

# Stringy      Resolutions      of      Null      Singularities

A. Maloney

I)

Dabholkar, Kallosh & Maloney  
Hubeny, Maloney & Ramgoolam

- Cordova, DelWitt, Mohaupt & Sparks
  - Dabholkar
  - Sen
- 

II)

Maloney (to appear)

- Horowitz & Marolf
- Maldacena & Maoz

String Theory can successfully resolve many singularities of classical gravity

Time like  $\Rightarrow$  Orbifold  
BPS BH

But other singularities remain mysterious...

Space like or Null  $\Rightarrow$  Schwarzschild BH  
Cosmological Sing.

Today, I'll focus on two simple Null sing.

I) "Naked" BPS BH

$\rightarrow$  Quantum corrections generate a horizon  
& make singularity time like

II) Cosmological Orbifolds of AdS

$\rightarrow$  Euclidean AdS/CFT to study B.B./B.C.

## Quantum Horizons for Naked Sing.

Compactifications of S.T. to  $D=4$  have many BPS Sol'n at low energy.

S.T.

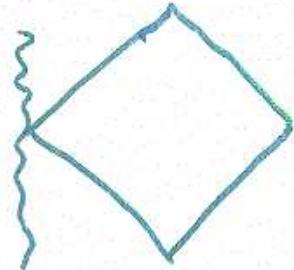
Brane & Strings



$$\log N \sim Q^2$$

SUGRA

SUSY BH



$$S_{BH} = \frac{1}{4} A$$

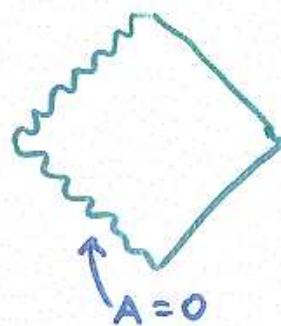
But for other objects, when  $\log N \sim Q$ , the SUGRA sol'n is singular:

Conjecture: Quantum Corrections

$$\downarrow$$

$$A \neq 0$$

Sen  
Susskind



Q: Can we see this by studying quantum, higher curvature corrections to SUGRA?

Setup: Type II on a CY

→  $N=2$  SUGRA in  $D=4$ , with

Gravity Multiplet +  $N$  Vector multiplets

Fields: Vector fields  $A_\mu^I$   $I=0 \dots N$

Vector Mult. Scalars  $X^I$  projective

Action: Determined by a prepotential

$$F(X^I, \omega^2) = F_0(X^I) + F_1(X^I)\omega^2 + \dots$$

↑  
chiral multiplet  
containing GR  
curvatures

↑  
vector  
interactions

↑  
higher curvature  
terms

$$F_0 \sim \text{string tree} \sim D_{ABC} \frac{X^A X^B X^C}{X^0} + \dots$$

$$F_1 \sim \text{string one-loop} \sim d_A \frac{X^A}{X^0} + \dots$$

⋮

## Black Hole Solutions

BPS BHs are specified by  $N+1$  electric  
+ magnetic charges,  $p^I + g_I$

Of these, one special linear combination

$$Z = p^I F_I - g_I X^I$$

$\uparrow$   
 $\frac{\partial F}{\partial X^I}$

is the graviphoton charge. It is the central charge appearing in the SUSY algebra.

$\Rightarrow$  the Area of the BH is

$$A = 4\pi |Z|^2$$

Attractor Mechanism : the moduli fields

$X^I(r)$  flow to values at the horizon that are fixed only by the charges

$$\rightarrow \text{minimize } Z(p, g; X^I)$$

Ferrara  
Kallosh  
Strominger

# Quantum Generated Horizons

with Dabholkar & Kallash

To study quantum corrected BH, we just need  
to study attractor eq. in presence of  
higher curvature terms ...

CdWKM

Tree (i.e. classical) level :

$$A = 4\pi \int D_{ABC} p^A p^B p^C g_0$$

is finite for some B.H., but vanishes for  
others. These objects appear to be naked  
singularities, classically... For such objects

One loop (i.e. quantum) result :

$$A = 4\pi \int d_A p^A g_0$$

So quantum effects generate a horizon ! For  
large p, g this horizon is macroscopic.

Comments:

One might worry that we have no right to neglect higher corrections, but this is not the case — we're doing a well defined perturbative expansion in  $\frac{1}{Q}$

$$A = Q^2 + Q + \dots$$

$\uparrow$              $\uparrow$              $\nearrow$   
tree        one-loop      higher order

For our BH, the leading term vanishes, so we must go to next order. But higher (e.g. 2+ loop) terms are still negligible if  $Q$  is large.

Nevertheless, this is still a very quantum system . . .

## Quantum Area / Entropy Relations

The classical B.-H. relation  $S = \frac{1}{4} A$  is altered by these higher curvature terms...

CdWKM

For our BH, the one-loop corrected entropy and area are related by

$$S = \frac{1}{2} A$$

Moral: some quantum corrections are important

In fact, there are other examples with

$$S = c A$$

where  $c = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \dots$

Although  $c$  appears to be bounded from above by charge quantization.

→ is there still a useful holographic bound?

## The full metric

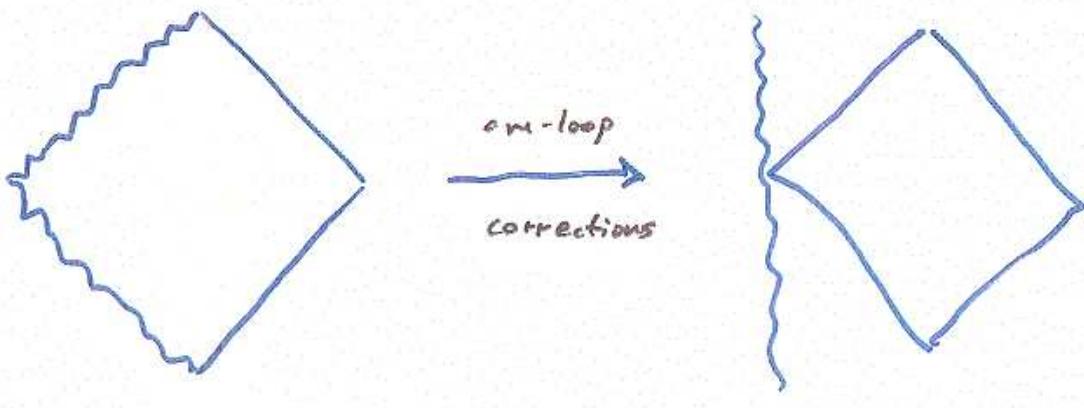
with Hobson & Pandey:

We've seen quantum corrections generate a horizon  
for our naked sing. How do they change  
the structure of the singularity itself?

The metric takes the form

$$ds^2 = -e^{2g} dt^2 + e^{2g} dx^2$$

where  $e^{2g}$  is found by solving a killing  
spinor equation. Once higher curvature terms  
are included it can't be solved analytically,  
but you can prove:

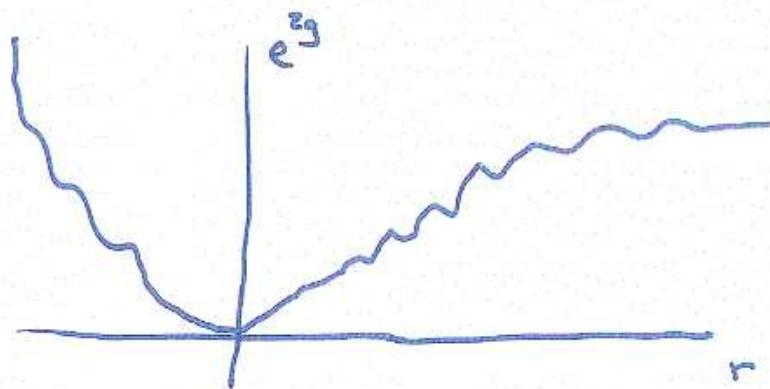


Null Singularity



Timelike

But the detailed form of the metric is rather surprising ...



Non-monotonic  $\Rightarrow$  gravitational potential not purely attractive!

- these non-monotonies are of order the compactification scale
- arise because, as is typical for quantum corrections, the higher curvature terms violate a Null Energy condition.

## Open Questions

- We've found "classical" sol'n to quantum effective action.  
Sen  $\Rightarrow$  Higher curvature  $\leftrightarrow$  Spurious sol'n?
- Can we understand  $S = \frac{1}{2} A$  in perturbative string theory, a la Das & Mathur?
- Can we understand  $S = \frac{1}{2} A$  via a Quantum Mechanics dual to near horizon AdS<sub>2</sub>, a la Gaiotto, Simons, Strominger & Yin?

## Summary

- Higher curvature corrections, found from String Theory, convert apparently pathological sol'n of classical gravity into regular BH with timelike singularity.
- Perturbative setup
  - Area large in Planck units
  - Higher curvature corrections suppressed
- Nevertheless, important qualitative differences
  - large corrections to Area/Eentropy relation

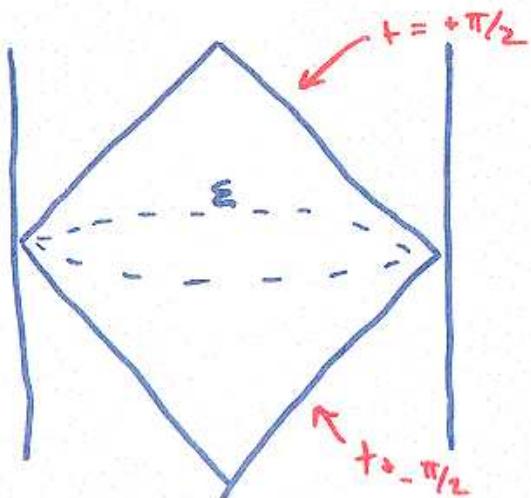
## Cosmological Singularities in AdS/CFT

We've seen how some naked sing. are converted into regular BH. in string theory.

Goal: To understand more serious, cosmological  
(i.e. B.B. / B.C.) sing.

How? AdS / CFT ...

In FRW coords, AdS looks like a cosmology



$$-dt^2 + \cos^2 t d\Sigma^2$$

↑  
unit hyperbola

Here  $t = \pm \frac{\pi}{2}$  look like B.B. & B.C., but they're just coordinate singularities.

But if we quotient  $\Sigma$  by some subgroup  $\Gamma$  of its isometry group

$$\Sigma = H_2 / \Gamma$$

then these null surfaces  $t = \pm \pi/2$  become genuine curvature singularities. This is a very simple cosmology :

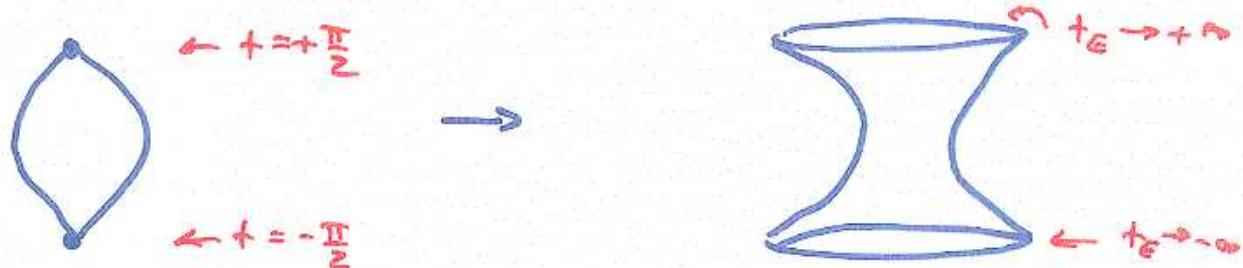
- locally AdS
- singularities of  $S^1 \times$  (Milne) type

There are two interesting cases :

- $\Sigma$  infinite volume (e.g. BTZ)  
→ still has bdy. still has holographic dual.
- $\Sigma$  finite volume  $\Rightarrow$  genuine closed FRW universe  
→ no bdy. how can we use holography?

But what if we Wick rotate  $t \rightarrow it_\epsilon$

$$-dt^2 + \cos^2 t d\Sigma^2 \rightarrow dt_\epsilon^2 + \cosh^2 t_\epsilon d\Sigma^2$$



- 2 curvature singularities
  - no boundary
- 
- smooth
  - 2 boundaries

On Euclidean side, expect two bdy CFTs ...

what can they tell us about cosmology ?

Goal: translate E AdS/CFT dictionary into

language of cosmology

→ Use BTZ as a guide

This is not the usual AdS/CFT dictionary...

## QFT

To work out the dictionary, consider QFT in  
this bkg...

Broken time translation inv.

$\Rightarrow \exists!$  vacuum state  $\Rightarrow$  particle production.

E.g. two natural vacuum states



$|out\rangle$ : no particles at  $t \rightarrow +\pi/2$

$|in\rangle$ : no particles at  $t \rightarrow -\pi/2$

which differ by a Bogoliubov transformation.

So if we start with no particles at  $t \rightarrow -\pi/2$

we end up in an excited state at

$t \rightarrow +\pi/2$ , with a distribution of particles

$N_k = \text{horrible function of } m_\phi, k, \dots$

## The Euclidean Vacuum

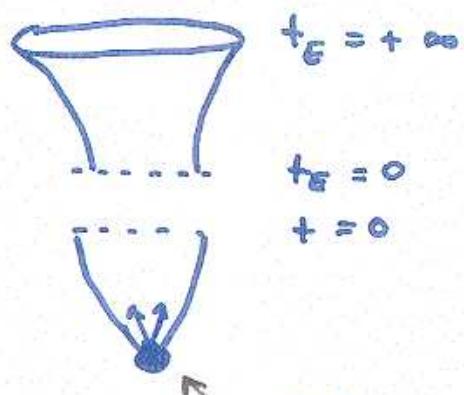
The Euclidean CFT will naturally compute  
in a vacuum state  $|E\rangle \dots$

Near a E. bdy (say  $t_E \rightarrow +\infty$ ) two possible b.c.

Normalizable:  $\phi \sim e^{-(1+\mu)t_E} \approx \langle 0 \dots 0 \rangle_{\text{CFT}}$

Non-Normalizable:  $\phi \sim e^{-(1-\mu)t_E} \propto \delta \rho_{\text{CFT}}$

Imposing normalizable b.c.  $\rightarrow$  particular / orientation  
state  $|E\rangle$



$|E\rangle$  describes particular config. of  
particles emerging from B.B....

In fact,  $|E\rangle$  describes a completely thermal config. of particles

$$N_k = \frac{1}{e^{H/T} - 1} \quad \text{Bose dist.}$$

With  $T = R_{AdS}^{-1}$  and  $H = \nabla_\Sigma^2$

→ totally feature less (indep of  $m_\phi$ )

→ surprising to find thermality w/o

Euclidean periodicity

As a check, this reproduces usual Hawking

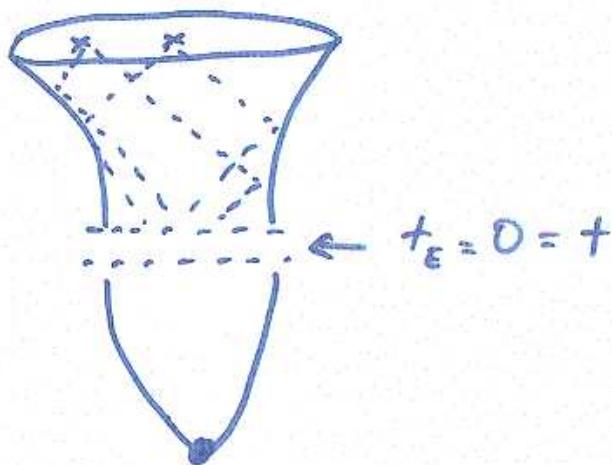
behavior of BTZ BH in the case

where  $\Sigma$  infinite volume

$$T \sim \left(\frac{M}{R_{AdS}}\right)^{1/2} \quad \text{w.r.t. Schwarzschild Hamiltonian}$$

This picture generalized to the full Hilbert space ...

Excited states are associated to inserting sources on bdy



⇒ two classes of states

- finite excitations of  $|E\rangle$ ;  $a_\epsilon^+ \dots a_\epsilon^+ |E\rangle$

$$\leftrightarrow \langle 0 \dots 0 \rangle_{\text{CFT}}$$

- squeezed states  $e^{\gamma a_\epsilon^{+2}} |E\rangle$

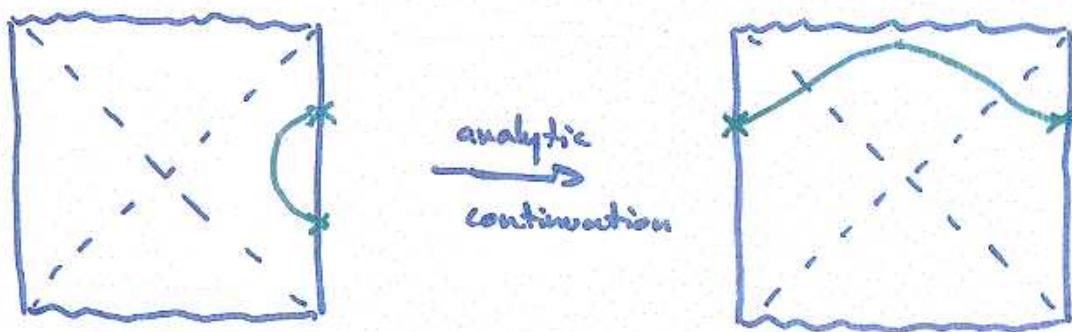
$$\leftrightarrow \delta \mathcal{L}_{\text{CFT}}$$

## Geodesic Probes of B.B. Sing.

Q: How does ECFT encode inf. about  
B.B. Sing?

Motivation: In BTZ case, one can analytically continue CFT correlators to go near the BH sing:

Maldacena  
Kraus, Dyer & Shenker  
Fidkowski, Hoban  
Kleban & Shenker

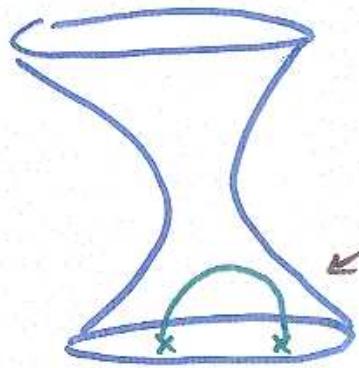


In our case, we must analytically continue correlators in a ECFT defined on  $\Sigma$

These correlators are, in the bulk picture, given by Euclidean geodesics:

$$\langle O_1 O_2 \rangle \sim e^{-\Delta L}$$

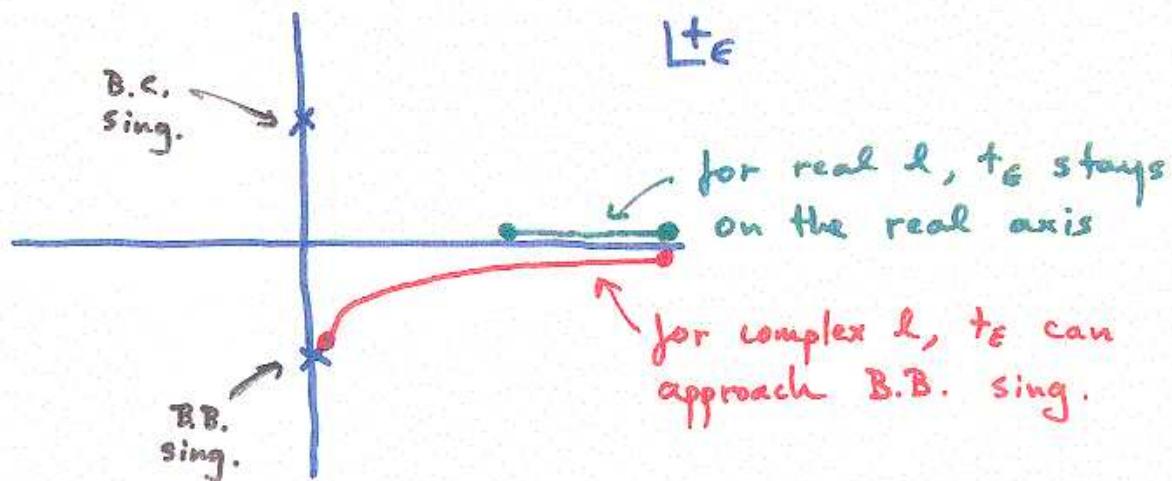
They look like



the profile  $t_\epsilon(l)$  depends on  
the length  $l$  separating the  
two points on the body

Now analytically continue coords of  $\Sigma$

$\Rightarrow$  continue  $l$



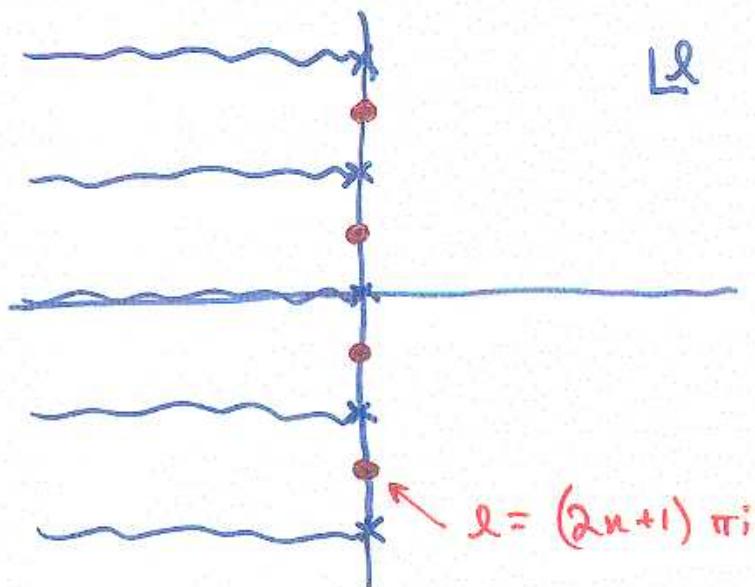
For certain complex values  $l$ , the E CFT should  
contain information about the geometry at the  
B.B. sing.

What do the correlators look like?

$$\begin{aligned} L &= \int \sqrt{t_E^2 + l^2 \cosh^2 t_E} \\ &= 2T + \ln \sinh \frac{l}{2} + \dots \uparrow \end{aligned}$$

↑  
cutoff

subleading in  
cutoff



So  $\langle 00 \rangle$  is a complicated,  $\infty$  sheeted function. But it seems perfectly well defined at the points  $l = (2n+1)\pi$  where geodesics go through singularities...

## Comments

- As before, we can check that this is consistent with the standard BTZ picture

In that case, the Euclidean body is  $T^2$ , and the Euclidean correlators we're studying are just those used to define the thermal two point functions

$$\langle O, O \rangle_{\text{CFT}}$$

## Open Problems

Can we understand more interesting (non  $\delta$  fn) singularities?

→ Higher dimensional Examples

(actually, for  $\Sigma$  compact, our cosmology will already develop more severe curvature sing.  
once back-reaction is included)

Can we actually do any CFT calculations?

→ See resolution of singularity

→ requires mathematical machinery

## Summary

Euclidean AdS/CFT has interesting things to say about simple cosmological singularities

→ Natural HH state, describing particular config. of particles exiting B.B.

→ Euclidean correlators contain information about how B.B. sing. resolved in S.T.

Essentially, we've seen this by rewriting BTZ in a language applicable to cosmology.

→ These considerations apply to BTZ & its less symmetric cousins.