The geometry and landscape of Supergravity

Antoine Van Proeyen
K.U. Leuven

Toronto, May 3, 2005

From strings to supergravity

- Landscape of vacua of string theories is a landscape of supergravities
- The basic string theories have a supergravity as field theory approximation
- Also after the choice of a compact manifold one is left with an effective lower dimensional supergravity with number of supersymmetries determined by the Killing spinors of the compact manifold
- Fluxes and non-perturbative effects lead to gauged supergravities
- Not every supergravity is interpretable in terms of strings and branes (yet).

This talk

- Overview of possibilities. Where are we ?
- Structure of supergravity theories
- Which data completely determine a supergravity theory ?
- The description of supergravities can be simplified.
  (rigid supersymmetry is so much simpler and transparant due to superspace description; the conformal approach allows one to re-interpret supergravity as a covariantized rigid theory)
- Examples:
  stable de Sitter vacua in 5 dimensions

Plan

1. The classification of ‘non-gauged’ supergravity theories
2. Gauged supergravity theories
3. Simplification by a parent rigid susy theory (using superconformal ideas)
4. Example: Stable de Sitter vacua from supergravity in 5 dimensions
5. Final remarks

1. The classification of ‘non-gauged’ supergravity theories

- Restrictions in the classification
- Supergravities by dimension and extension
- Structure of the action
- R-symmetry
- Kinetic terms :
  Geometry of the scalar manifolds

Restrictions for the classification

- Theories with an action
  - Other possibility: field equations determined by the supersymmetry algebra, but these cannot be obtained from a covariant action
  - IIB in 10 dimensions is such an example, but we include this for the systematics
- At most 2 spacetime derivatives in any term
- Signature of spacetime is Minkowski
- Positive definite kinetic terms for physical fields
- Poincarè-like algebra

Strathdee,1987

The map: dimensions and # of supersymmetries

d	susy	32		24	20	16		12	8	4
11	M	M								
10	MW	IIA	IIB			I				
9	M	N=2				N=1				
8	M	N=2				N=1				
7	S	N=4				N=2				
6	SW	(2,2)		(2,1)		(1,1)	(2,0)		(1,0)	
5	S	N=8		N=6		N=4			N=2	
4	M	N=8		N=6	N=5	N=4		N=3	N=2	N=1
		SUGRA				SUGRA/SUSY		SUGRA	SUGRA/SUSY	

See discussion in AVP, 0301005

vector multiplets

tensor multiplet

vector multiplets + multiplets up to spin 1/2

Structure of the action

- *(D=4)* with fields of spin 2, 1, 0, 3/2, 1/2

$$e_\mu^a,\ A_\mu^I,\ \phi^u,\ \psi_\mu,\ \lambda^A$$

$$\begin{aligned} e^{-1}\mathcal{L} \ = \ & \tfrac{1}{2}R \\ & +\tfrac{1}{4}(\mathrm{Im}\,\mathcal{N}_{IJ})\mathcal{F}_{\mu\nu}^I\mathcal{F}^{\mu\nu J} - \tfrac{1}{8}(\mathrm{Re}\,\mathcal{N}_{IJ})\varepsilon^{\mu\nu\rho\sigma}\mathcal{F}_{\mu\nu}^I\mathcal{F}_{\rho\sigma}^J \\ & -\tfrac{1}{2}g_{uv}D_\mu\phi^u D^\mu\phi^v - V \\ & \left\{-\bar{\psi}_{\mu i}\gamma^{\mu\nu\rho}D_\nu\psi_\rho{}^i - \tfrac{1}{2}g_A{}^B\bar{\lambda}^A \not{D}\lambda_B + \mathrm{h.c.}\right\} + \dots \end{aligned}$$

- Kinetic terms determined by some matrices.
  They describe the geometry
- Gauged supergravity:
  - What are the covariant derivatives ?
  - The field strenghts can be non-abelian.
  - Potential V ?

Are all antisymmetric tensors equivalent to scalars ?

What is determined / to be determined ?

- $32 \geq Q > 8$: Once the field content is determined: kinetic terms determined.
  Gauge group and its action on scalars (covariantization) to be determined.
  Potential depends on this gauging
- $Q = 8$: kinetic terms to be determined
  Gauge group and its action on the scalars to be determined.
  Potential depends on this gauging
- $Q = 4$: ($d=4$, $N=1$): potential depends moreover on a superpotential function W.

R-symmetry

- Supersymmetries are representations of the Lorentz group $[M_{\mu\nu}, Q^i_\alpha\ ] = -\frac{1}{4}(\gamma_{\mu\nu})_\alpha{}^\beta Q^i_\beta$
- They may rotate under an automorphism group $[T_A, Q^i_\alpha\ ] = (U_A)^i{}_j Q^j_\alpha$
- Jacobi identities [TTQ] , [TQQ], and reality conditions restrict possibilities

R-symmetry groups in different dimensions

- Majorana spinors in odd dimensions:
  SO(N) (d=3,9)
- Majorana spinors in even dimensions:
  U(N) (d=4,8)
- Majorana-Weyl spinors:
  $SO(N_L) \times SO(N_R)$ (d=2,10)
- Symplectic spinors:
  USp(N) (d=5,7)
- Symplectic Majorana-Weyl spinors:
  $USp(N_L) \times USp(N_R)$ (d=6)

The map of geometries

- With > 8 susys: symmetric spaces

d	32	24	20	16		12
9	$\frac{S\ell(2)}{SO(2)} \otimes O(1,1)$			$\frac{O(1,n)}{O(n)} \otimes O(1,1)$		
8	$\frac{S\ell(3)}{SU(2)} \otimes \frac{S\ell(2)}{U(1)}$			$\frac{O(2,n)}{U(1)\times O(n)} \otimes O(1,1)$		
7	$\frac{S\ell(5)}{USp(4)}$			$\frac{O(3,n)}{USp(2)\times O(n)} \otimes O(1,1)$		
6	$\frac{O(5,5)}{USp(4)\times USp(4)}$	$\frac{SO(5,1)}{SO(5)}$		$\frac{O(4,n)}{O(n)\times SO(4)} \otimes O(1,1)$	$\frac{O(5,n)}{O(n)\times USp(4)}$	
5	$\frac{E_6}{USp(8)}$	$\frac{SU^*(6)}{USp(6)}$		$\frac{O(5,n)}{USp(4)\times O(n)} \otimes O(1,1)$		
4	$\frac{E_7}{SU(8)}$	$\frac{SO^*(12)}{U(6)}$	$\frac{SU(1,5)}{U(5)}$	$\frac{SU(1,1)}{U(1)} \times \frac{SO(6,n)}{SU(4)\times SO(n)}$		$\frac{SU(3,n)}{U(3)\times SU(n)}$

- 8 susys: very special, special Kähler and quaternionic-Kähler

$d=6$	$d=5$	$d=4$
$\frac{O(1,n)}{O(n)} \times QK$	$VSR \times QK$	$SK \times QK$

U(1) part in holonomy group

SU(2)=USp(2) part in holonomy group

- 4 susys: Kähler: **U(1)** part in isotropy group

Kähler, hyper-Kähler, quaternionic-Kähler

- Complex structure $J_i{}^j$ (with $J\,J = -1$) or hypercomplex structure with $J^1 J^2 = J^3$
- If there is also a hermitian metric and $\nabla_k J_i{}^j = 0$: 'Kähler manifold'. **Holonomy group U(*n*)**
- Hypercomplex structure with metric and $\nabla_k \vec{J}_i{}^j = 0$ 'hyper-Kähler' **Holonomy group USp(2n)**
- If there is an SU(2) connection such that

$$\nabla_k \vec{J}_i{}^j + 2\vec{\omega}_k \times \vec{J}_i{}^j = 0$$

'quaternionic-Kähler' **Holonomy group USp(2n)×SU(2)**

(for positive definite metric)

i labels the scalar fields ϕ^i

2. Gauged supergravity theories

- Gauging and potential
  (see: fluxes for stabilisation of moduli)
- Isometries and their embedding in the gauge group
- Gauged R-symmetry
- The potential

Gauged supergravity for applications

- Essential progress: stabilisation of moduli by supergravity potentials
- $\mathcal{N}$=1: **superpotential** produced by fluxes
  - e.g. Gukov-Vafa-Witten result for CY in IIB

$$W = \int_{\mathrm{CY}_6} G \wedge \Omega$$

  - Analogous result in heterotic string
  - Or understand from

Structure of the action

- *(D=4)* with fields of spin 2, 1, 0, 3/2, 1/2

$$e_\mu^a,\ A_\mu^I,\ \phi^u,\ \psi_\mu,\ \lambda^A$$

$$
\begin{aligned}
e^{-1}\mathcal{L} \;=\; & \tfrac{1}{2}R \\
& +\tfrac{1}{4}(\mathrm{Im}\,\mathcal{N}_{IJ})\mathcal{F}_{\mu\nu}^{I}\mathcal{F}^{\mu\nu J} - \tfrac{1}{8}(\mathrm{Re}\,\mathcal{N}_{IJ})\varepsilon^{\mu\nu\rho\sigma}\mathcal{F}_{\mu\nu}^{I}\mathcal{F}_{\rho\sigma}^{J} \\
& -\tfrac{1}{2}g_{uv}D_\mu\phi^u D^\mu\phi^v - V \\
& \left\{-\bar{\psi}_{\mu i}\gamma^{\mu\nu\rho}D_\nu\psi_\rho{}^i - \tfrac{1}{2}g_A{}^B\bar{\lambda}^A \not{D}\lambda_B + \text{h.c.}\right\} + \ldots
\end{aligned}
$$

- Kinetic terms d
  They describe t
- Gauged superg
  - What are the covariant derivatives ?
  - The field strenghts can be non-abelian.
  - Potential *V* ?

If these (from a higher-dimensional theory) get scalar values, these terms produce new contributions to the potential *V*.

Gauged supergravity for applications

- Essential progress: stabilisation of moduli by supergravity potentials
- N=1: **superpotential** produced by fluxes
- or from non-perturbative effects or both

- higher N supergravity or in higher dimensions : Potential is generated only from gaugings.
  N=1 is special case: potential appears also from superpotential W

Gauge group

- Number of generators = number of vectors.
- This includes as well vectors in supergravity multiplet and those in vector multiplets (cannot be distinguished in general)
- The gauge group is arbitrary, but to have positive kinetic terms gives restrictions on possible non-compact gauge groups.

Isometries of the scalar manifold

- Diffeomorphisms $\phi'(\phi)$ of the scalar manifold $\supset$ **isometries**
  (symmetries of kinetic energy $\mathrm{d}s^2=g_{ij}\mathrm{d}\phi^i\,\mathrm{d}\phi^j$)
- A subset of the isometries can be gauged.
- If ϕ^i are all the scalar fields,
  $A_\mu{}^I$ all the vectors,
  and $k_\Lambda{}^i$ are all the Killing vectors, (isom.: $\delta\phi^i=\varepsilon^\Lambda k_\Lambda{}^i$)
  the 'embedding matrix' $t_I{}^\Lambda$ determines
  the subset of isometries $k_I{}^i = t_I{}^\Lambda k_\Lambda{}^i$ that are gauged.
- These should satisfy the gauge algebra of the vectors.

Isometries, complex structures and R-symmetry

- When the manifold has complex structures, the isometries should respect them.
- This implies e.g. that in hyper-Kähler manifolds the matrix $D_i\, k_I{}^j$ commutes with $\vec{J}_i{}^j$
- For a general isometry in quaternionic-Kähler:

$$D_i k_I^j = \vec{J}_i{}^j \cdot \vec{P}_I + \text{part related to USp}(2r)$$

- Moment map of an isometry is its SU(2) part in the decomposition in SU(2)×USp(2r)

Example: Independent quantities of an *N=1, D=4* supergravity

$$e^{-1}\mathcal{L}_{\mathrm{bos}} = -\tfrac{1}{2}M_P^2 R - \tfrac{1}{4}(\mathrm{Im}\,\mathcal{N}_{IJ})F^I_{\mu\nu}F^{\mu\nu\,J} - \tfrac{1}{8}(\mathrm{Im}\,\mathcal{N}_{IJ})e^{-1}\varepsilon^{\mu\nu\rho\sigma}F^I_{\mu\nu}\tilde{F}^J_{\rho\sigma}$$
$$-g_{\alpha\bar\beta}(D_\mu z^\alpha)(D^\mu \bar z^{\bar\beta}) - V(z,\bar z)$$

- kinetic holomorphic function $\mathcal{N}_{IJ}(z) = \mathrm{i} f_{IJ}(z)$
- chiral multiplet kinetic terms: Kähler potential *K*

$$g_{\alpha\bar\beta} = \partial_\alpha \partial_{\bar\beta} K(z,\bar z)$$

- gauge group such that $\mathcal{N}_{IJ}$ is in (adj x adj)$_{\mathrm{symm}}$
- chiral multiplet: representation of gauge group:
  - should act as isometries of Kähler metric
  - embedding in isometry group such that algebra is satisfied
- (holomorphic) superpotential *W*(*z*) only for *N=1 !!*
- for any *U(1)* a Fayet-Iliopoulos (FI) constant ξ_I

Gauged R-symmetry

In supergravity:

In the susy transformation of the gravitino appears a gauge vector for the R-transformation: e.g. *N*=1:

$$\delta\psi_{\mu L} = \left(\partial_\mu + \tfrac{1}{4}\omega_\mu{}^{ab}(e)\gamma_{ab} + \tfrac{1}{2}\mathrm{i}\mathcal{V}_\mu\right)\epsilon_L + \tfrac{1}{2}M_P^{-2}\gamma_\mu F_0\epsilon_R$$

$$\mathcal{V}_\mu = (\partial_\mu\phi^i)\omega_i + \kappa^2 A_\mu^I \mathcal{P}_I$$

$$\omega_i \equiv J_i{}^j\partial_j K$$

composite gauge field for U(1)

'R-symmetry' $\epsilon \to e^{i\alpha\gamma_5}\epsilon$

- Holonomy group of scalar manifold includes R-symmetry as factor: first term is pull-back of the connection on the scalar manifold
- The amount in which the gauge symmetry contributes to the R-symmetry is determined by the **moment map**

Potential

- General fact in supergravity ("Ward identity")

$$V = \sum_{\text{fermions}} (\delta \text{ fermion}) (\text{metric}) (\delta \text{ fermion})$$

N=1

δ gravitino

δ chiral fermions

δ gaugino

$$V = \underbrace{-3M_P^{-2} F_0 \bar{F}_0 + F_\alpha g^{\alpha\bar{\beta}} \bar{F}_{\bar{\beta}}}_{\textit{F-term}} + \underbrace{\tfrac{1}{2} D^I (\mathrm{Im}\mathcal{N}_{IJ}) D^J}_{\textit{D-term}}$$

depends on superpotential *W*

depends on gauge transformations + arbitrary FI constants ξ_I (for U(1) factors)

- In higher *N*: all determined by gauge transformations
- E.g. *N*=2: complex *F* and real *D* are combined in triplet moment map *P*

3. Simplification by a parent rigid supersymmetric theory

- Superconformal idea
- The geometries of supergravity by gauge fixing those of rigid supersymmetry
- Isometries and R-symmetry
- The potential

Rigid supersymmetry

- Difference: the concept of multiplets is clear in susy, they are mixed in supergravity
- Superfields are an easy conceptual tool
- Gravity can be obtained by starting with conformal symmetry and gauge fixing.
- Before gauge fixing: everything looks like in rigid supersymmetry + covariantizations

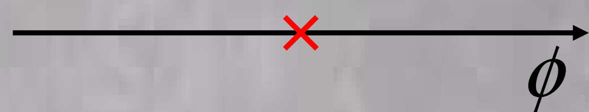
Poincaré gravity by gauge fixing

- scalar field (compensator) $\mathcal{L} = \phi\Box\phi$

conformal gravity: $\mathcal{L} = -\sqrt{g}\phi\Box^{C}\phi = -\sqrt{g}\phi\Box\phi + \frac{1}{6}\sqrt{g}R\phi^{2}$

dilatational gauge fixing $\phi = \sqrt{3}M_P \rightarrow \mathcal{L} = \frac{M_P^2}{2}\sqrt{g}R$

- First action is conformal invariant,
- Scalar field had scale transformation $\delta\phi(x) = \Lambda_D(x)\phi(x)$



choice determines M_{P}

See: negative signature of scalars !
Thus: if more physical scalars: start with (– ++...+)

Very special real manifolds

- *VSR* : very special real manifold: an embedding of a *n*-dimensional manifold in an *(n+1)*-dimensional manifold with metric $C_{IJK}\, h^I\, \mathrm{d}h^J\, \mathrm{d}h^K$ by constraint $C_{IJK}\, h^I h^J h^K = const.$
- The cubic homogeneous structure is the conformal symmetry of the theory
- The constraint is the gauge fixing condition

Geometries from supersymmetric theories with 8 real supercharges

	d=5, N=2 vector multiplets	Kähler manifolds	hyper-multiplets
rigid (affine) +conform	**affine very special real**	**affine (special) Kähler**	**hyper-Kähler**
	$\downarrow$ gauge fix D	$\downarrow$ gauge fix D, U(1)	$\downarrow$ gauge fix D, SU(2)
local (projective)	**(projective) very special real**	**(projective) (special) Kähler**	**quaternionic-Kähler**

Superconformal formulation for $N=1$, $d=4$

- superconformal group includes dilatations and U(1) R-symmetry

- Super-Poincaré gravity =
  Weyl multiplet: includes (auxiliary) U(1) gauge field
  + compensating chiral multiplet
- Corresponding scalar is called 'conformon': Y
- Fixing value gives rise to M_P :

$$Y Y^* \mathrm{e}^{-\kappa^2 K/3} = \kappa^{-2} = M_P^2$$

- U(1) is gauge fixed by fixing the imaginary part of Y, e.g. $Y=Y^*$

Superconformal methods for N=1 d=4

$(n+1)$ – dimensional Kähler manifold
with conformal symmetry
(a closed homothetic Killing vector k^i)
(implies a U(1) generated by $k^j \boldsymbol{J}_j{}^i$)

Gauge fix dilatations and U(1)

n-dimensional Hodge-Kähler manifold

Quaternionic-Kähler from hyper-Kähler

4(r+1) – dimensional hyper-Kähler manifold with conformal symmetry
(a closed homothetic Killing vector k^i)
(implies an SU(2) generated by $\vec{k}^i = k^j \vec{J}_j{}^i$)

USp(2r,2)
$\Downarrow$
USp(2r) $\times$ SU(2)

Gauge fix dilatations and SU(2)

4r – dimensional quaternionic-Kähler manifold

USp(2r) $\times$ SU(2)

B. de Wit, S.Vandoren, series of papers, e.g. 9909228

E. Bergshoeff, S. Cucu, T. de Wit, J. Gheerardyn, S. Vandoren, AVP, ‘The map between conformal hypercomplex/hyper-Kähler and quaternionic(-Kähler) geometry’, 0411209

(also for hypercomplex → quaternionic)

R-symmetry in conformal approach

- R-group is part of the superconformal group.
- Should then be gauge-fixed

- Isometries act also on compensating scalars
- The moment map is the transformation of the compensating scalars under the isometry
- In conformal formulation: R $\times$ Isom
- Gauge fixing of R : R $\times$ Isom $\to$ Isom. But the remaining isometries have contribution from R-symmetry

$$\delta_{\mathsf{isom}}(\epsilon^I) + \delta_{\mathsf{SU(2)}}\left[\epsilon^I\left(\vec{\omega}_i k_I^i + \vec{P}_I\right)\right]$$

Potential (in example *d=4, N=1*)

$V = V_F + V_D$

- F-term potential is unified by including the extra chiral multiplet: $Z^A = \{Y, z^\alpha\}$

$$\begin{aligned} V &= \mathrm{e}^K \left[-3W\bar{W} + (D_\alpha W) g^{\alpha\bar\beta} (D_{\bar\beta}\bar{W}) \right] \\ D_\alpha W &= \partial_\alpha W + (\partial_\alpha K) W \end{aligned}$$

$$\downarrow$$

$$\begin{aligned} V &= (\partial_A \mathcal{W}) G^{A\bar B} (\partial_{\bar B} \mathcal{W}) \Big|_{\mathcal{K}=-3} \\ \mathcal{K} &= -3 Y Y^* \mathrm{e}^{-K(z,\bar z)/3}, \quad G_{A\bar B} = \partial_A \partial_{\bar B} \mathcal{K} \\ \mathcal{W} &= Y^3 W(z) \end{aligned}$$

- D-term potential: is unified as FI is the gauge transformation of the compensating scalar:

$V_D = \frac{1}{2} D^2$

$k^Y = 3\mathrm{i} g \xi Y$

$$\begin{aligned} D &= \tfrac{1}{2}\mathrm{i} k^\alpha \partial_\alpha K - \tfrac{1}{2}\mathrm{i} k^{\bar\alpha} \partial_{\bar\alpha} K + g\xi \\ &= \tfrac{1}{2}\mathrm{i} k^A \partial_A \mathcal{K} - \tfrac{1}{2}\mathrm{i} k^{\bar A} \partial_{\bar A} K \Big|_{\mathcal{K}=-3} \end{aligned}$$

4. Example: Stable de Sitter vacua from 5 d supergravity

- A few models have been found in **d=5, N=2** (Q=8) that allow de Sitter vacua where the potential has a minimum at the critical point

  B. Cosemans, G. Smet, 0502202

- A few years ago similar models where found for **d=4, N=2**

  P. Frè, M. Trigiante, AVP, 0205119

- These are exceptional: apart from these constructions all de Sitter extrema in theories with 8 or more supersymmetries are at most saddle points of the potential

Main ingredients

- Tensor multiplets.
  Abelian case: tensors (2-forms) dual to vector multiplets (in d=5).
  However, they allow gaugings that are not possible for vector multiplets
- Non-compact gauging:
  SO(1,1) gauge group involved
- Fayet-Iliopoulos terms: the compensating hypermultiplet transforms under the gauge group.

The model

- 1 vector multiplet (+ 1 compensating)
- 2 tensor multiplets
- No hypermultiplet   (1 compensating)
- Vectors (A^0, A^3) and tensors (B^1,B^2) in one 'very special real' structure' determined by

$$h^0\left[(h^1)^2-(h^2)^2-(h^3)^2\right]$$

- 3 physical scalars form very special real space

$$\frac{\mathsf{SO}(2,1)\times\mathsf{SO}(1,1)}{\mathsf{SO}(2)}$$

- Embedded in 2 vector mult. + 2 tensor mult. with conformal symmetry

This is a minimal version of the models in B. Cosemans, G. Smet, 0502202

Gauging

- Vector A^0 and A^3 can be used for gauging: 2 Abelian factors.
- Isometries that are gauged:
  - rotation between tensors B^1 and B^2 (scalars h^1 and h^2) : isometry in part SO(1,1)⊂ SO(2,1). This is only possible because they are dualized to tensors. As vectors they should have been in adjoint representation. *Tensor contribution to potential*: square of transformation of tensorinos
  - FI term = gauging of SO(2) subgroup of the SU(2) in compensating hypermultiplet. Here the embedding matrix is the moment map Potential: square of transformations of gravitini and gaugini: simpler at once in extended space:

$$V^{(V)} = -4C^{IJK} h_K \vec{P}_I \cdot \vec{P}_J , \qquad \vec{P}_3 = (0,0,g\xi)$$

Vacuum

- The constraint $h^0\left[(h^1)^2-(h^2)^2-(h^3)^2\right]=1$ solved for h^0 in terms of scalars $\phi^i = h^i$ *(i=1,2,3).*
- Potential
$$\begin{aligned} V &= V^{(T)}+V^{(V)} \\ &= \frac{g^2}{18}\frac{(\phi^1)^2-(\phi^2)^2}{||\phi||^6}+3g^2\xi^2||\phi||^2 \\ ||\phi||^2 &\equiv (\phi^1)^2-(\phi^2)^2-(\phi^3)^2 \end{aligned}$$
- Extremum: $\phi^3=0, \qquad (\phi^1)^2-(\phi^2)^2=\frac{1}{3}\xi^{-2/3}$
$$V_{\text{min}}=\frac{3}{2}g^2\xi^{4/3}$$
- Second derivatives, diagonalized: $m^2=g^2\left\{0,\frac{8}{3},\frac{2}{3}\right\}V_{\text{min}}$
- BEH effect: Spontaneously broken SO(1,1).
  Vector becomes massive. Goldstone boson absorbed.

5. Final remarks

- A classification of the landscape of all supergravities is not yet completed, but basic principles are known.
- With the restrictions that we have imposed, the possible gaugings still have to be discussed.
  Which gaugings produce positive definite kinetic terms ?
- Dualities between multiplets do not hold for gauged supergravities. What is a complete set of theories ?
- It may be good to construct an inventory of known theories: actions – transformation laws – field equations
- More ambitious: solutions, relations by compactification