Bayesian Analyses for Dyadic Data

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Relevant Papers:

- Gill and Swartz (2001), CJS
- Gill and Swartz (2004), JRSSC
- Gill and Swartz (2006), AJMMS

Outline

- Bayesian Computation
- Examples of Dyadic Data
- Round Robin Models
- Directed Graphs Models
- Concluding Remarks

Bayesian Computation

- turn the Bayesian crank

$$\pi(\theta \mid x) \;\propto\; f(x \mid \theta)\, \pi(\theta)$$

- obtain posterior characteristics

$$I(m) \;=\; \int m(\theta)\, \pi(\theta \mid x)\, d\theta$$

- for example: $m(\theta) = \theta_i$

Strategies for Bayesian Computation

(1) asymptotic approximations

(2) quadrature

(3) direct simulation from the posterior

$$\hat{I}(m) \;=\; \sum_{i=1}^{N} \frac{m(\theta^{(i)})}{N} \quad \text{where } \theta^{(i)} \sim \pi(\theta \mid x)$$

(4) importance sampling

$$\hat{I}(m) \;=\; \sum_{i=1}^{N} \frac{m(\theta^{(i)})\;\pi(\theta^{(i)} \mid x)}{N\;w(\theta^{(i)})} \quad \text{where } \theta^{(i)} \sim w(\theta)$$

(5) Markov chain Monte Carlo (MCMC)

$$\theta^{(1)} \longrightarrow \theta^{(2)} \longrightarrow \theta^{(3)} \longrightarrow \cdots \quad \text{where } \theta^{(N)} \longrightarrow \pi(\theta \mid x)$$

Typical MCMC Approach

- given $\theta = (\theta_1, \ldots, \theta_k)'$, obtain full conditional distributions

$$[\theta_i \mid \theta_1, \ldots, \theta_{i-1}, \theta_{i+1}, \ldots, \theta_k, X]$$

- sample from full conditionals or imbed a Metropolis step

$\longrightarrow$ considerable programming and smarts required

The Beauty of WinBUGS

- only model specification required

- MCMC done in the background

- its free

Sample WinBUGS Code:

```
for (i in 1:m-1) {for (j in i+1:m) {
gamma[i,j] <- gg[i,j,1]
gamma[j,i] <- gg[i,j,2]
gg[i,j,1:2] ~ dmnorm(zero[1:2],S2[,]) }}
```

Examples of Dyadic Data

- migration between two regions
- influence between journals
- brand switching
- sports
- social psychology

Who Likes Whom and by How Much?

Round Robin Models

- basic model: Warner, Kenny and Stoto (1979)
  - $y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_{ij} + \epsilon_{ijk}$
  - $y_{jik} = \mu + \alpha_j + \beta_i + \gamma_{ji} + \epsilon_{jik}$
  - $\mathrm{E}(\alpha_i) = \mathrm{E}(\beta_j) = \mathrm{E}(\gamma_{ij}) = \mathrm{E}(\epsilon_{ijk}) = 0$
  - $\mathrm{Var}\begin{pmatrix} \alpha_i \\ \beta_i \end{pmatrix} = \begin{pmatrix} \sigma_\alpha^2 & \sigma_{\alpha\beta} \\ \sigma_{\alpha\beta} & \sigma_\beta^2 \end{pmatrix}$
  - $\mathrm{Var}\begin{pmatrix} \gamma_{ij} \\ \gamma_{ji} \end{pmatrix} = \begin{pmatrix} \sigma_\gamma^2 & \sigma_{\gamma\gamma} \\ \sigma_{\gamma\gamma} & \sigma_\gamma^2 \end{pmatrix}$
  - $\mathrm{Var}\begin{pmatrix} \epsilon_{ijk} \\ \epsilon_{jik} \end{pmatrix} = \begin{pmatrix} \sigma_\epsilon^2 & \sigma_{\epsilon\epsilon} \\ \sigma_{\epsilon\epsilon} & \sigma_\epsilon^2 \end{pmatrix}$

Bayesian Version of Basic Model

- normal distributions for μ, $\begin{pmatrix} \alpha_i \\ \beta_i \end{pmatrix}$, $\begin{pmatrix} \gamma_{ij} \\ \gamma_{ji} \end{pmatrix}$, $\begin{pmatrix} \epsilon_{ijk} \\ \epsilon_{jik} \end{pmatrix}$

- diffuse distributions for hyperparameters

- one subjective hyperparameter r_0
  - $\mathrm{E}(\sigma_\alpha^2) = \mathrm{E}(\sigma_\beta^2) \approx \mathrm{E}(\sigma_\gamma^2) \approx \mathrm{E}(\sigma_\epsilon^2) = r_0$

- $m^2 + m + 10$ dimensional posterior distribution

A More Complex Bayesian Model

- interest in the evolution of interpersonal attraction
- Curry and Emerson (1970)
- six 8-person groups with subgroups of roommates
- ratings on 100-point scale in weeks 1,2,4,6,8
- longitudinal model:
  - $y_{gijk} = \mu_k + \alpha_{gik} + \beta_{gjk} + \delta_k \, \mathrm{room}_{gij} + \epsilon_{gijk}$
  - $y_{gjik} = \mu_k + \alpha_{gjk} + \beta_{gik} + \delta_k \, \mathrm{room}_{gij} + \epsilon_{gjik}$
- normality assumption for the 490 first-order parameters

Some Summary Results

	Week 1		Week 8	
Parameter	Mean	SD	Mean	SD
General mean μ_k	75.0	2.8	78.0	1.4
Roommate effect δ_k	8.9	1.7	8.0	1.3
Actor variance $\sigma^2_{k\alpha}$	88.0	28.0	71.0	20.0
Partner variance $\sigma^2_{k\beta}$	74.0	24.0	86.0	24.0
Actor-partner correlation $\rho_{k\alpha\beta}$	0.5	0.2	0.0	0.2

Directed Graphs Models

- various models
  - p_1 (Holland and Leinhardt, 1981)
  - p^* (Wasserman and Pattison, 1996)
- various parameterizations

Bayesian Version of the Basic p_1-Model

- $X_{ij} = 1(0) \quad$ if node i relates (does not relate) to node j
- $\phi_{ij} = \log\left(\frac{\Pr(X_{ij}=1 \mid X_{ji}=1)\,\Pr(X_{ij}=0 \mid X_{ji}=0)}{\Pr(X_{ij}=0 \mid X_{ji}=1)\,\Pr(X_{ij}=1 \mid X_{ji}=0)}\right)$
- $\theta_{ij} = \log\left(\frac{\Pr(X_{ij}=1 \mid X_{ji}=0)}{\Pr(X_{ij}=0 \mid X_{ji}=0)}\right)$
- model
  - $\text{Prob}(\mathbf{X}) \propto \exp\left(\sum_{i<j} \phi_{ij} X_{ij} X_{ji} + \sum_{i \neq j} \theta_{ij} X_{ij}\right)$
  - $\phi_{ij} = \phi$
  - $\theta_{ij} = \theta + \alpha_i + \beta_j$
- normality assumption for ϕ, θ and $\begin{pmatrix} \alpha_i \\ \beta_i \end{pmatrix}$
- diffuse distributions on hyperparameters

Corporate Law Firm Example

- interest in “advice” variable
- Lazega and Pattison (1999)
- 71 lawyers (36 partners and 35 associates)
- $G_{ij} = 1$ if lawyers i and j both partners or both associates
- $\phi_{ij} = \phi + \nu G_{ij}$
- $\theta_{ij} = \theta + \alpha_i + \beta_j + \delta G_{ij}$
- normality assumption for ν and δ
- block specific covariance matrices introduced
- multigraphs considered using “co-working” variable

Some Summary Results

Parameter	Mean	SD
$\sigma^2_{P\alpha}$	0.66	0.21
$\sigma^2_{A\alpha}$	2.58	1.00
$\sigma^2_{P\beta}$	1.21	0.39
$\sigma^2_{A\beta}$	0.66	0.21
ρ_P	-0.33	0.18
ρ_A	-0.41	0.19
θ	-2.60	0.21
ϕ	1.74	0.26
δ	0.94	0.10
ν	0.02	0.27

Concluding Remarks

- exact inference
- unbalanced designs (i.e. $n_{ij} \neq n$ for all i, j)
- missing data (i.e. $n_{ij} \neq n_{ji}$ for some i, j)
- not restricted to normal data
- introduction of covariates is often straightforward
- computational simplicity via WinBUGS