Investing Confidence
in the Ex Ante Equity Premium:
A New Methodology and
A Narrower Range of Estimates

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Synopsis

- Simulate thousands of economies calibrated to the US economy based on each of several values of the ex ante equity premium

  - Generate streams of future dividends and interest rates

  - Discount the dividends to get prices at various points in time

  - Calculate financial statistics for each economy

Synopsis Continued

- By considering thousands of these economies, we obtain *joint distributions* of key financial statistics

- Compare moments of simulated data to those of the US economy

  $\rightarrow$ Our results suggest the true value of the ex ante equity premium is in the close vicinity of 3.5%

  $\rightarrow$ Findings are robust to changes in the parameters underlying our simulations

Basic Definitions

- The equity premium, π_e, is the premium investors anticipate *ex ante*:

$$\pi_e \equiv \mathcal{E}\{R\} - \mathcal{E}\{r_f\}, \quad \text{where}$$

$\mathcal{E}\{R\}$ is the expected market return and $\mathcal{E}\{r_f\}$ is the expected risk-free return.

Basic Definitions Continued

- The equity premium we observe, $\hat{\pi}_e$, is the return investors actually received *ex post*:

$$\hat{\pi}_e \equiv \overline{R} - \overline{r_f}, \quad \text{where}$$

$\overline{R}$ is the average annual return on the S&P 500,

$\overline{r_f}$ is the average return on US T-bills.

- Historically in the US, $\hat{\pi}_e$ has been too high.

Context within the Literature

Common approaches to resolving the equity premium puzzle:

- Extend theoretical models to rationalize a high ex ante equity premium π_e.

- Develop alternative lower estimates $\hat{\pi}_e$.

  – These lower estimates come with confidence intervals that easily encompass troublingly high equity premia.

Context within the Literature Continued

- We develop an approach of looking at combinations of financial statistics that emerge given various values of the ex ante equity premium, and we determine a range of ex ante equity premia most consistent with what has been observed in the US.

Overview of Methodology

We consider: based on an ex ante premium of $X\%$, what combinations of financial statistics are we likely to see?

The basic methodology:

- Assume a value for the equity premium

- Estimate models for the processes driving dividends and interest rates in the US economy.

Overview of Methodology Continued

- Use these models to simulate a variety of potential paths for US dividends and interest rates.

- Use a discounted-dividend model to calculate a time series of ex post fundamental stock returns for each simulated economy

- Compare the range of simulated statistics with the actual US outcome.

Fundamental Prices

$$P_t = \mathcal{E}_t \left\{ \frac{P_{t+1} + D_{t+1}}{1 + r_{f,t} + \pi_d} \right\} \tag{4}$$

$$P_t = D_t \mathcal{E}_t \left\{ \sum_{i=1}^{\infty} \left(\Pi_{k=1}^i \left[\frac{1 + g_{t+k-1}}{1 + r_{t+k-1}} \right] \right) \right\} \tag{5}$$

$$g_t \equiv (D_{t+1} - D_t)/D_t \qquad y_t \equiv \frac{(1 + g_t)}{(1 + r_t)}$$

Fundamental Prices Continued

$$P_t = D_t \mathcal{E}_t \left\{ \sum_{k=0}^{\infty} \left(\Pi_{i=0}^{k} y_{t+i} \right) \right\} \qquad (3)$$

or $P_t = D_t \mathcal{E}_t \left\{ y_t + y_t y_{t+1} + y_t y_{t+1} y_{t+2} + \cdots \right\}$

$$R_t = (P_{t+1} + D_{t+1} - P_t)/P_t$$

Simulation Details and Discussion

- There is a large body of work that simulates stock prices and dividends to investigate price and dividend behavior.

    - These studies typically obtain returns from some variant of the Gordon [1962] model and/or a log-linear approximating framework.

    - These models almost always lead to constant dividend yields and very smooth returns.

Simulation Details and Discussion Continued

- Instead we simulate the dividend growth and discount rate processes directly, and evaluate the expectation through numerical integration.

    - Using models that capture the serial dependence of dividend growth rates and interest rates observed in the data leads to time-varying dividend yields and variable returns matching the S&P 500.

Exhibit 1

Diagram of a Simple Market Price Calculation for the tth observation of the nth economy

$$P_t^n = \sum_{j=1}^{J} P_t^{j,n} / J$$

$D_t^n, g_{t-1}^n, r_{t-1}^n$

$g_{t+i}^{1,n}, r_{t+i}^{1,n}$

$g_{t+i}^{2,n}, r_{t+i}^{2,n}$

$g_{t+i}^{J,n}, r_{t+i}^{J,n}$

$P_t^{1,n}$

$P_t^{2,n}$

$P_t^{J,n}$

$i = 0, \cdots, I$

J Possible Paths of Economy n
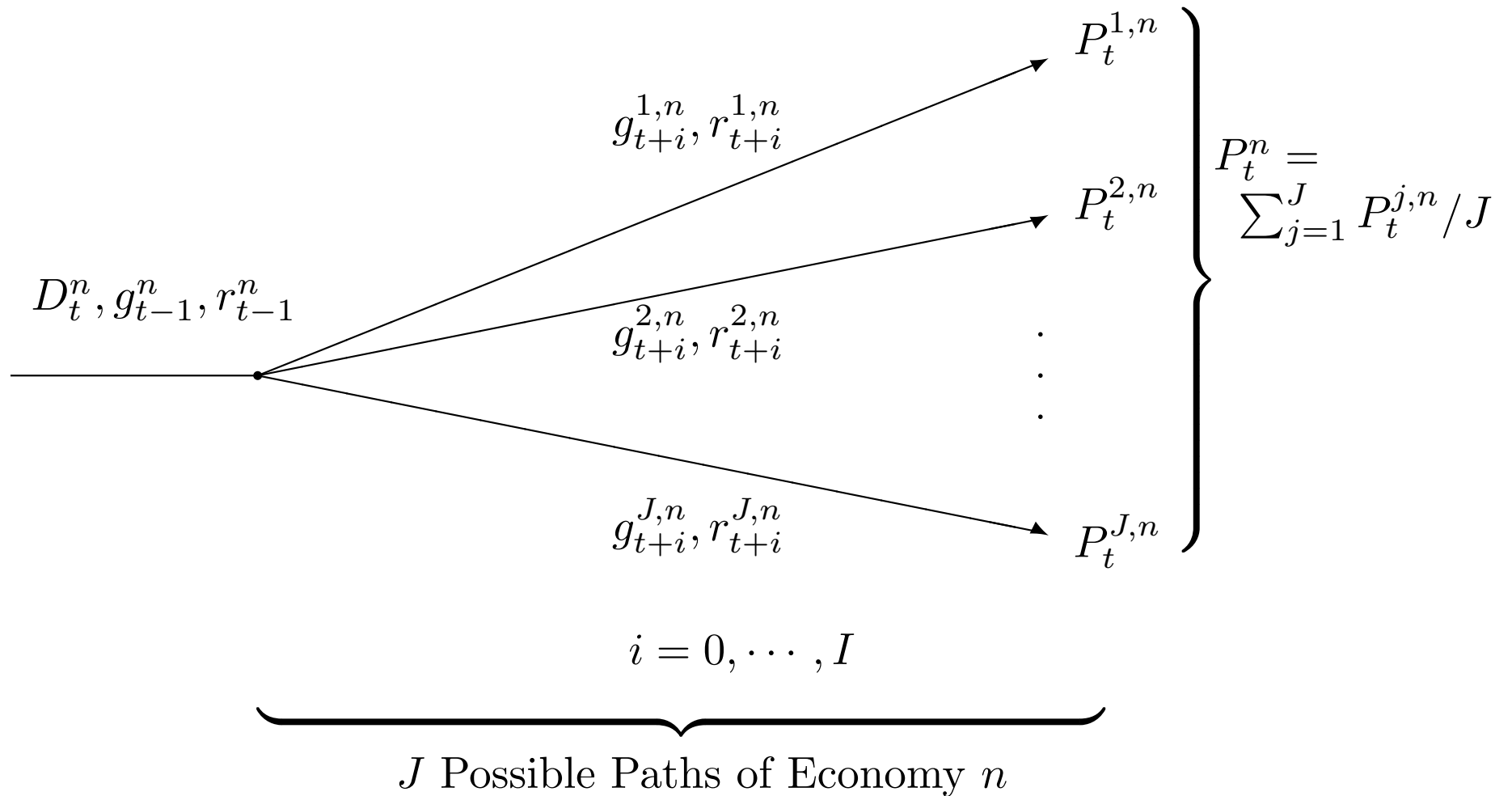
Data

- Calibrate simulations to US dividend and interest rate data, not returns.

- We use annual S&P 500 dividend data and 1-year T-bill rates over 1952-2002.

Models for Interest rates and Dividends

- We estimate either an AR(1) or ARMA(1,1) model of the logarithm of interest rates and the logarithm of (1 + dividend growth rate).

- Standard specification tests for normality, autocorrelation and ARCH do not reject the null of no misspecification.

- The error terms from these two models are correlated.

Time-Varying Equity Premia

- We also consider various forms of time-varying equity premia linked to a conditional version of Merton's [1980] CAPM.

  – Autocorrelated Equity Premia

  – Downward Trending Equity Premia

  – Structural Breaks in Equity Premia

Parameter Uncertainty

- If we aren't certain of the equity premium, can we be certain of our model or parameter estimates? We also incorporate parameter uncertainty and estimation error.

Quick Look at DGPs

r & g	AR(1) EQ	Trended EQ	Break in EQ	Param. Uncert.	Agents Uncert.
ARMA	Yes	No	No	Yes	Yes
ARMA	Yes	Yes (1%)	Yes (0.5% drop)	Yes	Yes
ARMA	Yes	Yes (2%)	No	Yes	Yes
ARMA	Yes	No	Yes (0.5% drop)	Yes	Yes
ARMA	No	No	No	Yes	Yes
AR	No	No	No	Yes	Yes
AR	No	No	No	No	No

Statistics on Ex Post Equity Premium Estimates
for the Simulated Market Economies
(Based on Various Ex Ante Equity Premia)

Ex Ante Equity Prem.	Mean of Sim. $\hat{\pi}_e$	Percentiles of Simulated $\hat{\pi}_e$				
		1%	5%	50%	95%	99%
2 %	2.032	-7.008	-2.977	2.452	5.316	6.244
2.5 %	2.516	-5.230	-1.773	2.907	5.626	6.847
3 %	2.954	-5.198	-1.518	3.345	6.076	7.107
3.5 %	3.498	-3.911	-0.625	3.872	6.494	7.278
4 %	3.980	-3.319	0.027	4.344	6.825	7.618
4.5 %	4.532	-2.268	0.705	4.934	7.317	8.200
5 %	5.024	-1.610	1.334	5.397	7.662	8.522
6 %	6.040	-0.099	2.362	6.304	8.561	9.248

Figure 1: Probability Distribution Functions
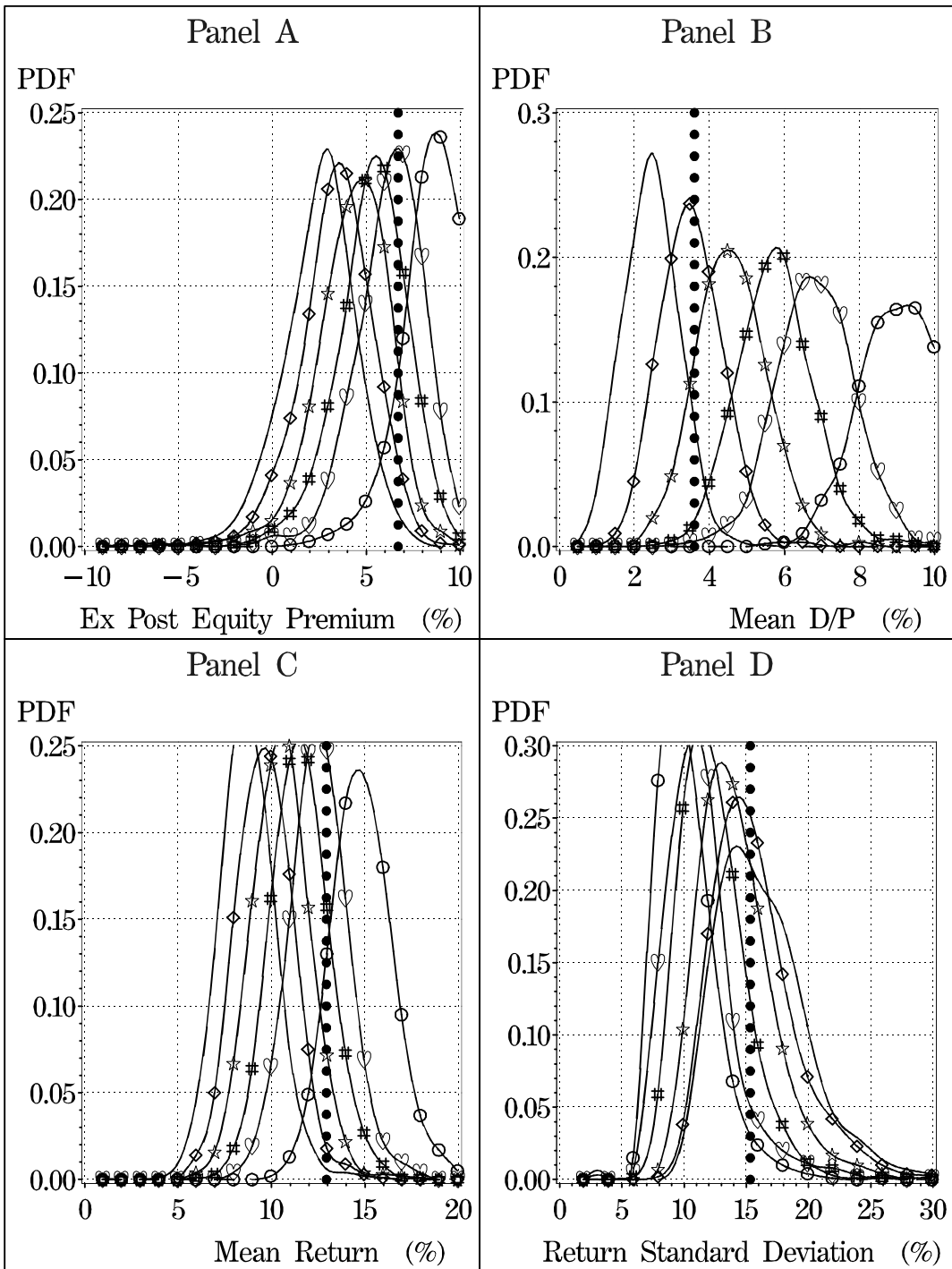
Figure 2: Probability Distribution Functions
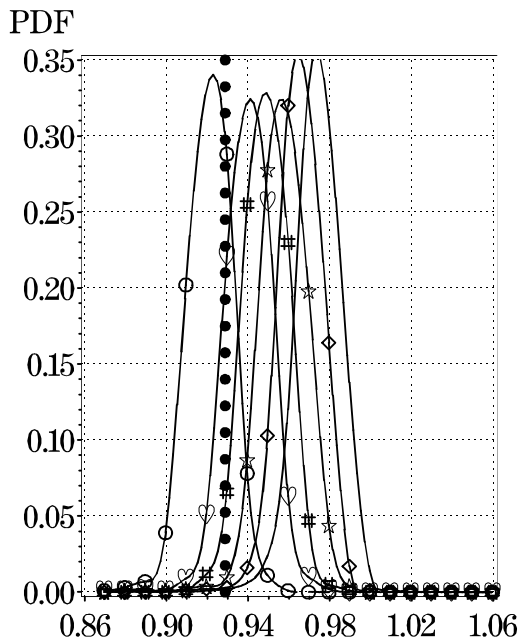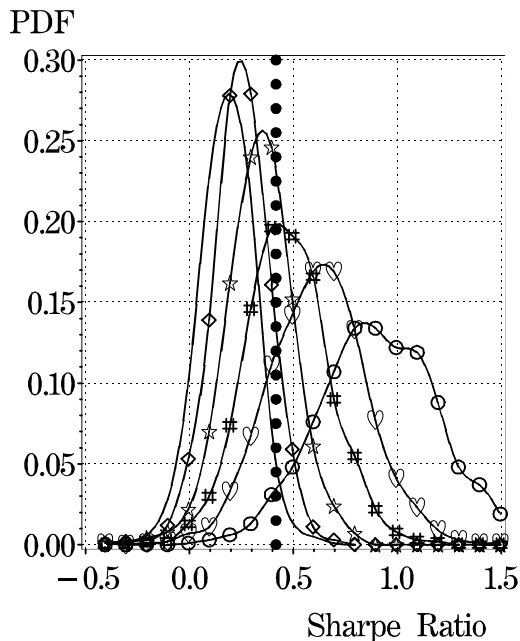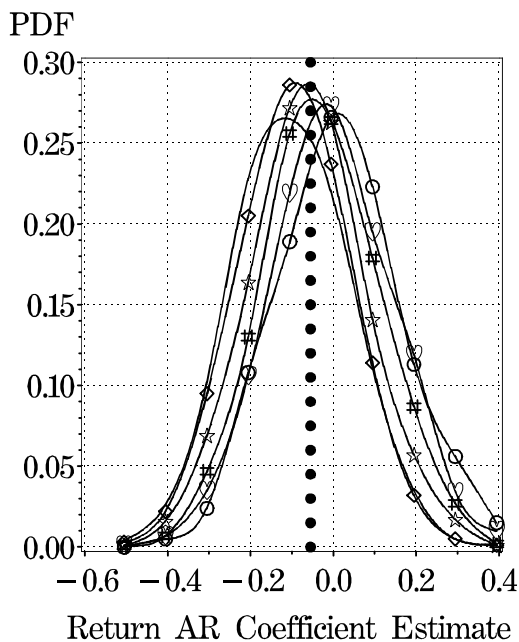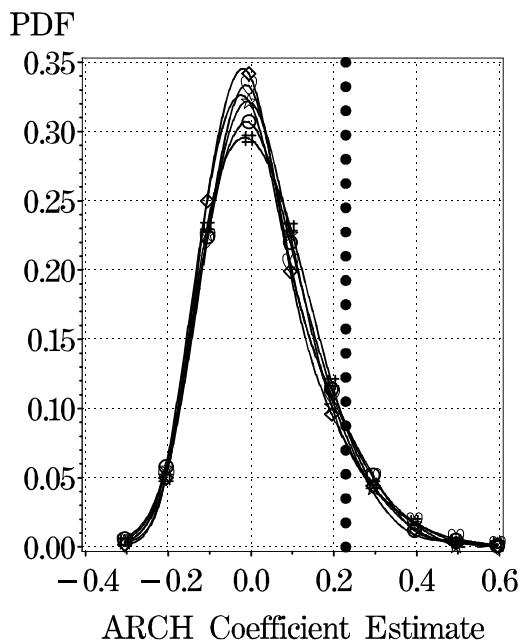
Univariate versus Multivariate Distributions

- From univariate plots shown in Figures 1 and 2, it is clear that the US data are consistent with ex ante equity premia between 2 and 6%.

- Next we consider whether joint distributions of the simulated financial statistics help us to further narrow the range of plausible ex ante equity premia.

  - We consider χ^2 statistics to test whether joint distributions of the simulated data are consistent with values observed in the US.

Univariate vs Multivariate Distributions Cont'd

- We do not consider variables to which we calibrate our experiments: interest rates and dividend growth rates.

- The χ^2 statistics:
  * Tests on individual moments of the data (mean return, etc.);
  * a joint test based on the mean return, return standard deviation, mean dividend yield, and ex post equity premium.
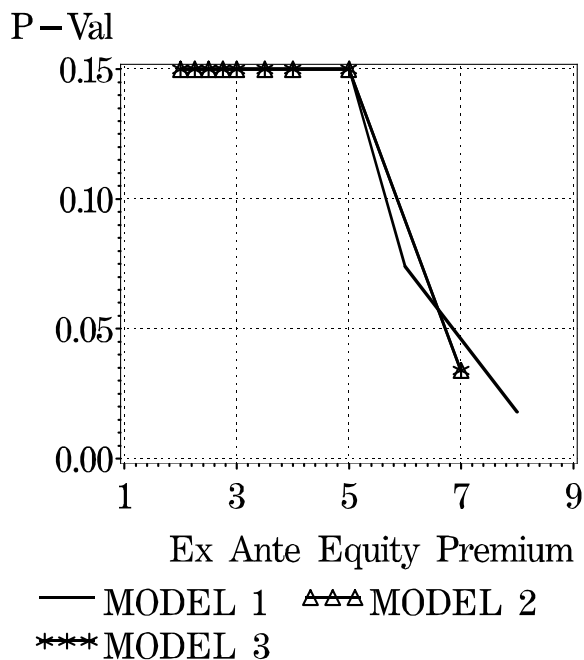
Figure 3: P-Values for Models 1, − 3

Panel A:
Joint Test
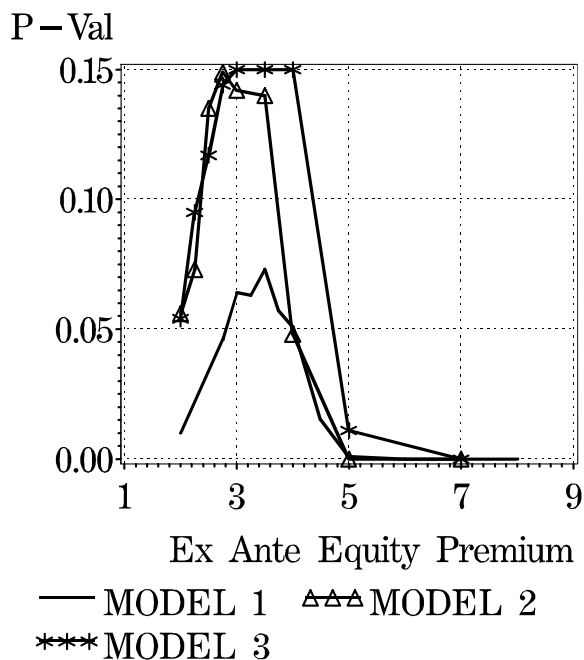
Panel B:
Test on Return Mean
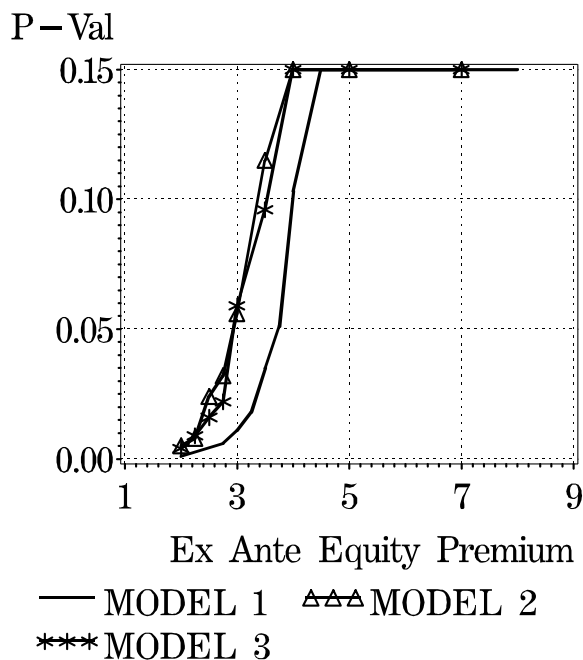
Panel C:
Test on Return Std Dev.

Panel D:
Test on Dividend Yield
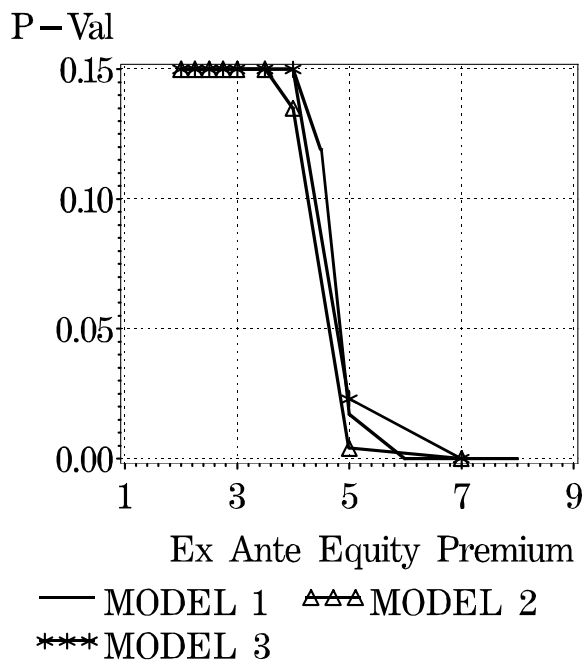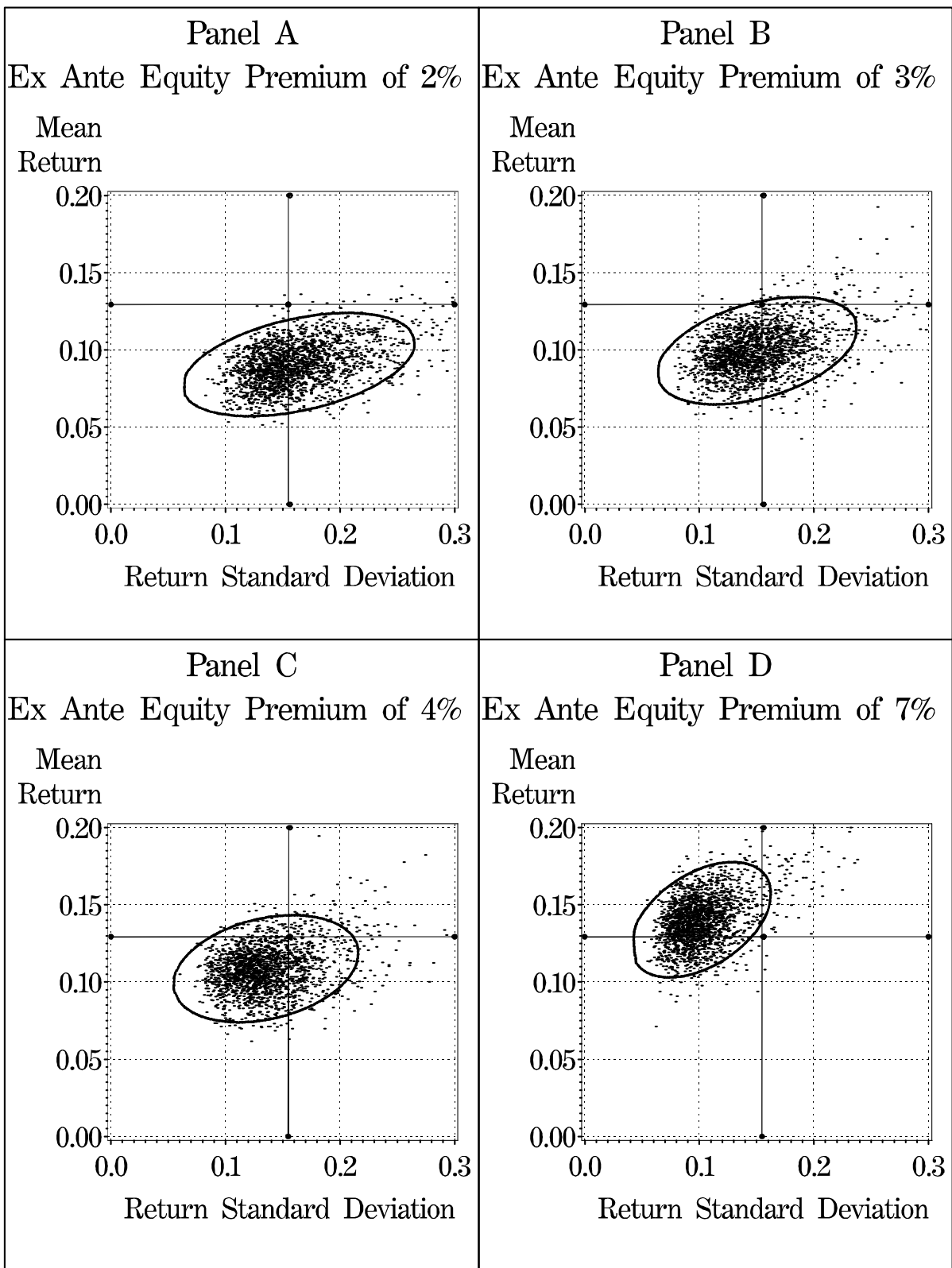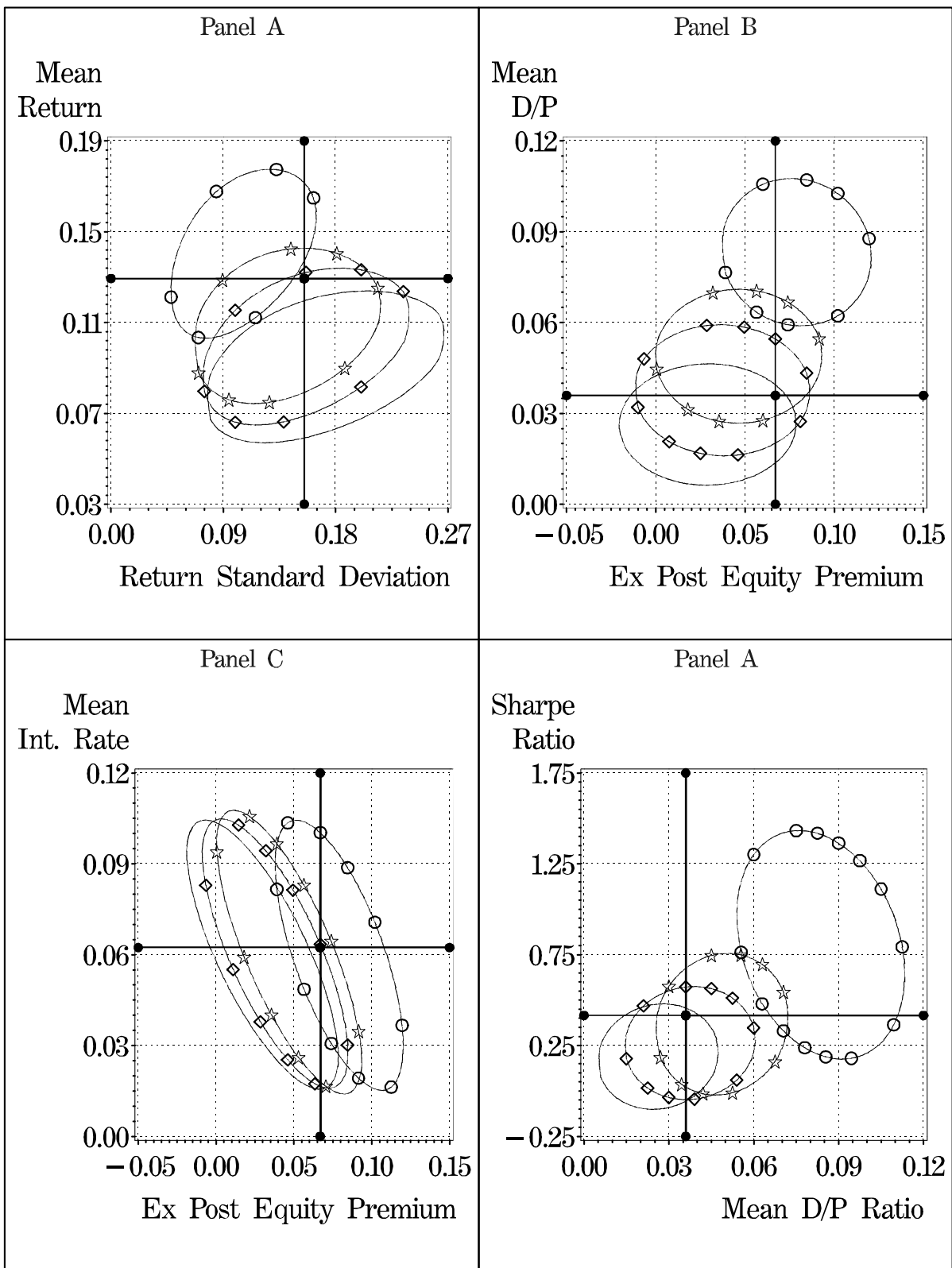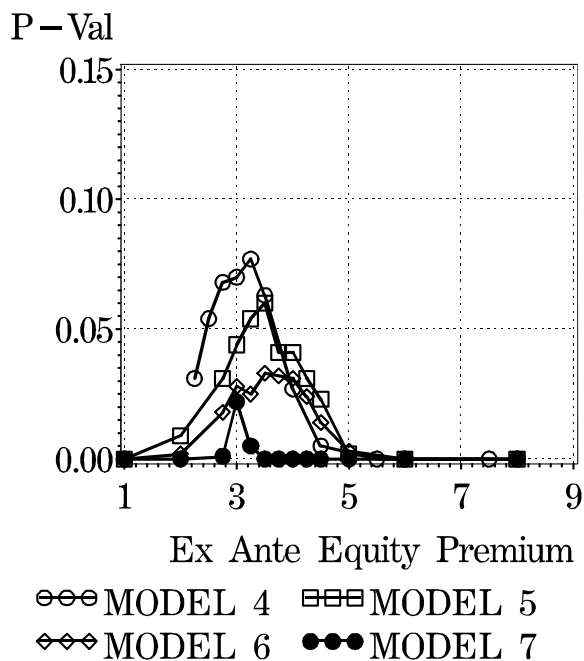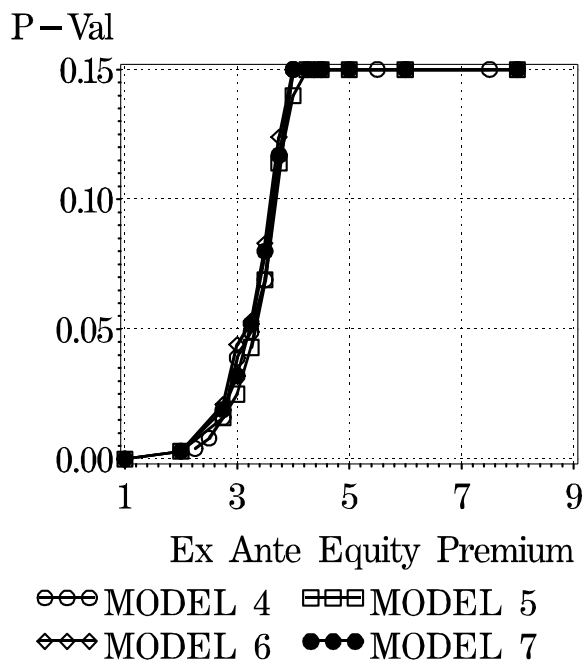
Figure 4: Bivariate Distributions

Panel A
Ex Ante Equity Premium of 2%

Panel B
Ex Ante Equity Premium of 3%

Panel C
Ex Ante Equity Premium of 4%

Panel D
Ex Ante Equity Premium of 7%

Figure 5: Bivariate Distributions

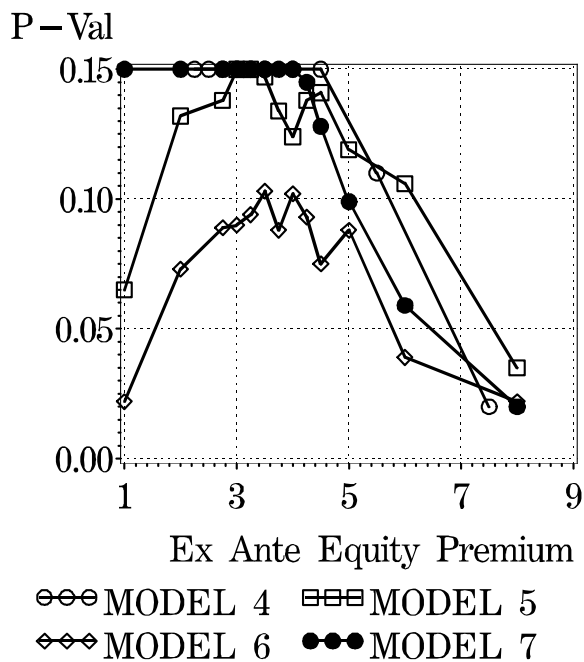# Figure 6: P-Values for Models 4 − 7
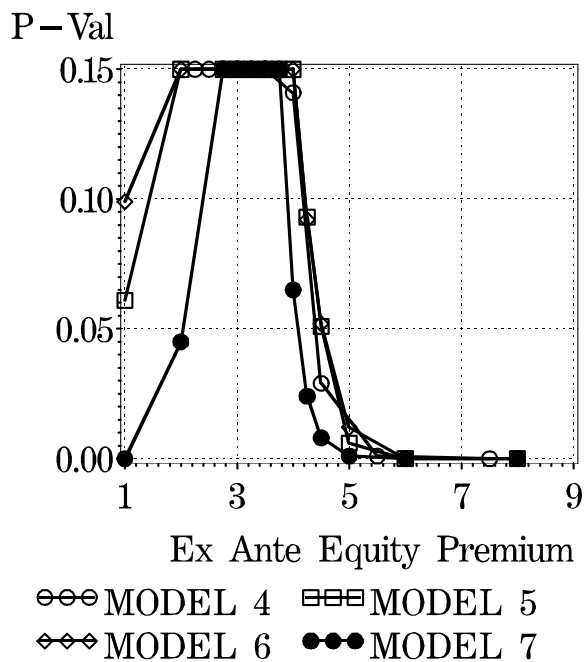

Panel A:
Joint Test

Panel B:
Test on Return Mean

Panel C:
Test on Return Std Dev.

Panel D:
Test on Dividend Yield

Conclusions

- We have devised a simulation method to evaluate the distribution of various financial statistics based on different values of the ex ante premia.

- Our findings are fairly robust to changes in the values to which we calibrate our simulations and to time-varying equity premia.

- Extensions underway explore a richer set of models and model uncertainty on the part of the agents.