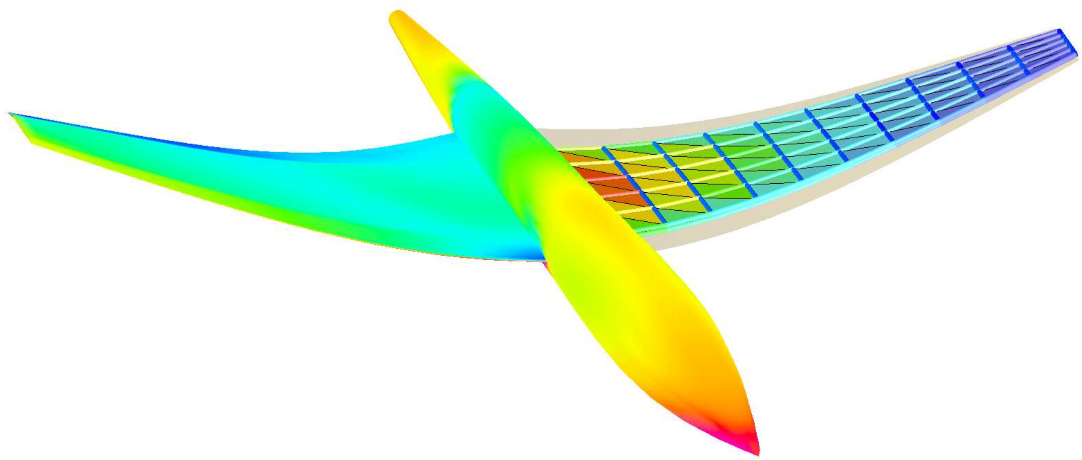
University of Toronto Institute for Aerospace Studies

Aero-Structural Wing Design using Coupled Sensitivity Analysis



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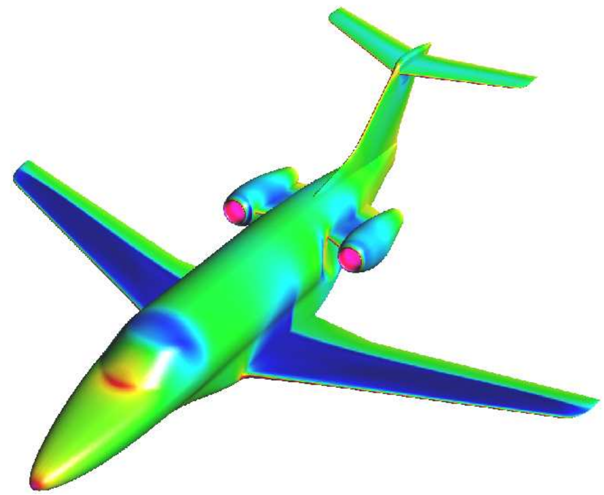
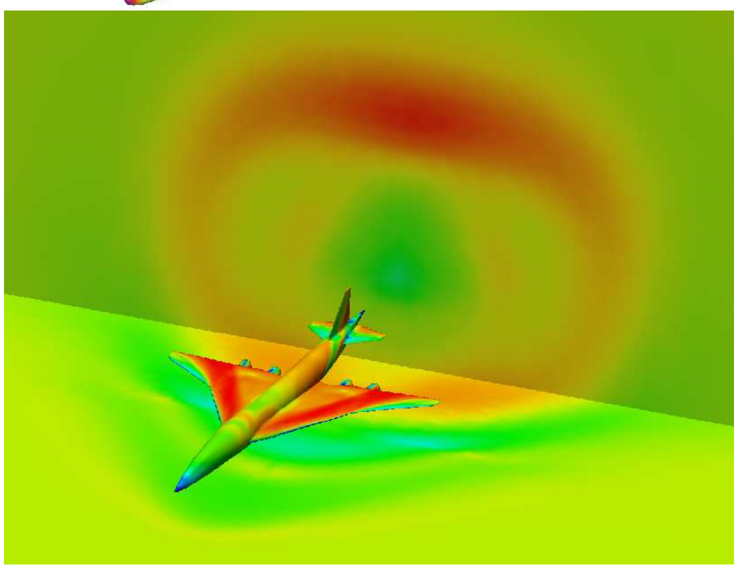
Outline

- Introduction
  - Motivation
  - Sensitivity analysis methods
- The complex-step derivative approximation
- Coupled-adjoint method
  - Sensitivity equations for multidisciplinary systems
  - Lagged aero-structural adjoint equations
- Results
  - Aero-structural sensitivity validation
  - Optimization results
- Conclusions

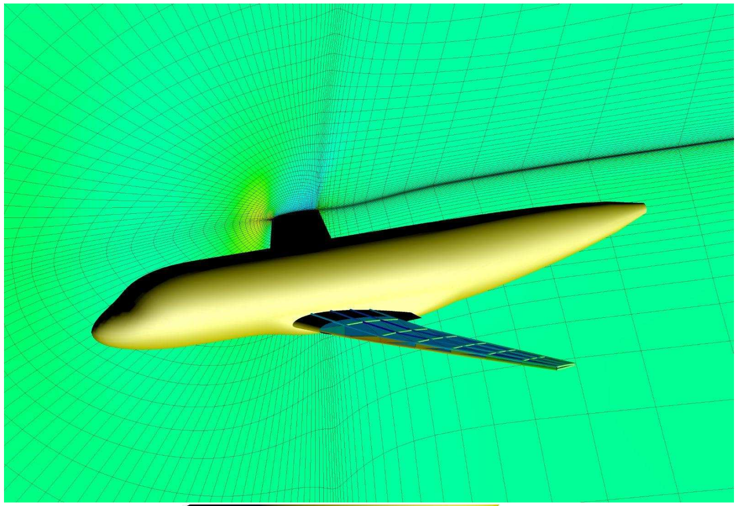
High-Fidelity Aerodynamic Shape Optimization

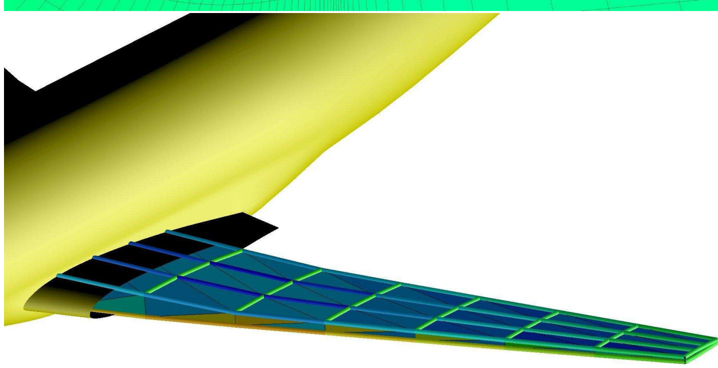




- Start from a baseline geometry provided by a conceptual design tool.
- High-fidelity models required for transonic configurations where shocks are present, high-dimensionality required to smooth these shocks.
- Accurate models also required for complex supersonic configurations, subtle shape variations required to take advantage of favorable shock interference.
- Large numbers of design variables and high-fidelity models incur a large cost.

Aero-Structural Aircraft Design Optimization





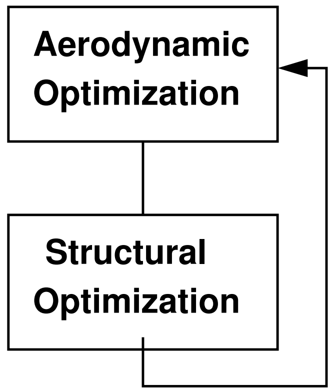
- Aerodynamics and structures are core disciplines in aircraft design and are very tightly coupled.

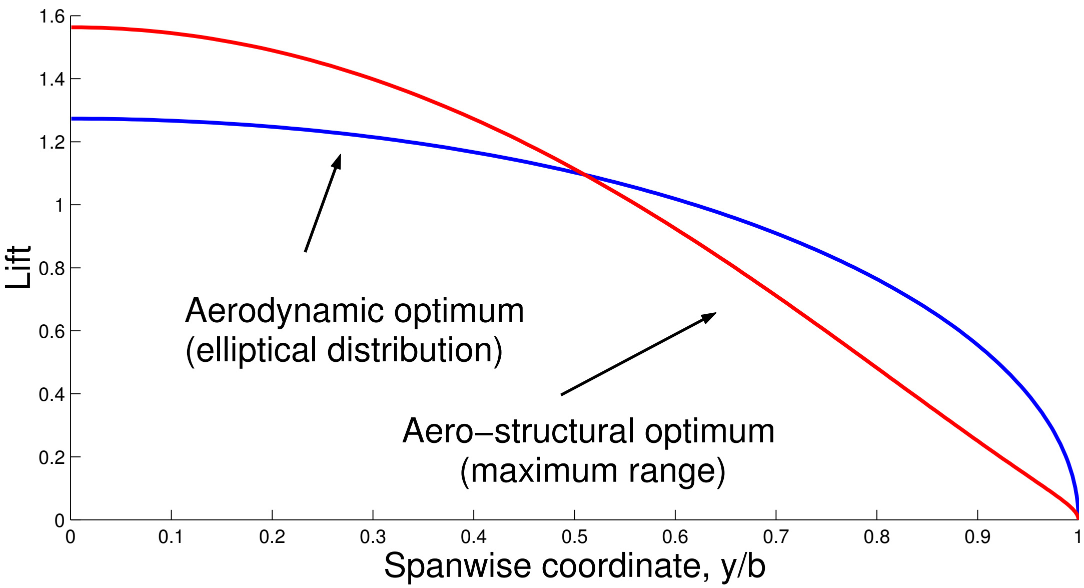
- By including structural analysis and design there is no need to impose artificial wing thickness constraints.

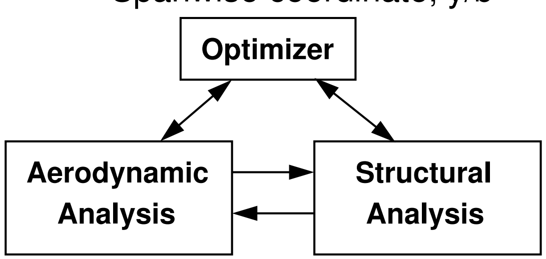
- Want to simultaneously optimize the aerodynamic shape and structure, since there is a trade-off between aerodynamic performance and structural weight, e.g.,

$$\text{Range} \propto \frac{L}{D} \ln\left(\frac{W_i}{W_f}\right)$$

The Need for Aero-Structural Sensitivities





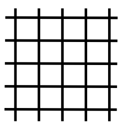


- Sequential optimization does not lead to the true optimum.
- Aero-structural optimization requires coupled sensitivities.
- Add structural element sizes to the design variables.
- Including structures in the high-fidelity wing optimization will allow larger changes in the design.

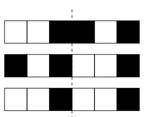
The Need for Aero-Structural Sensitivities

Optimization Methods

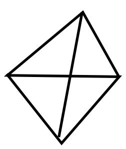
- **Intuition:** decreases with increasing dimensionality.



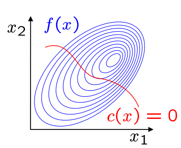
- **Grid or random search:** the cost of searching the design space increases rapidly with the number of design variables.



- **Genetic algorithms:** good for discrete design variables and very robust; but infeasible when using a large number of design variables.



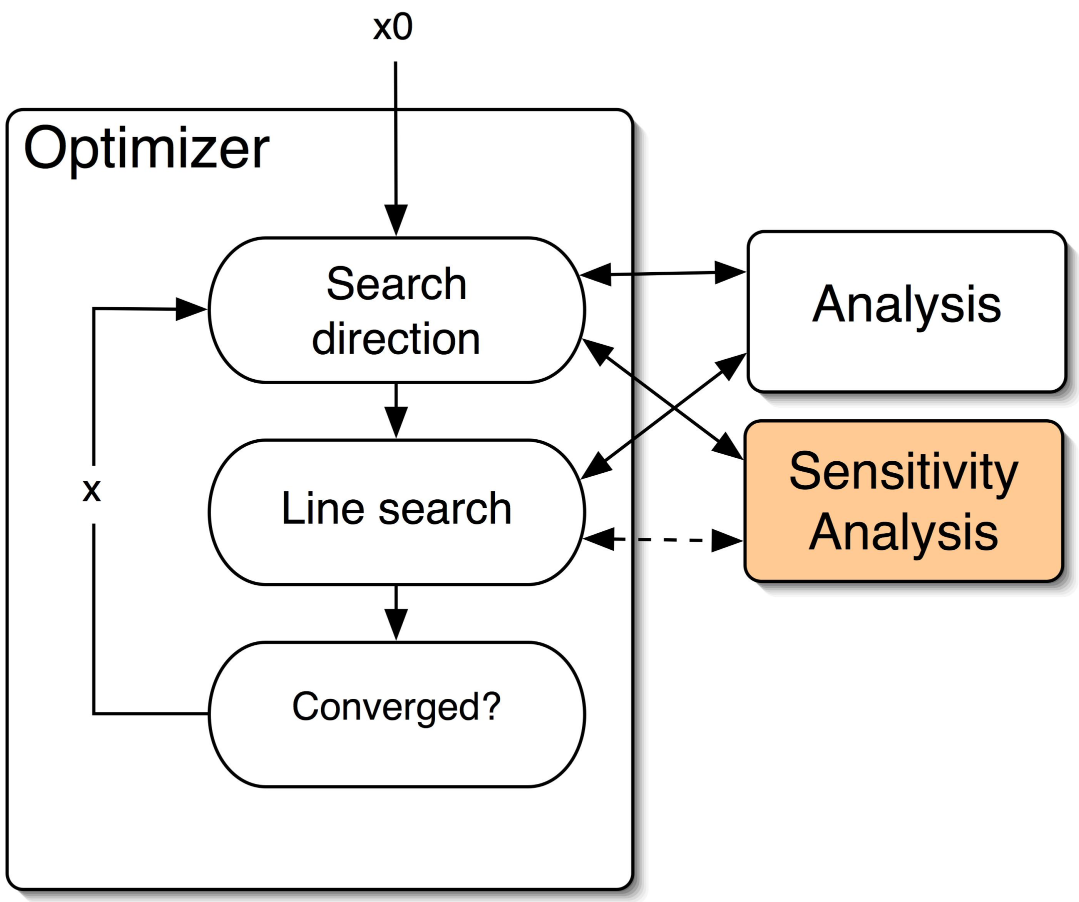
- **Nonlinear simplex:** simple and robust but inefficient for more than a few design variables.



- **Gradient-based:** the most efficient for a large number of design variables; assumes the objective function is "well-behaved".

Motivation

- By default, most gradient-based optimizers use finite-differences for sensitivity analysis.
- When the cost of calculating the sensitivities is proportional to the number of design variables, and this number is large, sensitivity analysis is the bottleneck.
- Accurate sensitivities are required for convergence.



Sensitivity Analysis Methods

- **Finite Differences:** very popular; easy, but lacks robustness and accuracy; run solver N_x times.

$$\boxed{\frac{\mathrm{d}f}{\mathrm{d}x_n} \approx \frac{f(x_n + h) - f(x)}{h} + \mathcal{O}(h)}$$

- **Complex-Step Method:** relatively new; accurate and robust; easy to implement and maintain; run solver N_x times.

$$\boxed{\frac{\mathrm{d}f}{\mathrm{d}x_n} \approx \frac{\mathrm{Im}\left[f(x_n + ih)\right]}{h} + \mathcal{O}(h^2)}$$

- **Automatic Differentiation:** accurate; ease of implementation and cost varies.
- **(Semi)-Analytic Methods:** efficient and accurate; long development time; cost can be independent of N_x.

Complex-Step Derivative Approximation

Can also be derived from a Taylor series expansion about x with a complex step ih:

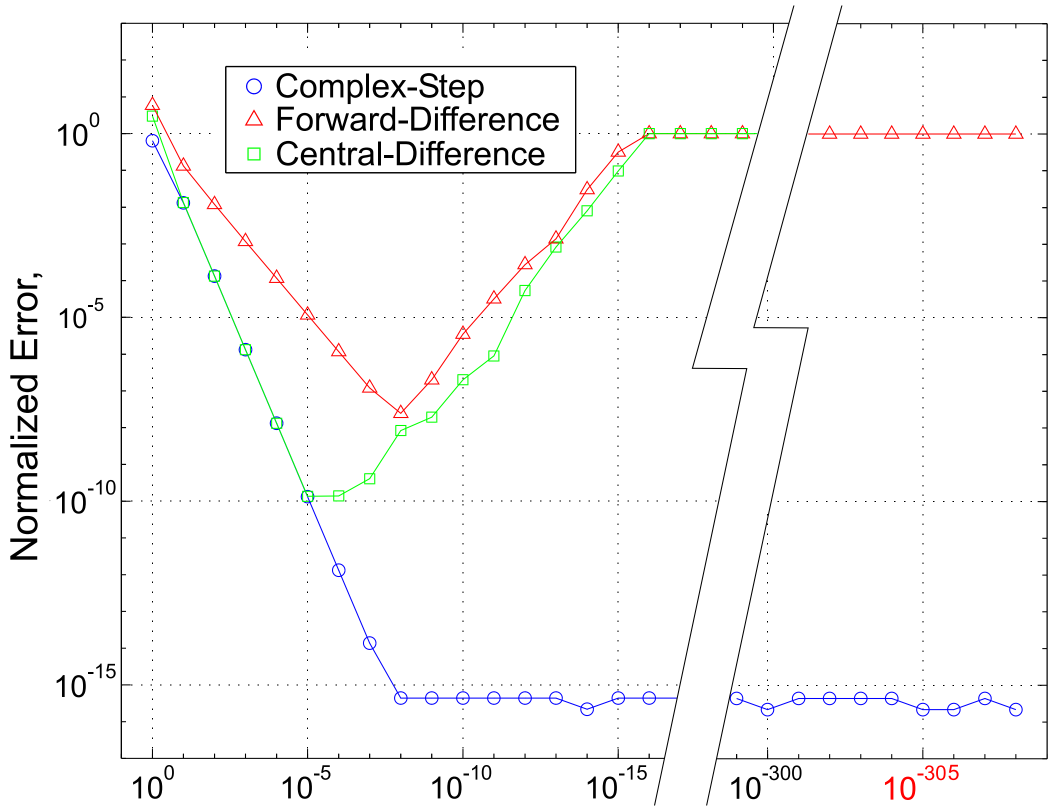
$$f(x+ih) = f(x) + ihf'(x) - h^2\frac{f''(x)}{2!} - ih^3\frac{f'''(x)}{3!} + \ldots$$

$$\Rightarrow f'(x) = \frac{\text{Im}\left[f(x+ih)\right]}{h} + h^2\frac{f'''(x)}{3!} + \ldots$$

$$\Rightarrow \boxed{f'(x) \approx \frac{\text{Im}\left[f(x+ih)\right]}{h}}$$

No subtraction! Second order approximation.

Simple Numerical Example



Estimate derivative at $x = 1.5$ of the function,

$$f(x) = \frac{e^x}{\sqrt{sin^3 x + cos^3 x}}$$

Relative error defined as:

$$\varepsilon = \frac{\left| f' - f'_{ref} \right|}{\left| f'_{ref} \right|}$$

Connection to Automatic Differentiation

Same example as previous talk: $f = (xy + \sin x + 4)(3y^2 + 6)$,

$$
\begin{aligned}
t_1 &= x + ih, \quad t_2 = y \\
t_3 &= xy + iyh \\
t_4 &= \sin x \cosh h + i\cos x \sinh h \\
t_5 &= xy + \sin x \cosh h + i(yh + \cos x \sinh h) \\
t_6 &= xy + \sin x \cosh h + 4 + i(yh + \cos x \sinh h) \\
t_7 &= y^2, \quad t_8 = 3y^2, \quad t_9 = 3y^2 + 6 \\
t_{10} &= (xy + \sin x \cosh h + 4)\left(3y^2 + 6\right) + i(yh + \cos x \sinh h)\left(3y^2 + 6\right) \\
\frac{\mathrm{d}f}{\mathrm{d}x} &\approx \frac{\mathrm{Im}\left[f(x + ih, y)\right]}{h} = \left(y + \cos x \frac{\sinh h}{h}\right)\left(3y^2 + 6\right)
\end{aligned}
$$

Superfluous calculations are made.

For sufficiently small h they vanish but still affect speed.

Objective Function and Governing Equations

Want to minimize scalar objective function,

$$I = I(x_n, y_i),$$

which depends on:

- x_n: vector of design variables, e.g. structural plate thickness.
- y_i: state vector, e.g. flow variables.

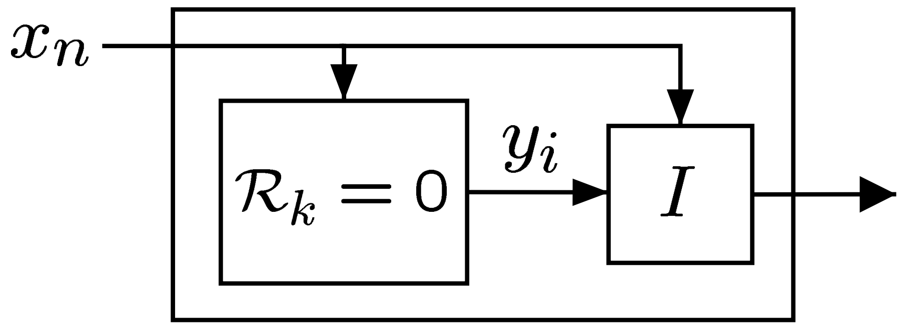
Physical system is modeled by a set of governing equations:

$$\mathcal{R}_k\left(x_n, y_i\left(x_n\right)\right) = 0,$$

where:

- Same number of state and governing equations, $i, k = 1, \ldots, N_{\mathcal{R}}$
- N_x design variables.

Sensitivity Equations



Total sensitivity of the objective function:

$$\frac{\mathrm{d}I}{\mathrm{d}x_n} = \frac{\partial I}{\partial x_n} + \frac{\partial I}{\partial y_i}\frac{\mathrm{d}y_i}{\mathrm{d}x_n}.$$

Total sensitivity of the governing equations:

$$\frac{\mathrm{d}\mathcal{R}_k}{\mathrm{d}x_n} = \frac{\partial \mathcal{R}_k}{\partial x_n} + \frac{\partial \mathcal{R}_k}{\partial y_i}\frac{\mathrm{d}y_i}{\mathrm{d}x_n} = 0.$$

Solving the Sensitivity Equations

Solve the total sensitivity of the governing equations

$$\frac{\partial \mathcal{R}_k}{\partial y_i}\frac{\mathrm{d}y_i}{\mathrm{d}x_n} = -\frac{\partial \mathcal{R}_k}{\partial x_n}.$$

Substitute this result into the total sensitivity equation

$$\frac{\mathrm{d}I}{\mathrm{d}x_n} = \frac{\partial I}{\partial x_n} - \underbrace{\frac{\partial I}{\partial y_i}\left[\frac{\partial \mathcal{R}_k}{\partial y_i}\right]^{-1}}_{-\Psi_k} \overbrace{\left[\frac{\partial \mathcal{R}_k}{\partial y_i}\right]^{-1}\frac{\partial \mathcal{R}_k}{\partial x_n}}^{-\,\mathrm{d}y_i/\,\mathrm{d}x_n},$$

where Ψ_k is the *adjoint vector*.

Adjoint Sensitivity Equations

Solve the adjoint equations

$$\frac{\partial \mathcal{R}_k}{\partial y_i}\Psi_k = -\frac{\partial I}{\partial y_i}.$$

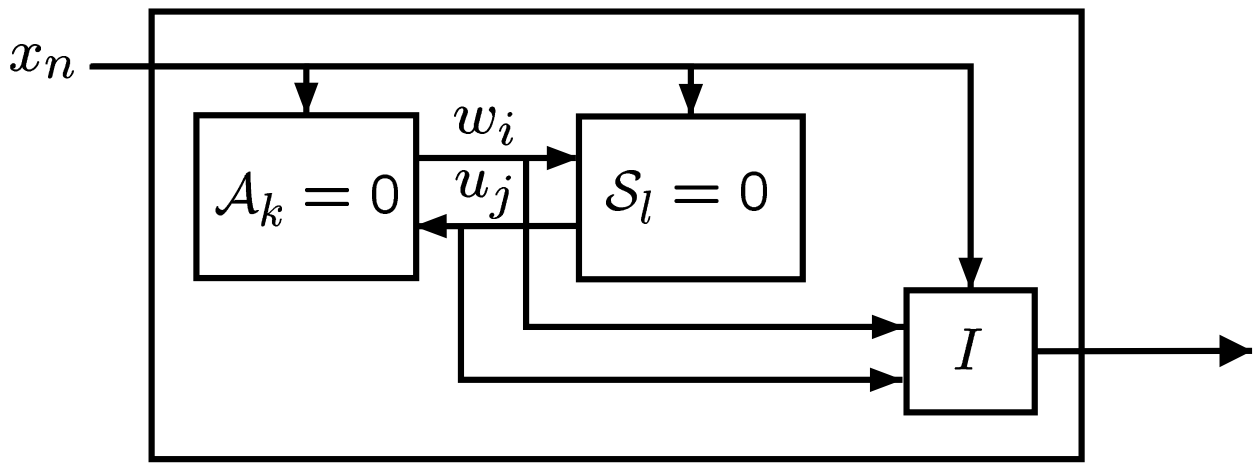
Adjoint vector is valid for all design variables.

Now the total sensitivity of the the function of interest I is:

$$\frac{\mathrm{d}I}{\mathrm{d}x_n} = \frac{\partial I}{\partial x_n} + \Psi_k \frac{\partial \mathcal{R}_k}{\partial x_n}$$

The partial derivatives are inexpensive, since they don't require the solution of the governing equations.

Aero-Structural Adjoint Equations



Two coupled disciplines: Aerodynamics ($\mathcal{A}_k$) and Structures ($\mathcal{S}_l$).

$$\mathcal{R}_{k'} = \begin{bmatrix} \mathcal{A}_k \\ \mathcal{S}_l \end{bmatrix}, \quad y_{i'} = \begin{bmatrix} w_i \\ u_j \end{bmatrix}, \quad \Psi_{k'} = \begin{bmatrix} \psi_k \\ \phi_l \end{bmatrix}.$$

Flow variables, w_i, five for each grid point.

Structural displacements, u_j, three for each structural node.

Aero-Structural Adjoint Equations

$$\begin{bmatrix} \frac{\partial \mathcal{A}_k}{\partial w_i} & \frac{\partial \mathcal{A}_k}{\partial u_j} \\ \frac{\partial \mathcal{S}_l}{\partial w_i} & \frac{\partial \mathcal{S}_l}{\partial u_j} \end{bmatrix}^T \begin{bmatrix} \psi_k \\ \phi_l \end{bmatrix} = - \begin{bmatrix} \frac{\partial I}{\partial w_i} \\ \frac{\partial I}{\partial u_j} \end{bmatrix}.$$

- $\partial \mathcal{A}_k / \partial w_i$: a change in one of the flow variables affects only the residuals of its cell and the neighboring ones.
- $\partial \mathcal{A}_k / \partial u_j$: wing deflections cause the mesh to warp, affecting the residuals.
- $\partial \mathcal{S}_l / \partial w_i$: since $\mathcal{S}_l = K_{lj} u_j - f_l$, this is equal to $-\partial f_l / \partial w_i$.
- $\partial \mathcal{S}_l / \partial u_j$: equal to the stiffness matrix, K_{lj}.
- $\partial I / \partial w_i$: for C_D, obtained from the integration of pressures; for stresses, its zero.
- $\partial I / \partial u_j$: for C_D, wing displacement changes the surface boundary over which drag is integrated; for stresses, related to $\sigma_m = S_{mj} u_j$.

Lagged Aero-Structural Adjoint Equations

Since the factorization of the complete residual sensitivity matrix is impractical, decouple the system and lag the adjoint variables,

$$\underbrace{\frac{\partial \mathcal{A}_k}{\partial w_i}\psi_k = -\frac{\partial I}{\partial w_i}}_{\text{Aerodynamic adjoint}} - \frac{\partial \mathcal{S}_l}{\partial w_i}\tilde{\phi}_l,$$

$$\underbrace{\frac{\partial \mathcal{S}_l}{\partial u_j}\phi_l = -\frac{\partial I}{\partial u_j}}_{\text{Structural adjoint}} - \frac{\partial \mathcal{A}_k}{\partial u_j}\tilde{\psi}_k,$$

Lagged adjoint equations are the single discipline ones with an added forcing term that takes the coupling into account.

System is solved iteratively, much like the aero-structural analysis.

Total Sensitivity

The aero-structural sensitivities of the drag coefficient with respect to wing shape perturbations are,

$$\frac{\mathrm{d}I}{\mathrm{d}x_n} = \frac{\partial I}{\partial x_n} + \psi_k \frac{\partial \mathcal{A}_k}{\partial x_n} + \phi_l \frac{\partial \mathcal{S}_l}{\partial x_n}.$$

- $\partial I/\partial x_n$: C_D changes when the boundary over which the pressures are integrated is perturbed; stresses change when nodes are moved.
- $\partial \mathcal{A}_k/\partial x_n$: the shape perturbations affect the grid, which in turn changes the residuals; structural variables have no effect on this term.
- $\mathcal{S}_l/\partial x_n$: shape perturbations affect the structural equations, so this term is equal to $\partial K_{lj}/\partial x_n u_j - \partial f_l/\partial x_n$.

Coupled Direct Methods

The single discipline direct method equations yield,

$$\begin{bmatrix} \frac{\partial \mathcal{A}_k}{\partial w_i} & \frac{\partial \mathcal{A}_k}{\partial u_j} \\ \frac{\partial \mathcal{S}_l}{\partial w_i} & \frac{\partial \mathcal{S}_l}{\partial u_j} \end{bmatrix} \begin{bmatrix} \frac{\mathrm{d} w_i}{\mathrm{d} x_n} \\ \frac{\mathrm{d} u_j}{\mathrm{d} x_n} \end{bmatrix} = - \begin{bmatrix} \frac{\partial \mathcal{A}_k}{\partial x_n} \\ \frac{\partial \mathcal{S}_l}{\partial x_n} \end{bmatrix}.$$

An equivalent alternate approach is,

$$\begin{bmatrix} \mathcal{I} & -\frac{\partial w_i}{\partial u_j} \\ -\frac{\partial u_j}{\partial w_i} & \mathcal{I} \end{bmatrix} \begin{bmatrix} \frac{\mathrm{d} w_i}{\mathrm{d} x_n} \\ \frac{\mathrm{d} u_j}{\mathrm{d} x_n} \end{bmatrix} = \begin{bmatrix} \frac{\partial w_i}{\partial x_n} \\ \frac{\partial u_j}{\partial x_n} \end{bmatrix}.$$

Solving either of these, we then use the total sensitivity equation

$$\frac{\mathrm{d} f}{\mathrm{d} x_n} = \frac{\partial f}{\partial x_n} + \frac{\partial f}{\partial u_j}\frac{\mathrm{d} u_j}{\mathrm{d} x_n} + \frac{\partial f}{\partial w_i}\frac{\mathrm{d} w_i}{\mathrm{d} x_n}.$$

Alternate Coupled Adjoint Method

Similarly to the alternate coupled direct method, there is an alternate coupled adjoint method.

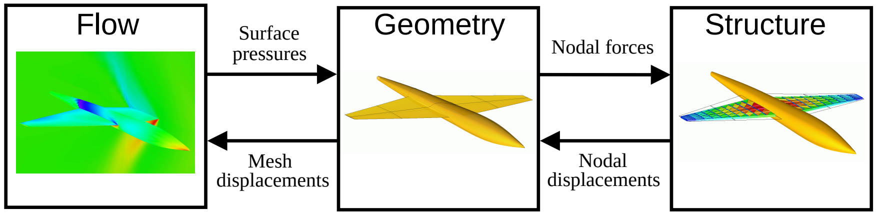
$$\begin{bmatrix} \mathcal{I} & -\frac{\partial w_i}{\partial u_j} \\ -\frac{\partial u_j}{\partial w_i} & \mathcal{I} \end{bmatrix}^T \begin{bmatrix} \bar{\psi}_i \\ \bar{\phi}_j \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial w_i} \\ \frac{\partial f}{\partial u_j} \end{bmatrix},$$

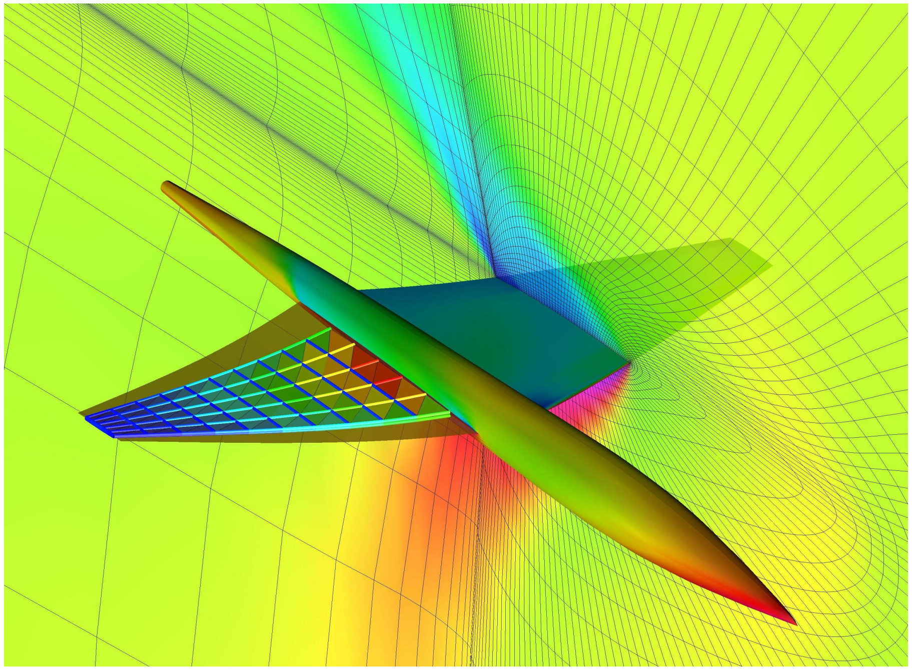
$\bar{\psi}_k$ has a different meaning from the standard adjoint and therefore requires a different total sensitivity equation,

$$\frac{\mathrm{d}f}{\mathrm{d}x_n} = \frac{\partial f}{\partial x_n} + \bar{\psi}_i \frac{\partial w_i}{\partial x_n} + \bar{\phi}_j \frac{\partial u_j}{\partial x_n},$$

where the partial derivatives of the state variables $(\partial w_i/\partial x_n, \partial u_j/\partial x_n)$ also require the solution of the corresponding governing equations.

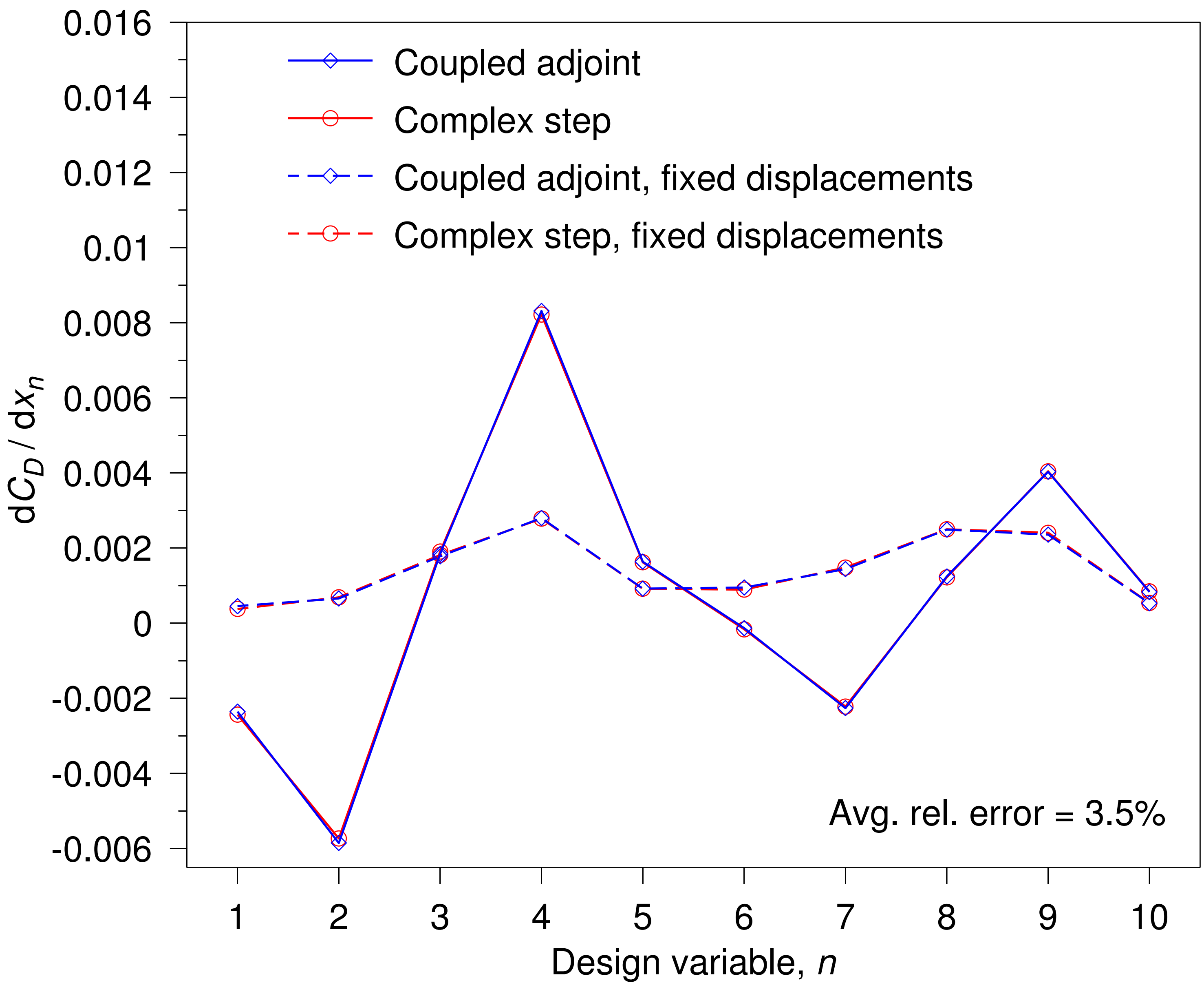
3D Aero-Structural Design Optimization Framework



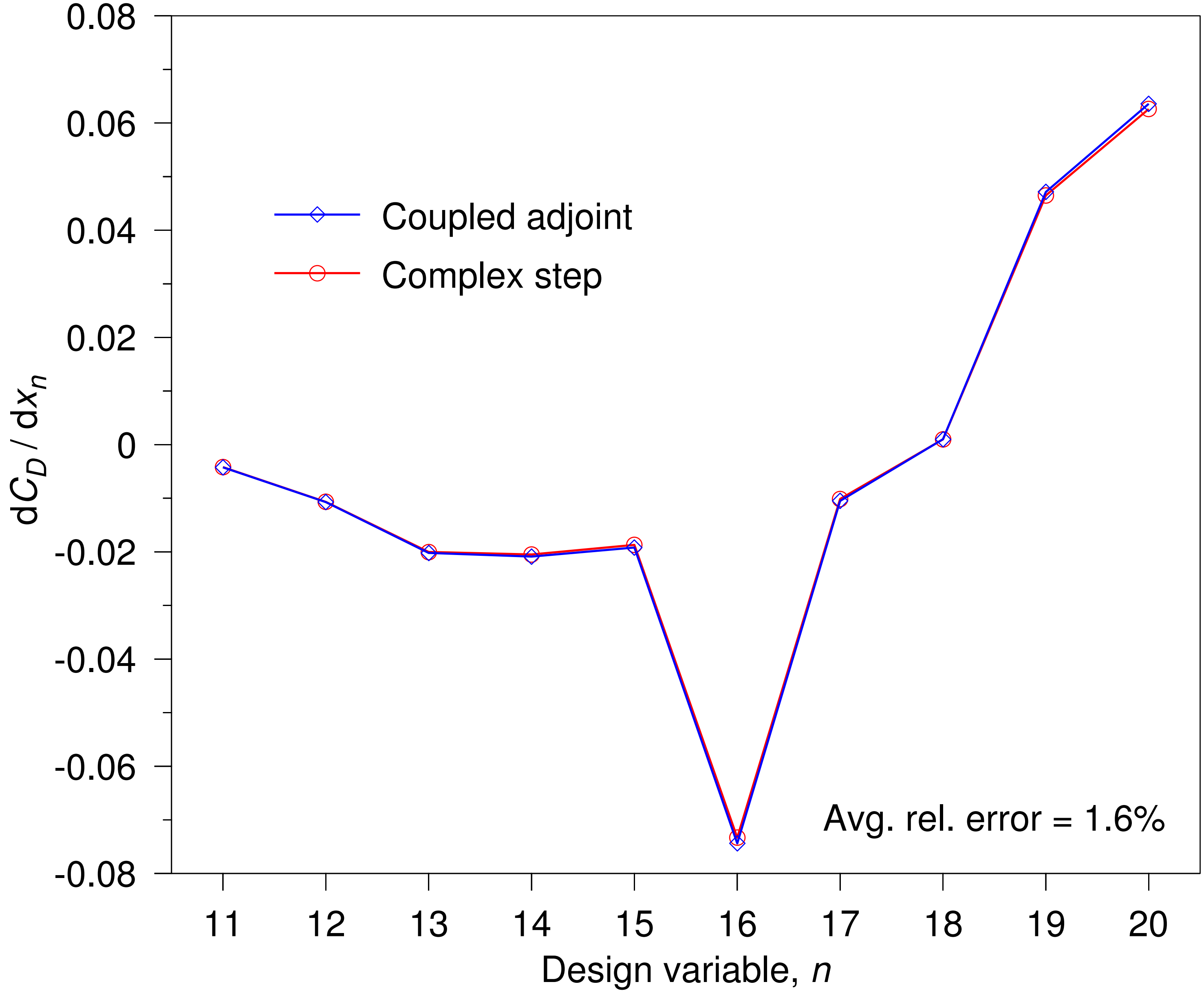


- Aerodynamics: SYN107-MB, a parallel, multiblock Navier–Stokes flow solver.
- Structures: detailed finite element model with plates and trusses.
- Coupling: high-fidelity, consistent and conservative.
- Geometry: centralized database for exchanges (jig shape, pressure distributions, displacements.)
- Coupled-adjoint sensitivity analysis: aerodynamic and structural design variables.

Sensitivity of C_D wrt Shape



Sensitivity of C_D wrt Structural Thickness



Structural Stress Constraint Lumping

To perform structural optimization, we need the sensitivities of all the stresses in the finite-element model with respect to many design variables.

There is no method to calculate this matrix of sensitivities efficiently.

Therefore, lump stress constraints

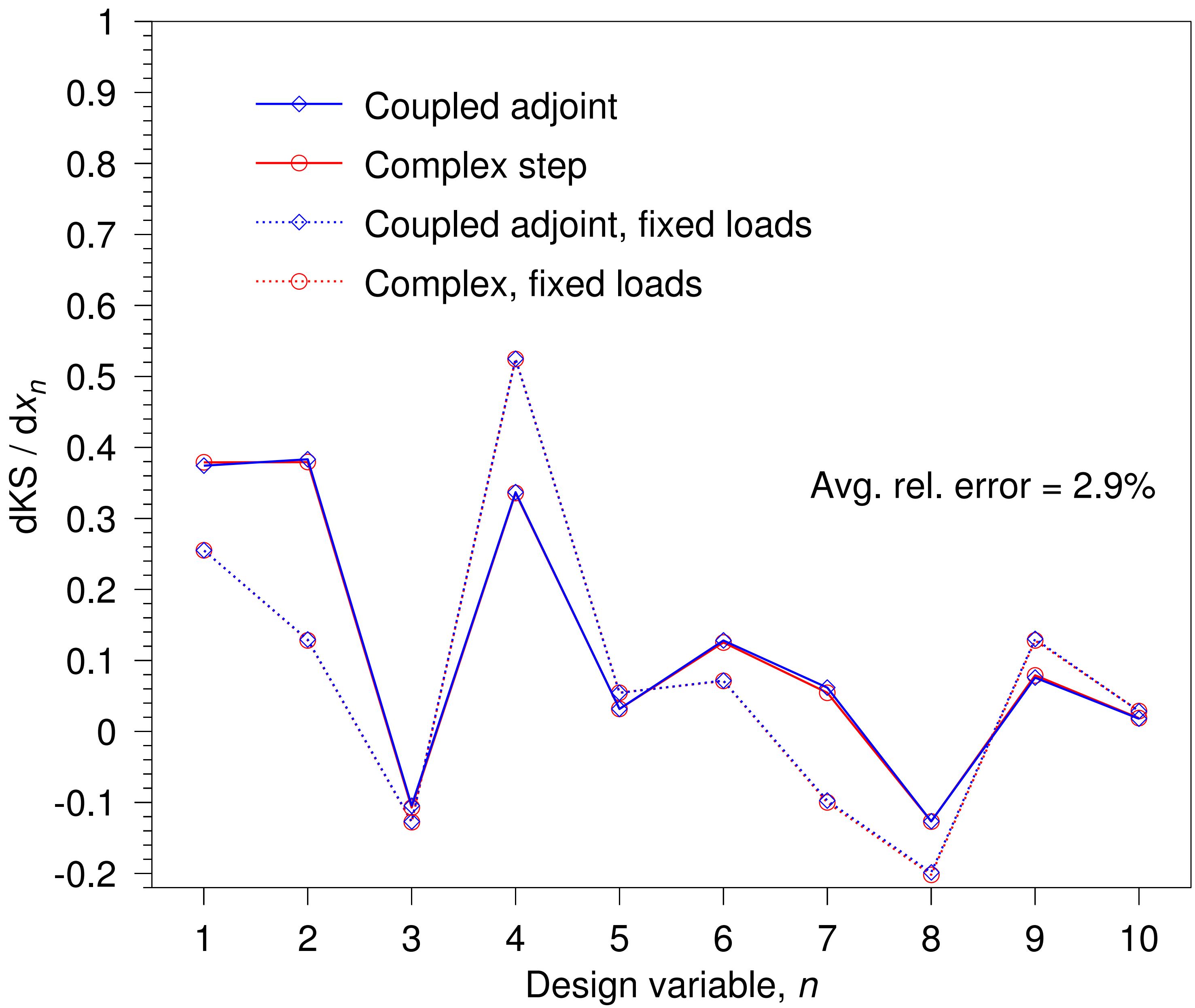
$$g_m = 1 - \frac{\sigma_m}{\sigma_{\text{yield}}} \geq 0,$$

using the Kreisselmeier–Steinhauser function

$$\text{KS}\left(g_m\right) = -\frac{1}{\rho} \ln \left(\sum_m \mathrm{e}^{-\rho g_m} \right),$$

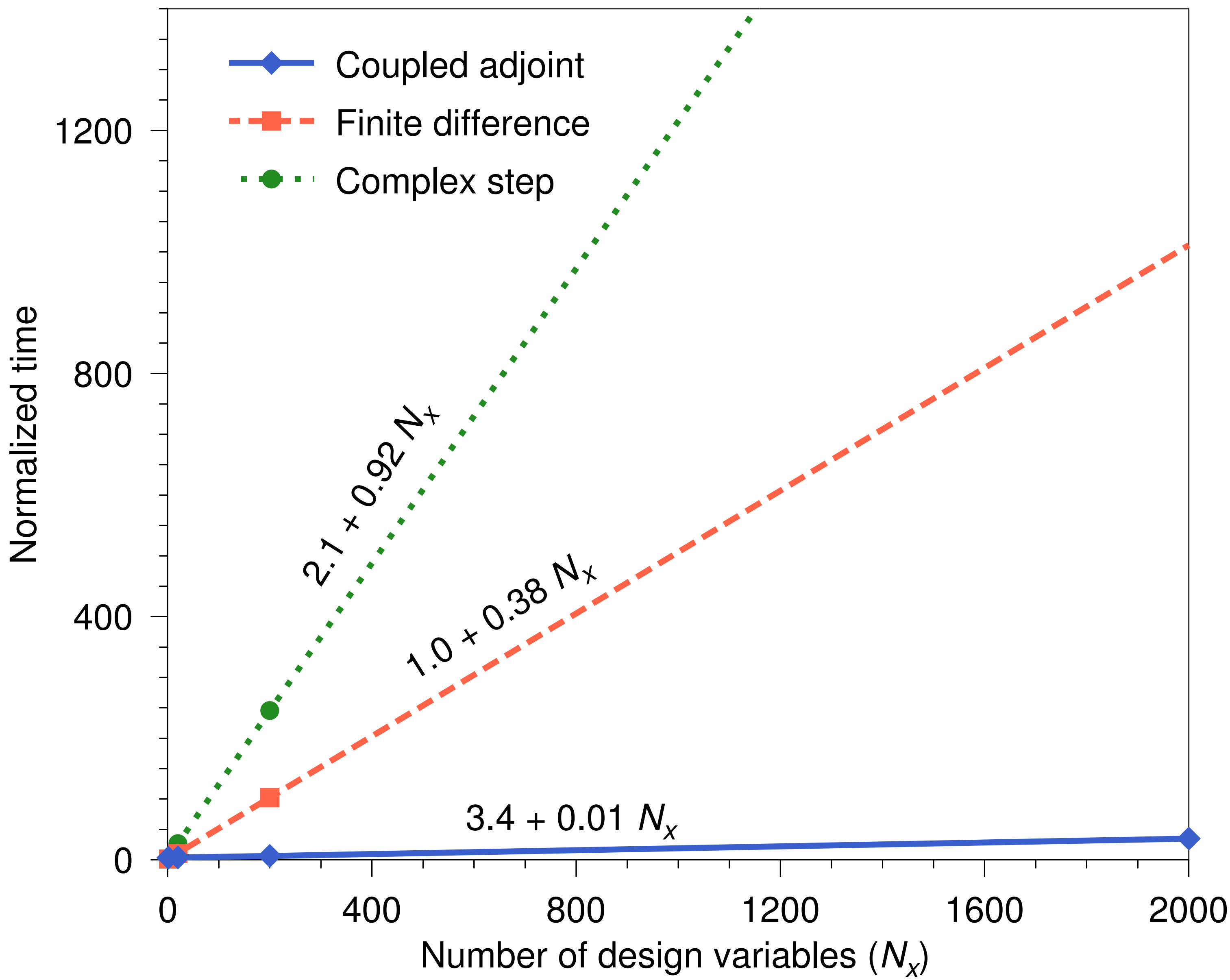
where ρ controls how close the function is to the minimum of the stress constraints.

Sensitivity of KS wrt Shape



Sensitivity of KS wrt Structural Thickness

Computational Cost vs. Number of Variables



Computational Cost Breakdown

$$\frac{\partial \mathcal{A}_k}{\partial w_i}\psi_k = -\frac{\partial I}{\partial w_i} - \boxed{\frac{\partial \mathcal{S}_l}{\partial w_i}\tilde{\phi}_l} \quad 0.60$$

0.64

$$\frac{\partial \mathcal{S}_l}{\partial u_j}\phi_l = -\frac{\partial I}{\partial u_j} - \boxed{\frac{\partial \mathcal{A}_k}{\partial u_j}\tilde{\psi}_k} \quad < 0.001$$

1.20

2.4

$$\frac{\mathrm{d}I}{\mathrm{d}x_n} = \frac{\partial I}{\partial x_n} + \psi_k \frac{\partial \mathcal{A}_k}{\partial x_n} + \phi_l \frac{\partial \mathcal{S}_l}{\partial x_n} \quad 0.01N_x$$

Supersonic Business Jet Optimization Problem

Minimize:

$$I = \alpha C_D + \beta W$$

where C_D is that of the cruise condition.

Subject to:

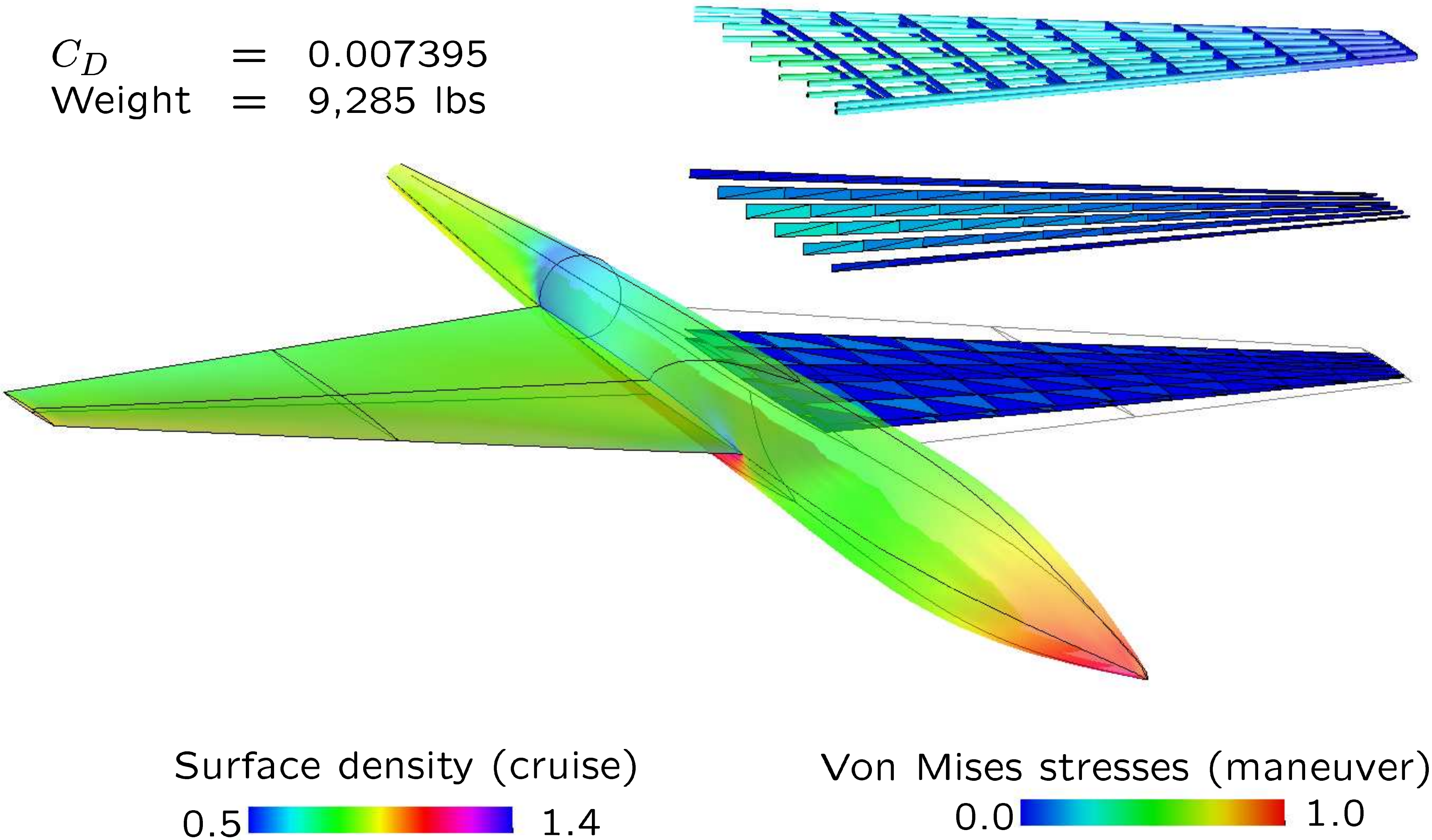
$$\text{KS}(\sigma_m) \geq 0$$

where KS is taken from a maneuver condition.

With respect to: external shape and internal structural sizes.

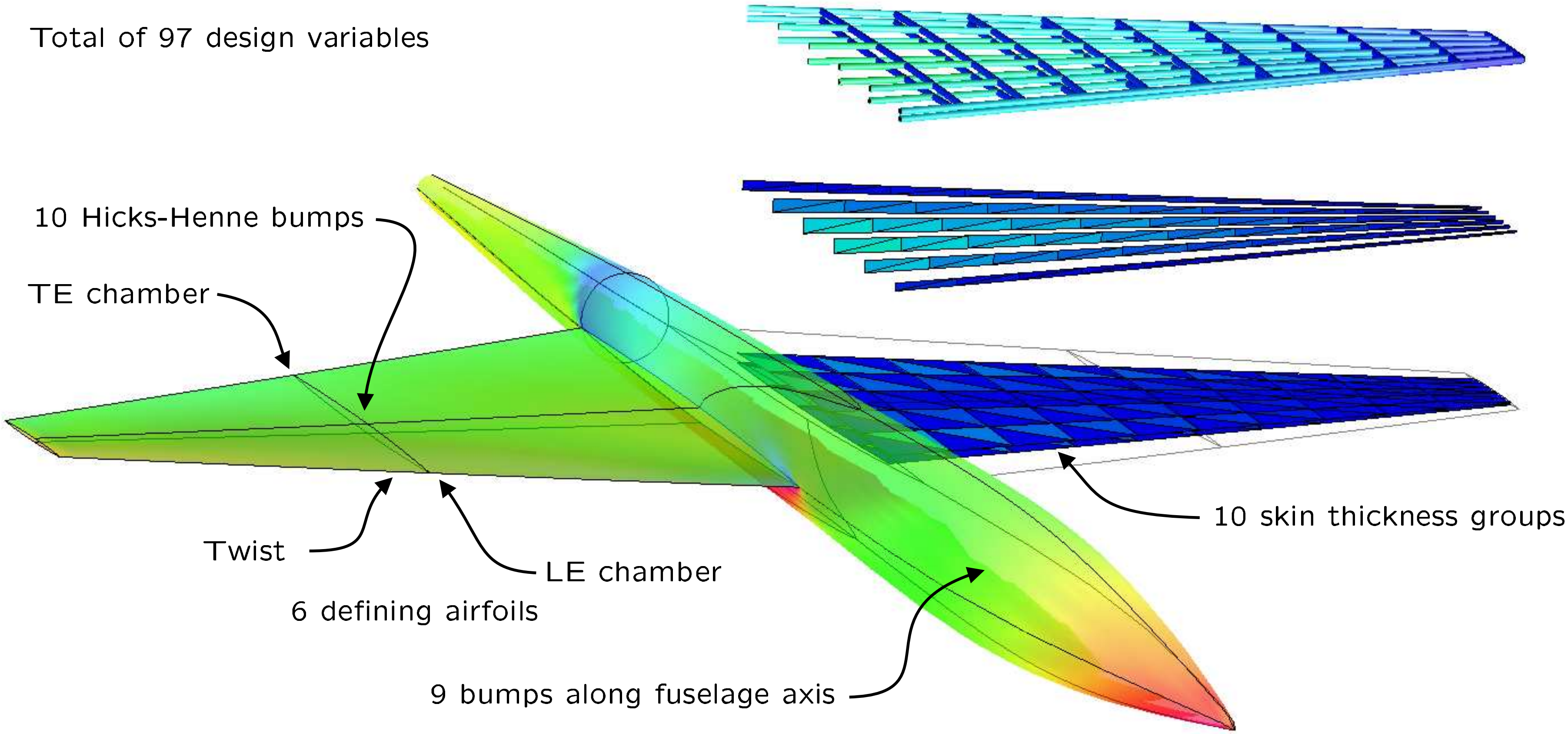
Baseline Design

$$C_D = 0.007395$$
$$\text{Weight} = 9{,}285 \text{ lbs}$$



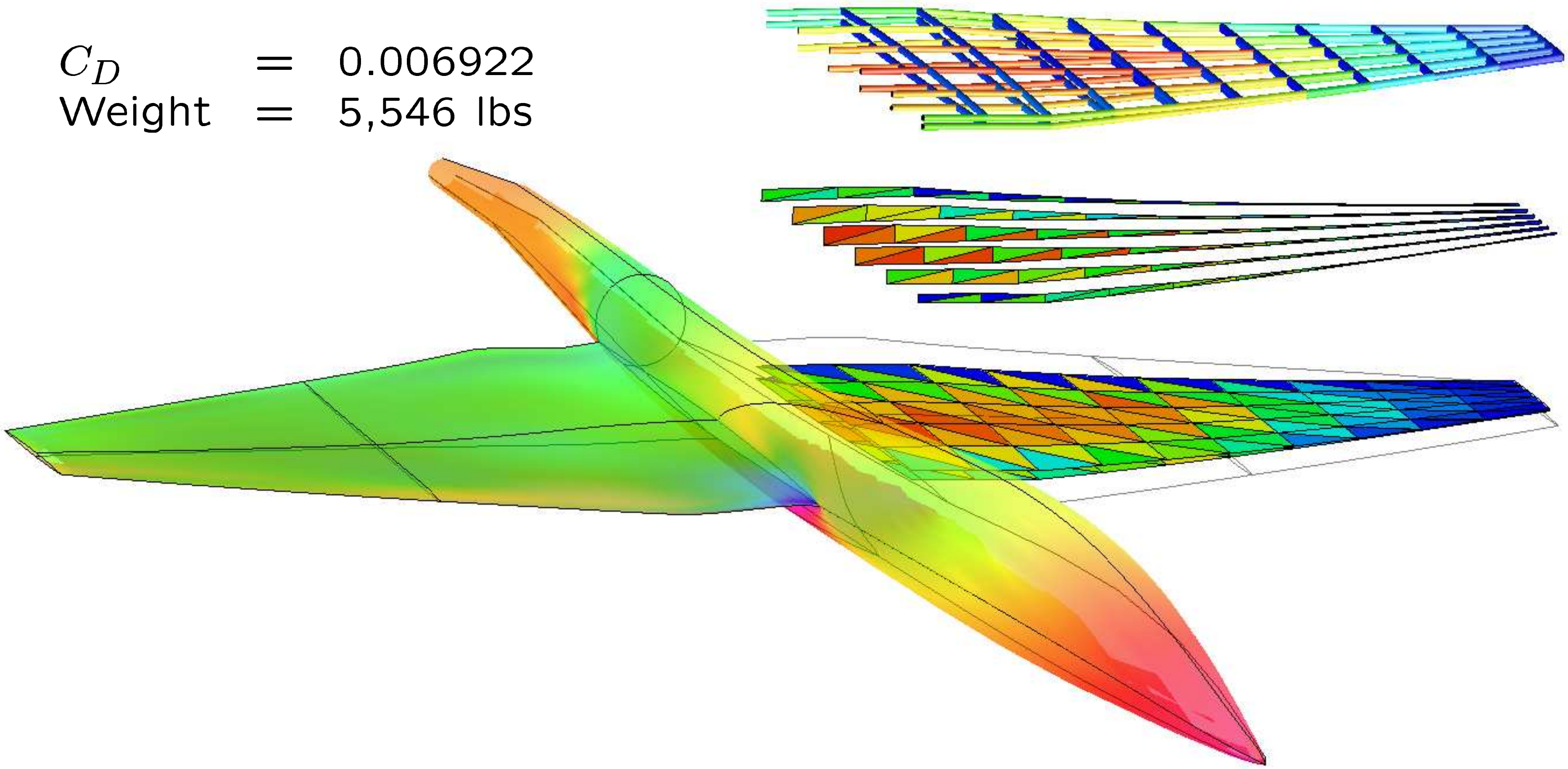
Design Variables

Total of 97 design variables



Aero-Structural Optimization Results

C_D = 0.006922
Weight = 5,546 lbs

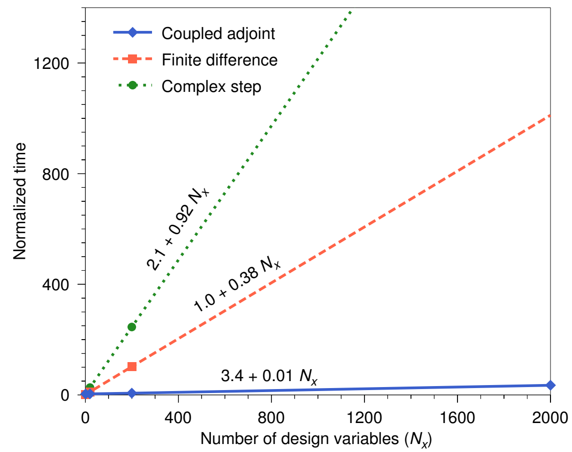


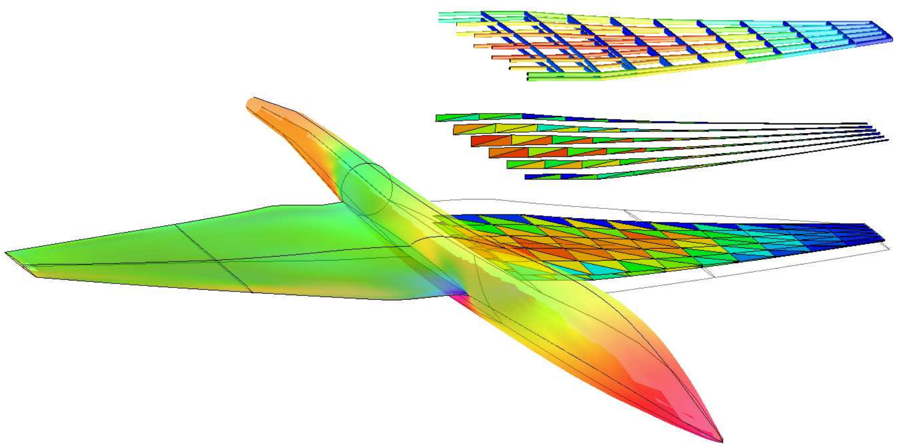
Surface density (cruise)
0.5 — 1.4

Von Mises stress (maneuver)
0.0 — 1.0

Conclusions

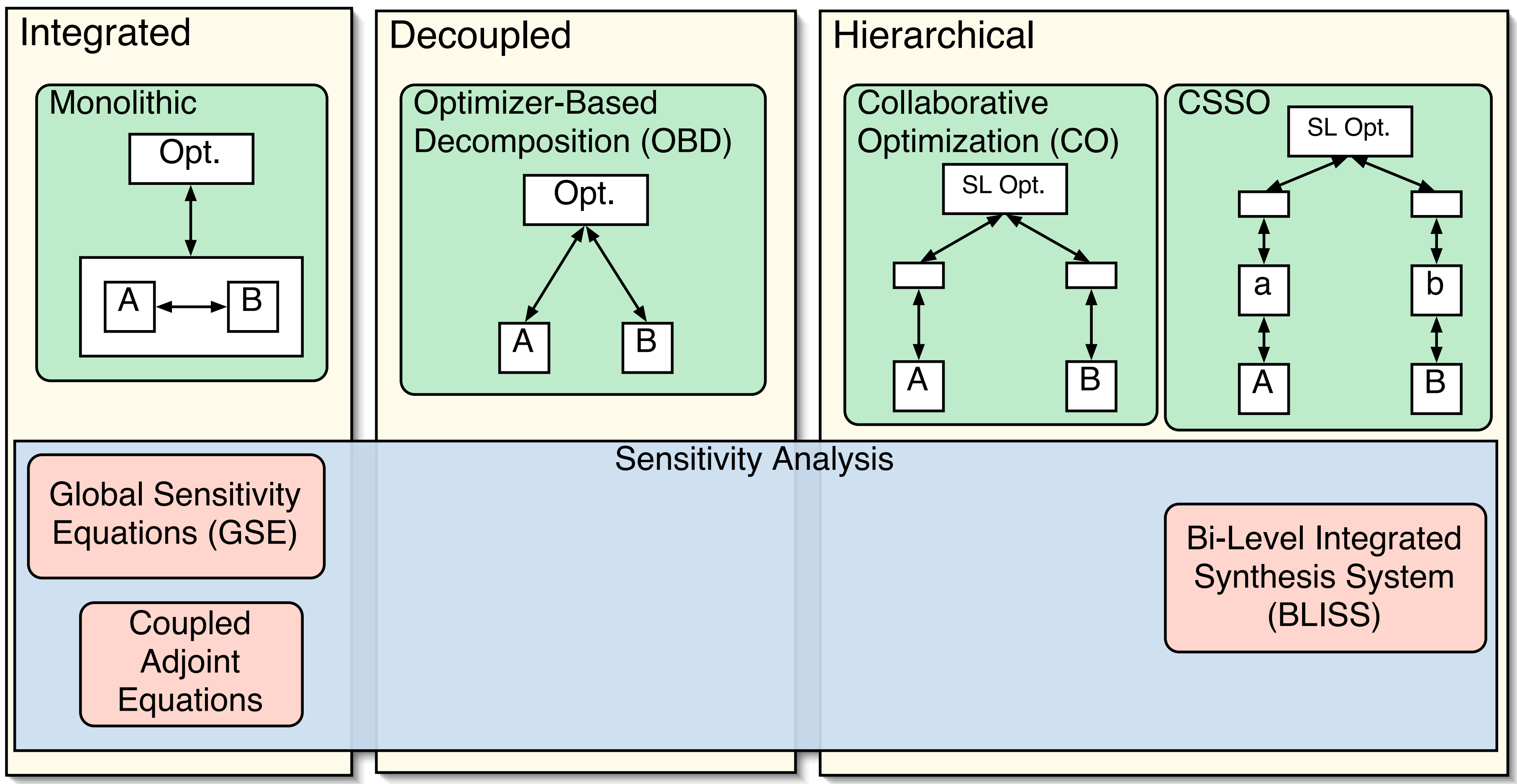
- Developed the general formulation for a coupled-adjoint method for multidisciplinary systems.
- Applied this method to a high-fidelity aero-structural solver.
- Showed that the computation of sensitivities using the aero-structural adjoint is extremely accurate and efficient.
- Demonstrated the usefulness of the coupled adjoint by optimizing a supersonic business jet configuration.



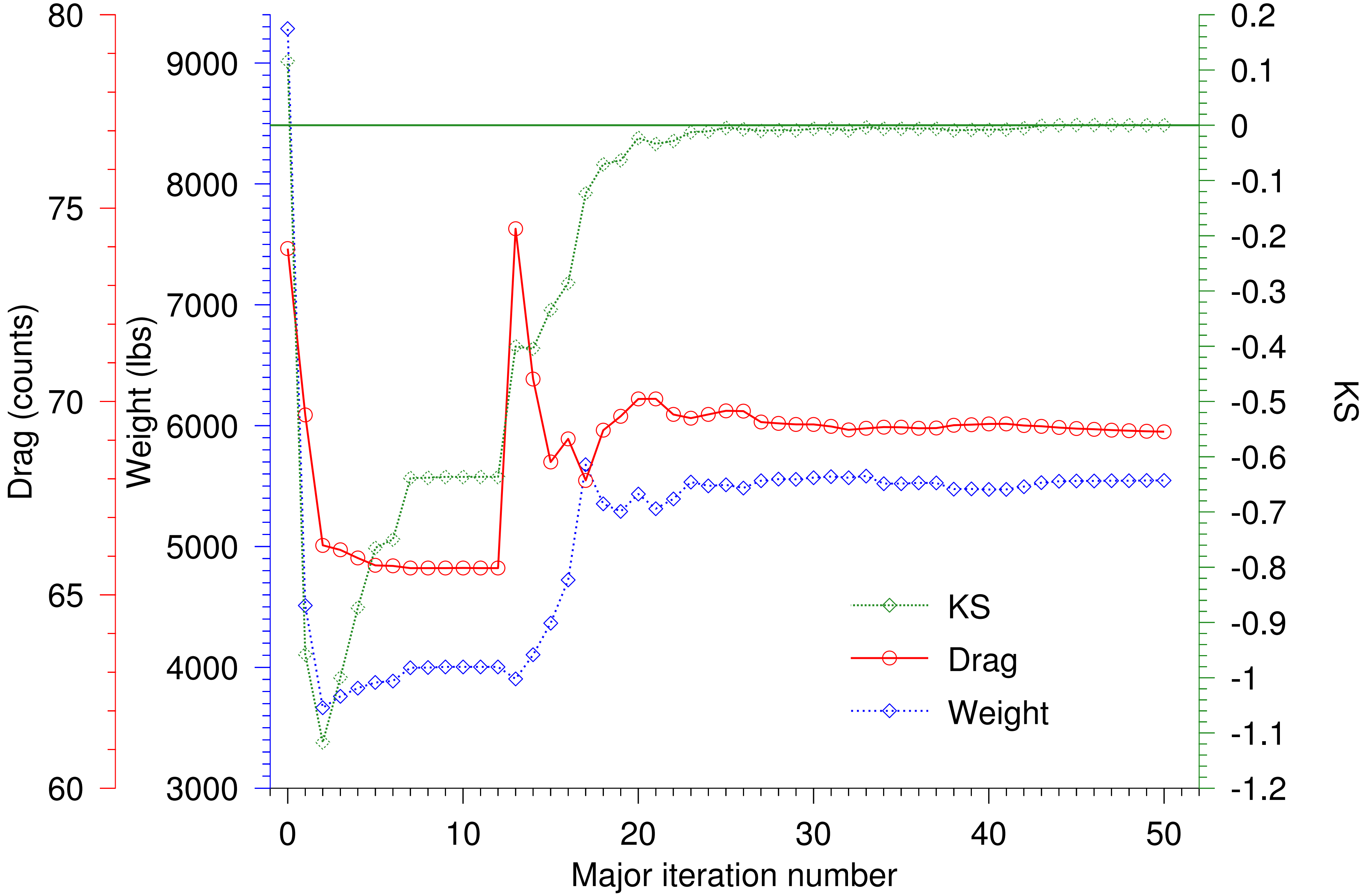


Questions?

MDO Architectures



Aero-Structural Optimization Convergence History



Long Term Vision

Build a large-scale, versatile MDO framework for aircraft design