NON CRITICAL SUGRA and GAUGE GRAVITY DUALITY

Cobi Sonnenschein

Stanislav Kuperstein

Introduction

- Critical supergravity/gauge duality enables us to determine qualitatively as well as quantitatively several important phenomena of guage dynamics like Wilson lines, 't Hooft lines, Polyakov lines, glueball spectrum, external baryons, domain walls etc.
- However, the anitholographic descriptions of gauge theories suffers from a severe limitation which is the fact that generically their spectrum includes KK states with the same mass scale as that of the hadronic states.
- To date there is no mechanism to disentangle the KK states from the hadrons.
- The most naive way to overcome this problem is to study the holographic duality in non-critical string theory of dimension four or close to four.
- Are there such consistent string theories?
- The simplest non-critical string theory is the Linear dilaton string where

$$X^\mu, \mu = 1, ..., d < 10 \qquad \phi = V_\mu X^\mu \qquad V_\mu V^\mu = \frac{26 - d}{6}$$

- There is no no-go theorem that states that superstring theories can not exist in the window of $3/2 < d < 10$. Kutasov and Seiberg showed that the superLiouville theory in even dimension is consistent (tachyon free).

- The study of gauge/gravity duality of non conformal gauge theory taught us that the renormalization scale is naturally identified with a fifth dimension.

- Therefore for (non-supersymmetric) gauge theories we look for a curved background metric with a warp factor

$$ds^2 = e^{2\lambda}(\tau)d_{II}x^2 + d\tau^2$$

- R symmetries of supersymmetric gauge theories are expected to be the duals of the isometries of the transverse space. To accommodate the $SO(6)$ R symmetry of $N = 4$ SYM we need the S^5 of the $AdS_5 \times S^5$. Theories with $U(1)_R$ call for S^1 transverse dimension etc. for instance for $\mathcal{N} = 1$ SYM our ansaz metric will be

$$ds^2 = e^{2\lambda}(\tau)d_{II}x^2 + d\tau^2 + e^{2\nu}(\tau)d\Omega^2_{s^1}$$

- A basic ingredient in the gauge/gravity duality are the RR forms. We do not know how to quantize critical NSR string theories on such backgrounds, let alone non-critical ones.

- Hence our strategy is to first address the non-critical holography in the SUGRA limit.
- The starting point are the equations of the vanishing β functions. These determine the low energy SUGRA effective action.
- The non critical SUGRA solutions are characterized by finite curvature (in units of α') and hence one cannot a priori ignore higher order curvature corrections.
- However, there are non critical string theories with high curvature that are exactly solvable.

Outline

- The non-critical SUGRA equations of motion and BPS equations
- Families of solutions:
  - The linear dilaton; The Cigar and Trumpet as T-duals
  - The non critical $AdS_3 \times S^3$ string theory.
  - Conformal $AdS_{n+1} \times S^k$ backgrounds
  - AdS black hole solutions
  - The RR deformed two dimensional black hole
  - Backgrounds with non-zero RR charge $Q \neq 0$ that asymptote to the linear dilaton solution
- Holographic dual gauge theories: The entropy; A novel large N limit; The gauge theories duals of the $AdS_{p+2} \times S^{d-p-2}$ SUGRA backgrounds and Ads BH backgrounds;
- The phenomenology of the AdS_6 black hole : Wilson line, 't Hooft line, glueball spectra, spinning strings, flavored quarks
- SUGRA backgrounds duals of flavored gauge theories.
- Anomalies
- Toward the exact non critical string theory with RR background.

The non-critical SUGRA equations of motion

- The bosonic part of the non-critical SUGRA action in d dimensions that follows from the vanishing β functions is

$$S = \int d^{n+k+1}x\sqrt{G}e^{-2\phi}\left(R + 4(\partial\phi)^2 + \frac{c}{\alpha'}\right)$$
$$-\frac{e^{-2\phi}}{2}\int H_{(3)} \wedge \star H_{(3)} - \sum_p \frac{1}{2}\int F_{(p+2)} \wedge \star F_{(p+2)},$$

where

$$\frac{c}{\alpha'} = \frac{10-d}{\alpha'}$$

is the non-criticality term

The background includes

- The metric in the *string* frame is taken to depend only on the radial coordinate τ.

$$l_s^{-2}ds^2 = d\tau^2 + e^{2\lambda(\tau)}dx_{\|}^2 + e^{2\nu(\tau)}d\Omega_k^2$$

where $dx_{\|}^2$ is n dimensional flat metric, and $d\Omega_k^2$ is a k dimensional sphere.

- The F_{p+2} RR form that corresponds to a D_p brane with $n = p+1$ dimensional world volume. Only a single F_{p+2} form will be considered

- Upon substituting the metric into the action and performing the integration one finds

$$S = l_s^{-2} \int d\rho \left(\left[-n(\lambda')^2 - k(\nu')^2 + (\varphi')^2 + ce^{-2\varphi} + (k-1)ke^{-2\nu-2\varphi} \right] \right) + S_{RR}$$

where $d\tau = -e^{-\varphi} d\rho$, $(A)' = \partial_\rho A$ and

$$\varphi = 2\phi - n\lambda - k\nu$$

is the “shifted” dilaton.

- Assuming that the RR form also depends only on the radial direction , namely, $F = \partial_\tau A dx^0 \wedge \ldots dx^p \wedge d\tau$, the RR part of the action reads

$$S_{RR} = -\int d\rho \left(\frac{1}{4} e^{-n\lambda + k\nu + \varphi} (A')^2 \right) = -Q^2 \int d\rho e^{n\lambda - k\nu - \varphi},$$

where we made the substitution

$$A' = 2Q e^{n\lambda - k\nu - \varphi}$$

which is the solution of the equation of motion of

$$A'' - A'(n\lambda' - k\nu' - \varphi') = 0$$

- The second order equations of motion are:

$$\partial_\rho^2 \lambda - \frac{1}{2} Q_{RR}^2 e^{n\lambda - k\nu - \varphi} = 0,$$

$$\partial_\rho^2 \nu - (k-1) e^{-2\nu - 2\varphi} + \frac{1}{2} Q_{RR}^2 e^{n\lambda - k\nu - \varphi} + Q_{NS}^2 e^{-2k\nu - 2\varphi} = 0,$$

$$\partial_\rho^2 \varphi + (c + (k-1) k e^{-2\nu}) e^{-2\varphi} - \frac{1}{2} Q_{RR}^2 e^{n\lambda - k\nu - \varphi} - Q_{NS}^2 e^{-2k\nu - 2\varphi} = 0$$

- Any solution has to obey the zero-energy constraint ,

$$n(\partial_\tau \lambda)^2 + k(\partial_\tau \nu)^2 - (\partial_\tau \varphi)^2 + c + (k-1) k e^{-2\nu} - Q_{RR}^2 e^{n\lambda - k\nu + \varphi} - Q_{NS}^2 e^{-2k\nu} = 0$$

The superpotential and BPS equations

- For certain backgrounds one can avoid the hurdle of solving second order differential equations and instead solve first order BPS equations.
- Consider the following general form of a background action

$$S = \int d\rho \left(-\frac{1}{2} G_{ab} f^{a\prime} f^{b\prime} - V(f) \right)$$

If the potential is related to a superpotential W as follows

$$V = \frac{1}{8} G^{ab} \partial_a W \partial_b W,$$

then the BPS equations are

$$f^{a\prime} = \frac{1}{2} G^{ab} \partial_b W.$$

- The BPS equations are compatible with the equations of motion and with the zero energy condition.
- Applying this procedure to our case we get

$$G_{\lambda\lambda} = 2n \qquad G_{\nu\nu} = 2k \qquad G_{\varphi\varphi} = -2$$

and

$$V = Q^2 e^{n\lambda - k\nu - \varphi} - (c + (k-1)ke^{-2\nu})e^{-2\varphi}.$$

and therefore the relation between the potential and the superpotential reads

$$\frac{1}{n}(\partial_\lambda W)^2 + \frac{1}{k}(\partial_\nu W)^2 - (\partial_\varphi W)^2 = 16V$$

and the BPS equations are

$$\lambda' = \frac{1}{4n}\partial_\lambda W, \qquad \nu' = \frac{1}{4k}\partial_\nu W, \qquad \varphi' = -\frac{1}{4}\partial_\varphi W.$$

Families of non critical backgrounds

- The linear dilaton; The cylinder; Cigar and Trumpet as T-duals
- Conformal $AdS_{n+1} \times S^k$ backgrounds
- AdS black hole solutions
- The RR deformed two dimensional black hole
- Backgrounds with non-zero RR charge $Q \neq 0$ that asymptote to the linear dilaton solution

The linear dilaton; The Cigar and Trumpet as T-duals

- The linear dilaton solution with no S^k reads

$$ds^2 = -dt^2 + \ldots + dx_{n-1}^2 + d\tau^2$$

and a linear dilaton

$$e^{\varphi} = \pm\sqrt{c}\rho \quad \rightarrow \qquad \varphi = \pm\sqrt{c}\tau \quad \rightarrow \qquad \phi = \pm\frac{\sqrt{c}}{2}\tau.$$

- Note that in 10d the dilaton becomes constant and the 10d flat space-time is retrieved.
- In fact the background with the linear dilaton corresponds to an exact 2d conformal theory on the world-sheet.
- A similar solution with S^1 in the background is the cylinder background

$$ds^2 = -dt^2 + \ldots + dx_{n-1}^2 + d\tau^2 + d\theta^2$$

➠ A more interesting background is the cigar background

$$ds^2 = -dt^2 + \ldots + dx_{n-1}^2 + d\tau^2 + \tanh^2\left(\frac{1}{2}\sqrt{c}\tau\right) d\theta^2$$

with a dilaton of the form

$$e^{2\phi} = \frac{1}{2a}\frac{1}{\cosh^2\left(\frac{1}{2}\sqrt{c}\tau\right)}.$$

➠ The radius of the compact direction is equal to

$$R_\theta = \frac{2}{\sqrt{c}}$$

It is fixed by requiring the space to be regular at $\tau = 0$.

➠ The scalar curvature of this "cigar" background is

$$l_s^2\mathcal{R} = -\frac{c}{\cosh^2\left(\frac{1}{2}\sqrt{c}\tau\right)}.$$

➠ The cigar background like the cylinder one corresponds to an exact string solution .

- We have discussed solutions characterized by a compact S_1 transverse space. This naturally calls for the implementation of T-duality to generate new solutions of the equations of motion.
- In the present context T-duality acts on e^{ν} and on e^{ϕ} as follows

$$e^{2\nu} \;\to e^{-2\nu} \qquad e^{2\phi} \;\to e^{2\phi-2\nu}$$

where we still use $\alpha' = 1$.

- Applying T duality to the cigar solution one finds a *trumpet* solution of the form

$$e^{\nu} = \coth\left(\frac{1}{2}\sqrt{c}\tau\right) \qquad e^{2\phi} = \frac{1}{2a}\frac{1}{\sinh^2\left(\frac{1}{2}\sqrt{c}\tau\right)}$$

The $AdS_3 \times S^3$

- An important class of non critical strings are the strings on group manifolds
- As an example consider the superstring on the group manilfods $SU(2)_k \times SL_{\tilde{k}}(2)$. In the corresponding SUGRA one turns on two NS three forms along the three dimensions associated with $e^{2\lambda}$ dx$^2_{II}$ and the S^3.
- The NS terms in the action read

$$S_{NS} = Q^2 \int d\rho e^{-6\nu - 2\varphi} \qquad S_{\tilde{NS}} = \tilde{Q}^2 \int d\rho e^{4\lambda}$$

- The solution of the equations of motion for this case reads

$$R_S^2 = \frac{Q}{\sqrt{2}} \qquad \frac{1}{R_{AdS}^2} - \frac{1}{R_S^2} = \frac{c}{4} = 1$$

- This is exactly the result that follows from the string calculation since

$$\frac{3(\tilde{k}-2)}{\tilde{k}} - \frac{3(k-2)}{k} + 3 = 15 \rightarrow \qquad \frac{1}{\tilde{k}} - \frac{1}{k} = 1$$

Conformal $AdS_{n+1} \times S^k$ backgrounds

- The analog of the critical $AdS_5 \times S^5$ background are the family of "conformal non-critical backgrounds that incorporate RR forms. and a constant dilaton.
- A brief glance over the equations of motion tells us that requiring a constant dilaton implies also a constant ν

$$\partial_\rho \phi = 0 \qquad \rightarrow \qquad \partial_\rho \nu = 0$$

and the solution of this condition takes the form

$$\begin{aligned} e^{2\phi_0} &= \frac{1}{n+1-k}\left(\frac{(n+1-k)(k-1)}{c}\right)^k \frac{2c}{Q^2} \\ e^{2\nu_0} &= \frac{(n+1-k)(k-1)}{c}. \end{aligned} \tag{1}$$

- In order not to have vanishing warp factor of the world-volume coordinates, we must require

$$n+1-k \neq 0 \qquad k \neq 1.$$

- It is convenient at this stage to switch from ρ to τ dependence. Recalling that $\varphi = 2\phi - n\lambda - k\nu$ we find that the equation for λ is

$$\partial_\tau^2\lambda + n(\partial_\tau\lambda)^2 = \frac{Q^2}{2}e^{2\phi_0 - 2k\nu_0}.$$

This is solved by

$$\lambda = \left(\frac{c}{n(n+1-k)}\right)^{1/2}\tau + \lambda_0.$$

➠ Defining $R_{AdS}u = e^{\tau R_{AdS}^{-1}}$ we end up with the following metric:

$$l_2^{-2}ds^2 = ds^2_{AdS_{n+1}} + ds^2_{S^k} = \left(\frac{u}{R_{AdS}}\right)^2 dx_\parallel^2 + \left(\frac{R_{AdS}}{u}\right)^2 du^2 + R^2_{S^k}d\Omega_k^2,$$

where

$$R_{AdS} = \left(\frac{n(n+1-k)}{c}\right)^{1/2} \quad \text{and} \quad R_{S^k} = \left(\frac{(n+1-k)(k-1)}{c}\right)^{1/2}.$$

AdS black hole solutions

- In is well known that in addition to the extremal SUGRA backgrounds, one can also construct near extremal solutions which correspond to boundary field theories at finite temperature. For $D3$ brane in the near horizon limit the near extremal solution is the AdS black hole solution.
- Since we have identified a family of $AdS_{n+1} \times S^k$ backgrounds it is natural to anticipate that there are also non-critical AdS black hole solutions.
- Indeed it is straightforward to derive these solutions

$$l_s^{-2} ds^2 = \left(\frac{u}{R_{AdS}}\right)^2 \left[-\left(1-\left(\frac{u_0}{u}\right)^n\right) dt^2 + dx_i^2\right] + \left(\frac{R_{AdS}}{u}\right)^2 \frac{du^2}{\left(1-\left(\frac{u_0}{u}\right)^n\right)} + R_{S^k}^2 d\Omega_k^2,$$

where the energy density on the brane is given by $u_0^n = 2aR_{ADS}^n$.

- Note that the thermal factor here is different from the one of near extremal D_p branes apart from the case $p = 3$ where they coincide.

The RR deformed two dimensional black hole

- In 2d with no transverse s^k the potential reads

$$V = Q^2 e^{n\lambda-\varphi} - ce^{-2\varphi}.$$

- The superpotential equation

$$w'(\phi)w(\phi) - w(\phi)^2 = Q^2 e^{2\phi} - c.$$

where $W = 4e^{-\varphi}w(\phi)$

This equation has an analytic solution Berkovitz, Gukov, vallilo

$$w(\phi) = \sqrt{2\phi Q^2 e^{2\phi} - 4me^{2\phi} + c},$$

where m is an integration constant.

- The background metric takes the form

$$l_s^{-2} ds^2 = -\frac{1}{4}w^2(\phi)dt^2 + \frac{d\phi^2}{\frac{1}{4}w^2(\phi)},$$

- It was shown that this solution can be interpreted as a two dimensional black hole with an ADM mass $M_{ADM} = \frac{2}{\sqrt{c}}m$. The scalar curvature was found to be $\alpha'\mathcal{R} = e^{2\phi}[Q^2(\phi+1) - m]$.

➠ In the near horizon limit, around ϕ_0 where $w(\phi_0) = 0$ with $u = \phi - \phi_0$ the space-time turns into an AdS_2 one

$$l_s^{-2} ds^2 = -\left(\frac{u}{R_{AdS}}\right)^2 dt^2 + \left(\frac{R_{AdS}}{u}\right)^2 du^2,$$

where $R_{AdS} = \sqrt{\frac{2}{c}}$ and the scalar curvature is $\alpha' \mathcal{R}_{AdS} = c$.

Backgrounds with non-zero RR charge $Q \neq 0$ that asymptote to the linear dilaton solution

- Upon turning on a RR flux in the cylindrical geometries the superpotential equaion does not admit an anlytic solution, only a numerical one.
- The typical form of the functions e^{λ} and e^{ν} is .

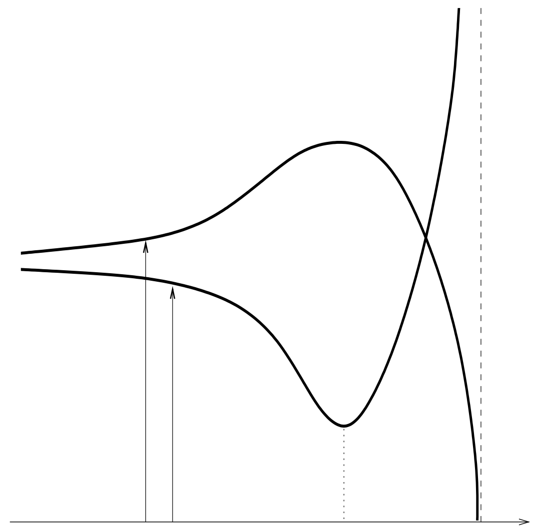

Figure 1: The picture represents the typical form of $g_{ii} = e^{2\lambda}$ and $g_{\theta\theta} = e^{2\nu}$. For $\tau \longrightarrow -\infty$ we approach the cylinder geometry, while at $\tau \longrightarrow 0$ the background becomes singular.

- Since e^{λ} has a global minimum where it does not vanish it is a confining background.

➠ Our original goal in this section was to construct the RR perturbation of the cigar geometry. As it evident, however, from all the solutions the RR form changes dramatically the metric.

Holographic dual gauge theories

- We conjecture here that the concept of holographic duality holds also for non-critical SUGRA backgrounds
- In critical dimensions a useful way to understand the D_p brane SUGRA backgrounds is via the back-reaction of a stack of N D_p branes on a background of flat space-time and a constant dilaton .
- In non-critical backgrounds one starts with a flat d dimensional Minkowski space-time with a linear dilaton The back-reaction of adding N D_p branes generates the $AdS_{p+2} \times S^{d-p-2}$ backgrounds with $p+2$ RR forms which again have N units of flux. However, unlike the critical cases, here the dilaton is constant for any p.
- The non-critical solutions we have found with an S^1 factor can also be thought of as the back-reaction of N D_p branes placed in manifolds of $R^{1,p+1} \times S^1$ geometry. Recall that this background is equivalent to the $\mathcal{N} = 2$ super Liuville theory.

The entropy and the duality to gauge degrees of freedom

- If the boundary field theory is a gauge theory the SUGRA entropy should match that of a gauge theory.
- The entropy of the boundary field theory scales like

$$S_{\text{gauge}} \sim \frac{N^2}{\delta^3},$$

where δ is a UV cutoff.

- A way to evaluate it is to compute the area in units of G_N in the Einstein frame. The area of the boundary diverges and similarly to the field theory calculation a cutoff δ has to be introduced. The area takes the form

$$\begin{aligned} S_{\text{SUGRA}} &\sim \text{Area} \sim V_{S^k}\left(\frac{R_{AdS}}{\delta}\right)^{n-1} \sim (R_{S^k})^k\left(\frac{R_{AdS}}{\delta}\right)^{n-1} \\ &\sim N^{\frac{2}{d-2}(k+n-1)}\delta^{-(n-1)} \sim N^2\delta^{-(n-1)}, \end{aligned}$$

- In particular in four dimensions with $n = 4$ we find an agreement with S_{gauge}.
- It turns out that the same result applies also to the non conformal solutions.

A novel large N limit

- The radii of the AdS_{n+1} and of the S^k are Q (and hence N) independent constants of order unity and the curvature is

$$\alpha' \mathcal{R} = -c.$$

- Hence unlike the critical AdS/CFT duality, here the curvature is fixed, of order unity and cannot be reduced by taking a large N limit.

- This is of course a problem of the whole analysis since high order curvature corrections can modify the general picture.

- Unlike the critical case, in the non-critical case the string coupling is N dependent

$$g_s \sim e^{\phi_0} = \left[\frac{1}{n+1-k} \left(\frac{(n+1-k)(k-1)}{c} \right)^k \frac{2c}{Q^2} \right]^{1/2}$$

and hence

$$g_s \to 0 \qquad N \to \infty$$

and therefore small string coupling means large N.

- Moreover, if we adopt the conventional correspondence between g_s and g_{YM}^2 then since $g_s \sim \frac{1}{N}$, we find that the 't Hooft coupling is a constant of order unity

$$\lambda_{'tHooft} = g_{YM}^2 N \sim \left(\frac{2c}{n+1-k} \left(\frac{(n+1-k)(k-1)}{c} \right)^k \right)^{1/2}.$$

To summarize, the large N limit that one has to take in the boundary gauge theory dual to the non-critical SUGRA is different than the one taken in the critical case:

$$\begin{aligned} \text{critical} &: \quad N \to \infty, \quad g_{YM}^2 N \gg 1 \\ \text{non-critical} &: \quad N \to \infty, \quad g_{YM}^2 N \sim 1. \end{aligned}$$

The gauge duals of the $AdS_{p+2} \times S^{d-p-2}$ backgrounds

- The SUGRA models and their corresponding global symmetries (for even d) are given in the following table.

Gauge theory in n dimensions	The SUGRA manifold	The global symmetry
2	$AdS_3 \times S^5$	$SO(6)$
3	AdS_4	-
3	$AdS_4 \times S^2$	$SO(3)$
4	$AdS_5 \times S^3$	$SO(4)$
5	AdS_6	-
5	$AdS_6 \times S^2$	$SO(3)$
7	AdS_8	-

[b]

- We have not chekced how many supersymmetries each model has, if at all, however, if a model do admit supersymmetry, the isometry of the background should correspond to the gauge theory R symmetry.

- From a brief glance on the table it seems that the two models with $SO(3)$ isometry may correspond to superconformal gauge theories. It is known that in three dimensions a theory with $\mathcal{N}$ supersymmetries has an R symmetry of $SO(N)$. Hence the $AdS_4 \times S^2$ model may correspond to $\mathcal{N} = 3$ in three dimensions. There is a five dimensional superconformal theory with $SP(1)$ R symmetry. This may relate to the $AdS_6 \times S^2$ model].

- For the rest of the models we cannot relate the data given in the table with known superconformal gauge theories. There are several logical explanations to this situation:

  (i) It might be that the dual gauge theories are non-supersymmetric theories. For instance, one could imagine four dimensional theories with four additional matter fields in the adjoint that admit the $SO(4)$ global symmetry and strongly coupled fixed points.

  (ii) It might be that only part of the full isometry translates into an R symmetry of the gauge theory due to the fact, that the GSO projection is compatible only with a subgroup of the full isometry group. Such a case occurs for the $AdS_3 \times S^3$.

- Assuming that there are conformal gauge theories that correspond to these non-critical SUGRA backgrounds, one can turn on the known machinery of computing the conformal dimensions of chiral operators computing correlation functions etc. in a similar manner to what was done in the critical cases.

The AdS_6 black hole and non-supersymmetric YM theory

- The AdS radius of the non-critical case and the critical one are

$$\text{critical} \quad : \quad R_{AdS} = \left(g_{YM}\sqrt{N}\right)^{1/2}$$
$$\text{non-critical} \quad : \quad R_{AdS} = \left(\frac{n(n+1-k)}{c}\right)^{1/2}.$$

- Due to the similarity between the critical and non-critical near extremal solutions , we do not have to redo the calculations that correspond to the properties of the gauge theory but rather read them from the known results of the critical theory.

- In particular, we can implement the idea of imposing anti-periodic boundary conditions while taking the large temperature limit, which leads to a pure YM theory in space-time with one les dimensions.

- Thus the non-critical AdS_6 black hole background (with no S^k corresponds to pure YM in four dimensions

The Wilson line

- To determine the Wilson loop one can write down the NG action associated with the background metric and determine the classical configuration of the string . Instead we can check whether one of the two conditions for an area law Wilson law is obeyed

$$\begin{array}{rlllll} g_{00}g_{ii}(u) & \text{has a minimum at} & u_{\text{min}} & \text{with} & g_{00}g_{ii}(u_{\text{min}}) & > 0, \\ g_{00}g_{uu}(\tau) & \text{diverges at} & \tau_{\text{div}} & \text{with} & g_{00}g_{ii}(\tau_{\text{div}}) & > 0. \end{array}$$

- It is easy to check that after the reduction to 3d

$$g_{00}g_{uu} = [1 - (\frac{u_T}{u})^4]^{-1}$$

which diverges at $u = u_T$. The conclusion is therefore that indeed the Wilson loop in this background admits an area law behavior

- The string tensions of the non critical versus critical case are

$$\text{critical} \quad : \quad \frac{1}{2}\pi\sqrt{g_{YM}^2 N}T^2$$

$$\text{non-critical} \quad : \quad \frac{1}{2\pi}\left(\frac{u_0}{R_{AdS}}\right)^2 = \frac{1}{2}\pi\frac{8}{c}T^2$$

- One can show that the analogous calculation of the ’t Hooft loop which measures the potential between a monopole anti-monopole pair admits a screening behavior. This is done in the SUGRA by calculating the configuration of a D2 brane that ends on the boundary and wraps the thermal cycle, and realizing that its energy is larger than the sum of the energy of a monopole and anti-monopole.

The glue-ball spectra

- Next we consider the glue-ball spectrum. The analysis of the four dimensional glue-balls extracted from the non-critical AdS_6 bh model is similar to the one done in the near extremal limit of the $D4$ critical background.
- The spectrum of the 0^{++} glueball associates with the fluctuation of the dilaton $\phi = \phi_{cl} + \delta\phi$
- Unlike the critical case where the $\nabla^2\phi = 0$ here due to the coupling to the RR we get

$$\nabla^2\delta\phi = 4\delta\phi$$

so taht for $\delta\phi = b(u)e^{ikx}$ we get

$$\partial_u^2 b(u) + \frac{6 - \left(\frac{R}{u}\right)^2}{u(1 - \left(\frac{R}{u}\right)^2)}\partial_u b(u) + \left[M^2 \frac{\left(\frac{R}{u}\right)^4}{1 - \left(\frac{u_0}{u}\right)^5} - \frac{30}{u^2\left(1 - \left(\frac{u_0}{u}\right)^5\right)}\right] b(u) = 0$$

- This can be translated to a Schroedinger equation with a potential of the following form

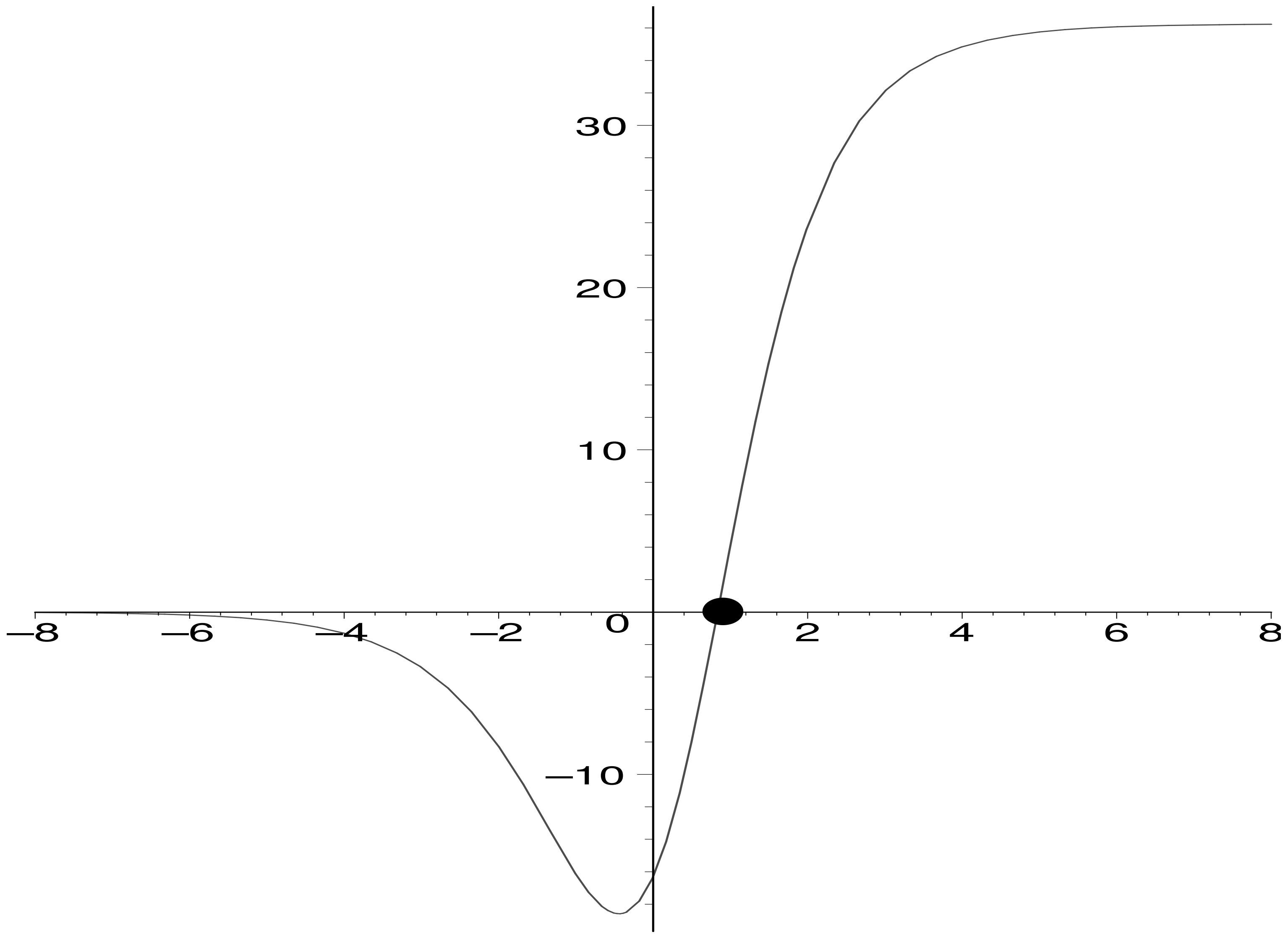

Figure 2: The effective potential (**??**) for $n = 5$ and $\frac{MR_{AdS}}{u_0} = 20$ The plot demonstrates that there are two classical turning point at $y = y_+$ and at $y = -\infty$.

[b]

The effective potential (**??**) for $n = 5$ and $\frac{MR_{AdS}}{u_0} = 20$ The plot demonstrates that there are two classical turning point at $y = y_+$ and at $y = -\infty$.

- Using techniques developed in the critical case Minaham we finally get the spectrum of the spin 0 glueballs

$$M_{0,\phi}^2 \approx \frac{39.66}{\beta^2} k(k + 6.02) + O(k^0).$$

- In a similar manner one computes the glueballs spectrum associated with the RR one forms
- The spectrum of excitations for both types of glueballs is compared to those extracted from the critical case

k	$M_{0^{++}}$	$M_{0^{+-}}$	M_{0,A_M}	$M_{0,\phi}$
1	9.85	11.8	9.96	16.7
2	15.6	17.8	16.7	25.2
3	21.2	23.5	23.1	32.8
4	26.7	29.1	29.5	39.9
5	32.2	34.6	35.9	46.7
6	37.7	40.1	42.2	53.5

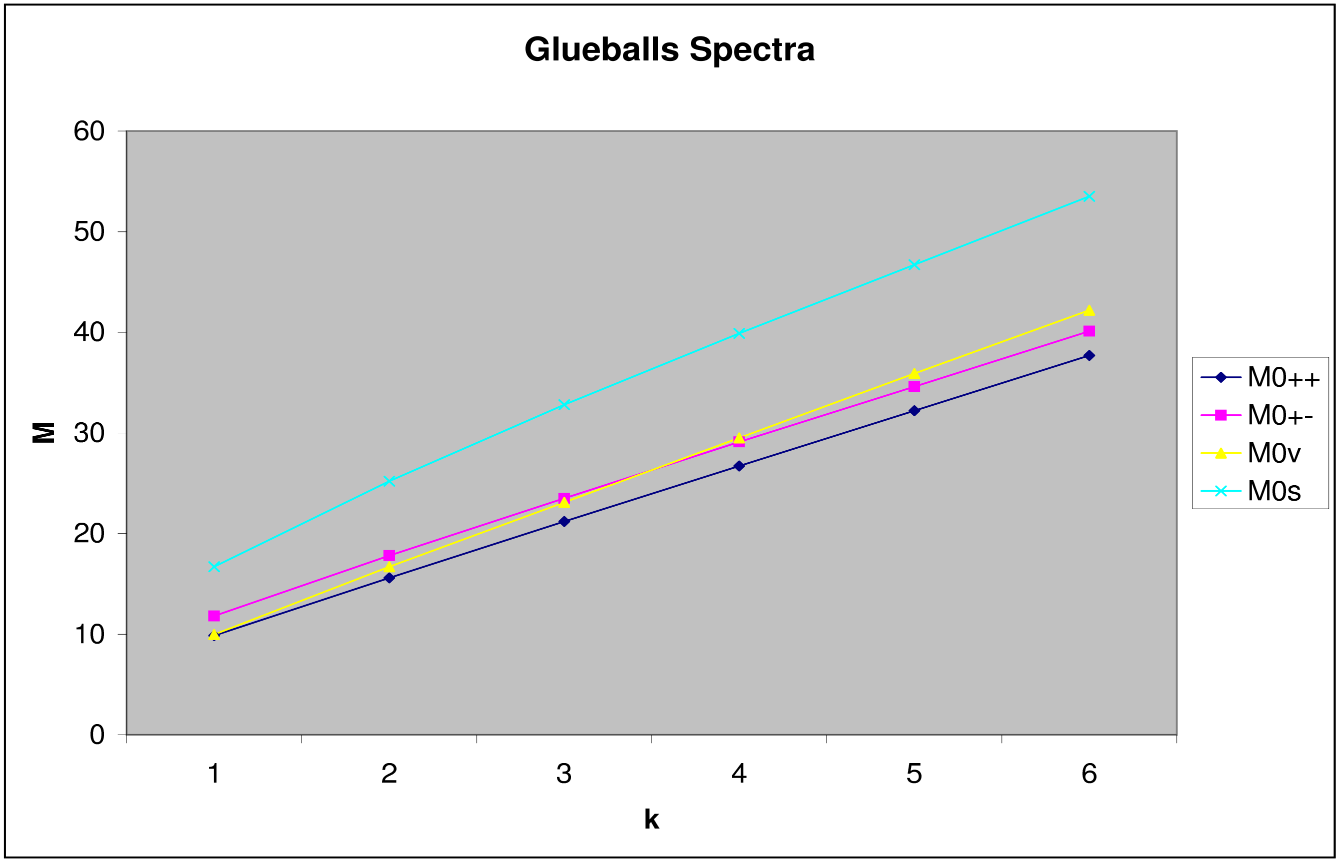

Figure 3: glueball spectra

Flavored SUGRA backgrounds

- Klebanov and Maldacena proposed to add N_f space filling branes anti-branes to introduce the flavor degrees of freedom of the dual gauge theories.
- There are several questions about this proposal in particular whether it can be supersymmetric or even stable. Here we will assume that it is consistent to add such a term
- The SUGRA action now reads

$$S = \int d^{n+k+1}x\sqrt{G}e^{-2\phi}\left(R + 4(\partial\phi)^2 + \frac{c}{\alpha'} - 2N_f e^{\phi}\right)$$
$$-\frac{e^{-2\phi}}{2}\int H_{(3)} \wedge \star H_{(3)} - \sum_p \frac{1}{2}\int F_{(p+2)} \wedge \star F_{(p+2)},$$

- The corresponding equations of motion are

$$\partial_\rho^2\lambda - \frac{1}{2}Q_{RR}^2 e^{n\lambda - k\nu - \varphi} - \frac{1}{2}N_f e^{1/2(-3\varphi + n\lambda + k\nu)} = 0,$$

$$\partial_\rho^2\nu - (k-1)e^{-2\nu-2\varphi} + \frac{1}{2}Q_{RR}^2 e^{n\lambda-k\nu-\varphi} + Q_{NS}^2 e^{-2k\nu-2\varphi} = -\frac{1}{2}N_f e^{1/2(-3\varphi+n\lambda+k\nu)}0,$$

$$\partial_\rho^2\varphi + (c+(k-1)ke^{-2\nu})e^{-2\varphi} - \frac{1}{2}Q_{RR}^2 e^{n\lambda-k\nu-\varphi} - Q_{NS}^2 e^{-2k\nu-2\varphi} - \frac{3}{2}N_f e^{1/2(-3\varphi+n\lambda+k\nu)} = 0.$$

- It is easy to check that these equations admit $AdS_{n+1}\times S^k$ solutions namely with constant dilaton and radii.
- The parameters of the solution, the string coupling g_s, the Ads radious R_{AdS} and the radius of the S^k are determined from the following algebraic relations.

$$\begin{aligned}
\frac{k-1}{R_S^2} - \frac{1}{2}\frac{(g_sN)^2}{R_S^{2k}} + \frac{1}{2}N_f g_s &= 0 \\
\frac{n}{R_{AdS}^2} - \frac{k-1}{R_S^2} &= g_s N_f \\
\frac{n(n+1)}{R_{AdS}^2} - \frac{k(k-1)}{R_S^2} &= c - g_s N_f
\end{aligned}$$

- Note that now, unlike the unflavored case, there is no restriction of the form $k\neq 1$ and $k\neq n+1$. In fact the cases with $k=1$ are easily determined to be

$$g_s = \frac{c}{n+2}\frac{1}{N_f} \qquad R_{Ads}^2 = \frac{n(n+2)}{c} \qquad R_S^2 = \frac{c}{n+2}\frac{N^2}{N_f^2}$$

- In particular for $k=1, n=4, c=4$ we get the Klebanov Maldacena solution.

$$g_s = \frac{2}{3}\frac{1}{N_f} \qquad R^2_{AdS} = 6 \qquad R^2_S = \frac{2}{3}\frac{N^2}{N_f^2}$$

- For the symmetric cases $AdS_{d/2} \times S^{d/2}$, namely $k = n+1$, we get

$$g_s = \frac{c}{n+2}\frac{1}{N_f} = \frac{2c}{c+2}\frac{1}{N_f}$$

The relation between the radii is

$$\frac{1}{R^2_{AdS}} - \frac{1}{R^2_S} = \frac{c}{n(n+2)}$$

Note that for $c=0$ we are back in the $AdS_5 \times S^5$ background.

The a anomaly function from SUGRA

- Supersymetric gauge theories have an analog of the 2d c-theorem the a theorem that states that

$$a \equiv \frac{2}{32}[3Tr(R^3) - Tr(R)]$$

is decreasing upon flowing from an UV to an IR fixed point.

- The $U_R(1)$ anomaly is related in susy theories to the confomal anomaly via

$$\langle T_i^i \rangle = \frac{1}{16\pi^2}(c\, C^2 - a\, E_4)$$

- Skenderis and Henningson taught us how to compute the conformal anomaly from the conformal variation of the bounday action.

$$\langle T_i^i \rangle = \frac{1}{16\pi G_N^{(n+1)}} \frac{R_{AdS}^3}{8}(C^2 - E_4)$$

and hence

$$a = c = \frac{\pi}{8} \frac{\mathrm{Vol}(S^k)}{\gamma\, g_s^2\, l_s^{D-2}} R_{AdS}^3 \cdot R_s^k \sim \frac{R_{AdS}^3 R_s^5}{g_s^2}$$

- The generaliztion to $AdS_{n+1}\times S^k$ is

$$a\sim\frac{R_{AdS}^{n-1}}{G_N^{(n+1)}}=\frac{R_{AdS}^{n-1}R_s^k}{G_N^{(d)}}=\frac{R_{AdS}^{n-1}R_s^k}{g_s^2}$$

- For the un-flavored $AdS_p\times S^k$ backgrounds the anomaly function a is

$$a=(\frac{c}{n+1-k})^{(n-k-1)/2}\frac{n^{(n-1)/2}}{(k-1)^{k/2}}\;N^2$$

Note that the result is proportional to N^2 for any s unlike the critical case where for instance in $d=6$ $a\sim N^3$

- Upon substituting the relations between the radii for the flavored cases we get

$$a=\frac{R_{AdS}^{n-1}N}{g_s}\left[\frac{2(k-1)}{R_S^2}+g_sN_f\right]^{-\frac{1}{2}}$$

- For all cases where the transverse part is an S^1 we get

$$a=NN_f(\frac{n+2}{c})^{\frac{n+2}{2}}n^{\frac{n-1}{2}}$$

- In particular for the KM model which is claimed to be dual to $\mathcal{N}=1$ SQCD we get

$$a \sim 3^3 N N_f$$

- The a function of $\mathcal{N}=1$ SQCD at the IR fixed point is

$$a_{SQCD} = 4N^2(1 - 3/2\frac{N^2}{N_f^2})$$

which seems quite different from the SUGRA result. However, for the relevant region where $3N \geq N_f \geq 3/2N$ the two results are of the same order in N .

➠ An other interesting case is the $AdS_3 \times S^3$ background with RR flux. For $\frac{N_f}{N} >> \frac{1}{16}$ the radius is

$$R_S^2 = \frac{8}{16\frac{N_f}{N} + 1}$$

and then the expression for a is

$$a \sim NN_f \frac{1}{\sqrt{(2\frac{N_f}{N} + \frac{5}{8})(2\frac{N_f}{N} + \frac{3}{8})}}$$

This model has an isometry of $SU(2) \times SU(2)$ and hence may correspond to a 2d (4,4) SYM theory with N_f flavors which has an R symmetry of $SU(2) \times SU(2)$. For this model the anomaly was found to be $a = 6NN_f$ so we see again that in the region where $\frac{N}{N_f} \sim 1$ we get a similar behavior behavior.

Toward the non critical string with RR background

- There are exact models with "large curvature" where we know that theleading SUGRA result is not corrected . For instance the leading SUGRA relation between the radii of $AdS_3 \times S^3$ is

$$\frac{1}{R_{AdS}^2} - \frac{1}{R_S^2} = 1$$

  This is exactly the result of the string theory. The same is for the cigar solution.

- Our conjecture is that the structure of the $AdS_p \times S^k$ backgrounds is not change apart from potentially the radii.

- A suport to this conjecture come from the AdS_2 case. The most general higher curvature correction can be written as $\sum_{n=2} c_n R^n$. In that case the exact string constant and radius are given by

$$e^{2\phi_0} = \frac{8}{Q^2}\left[1 - \sum_{n=2}(n-1)c_n(\frac{-2}{R_{AdS}})^n\right]^{-1}$$

$$\frac{8}{\alpha'} - \frac{2}{R_{AdS}^2} + \sum_{n=2} c_n(\frac{-2}{R_{AdS}})^n = 0$$

- Recall also that in the stronges version of the conjecture of the Ads/CFT duality, the $AdS_5 \times S^5$ structure is assumed to remain valid even in the region of large curvature.
- We also consider the fact that the gauge properties extracted from the gravity side "are sensible" as a further evidence for this conjecture.

Summary and open questions

- Several families of solutions of thenon-critical type II equations of motion were constructed.
- The $Ads_p \times S^q$ solutions and their near extremal generalizations are useful for the gauge/gravity duality.
- The non critical gague/gravity duality implies a novel 't Hooft limit different that of the usual AdS/CFT duality.
- Using the near extremal AdS_6 as our lab we extracted the Wilson line , 't Hoof loop, glueball spectrum etc.
- Following KM we incorporated space filling N_f flavr branes anti-branes and derived new $Ads_p \times S^q$ solutions
- The anomaly a function can be computed from SUGRA for several models in various space-time dimensions.
- We have certain evidence to support our claim that the $Ads_p \times S^q$ structure survives higher order curvature corrections.
- However, controlling the higher order curvature corrections and the construction of string theories is obviously the most important open question.

- An analysis of the Killing spinors and the determinations of the space-time supersymmetries.
- Comparison between the gauge properties extracted from critical versus non critical SUGRA backgrounds.
- Constructing SUGRA duals of confining gauge theories with fundamental flavored quarks .
- Scattering amplitudes of the non critical strings should be closer to reality than the critical ones since the are no KK that can take part in the scattering in critical models.