Towards generalized complex mirror symmetry

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Mainly based on
hep-th/0406137
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with: M.Graña, M.Petrini, R.Minasian
and work in progress: A. Kashani-Poor

Introduction

- An old idea: use forms rather than a metric

six dimensions: SU(3) structure

spinor ϵ without zeros

or

ω real, Ω complex such that

$$\omega \wedge \Omega = 0 \ , \quad i\Omega \wedge \overline{\Omega} = \frac{1}{3!}\omega^3$$

It determines a metric (O(6) structure)

SU(3) < O(6)

$$\omega_{i\bar{j}} = i g_{i\bar{j}}$$

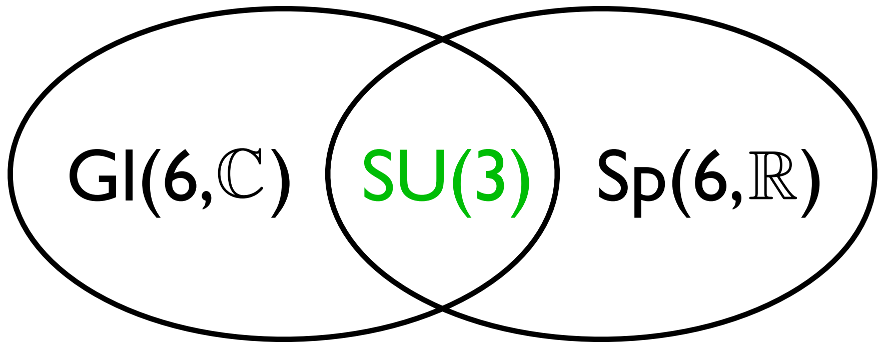
- To what extent can we rewrite string dynamics using forms?

Each half has a meaning:

$\Omega \longrightarrow {J_m}^n$ (no differential property yet) ω symplectic

Gl(6,$\mathbb{C}$) structure

Sp(6,$\mathbb{R}$) structure



Each half has its dynamics:

B model		A model
(imposes $d\Omega = 0$)	(not CY yet) $\longleftrightarrow$	(imposes $d\omega = 0$)
Complex	Mirror symmetry	Symplectic

[already Kontsevich, ICM '94]

Fluxes allow us to generalize Calabi-Yau

susy + only metric on $\Longrightarrow$ $\delta_\epsilon \psi_M = \boxed{\nabla_M \epsilon = 0}$

Calabi-Yau

With fluxes:

	IIA	IIB
NS & RR	symplectic	complex
NS only	complex	

Finer classification: Types of allowed manifolds $\longleftrightarrow$ Branes

Examples:	conformally CY	D3
	"Maldacena-Nuñez"	NS5

Mirror symmetry on branes is very well-understood

(on Calabi-Yau)

[Douglas, Kontsevich, Seidel-Thomas...]

Examples:

Brane	B: holomorphic	A: Special Lagrangian
F-term	$Im(e^{i\theta}e^{i\omega})=0$	$Im(e^{i\theta}\Omega)=0$
Central charge:	$\int_{CY} e^{i\omega}e^{b}ch(F_{\rm brane})\sqrt{Td(CY)}$	$\int_{\rm brane}\Omega$

- Can we use $e^{i\omega}$ and Ω also to characterize the flux (non CY) geometries?

⇒ generalized complex geometry

- If a mirror symmetry still exists, how does it act on topology?

- How are the topological rule and differential geometric mirror symmetry rules related? Is one encoded in the other? truncation to light modes of Hitchin functionals?

⇒ tough mathematical questions, in general; will see some examples

Plan

- Classify $\mathcal{N} = 1$ compactifications with SU(3) structures; generalized complex geometry
  - Big picture: spontaneous susy breaking; what saves us from instantons
    - Generalized mirror symmetry and topology; what replaces cohomology

Type II supergravity and SU(3) structure

We want to rewrite supergravity in terms of structures as much as possible

[Graña, Petrini, Minasian, AT]

The SU(3) structure can be viewed as the internal spinor $\epsilon_\pm$

Fastest way to see forms is in Clifford algebra:

$$\epsilon_+ \otimes \epsilon_+^\dagger = e^{-i\omega}$$

$$\epsilon_+ \otimes \epsilon_-^\dagger = \Omega$$

For example $\Omega = \frac{1}{3!}\Omega_{mnp}\gamma^{mnp}$

Fortunately there is a formulation in which RR appears as a single element of Clifford algebra:

[Bergshoeff, Kallosh, Ortin, Roest, Van Proeyen]

$$\delta\psi_M = \left(D_M + \frac{1}{8}\ \not{H}_M \mathcal{P}\right)\epsilon + \frac{1}{16}e^\phi \boxed{\sum_n \frac{1}{2n!}\ \not{G}^{(2n)}}\Gamma_M \mathcal{P}_n \epsilon$$

We can rewrite susy transformations using combinations of these elements of Clifford algebra

G Ramond-Ramond

$\Omega \, , \; e^{i\omega}$ geometry

$$\mathrm{Tr}(G e^{i\omega}) \qquad \mathrm{Tr}(G\Omega\gamma_m) \;\; \ldots$$

With some manipulations... all solutions

complex: $d\Omega = 0$

symplectic: $d\omega = 0$

	IIA	IIB
NS & RR	symplectic	complex
NS only	complex	

[Really: complex $\Longleftrightarrow$ $(d\Omega)_{2,2} = 0$]

slight modification for our purposes of G-structure techniques [Gauntlett, Martelli, Pakis, Waldram], [...]

Compatible with mirror symmetry?

IIA

$$(d + H\bullet)(f e^{i\omega}) = 0$$

$$(d + H\bullet)(f'\Omega) = \mathrm{stuff}(G)$$

IIB

$$(d + H\bullet)(f e^{i\omega}) = \mathrm{stuff}(G)$$

$$(d + H\bullet)(f'\Omega) = 0$$

$H\bullet$ is $H_{mnp}(dx^{mn}\iota^p - \frac{1}{3}\iota^{mnp})$

2 wedges,
1 contraction

3 contractions

Maybe we should take more seriously

$e^{i\omega}$ and Ω

$\Rightarrow$ generalized complex geometry

[Hitchin, Gualtieri]

Generalized complex geometry

Use direct sum of cotangent and tangent bundles

$T \oplus T^*$ rather than T

Structure: Natural to consider SU(3,3) rather than SU(3)

It turns out that $e^{i\omega}$ and Ω each defines indeed SU(3,3) structure

(more generally a pure spinor does that)

Together they define SU(3)xSU(3) structure on $T \oplus T^*$

think of this as the right definition of SU(3) structure

It makes it more reasonable to think that $e^{i\omega} \leftrightarrow \Omega$

is a mirror symmetry

Differential conditions:

Then: we can use Courant bracket rather than Lie

"Nijenhuis(pure spinor)=0" $\longleftrightarrow$ pure spinor is closed

Courant $[A,B] = dAB + AdB - ABd - BAd - (B \leftrightarrow A)$

A and B operators on forms $\longmapsto$ operator on forms

(examples: contractions i_v
wedges $\alpha\wedge$)

$\left\{ \begin{array}{c} \text{forms that annihilate} \\ \text{a pure spinor} \end{array} \right\}$

closed under this bracket

A large class of brackets ("derived") can be obtained by

$d \longrightarrow$ other differential (it squares to zero)

[Kosmann-Schwarzbach; Vinogradov]

in particular:

b two-form:

$$A \rightarrow e^{b} A e^{-b}$$

$$B \rightarrow e^{b} B e^{-b}$$

$$d \longrightarrow d + db\wedge$$

But we had $d + H\bullet$

mixed contractions and wedges

It does not square to zero! Open puzzle...

For eleven dimensional sugra on SU(3) structure (=spin) 7-manifolds

$$d\Phi + G\cdot\Phi + \Phi\cdot G = 0$$

$$\Phi = \{e^{i\omega}, e^{i\omega} v, \Omega, \Omega v\}$$

$$e^{i\omega}(1+v), \Omega(1+v)$$

together give SU(3)xSU(3)structure

worldsheet approach: [Lindstrom,Minasian, AT, Zabzine]

SU(3) structure compactifications

preserved $\mathcal{N}=2$	preserved $\mathcal{N}=1$	spontaneously broken $\mathcal{N}=2$
SU(3) holonomy (Calabi-Yau)	"twisted generalized" CY	SU(3) structure

- We can consider also SU(3) structure manifolds with no particular differential property
  - $e^{i\omega} \leftrightarrow \Omega$ is simply algebraic
    - Mirror symmetry can still make sense: same $\mathcal{N}=2$ action (not necessarily vacua)

Obviously this mirror symmetry is far from proven.
We can use some features of $\mathcal{N}=2$ actions to test it

Calabi-Yau effective action (IIA):

$h^{1,1}$ vector multiplets

$h^{2,1}+1$ hypermultiplets

(classical) dynamics:
prepotentials

$\mathcal{F}_0(e^{i\omega})$ $\mathcal{F}_0(\Omega)$

In presence of fluxes, hypermultiplet isometries get gauged

Example: H NS three-form $\int_{\alpha_a} H = p_a \quad \int_{\alpha_a} C_3 = \xi_a$

In other words:
H is a moment map on the space of Ω s

$$\boxed{D\xi_a = d\xi_a + p_a A}$$

gauging of a "translation" $\xi_a \longrightarrow \xi_a + \epsilon p_a$

One can view similarly $d\omega$ as a moment map on the space of Ω

hyper-moment map

$\Updownarrow$

$\mathcal{N}=2$ prepotentials

"Non simplecticity" also induces a gauging

[Hitchin; Grana, Louis, Waldram]

For the flux, the gauging extracts the integral part: $\int_{\alpha_a} H$

What is the integral part of $d\omega$?

Proposal:

[Gurrieri, Louis, Micu, Waldram]

$$d\omega = \sum_a e_a \beta^a$$

related to gauging of ξ_a

expand it in a basis of "massive harmonics"

$$\Delta\beta^a = m_a \beta^a$$

Local computation with T^3 fibrations

$$(\nabla\omega + H)_{ijk} \leftrightarrow (\nabla\omega + H)_{i\bar{j}\bar{k}}$$

The two types of gauging (flux and differential-geometric) are mixed by mirror symmetry

[GLMW; Fidanza, Minasian, AT]

Before expanding on the differential geometry of gaugings:

Are these gaugings only classical?

[work in progress with A. Kashani-Poor]

gauged supergravity requires
that the directions to be gauged be isometries

But generically isometries of hypermultiplets moduli spaces are
lifted by instantons

Quantum effects would
spoil previous slide

$\boxed{\text{Why?}}$

$$e^{\int_\Gamma C} = e^{\sum_a c_a \xi^a}$$

instanton

dependence
on the ξ^a

However $dF = H$

If H wraps Γ
there is a nonzero
tadpole for F on Γ $\Longrightarrow$ unless $\displaystyle\int_\Gamma H = \sum c_a p^a \neq 0$

isometry on
instanton: $\displaystyle\sum p^a \partial_{\xi^a}\left(\sum c_b \xi^b\right) = \sum c_a p^a$

Flux protects the isometry it wants to gauge
from quantum corrections

Topology of gaugings and mirror symmetry

By constructing an explicit example, we will test the proposals of [GLMW] on gaugings

The basic phenomenon is standard:

e.g. for circle fibrations

$$\boxed{\text{Chern class of the fibration} \longleftrightarrow H \text{ contracted on the fibre}}$$

a nontrivial fibration with no B-field goes into a trivial one with a B-field

We want actually to start from a Calabi-Yau with B-field

[Strominger-Yau-Zaslow]

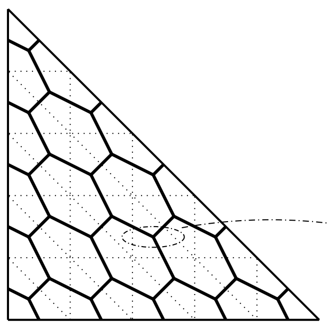
$$\begin{tikzcd} T^3 \arrow[r, hook] & \mathrm{CY} \arrow[d] \\ & S^3 \end{tikzcd}$$

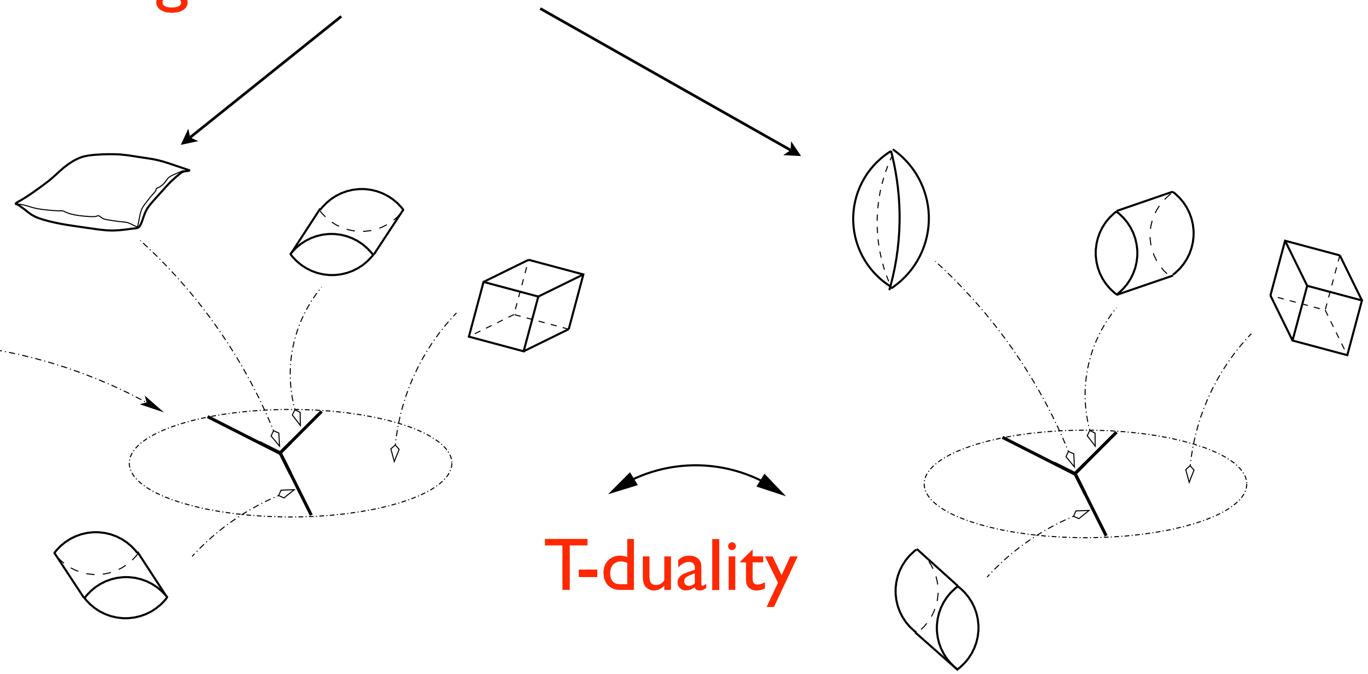
All CY are fibrations

Naively we would have suspicious Betti numbers:

$$\begin{array}{c|cccc} & 1 & 0 & 0 & 1 \\ & 3 & 0 & 0 & 3 \\ & 3 & 0 & 0 & 3 \\ \text{fibre} & 1 & 0 & 0 & 1 \\ \hline & & \text{base} & & \end{array}$$

So there must be singular fibres.





T-duality

The quintic has
10 such faces

which triangulate the base S^3

The cohomology becomes:

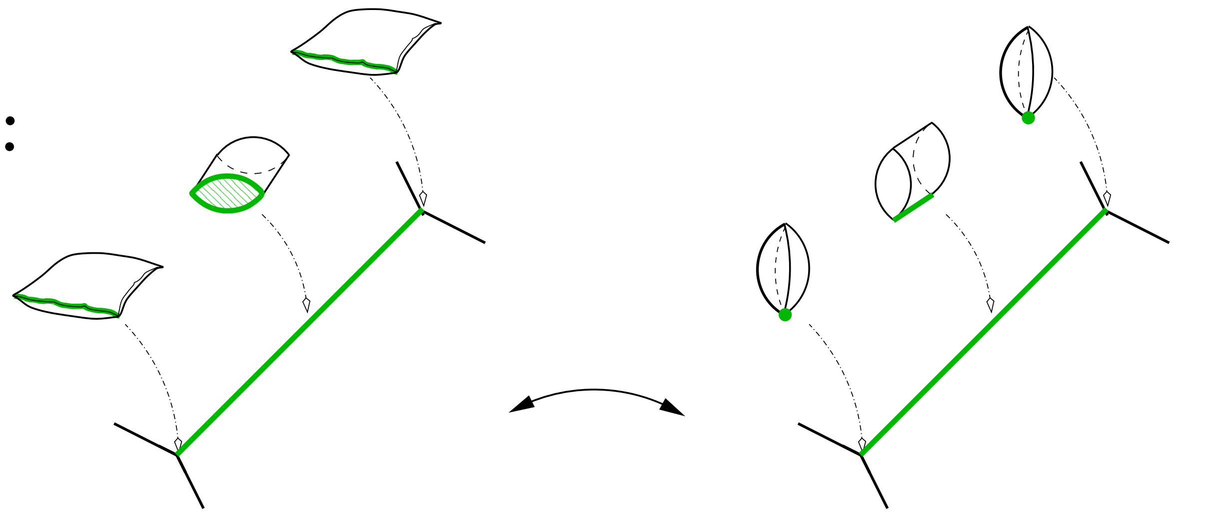
$$\begin{array}{cccc}
\mathbb{Z} & 0 & 0 & \mathbb{Z} \\
0 & H^1(B, R^2\pi_*\mathbb{Z}) & H^2(B, R^2\pi_*\mathbb{Z}) & 0 \\
0 & H^1(B, R^1\pi_*\mathbb{Z}) & H^2(B, R^1\pi_*\mathbb{Z}) & 0 \\
\mathbb{Z} & 0 & 0 & \mathbb{Z}
\end{array}$$

For a CY the nontriviality of the fibration is given by the monodromies around the singular loci these groups are cycles corresponding e.g. to paths:

When we add B-field by analogy with the case without monodromies we expect some "Chern class"

H contracted on the fibres is no longer in $H^2(\mathrm{base})$ (which vanishes)

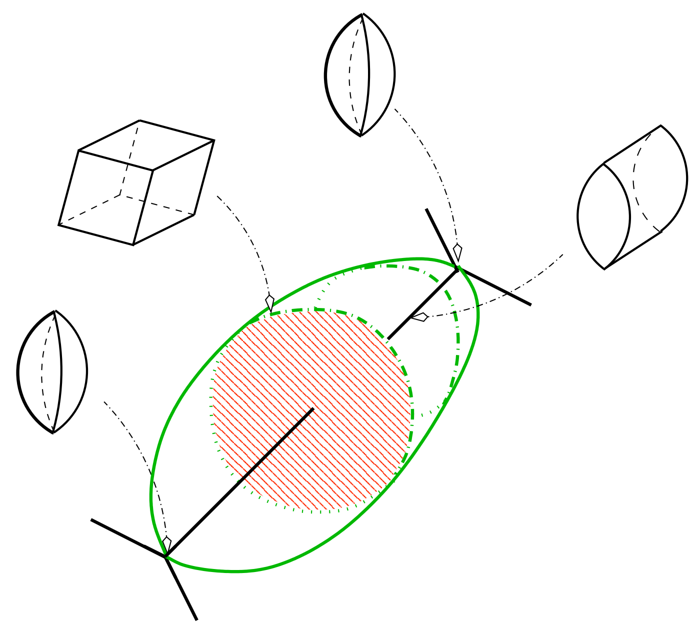
We can still define a "twisted Chern class" in $H^2(B, R^2\pi_*\mathbb{Z})$



3-cycle on CY

$H^1(B, R^2\pi_*\mathbb{Z})$

2-cycle on mirror CY



This defines the topology of a (half-flat) mirror to CY + flux

cohomology:

$$\begin{array}{cccc}
\mathbb{Z} & 0 & 0 & \mathbb{Z} \\
0 & H_2^1 & H_2^2 & 0 \\
0 & H_1^1 & H_1^2 & 0 \\
\mathbb{Z} & 0 & 0 & \mathbb{Z}
\end{array}
\longrightarrow
\boxed{\begin{array}{cccc}
0 & 0 & 0 & \mathbb{Z} \\
0 & H^1(B, R^2\pi_*\mathbb{Z}) & H^2(B, R^2\pi_*\mathbb{Z})/\mathbb{Z} \oplus \mathbb{Z}_N & 0 \\
0 & H^1(B, R^1\pi_*\mathbb{Z})/\mathbb{Z} & H^2(B, R^1\pi_*\mathbb{Z}) & 0 \\
\mathbb{Z} & 0 & 0 & \mathbb{Z}_N
\end{array}}$$

Notice: $\mathbb{Z}_N$

the torsion gives the mirror to H and thus the gauging

Compare: $S^2 \times S^1$ versus $S^3/\mathbb{Z}_N$

$$\begin{array}{ccc}
\mathbb{Z} & 0 & \mathbb{Z} \\
\mathbb{Z} & 0 & \mathbb{Z}
\end{array}
\longrightarrow
\begin{array}{ccc}
0 & 0 & \mathbb{Z} \\
\mathbb{Z} & 0 & \mathbb{Z}_N
\end{array}$$

What happens to the disappearing cycles?

$$d\omega_i = E_i \alpha_0 \ , \qquad d\alpha_A = 0$$
$$d\beta^A = \delta^{A0} E_i \widetilde{\omega}^i \ , \qquad d\widetilde{\omega}^i = 0$$

(conjectured in [GLMW])

Together with the remaining ones:

needed for KK reduction and for gaugings

For more general cases (not mirrors to CY + flux) $\Longrightarrow$ $E_i \to E_i{}^a$ [D'Auria, Ferrara, Trigiante, Vaula']

in the present case we see instead naturally rank one.

Conclusions

- Differential geometry of brackets useful for supersymmetry
  - Mirror symmetry can still be defined using spontaneously broken supersymmetry
    - Truncating the spectrum of SU(3) compactifications leads to interesting questions in topology vs. differential geometry