Asymmetric Beliefs, Agency Conflicts, and Venture Capital Investment

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Plan of Presentation

- Introduction and Motivation
- Verbal Description of Model
- Model Details
- Main Analytical Results
- Calibration of Model
- Results of Numerical Analysis
- Conclusions

Introduction and Motivation

Entrepreneurs tend to be wildly over-optimistic; if they were not, they would never get past their first crisis: The Economist, April 16, 2005

- Venture capital financing—financing of new ideas, economic growth
  - VCs have created nearly one-third of the total market value of all publicly traded companies in the U.S. (Gompers and Lerner, 2001)
- Important characteristics of VC relationships
  - High levels of risk with different attitudes towards risk—*agency conflicts*
  - *Imperfect information* about potential payoffs
  - Divergent views about payoffs—*asymmetric beliefs*
  - Staged investment and dynamic contracting

Research Objectives

- Develop theory of venture capital investment that incorporates these features in a dynamic setting

- Determine how risk, imperfect information, agency conflicts, and asymmetric beliefs affect VC-EN relationships
    - Economic value that they generate
    - Structure of long-term dynamic contracts between VCs and ENs
    - Staging of VC investment over time
    - Duration of VC-EN relationships

Main Findings

- Duration of relationship and expected payoff to the VC increase with *degree of asymmetry in beliefs*
    - VC has significant incentives to “feed” EN optimism
    - This incentive has a beneficial effect on firm value

- Depending on project’s *intrinsic* and *technical* risk and degree of asymmetry in beliefs
    - VC’s investments could increase over time, decrease over time, or initially increase and then decrease
    - Equilibrium long-term contract for the EN features decreasing pay-performance sensitivities

Main Findings

- Intrinsic and technical risks have *opposing* effects on the duration and economic value of the VC-EN relationship

- Duration, firm/project value, and VC's expected payoff
  - increase with technical risk
  - decrease with intrinsic risk

- Firm value and the expected payoff to the VC are actually *enhanced* when there is greater noise in the perception of project quality

Model Overview—The Players

- Continuous time framework with time horizon $[0, \infty)$
- At date 0, a cash-constrained entrepreneur (EN) with a project approaches a venture capitalist (VC) for funding
- Project could generate value via
  - physical capital investments by the VC
  - human capital (effort) investments by the EN
- VC and EN have imperfect information about the project
- VC and EN differ in their assessments of the project's quality

Model Overview—The Contract

- The VC offers the EN a long-term contract
  - Specifies VC's investments over time
  - Specifies EN's payoff

- Two-sided lack of commitment
  - VC or EN could terminate the relationship at any date
  - VC or EN could initiate a renegotiation of the contract

- Equilibrium contract is *renegotiation-proof*

- VC possesses bargaining power in negotiations with the EN

Model Overview—State Variable and Preferences

- Fundamental state variable—*termination value* $V(t)$ of the project
  - Market value of the project outside the VC-EN relationship
  - VC and EN possess non-transferrable project-specific skills
  - Termination value less than "rational expectations" value, that is, value under hypothetical full commitment
- Termination value is observable and verifiable and, therefore, contractible; the rational expectations value is non-contractible
- VC is risk-neutral and EN is risk-averse with inter-temporal CARA preferences—zero discount rates for simplicity
- For simplicity, payoffs upon termination—no intermediate cash flows

Model Overview—The Players' Actions

- The VC chooses the long-term renegotiation-proof contract for the EN and the termination time to maximize her expected payoff

- EN dynamically chooses his effort to maximize his expected utility payoff upon termination

- VC-EN contract, VC's dynamic investment policy, EN's dynamic effort policy, and the termination time determined *endogenously* in a *subgame-perfect* equilibrium of the VC-EN dynamic game.

Model Overview—Value Creation and Incentives

- The change in termination value over any time period depends on
  - VC’s investment
  - EN’s effort
  - Project’s intrinsic quality
  - Project’s intrinsic risk

- EN’s effort is observable
  - But is not verifiable and therefore not directly contractible

- EN must be provided with appropriate incentives to exert effort
  - VC offers contract contingent on the termination value

Evolution of Termination Value—Components

- Formulation

$$dV(t) = (c(t)^{\alpha}\eta(t)^{\beta} - l(t))dt + \Theta dt + sdB(t)$$

- *Net discretionary output* $(c(t)^{\alpha}\eta(t)^{\beta} - l(t))dt; \alpha, \beta > 0$
  - o VC's investment rate $c(t)$, EN's effort level $\eta(t)$, operating costs $l(t)$
  - o Operating costs—wages to salaried employees, depreciation expenses, decline in revenue due to competition, etc
  - o Deterministic—increasing and convex over time

- *Intrinsic risk* $sdB(t)$*:* component of project risk invariant over time

Evolution of Termination Value—Project Quality

- *Project quality* Θ*:* Growth rate of termination value arising from intrinsic quality of the project
  - o VC and EN have imperfect information about Θ and different beliefs
  - o Beliefs are common knowledge—"agree to disagree"
  - o Uncertainty in the value of Θ—project's *technical risk*
  - o Technical risk is resolved over time as VC and EN update their beliefs in a Bayesian manner based on observations of the project's termination value

- Initial beliefs about Θ: $N(\mu_0^{VC}, \sigma_0{}^2)$ and $N(\mu_0^{EN}, \sigma_0{}^2)$
- Posterior beliefs $\sigma_t{}^2 = \dfrac{s^2 \sigma_0{}^2}{s^2 + t\sigma_0{}^2}$; $\mu_t^{EN} - \mu_t^{EN} = \Delta_t = \dfrac{s^2}{s^2 + t\sigma_0{}^2} \Delta_0$

Feasible Contractual Payoffs

- Contract specifies payoffs upon termination, which is an $\{F_t\}-$stopping time—information filtration generated by termination value process

- Contract described by $\{F_t\}-$adapted stochastic process $P(.)$ describing EN's payoff if the relationship is terminated at any date

- $V(.) - P(.)$ = VC's payoff

- $P(0) = V(0)$ since EN owns the project at date 0

Contract—EN's Perspective

- EN's expected utility at date zero

$$-E\left[\exp\left\{-\lambda\left(P(\tau)-\int_0^\tau k\eta(t)^\gamma\,dt\right)\right\}\right]$$

  - τ—termination time
  - λ—EN's risk aversion
  - $k\eta(t)^\gamma\,dt$—disutility of effort

- EN can terminate the relationship at any date and receive $P(t)$
  - Continues only if expected utility from continuing is greater

Contract—VC's Perspective

- VC's continuation value at date t

$$CV(t) = E_t^{VC}\left[(V(\tau) - P(\tau) - (V(t) - P(t)) - \int_t^{\tau} c(u)du \right]$$

- VC continues relationship only if her continuation value is nonnegative

- Since VC possesses bargaining power
  - o Termination occurs the VC's behest in equilibrium
  - o EN indifferent between termination and continuation

The Equilibrium

- *Assumption:* $(1-\alpha)\gamma / \beta > 2$

- The optimal contract must have the following *affine* form:

$$dP(t) = a_t dt + b_t dV_t;\ a_t \in R; b_t > 0$$

$$P(\tau) = P(0) + \int_0^{\tau} [a_t dt + b_t dV(t)]$$

- Idea of proof
  - Consider any *admissible* effort process of the EN
  - Such a process is *implementable* (Holmstrom and Milgrom, 1987) if and only if contract has the above form

Proof Sketch

- Start with a fixed random process $V(.)$ on a probability space

- Investment and effort processes alter the probability distribution of this process—apply Girsanov's theorem

- *Cumulative value process for the EN*: conditional expected future utility to the EN at any date *including sunk disutility of prior effort* from a given contract $(P(.), c(.), \tau)$ and given effort process $\eta(.)$

$$\overline{U}_{P,c,\tau} = E_{c,\eta}^{EN}\left[-\exp\left(-\lambda\left\langle P(\tau)-\int_0^\tau k\eta(u)^\gamma du\right\rangle\right) | F_t\right]$$

Proof Sketch

- *Certainty equivalent process for the EN*

$$R_{P,c,\tau}(\eta(.),t) = -\frac{\log\left[-\overline{U}_{P,c,\tau}(t)\right]}{\lambda} + \int_0^t k\eta(u)^{\gamma} du$$

- *Key result*: A contract $(P(.), c(.), \tau)$ *implements* a given effort process $\eta^*(.)$ only if

$$P(0) = V(0), P(t) = R(\eta^*(t), t) \text{ a.s.}$$
$$dR(\eta^*(t), t) = a(t)dt + b(t)dV(t)$$
$; a(t), b(t)$ are functions of $\eta^*(t), c(t)$

- Use dynamic programming to derive evolution of the process $R(.)$ such that the EN's optimal effort process is $\eta^*(.)$

The Equilibrium Contract

- We derive the equilibrium contract by induction

- EN's equilibrium pay-performance sensitivity b_t^*, effort η_t^*, VC's investment rate c_t^* are all *deterministic* (conditional on continuation)

- The *fixed* portion of the EN's compensation in each period is chosen to satisfy his participation constraint
- VC's continuation value at date t

$$CV(t) = \underbrace{\Lambda_t(b_t^*, c_t^*)dt}_{\text{"within-period flow"}} \qquad + \qquad \underbrace{E_t^{VC}[\max(CV(t+dt), 0)]}_{\text{"future option value"}}$$

Derivation of Equilibrium

- First derive optimal effort for given investment and pay-performance sensitivity
- Derive optimal investment for given pay-performance sensitivity
- Derive optimal pay-performance sensitivity

The VC's Objective Function

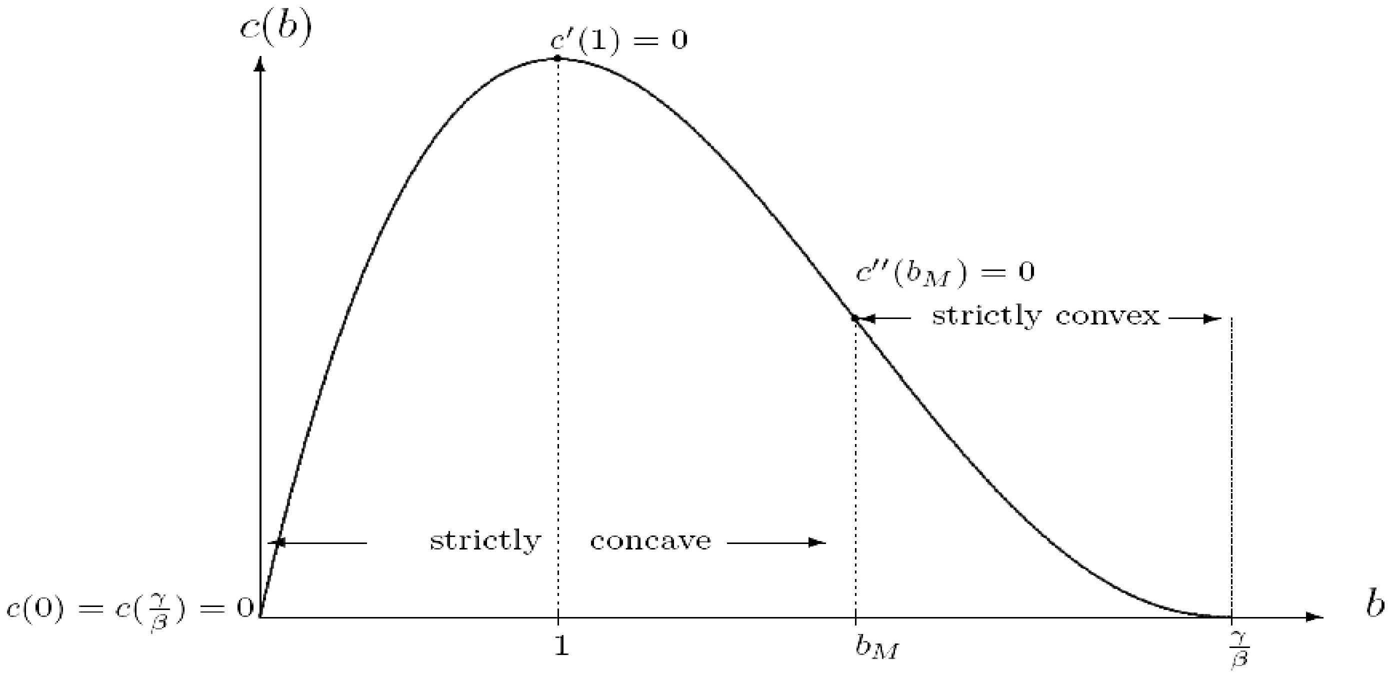
- Optimal pay-performance sensitivity b_t^* solves

$$b_t^* = \arg\max_{b>0} F_t(b) = \arg\max_{b>0} \left[\Delta_t b - \frac{1}{2} p b^2 + Kc(b) \right]$$

  - $\Delta_t b$: Economic rent from EN's optimism
  - $\frac{1}{2} p b^2 = \frac{1}{2} \lambda s^2 b^2$: Cost of risk-sharing with the risk-averse EN
  - $Kc(b)$: Return on investment

- Interplay between three "forces" determines equilibrium dynamics

The Optimal Investment Function



The Optimal Investment Function

- For a given pay-performance sensitivity, an increase in the investment affects the within-period flow to the VC in opposite ways:
  - On the positive side, the EN increases his effort, which increases output
  - On the negative side, since the EN's disutility of effort increases, VC's cost to maintain the EN's participation increases

- Below a threshold level of the pay-performance sensitivity, benefits of increased output predominate

- Above the threshold, EN's participation costs dominate

Benchmark Scenarios

Benchmark Scenario 1: Symmetric Risk Attitudes and Symmetric Beliefs ("No Agency")

- EN is risk-neutral and degree of asymmetry in beliefs is zero
- Pay-performance sensitivity equals one
- VC's investment and the EN's effort are constant—VC's investment attains its highest possible level

Benchmark Scenario 2: Asymmetric Risk Attitudes, but Symmetric Beliefs

- EN is risk-averse and degree of asymmetry in beliefs is zero
- Pay-performance sensitivity b_p^*, investment c_p^*, and effort η_p^* are constant, but lower than in the "no agency" scenario

Properties of the Equilibrium

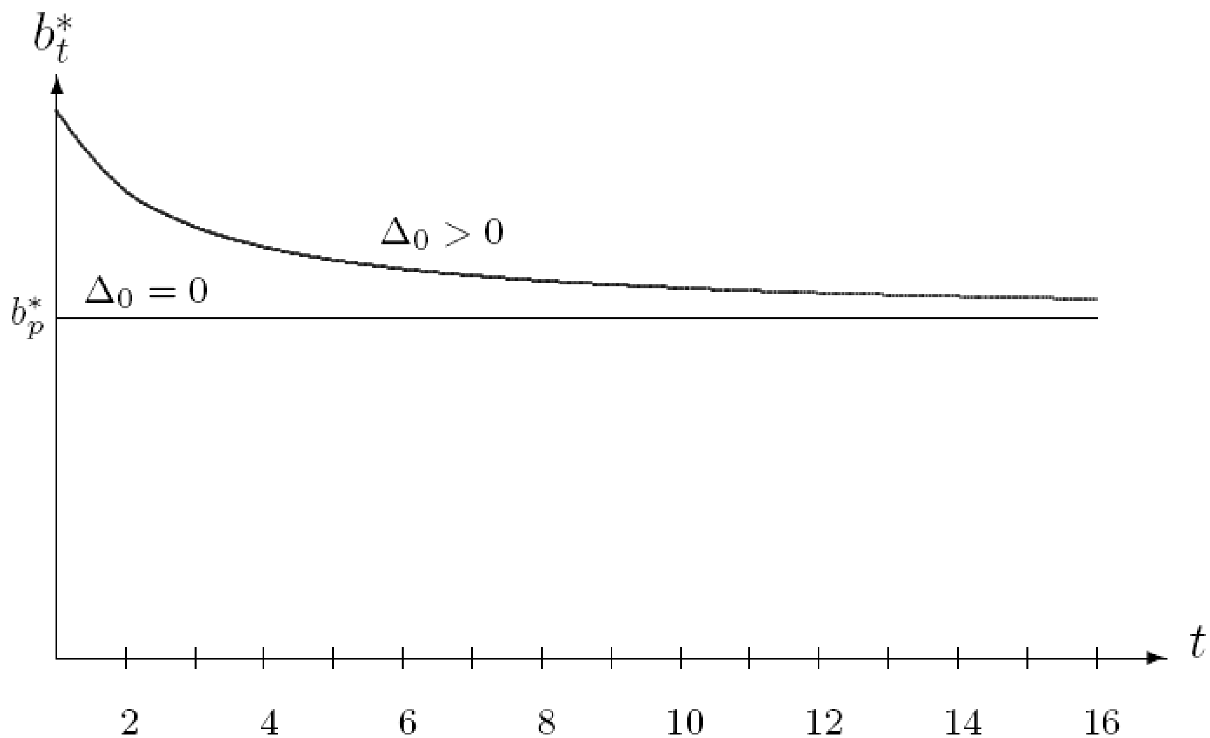


Figure 2: Possible equilibrium pay performance sensitivity paths

Properties of the Equilibrium

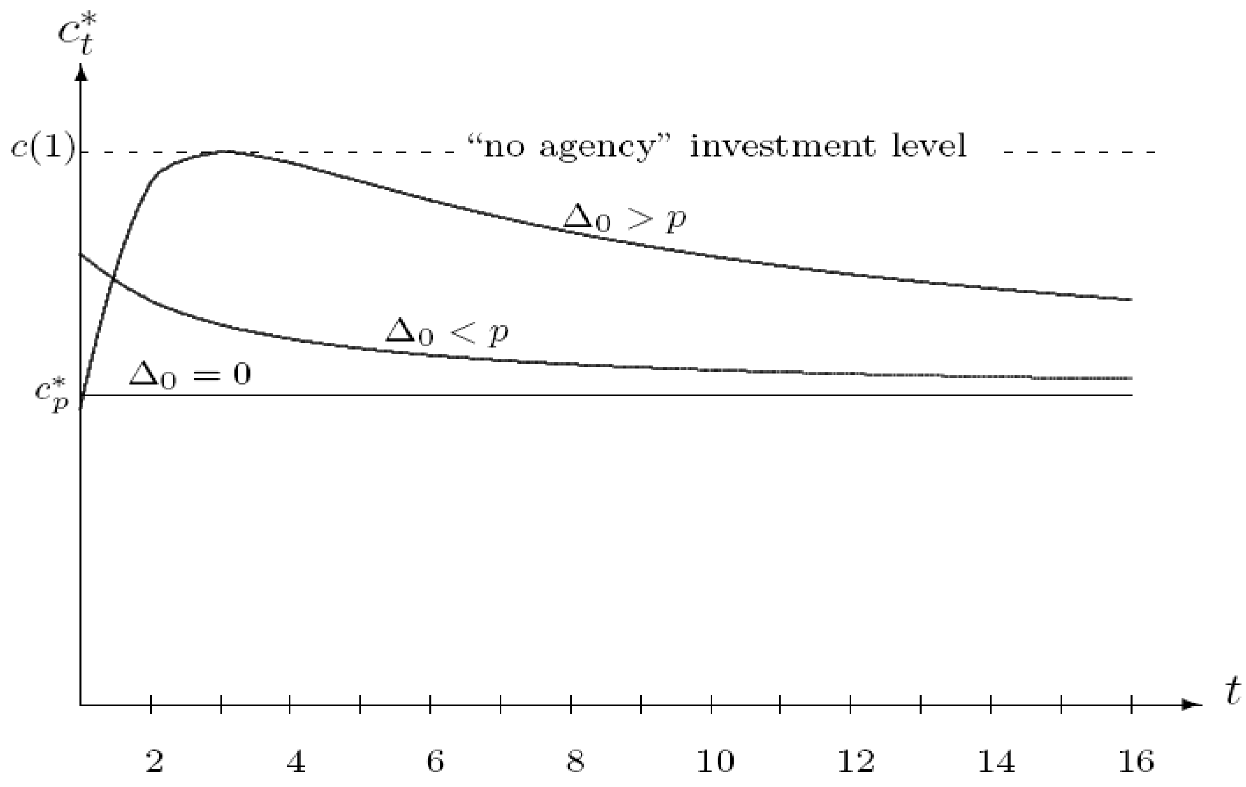


Figure 3: Possible equilibrium investment paths

Equilibrium Dynamics—Intuition

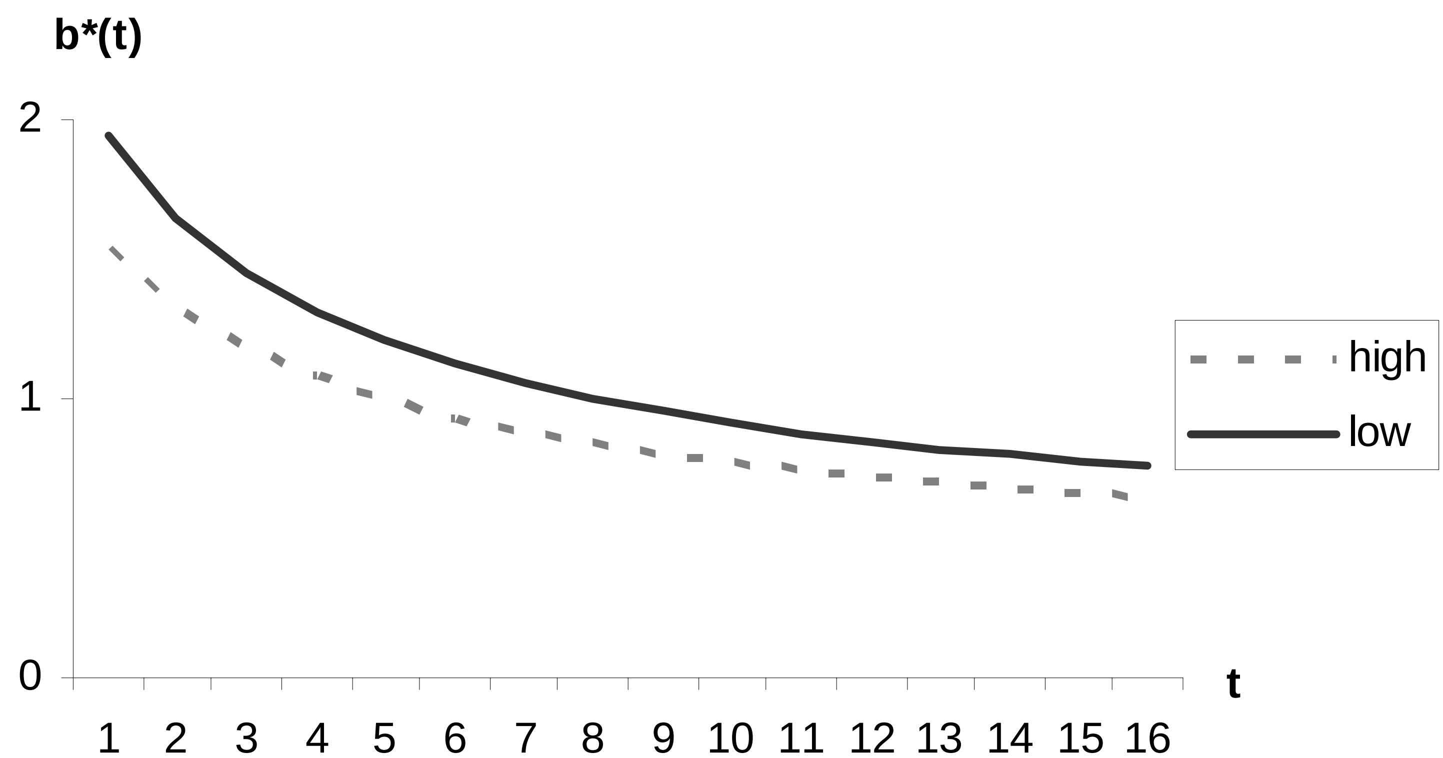
- Results hinge on interplay among
  - EN's effort—positively affected by optimism
  - Costs of risk-sharing—negatively affected by intrinsic risk
- Passage of time lowers degree of asymmetry in beliefs—EN revises optimistic assessments of project quality
  - Lowers economic rents from EN optimism—lowers pay-performance sensitivity and effort
  - Due to *non-monotonic* optimal investment function, VC's investment:
    - Initially increases to "compensate" for decline in EN's effort
    - Then VC's investments decrease over time

Positive Implications

- Decline of EN's pay-performance sensitivity over time
  - With successive capital infusions of VCs
  - EN optimism plays a key role in explaining this finding
  - With symmetric beliefs, no change in EN's equity stake

- VC's *observed* investment paths could either increase until termination, decrease until termination, or initially increase and then decrease
  - Model generates widely different paths of capital investment flows reported in Sahlman (1990) and Gompers (1995)

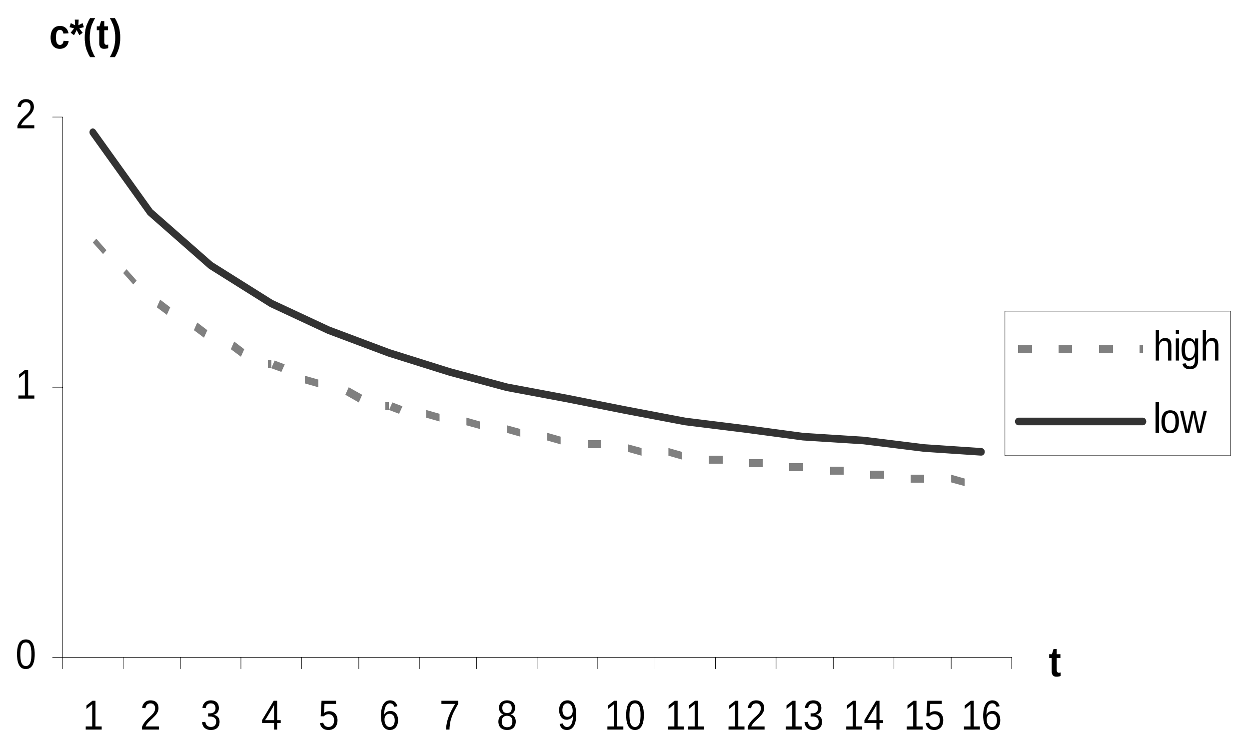
Variation of Pay Performance Sensitivity Path with Initial Technical Risk Intrinsic Risk, and EN Risk Aversion



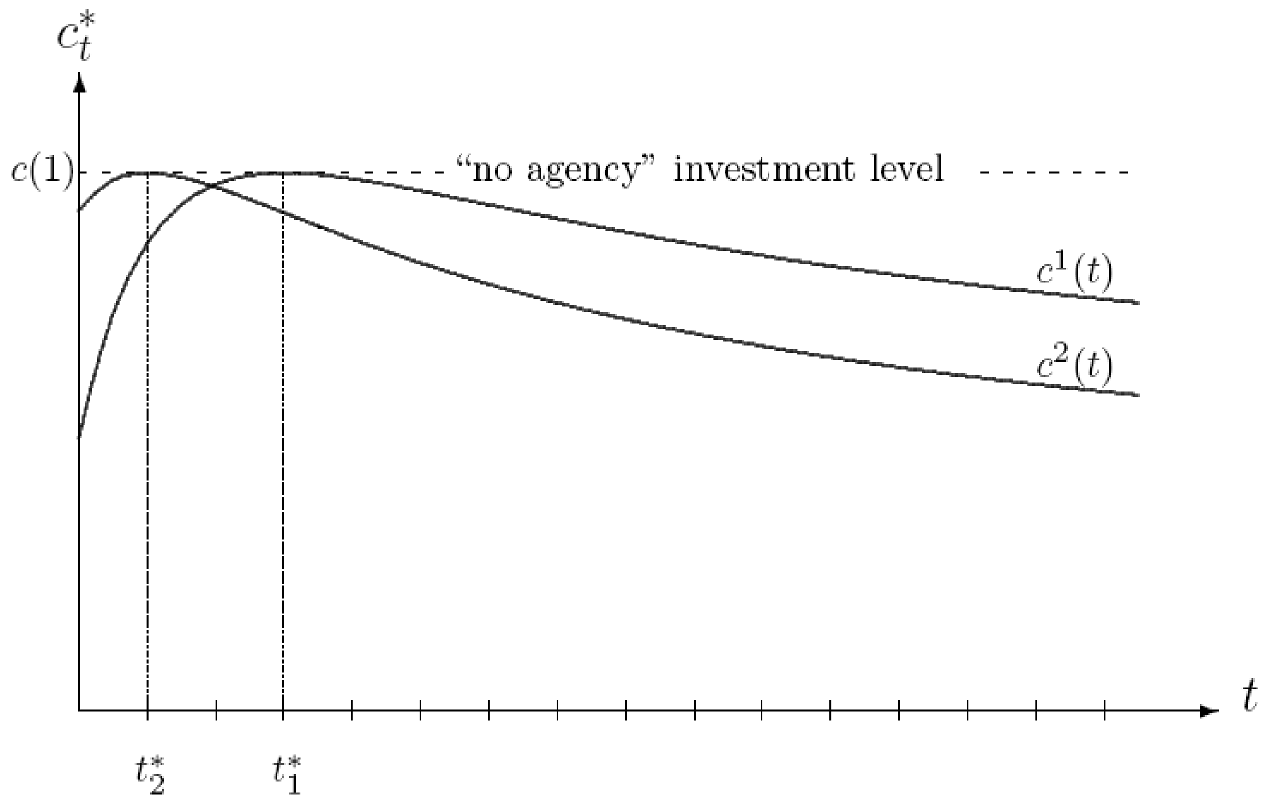
Variation of Pay-Performance Sensitivity Path

- An increase in EN's risk aversion increases costs of risk sharing relative to EN optimism
- Increase in initial technical risk lowers degree of asymmetry in beliefs because "signal to noise" ratio increases—EN "learns faster"
  - Rents from EN's optimism decline relative to costs of risk sharing
- Increase in intrinsic risk *increases* degree of asymmetry in beliefs *and* costs of risk sharing
  - "signal to noise" ratio decreases—costs of risk sharing dominate if $\Delta_0 < 4p$

Variation of Investment Path with Initial Technical Risk, Intrinsic Risk and EN Risk Aversion: EN reasonably optimistic ($\Delta_0 < p$)



Variation of Investment Path with Initial Technical Risk, Intrinsic Risk, and EN Risk Aversion: EN exuberant($\Delta_0 > p$)

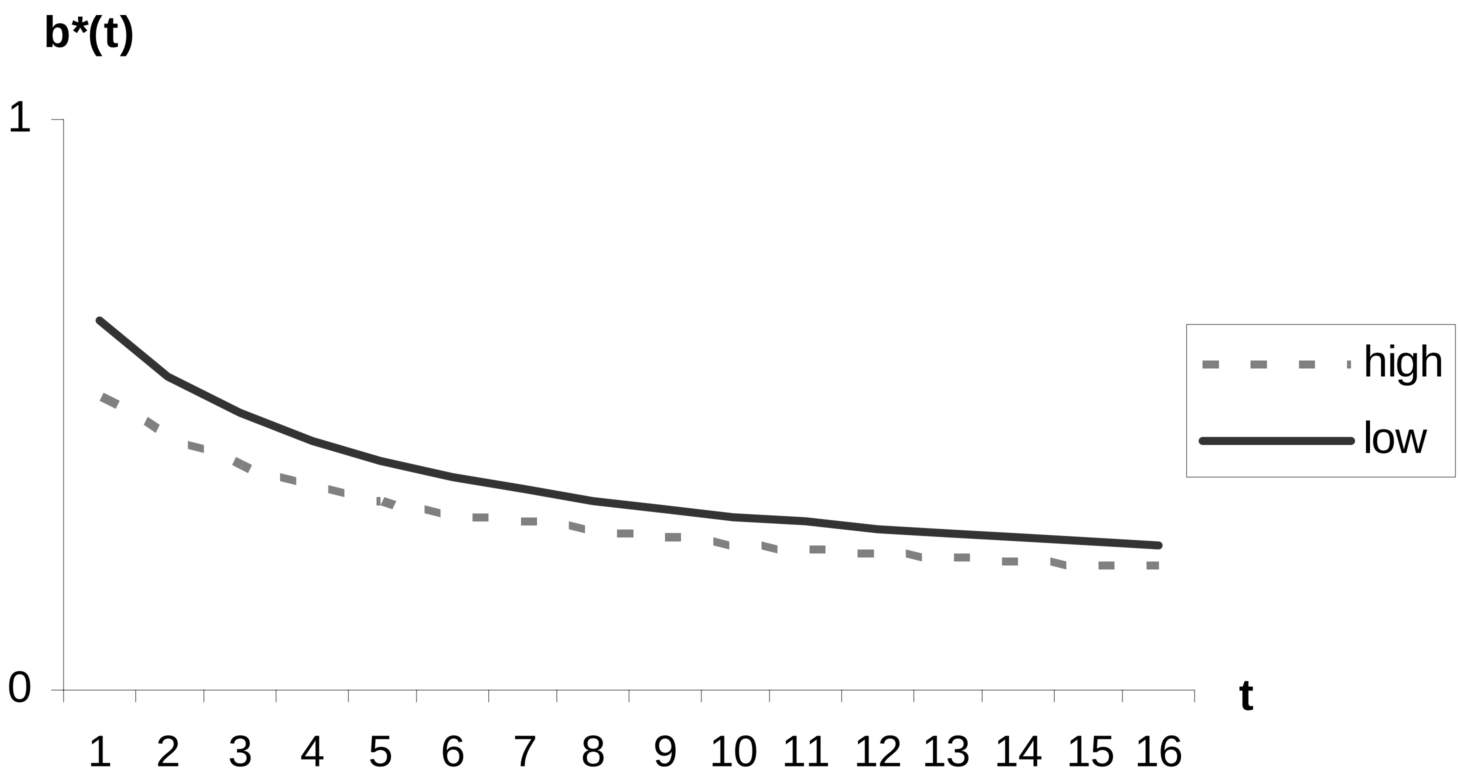


Variation of Investment Path

- Change in VC's equilibrium investment path depends on whether EN is initially "reasonably optimistic" ($\Delta_0 < p$) or "exuberant" ($\Delta_0 > p$)
- If EN is initially reasonably optimistic, costs of risk sharing dominate the effects of EN optimism
  - Investment path declines with EN's risk aversion, project's intrinsic and technical risks
- If the EN is initially exuberant, rents from EN optimism dominate costs of risk sharing in early periods, but costs of risk sharing dominate later

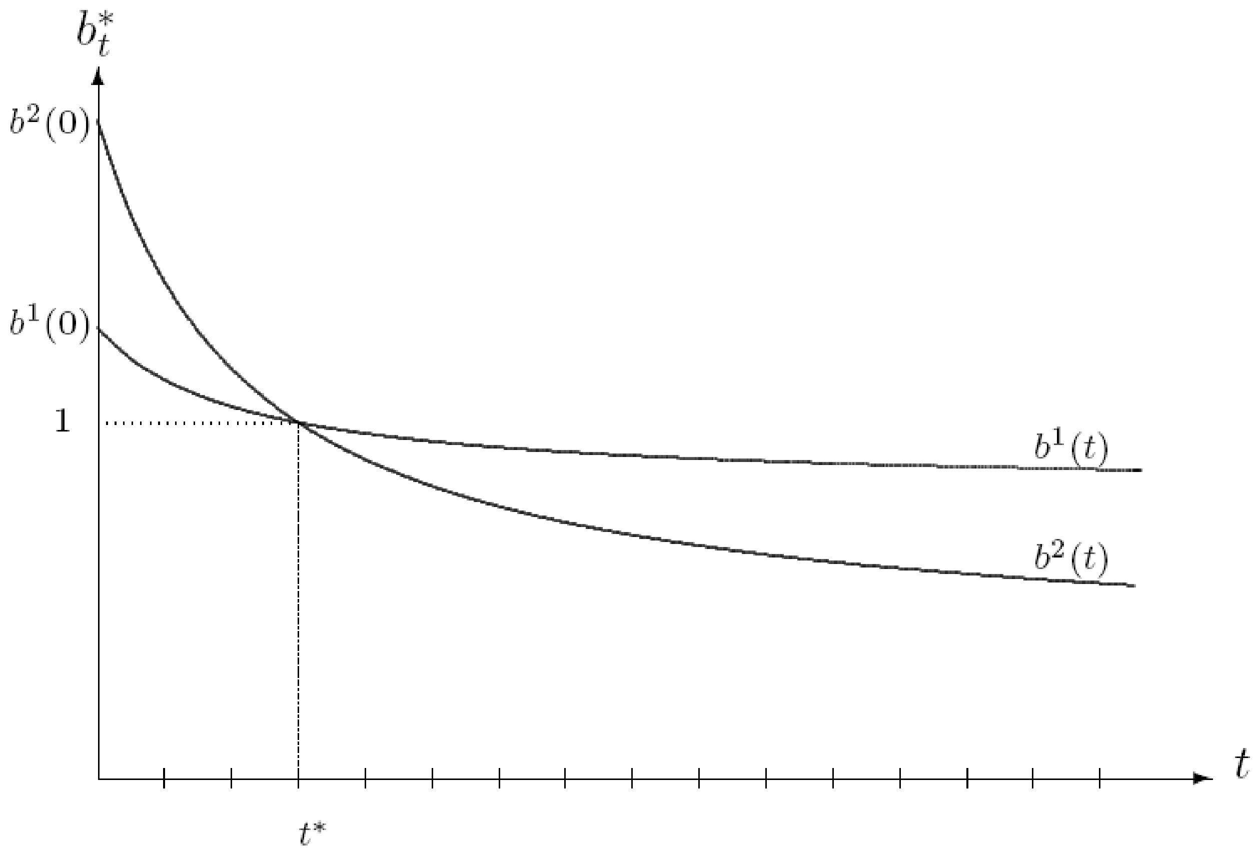
Variation of Pay Performance Sensitivity with the Cost of Effort

(EN reasonably optimistic)



Variation of Pay Performance Sensitivity with the Cost of Effort

(EN exuberant)



Project Duration

- The optimal termination policy for the VC is a trigger policy
  - There exists μ_t^* such that the VC terminates the project only if $\mu_t^{VC} < \mu_t^*$
  - $\mu_t^{VC} < \mu_t^*$ if and only if $V_t < V_t^*$
  - V_t^*'s are the *performance targets* that must be met to ensure continuation
- Project duration
  - increases with the initial degree of asymmetry in beliefs
  - decreases with the EN's risk aversion and cost of effort

Model Calibration: Approach #1

- Classify parameters as either “Direct” or “Indirect”
- Set Direct parameter values
- Find Indirect parameters to match model’s prediction to empirical evidence

Direct Parameters

- μ_0 – VC's initial assessment of project quality

  CAPM: $\mu_0 = r_f + \beta(r_M - r_f) = 0.06 + 1.0(0.10\text{-}0.06) = 0.10$

- s^2, σ_0^2 – Systematic and initial technical risk

  $s^2 + \sigma_0^2 = 1$ (Kerins, Smith and Smith, 2004)

  $s^2 = \sigma_0^2 = 0.5$

- Δ_0 – Initial degree of asymmetry of beliefs

  $\Delta_0 = 0.5$

- $k\eta^\gamma$ – Disutility of effort

  $\gamma = 2$

Indirect Parameters

λ - EN's risk aversion

k – EN's level of disutility of effort

α, β - Capital, effort intensities of firm's production function

L – loss function parameter [$l(t) = Lt^2$]

Empirical Data [Gompers, 1995, Sahlman, 1990]

Project duration:

2.7 = average number of investment periods

Distribution of returns from investment:

34.5% of total investment resulted in a negative return

49.8% of total investment resulted in a return between 0 and 5

15.7% of total investment resulted in a return greater than 5

Firm’s rate of success:

32.4% of companies failed to yield the amount invested

67.6% of companies yielded more than the amount invested

Firm value per unit of investment:

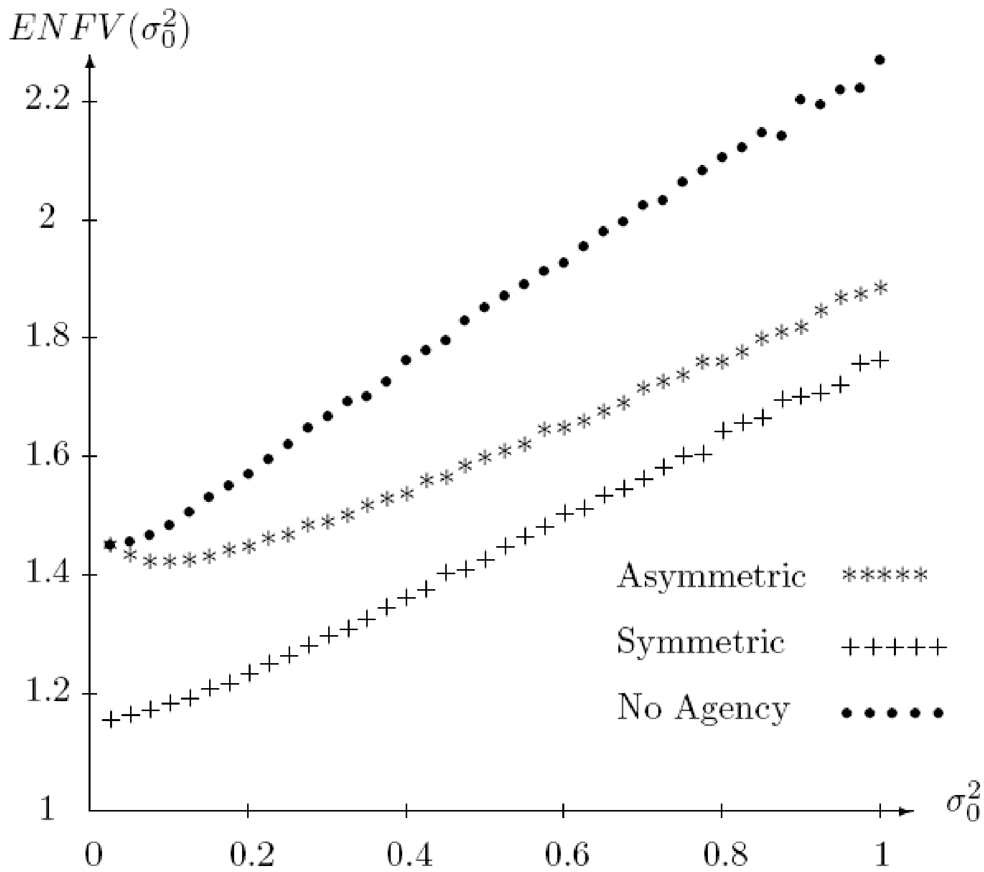
4.8 = total value of firms divided by total amount invested

Model Calibration: Approach 2

- Classify parameters as firm-specific or EN-specific
- Each choice of firm-specific parameters defines a firm *type*
- For each firm type t = 1, 2, …, N:
    - Find EN-specific parameters that best match empirical data
    - R_t —model's predictions for above combination of firm-specific and EN-specific parameters
- Find distribution p_t, t = 1, 2, …, N of firms so that $\{(p_t, R_t), t = 1, 2, \ldots, N\}$ matches empirical data *exactly*

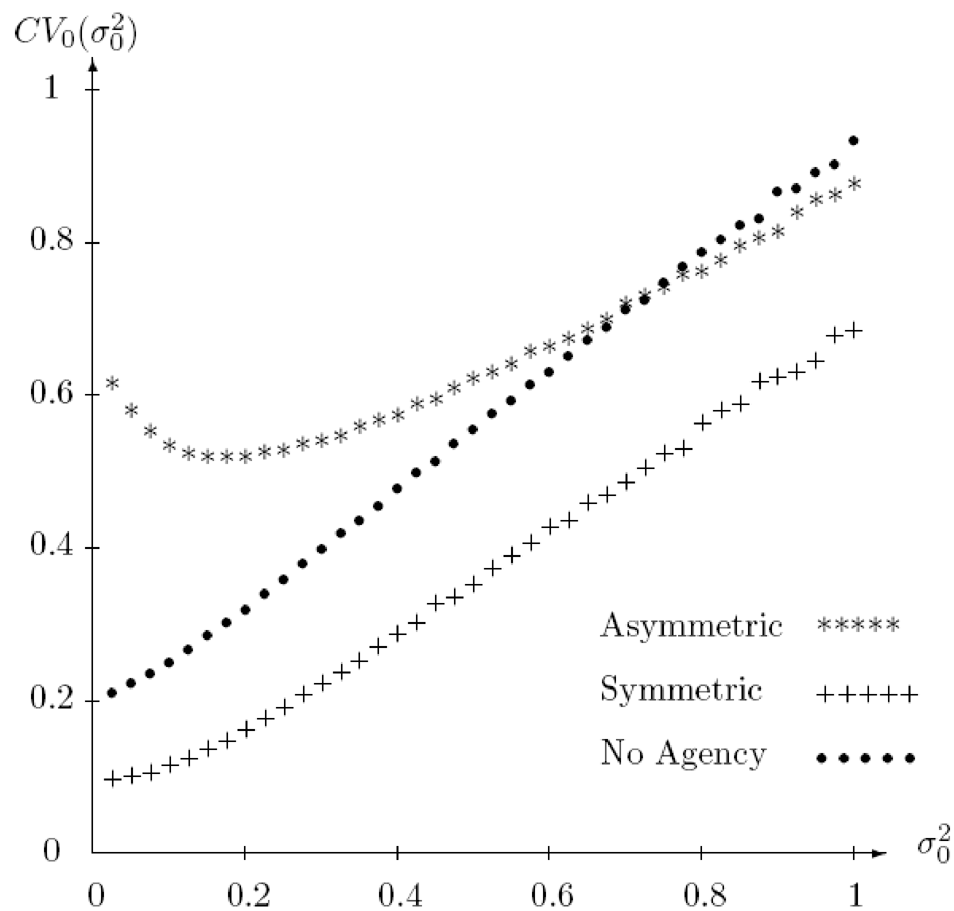
Numerical Analysis

Figure: The Effect of Technical Risk



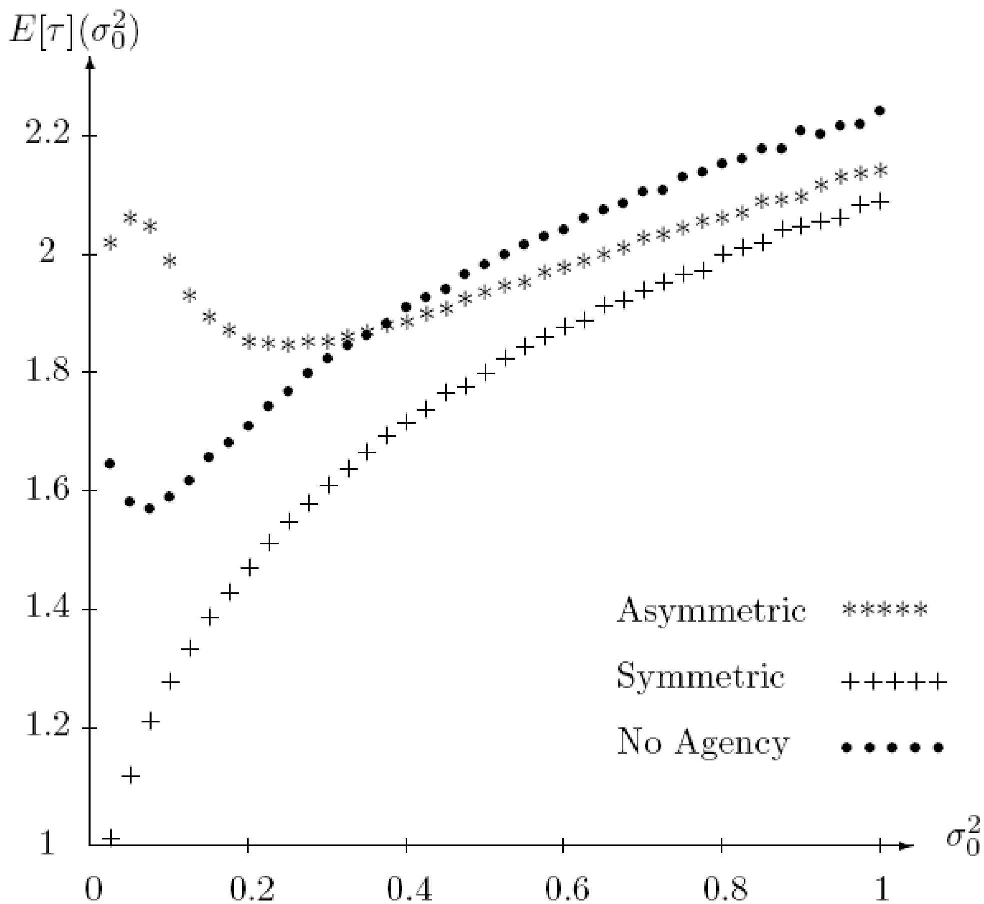
Numerical Analysis

Figure: The Effect of Technical Risk



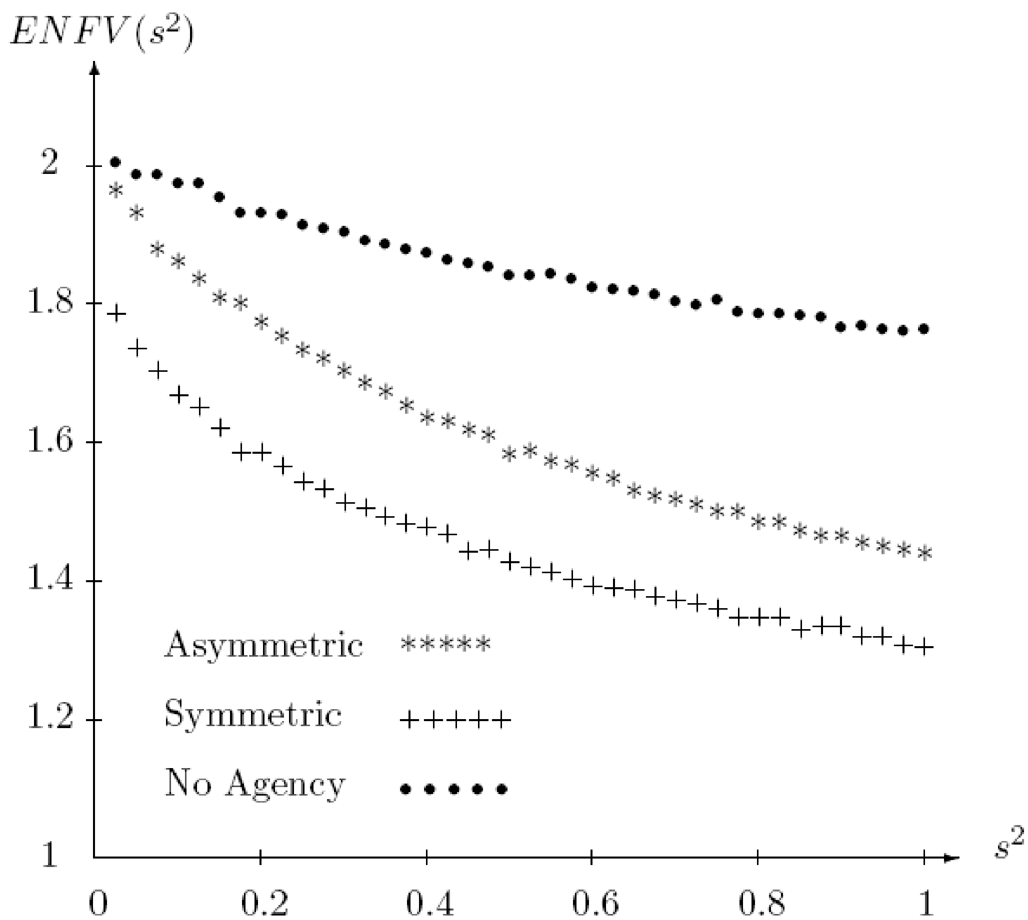
Numerical Analysis

Figure: The Effect of Technical Risk



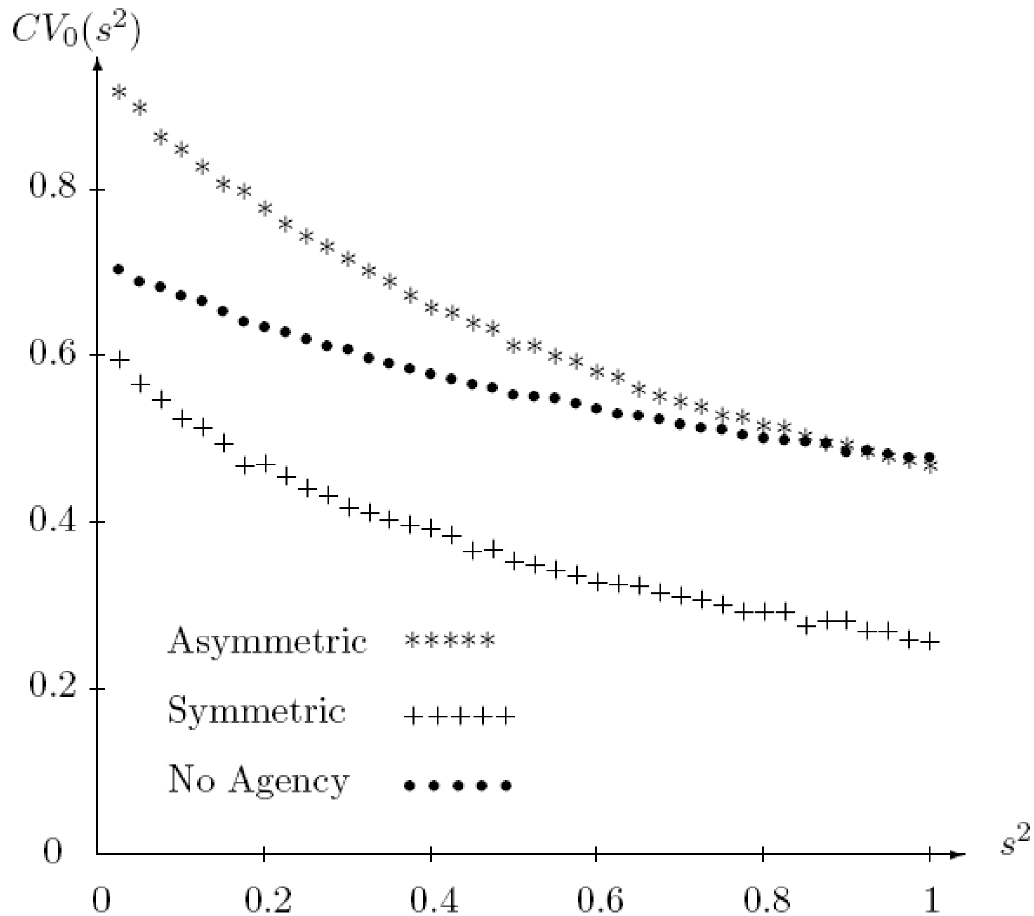
Numerical Analysis

Figure: The Effect of Intrinsic Risk



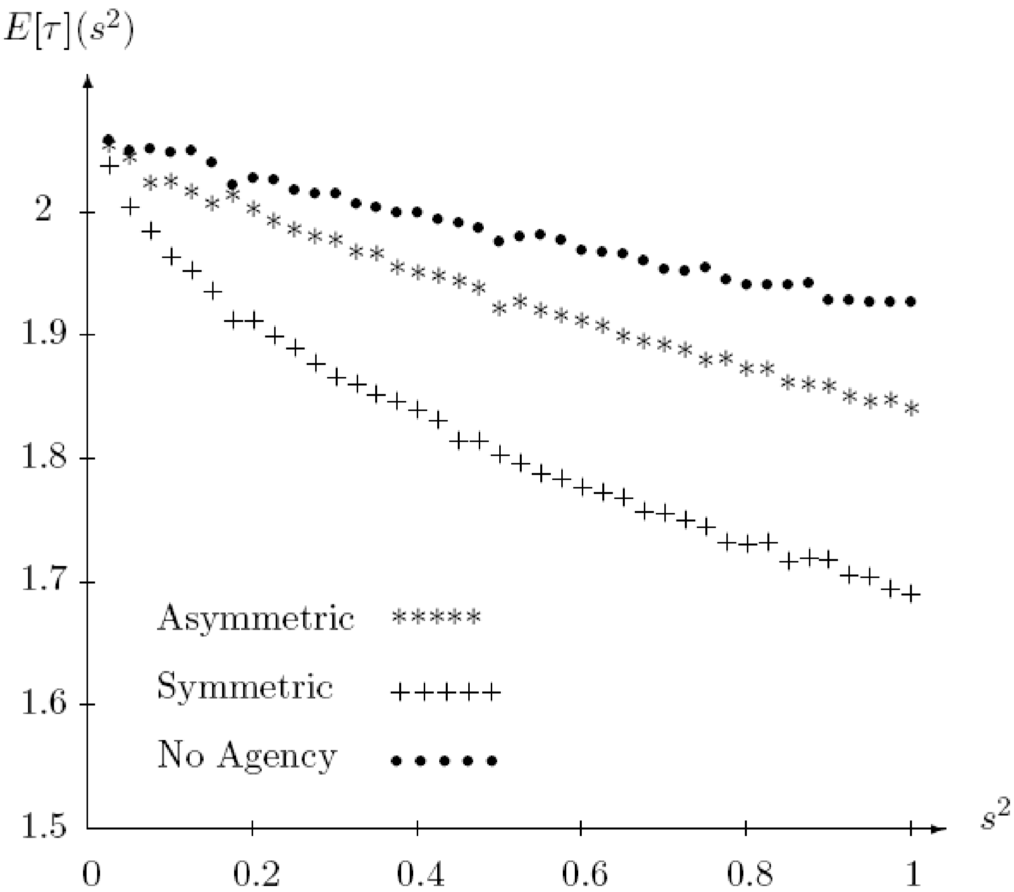
Numerical Analysis

Figure: The Effect of Intrinsic Risk



Numerical Analysis

Figure: The Effect of Intrinsic Risk



Project Quality Perception and Continuation Value

- *Evolution of Mean Posterior Assessment of Project Quality*

$$d\mu_t^{VC} = \frac{\sigma_0{}^2 s}{s^2 + t\sigma_0{}^2} dB_t^{VC} = \sigma_t^{\mu} dB_t^{VC}$$

- VC's Continuation Value

$$CV(t) = \Lambda_t(b_t^*, c_t^*)dt \quad + \quad E_t^{VC}[\max(CV(t+dt), 0)]$$

within period flow   future option value

$$= F_t^* dt \quad + \quad \mu_t^{VC} dt \quad + \quad E_t^{VC}[\max(CV(t+dt), 0)]$$

The Effects of Technical and Intrinsic Risk

No Agency and Symmetric Beliefs Scenarios

Factor	Technical Risk	Intrinsic Risk
Stdev of mean assessment of project quality	↑	↓
Deterministic component of within-period flow, $\mathbf{F_t*}$	—	↓
Future "option value" of continuation	↑	↓

The Effects of Technical and Intrinsic Risk

Actual Scenario

Factor	Technical Risk	Intrinsic Risk
Degree of asymmetry of beliefs	↓	↑
Deterministic component of within-period flow, $\mathbf{F_t^*}$	↓	Non-monotonic
Future "option value" of continuation	↑	↓

The Effects of Technical and Intrinsic Risk

- Below a threshold value of technical risk decrease in F_t^* dominates

- Above the threshold, increase in option value dominates

- In the case of intrinsic risk, decline in option value dominates effect on within-period flow

Conclusions

- Dynamic model of VC investment
  - High levels of intrinsic and technical risk
  - Agency conflicts, imperfect information, and asymmetric beliefs
  - Importance of staged investment and dynamic contracting in mitigating potential inefficiencies
- Positive Implications
  - Declining pay-performance sensitivity
  - Increasing, decreasing, or non-monotonic investment paths
  - Technical and intrinsic risks have opposing effects
  - EN optimism could be exploited by VC—improves firm value
- Normative implication
  - Society benefits from greater noise in project quality

Conclusions

- Dynamic principal-agent model with
  - double-sided moral hazard
  - risk
  - imperfect information
  - asymmetric beliefs

- Applicable in other economic settings
  - Manager-shareholder conflicts
  - Financing of R&D
  - Delegated portfolio management