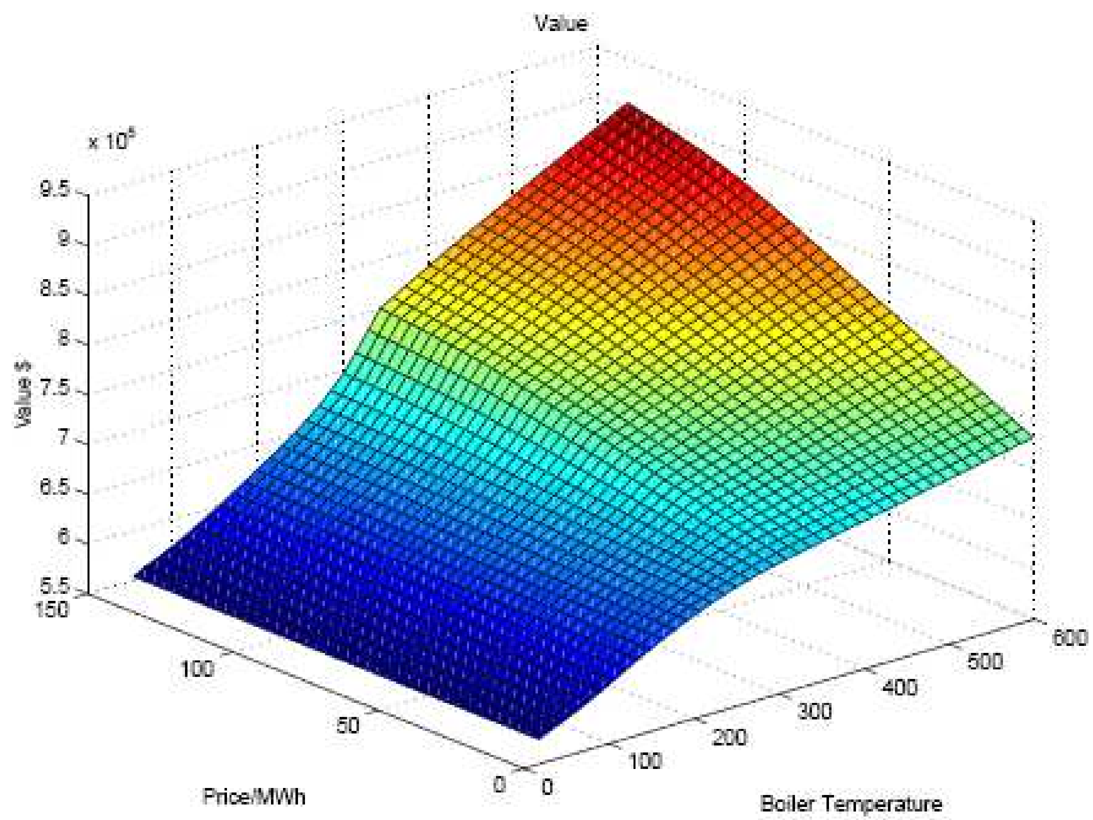
- where c maximizes

$$\max_{c}[0.1(\overline{L} - L)V_L - 3.5c]$$

Thermal Plant Control Surface



Thermal Plant Cash Flow



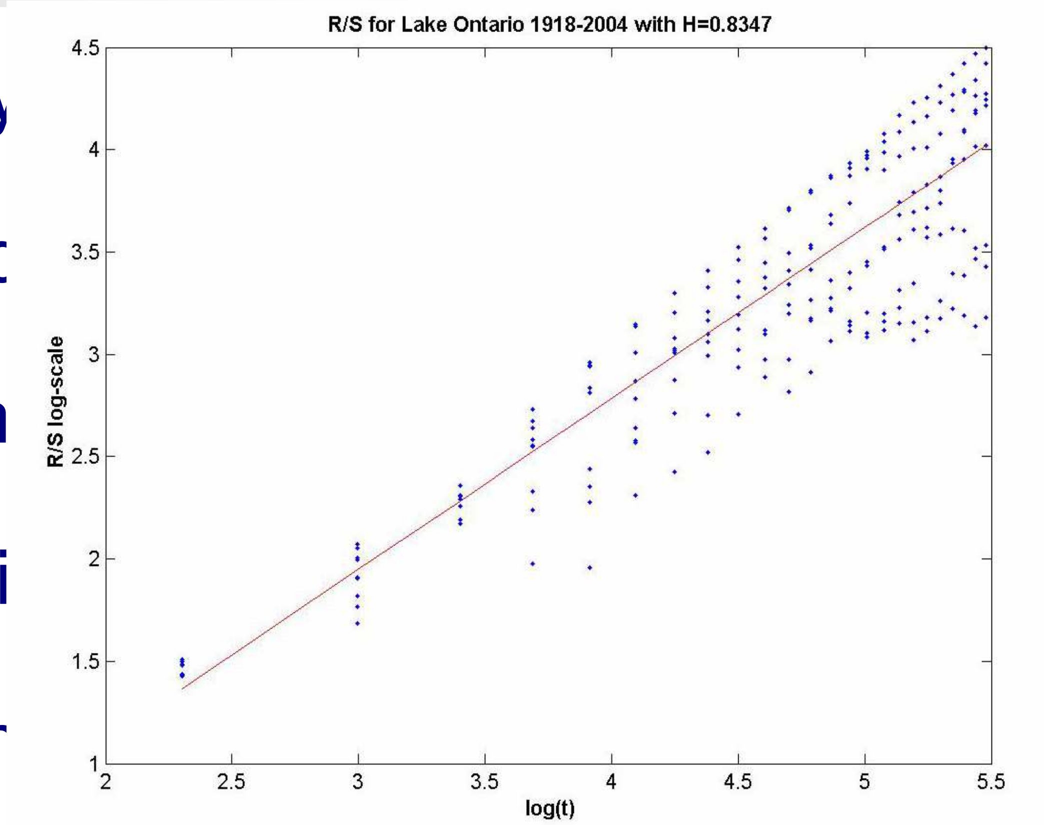
3. Thermal Plant

Future Work

- Incorporate realistic minimum down time constraints into thermal plant model – Thompson
- Use of more realistic Anderson/Davison electricity price model
- Hydroelectric
- Wind
- Exotic technologies

Microhydro

- Power sites formerly develop
- New technology and these into play
- Small scale watersh overflows
- Mathematical techni fractional Brownian
- Investigate joint sitir sites



Wind Power

Wind power promising but with a lot of problems

- Non-dispatchable
  - couple with microhydro or pump storage;
  - optimal construction of such pairs
- Uneconomic
  - use better economic metrics than total energy produced to optimize wind turbine design and siting
- Siting
  - Use investment under uncertainty techniques for optimal siting
  - Constraints on siting: aesthetic, bird/bat safety (with Brock Fenton)

Thank You !