Approximating the Power Dominating Set problem

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Dominating Set problem

Definition (DOMINATING RULES)

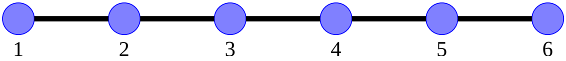
Given a graph $G = (V, E)$ and a subset $S \subseteq V$, the set of dominated nodes, $\mathcal{D}_S$, is defined by:

1. $v \in V$ is dominated if $v \in S$.
2. $v \in V$ is dominated if v is an neighbor of a node $u \in S$.

Problem (DOMSET)

Given a graph G find a min. size dominating set, i.e. min. size subset $S \subseteq V$ such that $\mathcal{D}_S = V(G)$.

First Example



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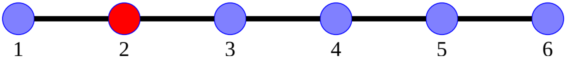

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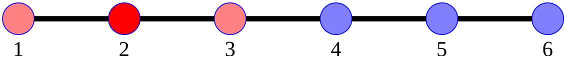

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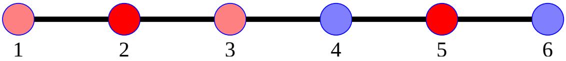

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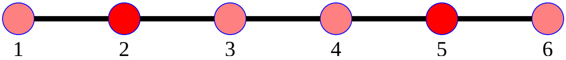
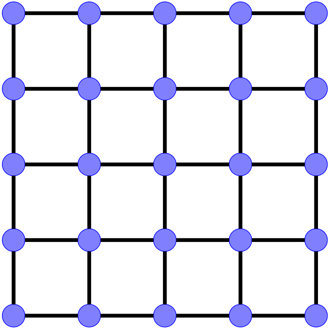


$S = \{2, 5\} \quad \mathcal{D}_S = V(G).$

Second Example - 5×5 grid



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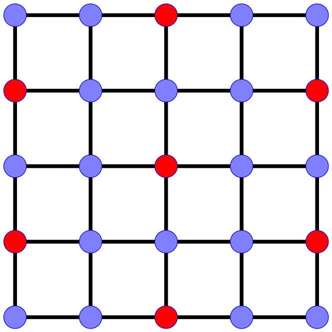

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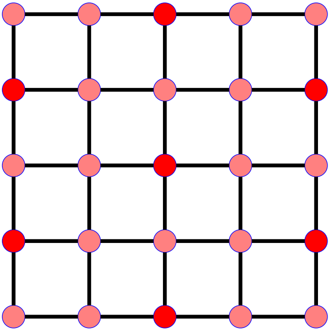


Approximating Set Cover

Problem (SetCover)

Given a universe U of n elements, a collection of subsets of U, $\mathcal{S} = \{S_1, \cdots, S_k\}$, find minimum number of sets from $\mathcal{S}$ that covers all elements of U.

Theorem ((Johnson), (Lovasz) and (Chvatal))

SetCover *can be approximated within* $\ln n$ *by a simple greedy algorithm.*

Theorem ((Lund,Yannakakis) and (Feige))

SetCover *cannot be approximated better than* $O(\log n)$ *unless* $P = NP$.

Approximating Dominating Set

- DomSet is a special case of the SetCover problem.
- Define $\mathcal{I}(G) = (\mathcal{S}, V)$: $\mathcal{S} = \{S_v = N_G(v) \cup \{v\} | \forall v \in V\}$, where $N_G(v)$ is the set of neighbors of v in G.
- DomSet can be approximated within $O(\log n)$ by the following greedy algorithm:

    $S \leftarrow \emptyset$
    while $\mathcal{D}_S \neq V(G)$ **do**
        Pick v which maximize $\left|\mathcal{D}_{S \cup \{v\}} \setminus \mathcal{D}_S\right|$.
        $S \leftarrow S \cup \{v\}$.
    end while
    Output S.

- DomSet can not be approximated better than $O(\log n)$

Power Dominating Set problem

Definition (Power Dominating Rules)

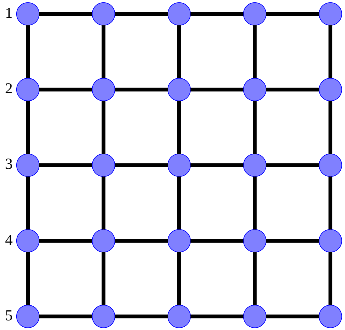
Given a graph $G = (V, E)$ and a subset $S \subseteq V$, the set of power dominated nodes, $\mathcal{P}_S$, is defined by:

1. $v \in V$ is power dominated if $v \in S$.
2. $v \in V$ is power dominated if v is an neighbor of a node $u \in S$.
3. $v \in V$ is power dominated if v has a neighbor u such that u and all of its neighbors except v are power dominated.

Problem (PDS)

Given a graph G find a min. size power dominating set, i.e. min. size subset $S \subseteq V$ such that $\mathcal{P}_S = V(G)$.

An Example - 5×5 grid



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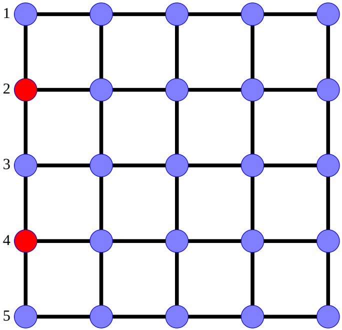
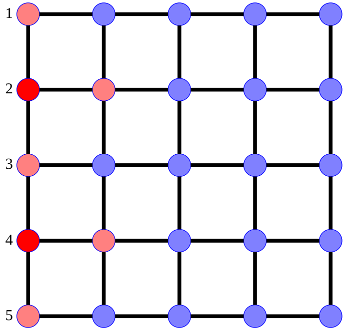


An Example - 5 $\times$ 5 grid



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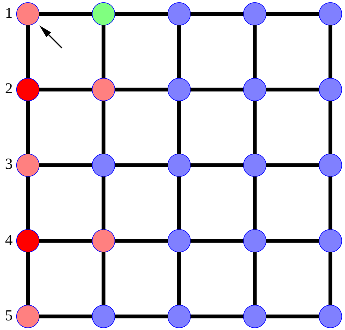

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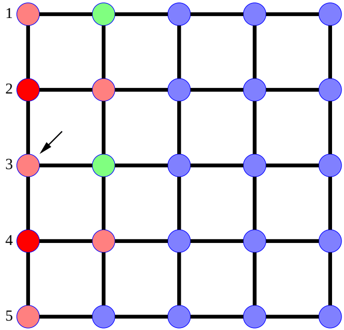

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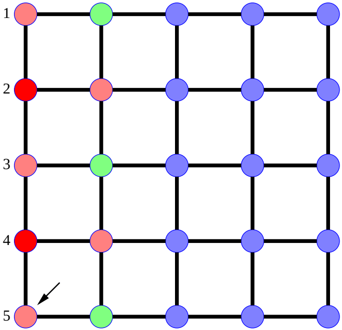

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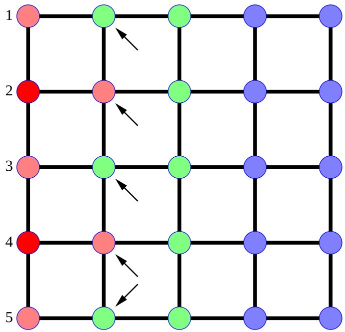

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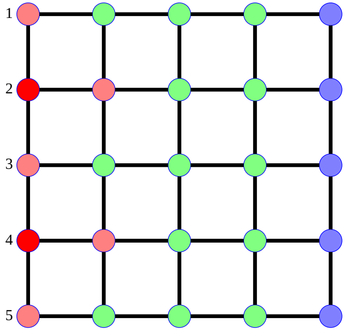

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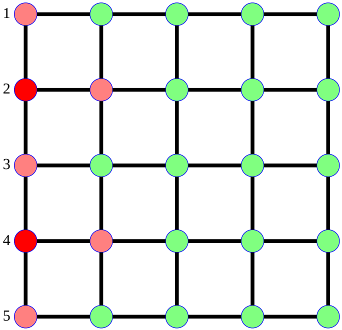

Generally

$\ell \times k$ grid:
$\mathtt{Opt} \leq \frac{1}{2}\min(\ell, k)$.

Hardness of PDS

- PDS is a generalization to DomSet.

Reduction:

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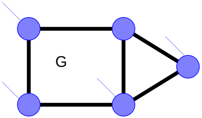

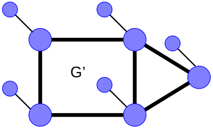

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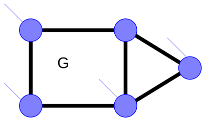

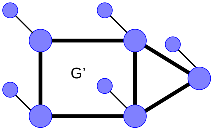

Theorem

Given graph G. Let G' be the graph constructed as above: S is a dominating set in G iff S is a power dominating set in G'.

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Theorem

Given graph G. Let G' be the graph constructed as above: S is a dominating set in G iff S is a power dominating set in G'.

Theorem ((Guo et al.), (Kneis et al.))

PDS cannot be approximated better than $O(\log(n))$ unless $P = NP$.

The Greedy Algorithm works poorly on PDS

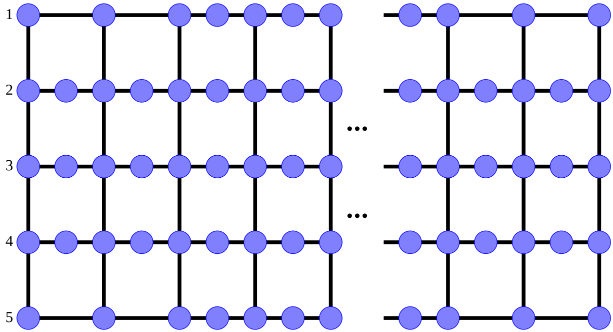
Greedy Algorithm

1: $S \leftarrow \emptyset$
2: **while** $\mathcal{P}_S \neq V(G)$ **do**
3: Pick v which maximize $\left|\mathcal{P}_{S\cup\{v\}} \backslash \mathcal{P}_S\right|$.
4: $S \leftarrow S \cup \{v\}$.
5: **end while**
6: Output S.

Theorem

Given graph $G = (V, E)$ with n nodes, the above greedy algorithm may find a solution S such that $|S| \geq \Omega(n) \times \mathcal{O}pt(G)$.

Proof of the theorem- $5 \times 5n$ subdivided grid



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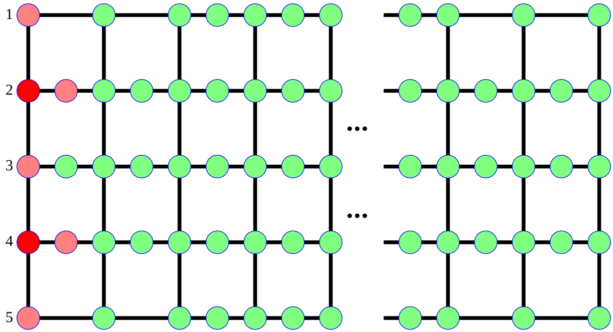

- $\mathtt{Opt} = 2$

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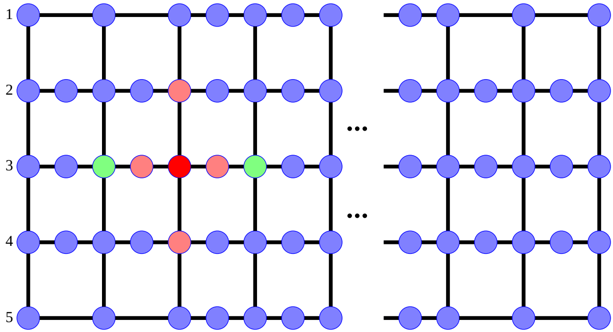

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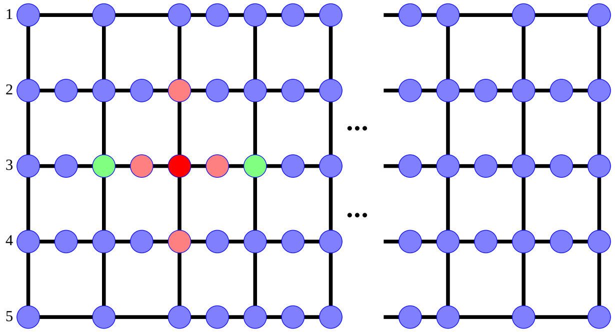
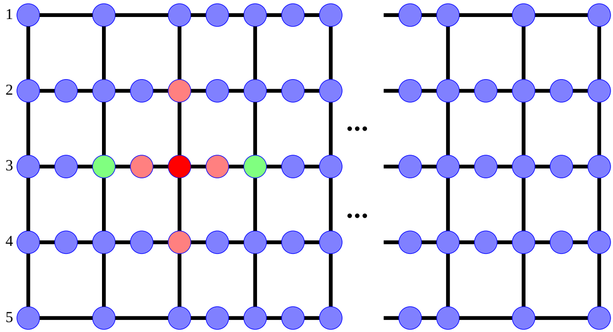


- $\texttt{Opt} = 2$
- Greedy Algorithm may take all of the middle nodes in the 5×5 subgrids.

Proof of the theorem- $5 \times 5n$ subdivided grid



- $\mathtt{Opt} = 2$
- Greedy Algorithm may take all of the middle nodes in the 5×5 subgrids.
- $|S| \geq n = \Omega(|V|).\mathtt{Opt}$

- Haynes et al. presented a linear-time algorithm to solve PDS optimally on trees.
- Guo et al. developed a poly-time dynamic programming to solve PDS optimally on graphs of bounded tree-width.
- PDS remains NP-hard even on planar graphs.
- We have a $\sqrt{n}$-approximation algorithm for planar graphs.
- We also show that there is no poly-log approximation for PDS by a reduction from MinRep problem.
- We also study the PDS problem in directed graphs and extend the result of Guo et al. to directed graphs where their underlying undirected graphs are bounded tree-width.

References

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