

Moduli Stabilization in F-Theory

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Fields Institute, Toronto, March 2005

Abstract

Based on

- hep-th/0404257 w/ F. Denef and M. R. Douglas
- hep-th/0503124 w/ F. Denef, M. R. Douglas, A. Grassi and S. Kachru

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1. Overview

- Brief review of KKLT mechanism for moduli stabilization in IIB orientifolds
- Work w/ **Denef** and **Douglas** on Kähler moduli stabilization in KKLT-type models
 - models
 - difficult to explicitly show that complex structure moduli and $D7$ -brane moduli are stabilized
- Simple model w/ all moduli explicitly stabilized

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2. Review of KKLT Construction

- Well-known proposal outlining a multi-step construction of metastable de Sitter vacua in IIB string theory
- KKLT argue that in this class of compactifications all moduli can be fixed and the physics that lifts the degeneracy of the moduli space is under good control.
- But they lacked explicit examples!

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2. Review of KKLT Construction

- Well-known proposal outlining a multi-step construction of metastable de Sitter vacua in IIB string theory
- KKLT argue that in this class of compactifications all moduli can be fixed and the physics that lifts the degeneracy of the moduli space is under good control.
- But they lacked explicit examples!
- Start w/ F-theory compactification on Calabi-Yau 4fold X

$$\begin{array}{ccc} T^2 & \longrightarrow & X \\ & & \downarrow \pi \\ & & B \end{array}$$

and turn on fluxes (KKLT step 1):

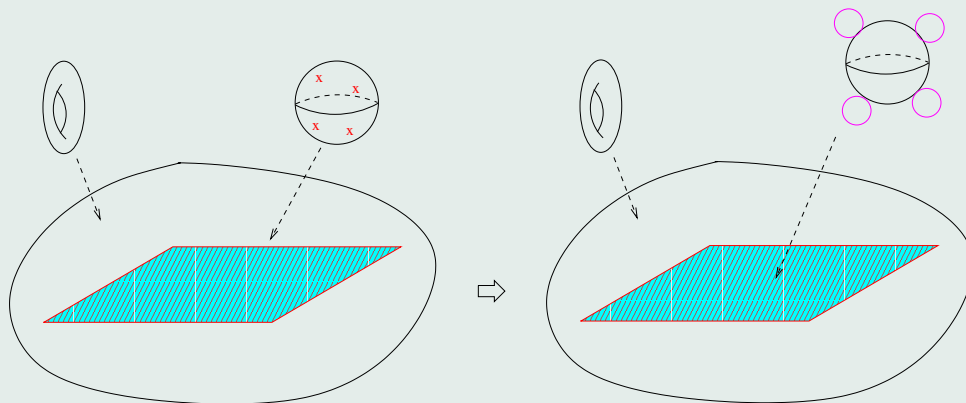
- generate a nontrivial superpotential for the complex structure moduli of the Calabi-Yau 4fold X ;
- go near the orientifold limit.

- Orientifold limit: X degenerates to $(Z \times T^2)/\mathcal{I}$, where
 - Z is a Calabi-Yau 3fold ('orientifold limit');
 - \mathcal{I} is a holomorphic involution on Z acting as -1 on T^2 ;
 - Z is a double cover of B

$$Z \xrightarrow{2:1} B$$

branched along the fixed locus of \mathcal{I} ;

- Fiber is smooth everywhere except on top of the branch locus, where degenerates to T^2/\mathbb{Z}_2 ;



- Complex structure moduli of X (type IIB perspective):
 - complex structure moduli of Z
 - axion-dilaton
 - branch locus data ($D7$ brane moduli)
- Kähler moduli of X (type IIB perspective): Kähler moduli of B

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- Moduli superpotentials from fluxes

- start with M-theory on X ;
- pick $[G^{(4)}] \in H^4(X, \mathbb{Z}) \rightsquigarrow$ get susy solutions if adjust complex structure and Kähler structure of X such that (Becker²):
 - * $G^{1,3} = 0$;
 - * $J \wedge G^{(4)} = 0$ (J is the Kähler form).
- constraints can be derived from superpotential interactions (Gukov, Vafa, Witten):

$$W_{\Omega} = \frac{1}{2\pi} \int_X \Omega \wedge G^{(4)};$$

$$W_J = \int_X J \wedge J \wedge G^{(4)}.$$

- turning on fluxes \rightsquigarrow complex structure of X is generically fixed ($h^{3,1}$ equations on $h^{3,1}$ moduli); constraints imposed on the Kähler moduli.

- What about F-Theory?

- $4D$ Lorentz invariance \rightsquigarrow constraints on $[G^{(4)}]$: the 4-cycle Poincaré dual to $[G^{(4)}]$ is neither contained in the base, nor is an elliptic fibration over a curve in the base;
 - $J = \sum_i t_i D_i$, D_i is a basis of the Kähler cone of X ;
 - the generators of the Kähler cone are $\{D_i\} = \{\Sigma, \pi^* C_j\}$, where Σ is the section of the elliptic fibration and $\{C_j\}$ is a basis for the Kähler cone of B ;
 - therefore $W_J \equiv 0 \rightsquigarrow$ **no constraints on Kähler moduli**;
- F-theory: turn on fluxes \rightsquigarrow generate a nontrivial superpotential for the complex structure moduli \rightsquigarrow go near the orientifold limit (weak string coupling);
 - Hard to pick a flux $[G^{(4)}]$ that realizes this explicitly! Calabi-Yau 4folds have typically **thousands** of complex structure moduli
 - At most, one can hope to fix the complex structure moduli of Z plus the axion-dilaton and try to argue that $D7$ -brane moduli are also stabilized.

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- Complex structure moduli stabilization for Z
 - flux superpotential: $W_\Omega = \int_Z \Omega \wedge G^{(3)}$, $G^{(3)} = F^{(3)} - \tau H^{(3)}$;
 - $D_i W = 0$, $i = 1, \dots, h^{2,1}(Z) \rightsquigarrow G^{1,2} = 0$;
 - $D_\tau W = 0 \rightsquigarrow G^{3,0} = 0$;
 - $G^{(3)} \in H^{2,1}(Z, \mathbb{Z}) \oplus H^{0,3}(Z, \mathbb{Z}) \rightsquigarrow$ complex structure moduli and axion-dilaton are generically stabilized.
- Potential for moduli fields ($\mathcal{N} = 1$ SUGRA formula):

$$V = e^{\mathcal{K}} \left(\sum_{a, \bar{b}} g^{a\bar{b}} D_a W D_{\bar{b}} \bar{W} - 3|W|^2 \right),$$

a, \bar{b} run over all moduli fields; \mathcal{K} is the Kähler potential,

$$\mathcal{K} = -\log \text{Im} \tau - \log \int_Z \Omega \wedge \bar{\Omega} - 3 \log[-i(\rho - \bar{\rho})],$$

where $\rho \rightsquigarrow$ single Kähler modulus (for simplicity).

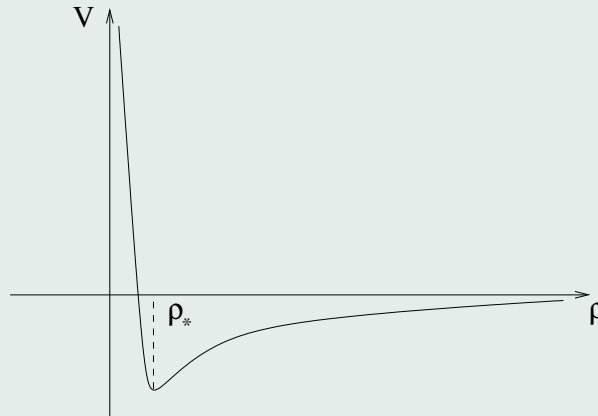
- easy to check that V is independent of $\rho \rightsquigarrow$ **the potential has a no-scale structure and Kähler moduli are not fixed.**

- **KKLT Step 2**: generate superpotential for the Kähler moduli and destroy the no-scale structure of the effective potential;
 - self-consistency: ignore α' corrections and stabilize the Kähler parameters at large values;
 - moreover, in this regime the effect of Kähler moduli variations on the complex structure moduli is exponentially suppressed \rightsquigarrow justified to stabilize complex structure moduli first and treat Kähler moduli separately.

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- Kähler moduli stabilization in an ideal scenario (1 Kähler parameter and (\exists) nonperturbative superpotential term depending on that parameter):

- $\mathcal{K} = -3 \log[-i(\rho - \bar{\rho})]$;
- $W = W_0 + B \cdot e^{-2\pi\rho}$, where $W_0 \rightsquigarrow$ flux contribution to the superpotential, independent of the size moduli;
- $D_\rho W = 0 \rightsquigarrow \rho_* = -i \log \frac{3W_0}{B}$;



- self-consistency: large volume $\rightsquigarrow W_0$ has to be small;

- Contributions to W come from $D3$ -brane instantons wrapping surfaces $D \subset B$ which satisfy the topological condition

$$\chi(\pi^{-1}(D)) = 1,$$

where $\chi(\mathcal{V}) = \sum_{i=0}^3 (-1)^i h^{0,i}(\mathcal{V})$ for \mathcal{V} a divisor in the Calabi-Yau 4fold X .

- This is the condition for an $M5$ instanton to have the right number of fermionic zero modes to contribute to the superpotential.

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- **Question:** How general is the above topological condition?
 - what if \mathcal{V} is not **spin**?
 - **Witten:** $M5$ worldvolume fermions are twisted; also spinors of the normal bundle;
 - possible to define fermions, even when \mathcal{V} is not spin

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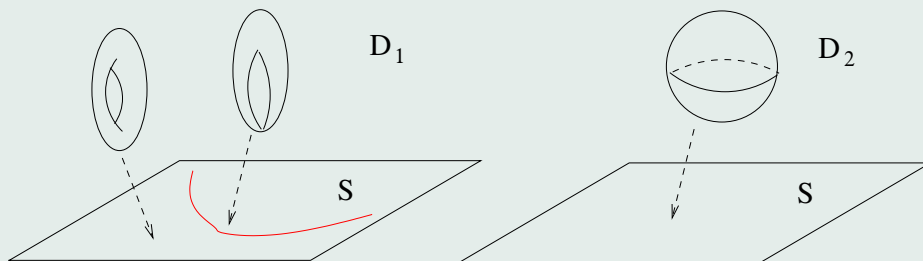
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 - fluxes can give mass to fermionic zero modes
 - **Görllich, Kachru, Tripathy, Trivedi:** $\chi > 1$ divisors can also contribute in the presence of fluxes
 - all our $M5$ -brane instantons are **rigid**: the higher cohomology groups vanish; they will always contribute to the superpotential

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- Search for vertical, holomorphic Euler characteristic 1 divisors in Calabi-Yau 4folds!

3. Kähler Moduli Stabilization

- **Program:** given X a Calabi-Yau 4fold, search for vertical divisors $\mathcal{V} \subset X$ such that $\chi(\mathcal{V}) = 1$.
- Such divisors fall into two classes:
 - $\mathcal{V} = \pi^*(D)$, where $D \subset B$ is a smooth divisor in the base;
 - components of the singular fibers.



- Conjecture (**Grassi**): divisors of the first type are always exceptional; (\exists) birational transformations of Calabi-Yau 4folds contracting these divisors; proved when B is Fano.
- B is Fano or toric \rightsquigarrow number of contributing divisors is finite:
 - they are the exceptional divisors associated w/ contracting one of the generators of the Mori cone, which is rational polyhedral;

- Kähler stabilization criterion: (\forall) Mori cone generator C , (\exists) a divisor D such that $\chi(\pi^*(D)) = 1$ and $D \cdot C < 0$.
 - in general, need to have $h^{1,1}(B)$ homologically distinct 4fold divisors of holomorphic Euler characteristic 1.
- Consider $\chi = 1$ divisors of the first kind: (\exists) examples that work: take B to be the Fano 3fold \mathcal{F}_{11} , $h^{1,1}(\mathcal{F}_{11}) = 3$. It has the following toric data:

| | D_1 | D_2 | D_3 | D_4 | D_5 | D_6 |
|-----------|-------|-------|-------|-------|-------|-------|
| $l^{(1)}$ | 0 | -2 | 1 | 1 | 1 | 0 |
| $l^{(2)}$ | -1 | 0 | 1 | 0 | 0 | 1 |
| $l^{(3)}$ | 1 | 1 | -1 | 0 | 0 | 0. |

- D_1, D_2, D_3 correspond to $\chi = 1$ divisors;
- take $W = W_0 + \sum_{i=1}^3 B_i \cdot e^{-2\pi\tau_i}$, where τ_i , $i = 1, 2, 3$ are the complexified volumes of $D_i \rightsquigarrow$ right holomorphic coordinates on the Kähler moduli space;
- take $W_0 = 10^{-30}$, $B_1 = B_2 = B_3 = 1$ and solve $D_{\tau_i} W = 0$, $i = 1, 2, 3 \rightsquigarrow \tau_i = (11.8, 11.9, 11.7)$, $V = 93.3 \rightsquigarrow$ achieved the proposed goal!

- loop correction to the Kähler potential (**Becker, Becker, Haack, Louis**):

$$\mathcal{K} = -2 \log \left[V + \frac{1}{2(2\pi)^3} \frac{\xi}{g_s^{3/2}} \right] + \text{const},$$

where $\xi = -\frac{\chi(Z)}{2}\zeta(3)$. This is at the percent level in our example; therefore ignoring α' corrections is justified.

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- What about complex structure moduli stabilization?
 - $h^{2,1}(Z) = 89 \rightsquigarrow$ impossible to find explicit flux vacua!
Need to compute 3fold periods: hypergeometric functions in 89 variables;
 - Ashok, Denef, Douglas developed techniques to estimate the number of flux vacua in a certain region of the moduli space; this involves computing the volume of that region in the moduli space;
 - by mirror symmetry, possible to compute the volume of an open set around large complex structure limit \rightsquigarrow compute volume of large radius limit region in the Kähler moduli space of the mirror (where only need classical part of the prepotential to describe geometry of the moduli space);
 - result: $\mathcal{N}_{vac}(LCS) \simeq 10^{-100} : ($
 - moving away from LCS $\rightsquigarrow (\exists)$ flux vacua, but stringy corrections become important; we argue that flux vacua do exist, but unable to provide explicit example;
- Little to say about $D7$ -brane moduli.

- What about examples with $\chi = 1$ divisors of the second type (components of the singular fibers)?

Start w/ X the elliptic fibration over \mathbb{P}^3 , $h^{3,1}(X) = 3878$: no $\chi = 1$ divisors, but can go to a codimension 2 locus in the complex structure moduli space where the Weierstrass model develops Kodaira type III* singularities along a $\mathbb{P}^2 \subset \mathbb{P}^3$.

$$\begin{array}{ccc} T^2 & \longrightarrow & X \\ & & \downarrow \pi \\ & & \mathbb{P}^3 \end{array}$$

- exceptional divisors appearing from resolving the elliptic fibration do have $\chi = 1$;
- can stabilize the (single) Kähler modulus of \mathbb{P}^3 ;
- again, hard to find fluxes that stabilize the complex structure of the Calabi-Yau 4fold X precisely at the singular locus of the Weierstrass model;
- also need to show that fluxes give mass to adjoint matter on $D7$ -branes worldvolume: very hard!

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 - also need to show that fluxes give mass to adjoint matter on $D7$ -branes worldvolume: very hard!
- Go to orientifold limit of a Calabi-Yau 4fold X by deforming the Kähler structure rather than the complex structure!

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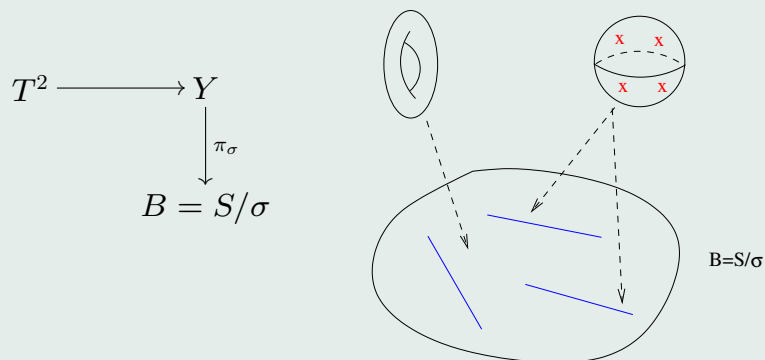
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4. Warm-up 6D Example

- Borcea-Voisin construction of elliptically fibered 3folds
 - Let S be a $K3$ surface and $\sigma : S \rightarrow S$ be a holomorphic involution, $\sigma^2 = \mathbb{1}$ such that $\sigma(\omega) = -\omega$, where ω is the holomorphic 2-form
 - Construct $S \times T^2 / (\sigma, -\mathbb{1})$ and resolve the singularities to obtain an elliptically fibered Calabi-Yau 3fold Y



- The fiber over B has I_0^* singularities along each component of the fixed locus of σ in B

- **Example:** take S to be a $K3$ orbifold, $S \simeq T^4/\mathbb{Z}_2$

- Y is the resolution of $(T^4/\mathbb{Z}_2 \times T^2)/(\sigma, -\mathbb{1}) \simeq T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$
- \mathbb{Z}_2 actions:

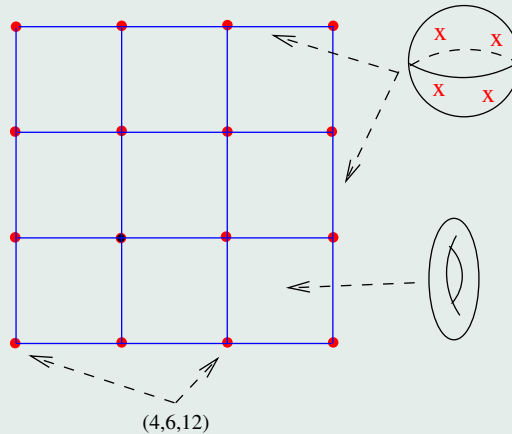
| | z_1 | z_2 | z_3 |
|----------------------|-------|-------|-------|
| α | + | – | – |
| β | – | – | + |
| $\alpha \circ \beta$ | – | + | – |

- $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ can be viewed as an torus fibration over $B = (T^4/\mathbb{Z}_2)/\sigma = T^2/\mathbb{Z}_2 \times T^2/\mathbb{Z}_2 \simeq \mathbb{P}^1 \times \mathbb{P}^1$
- the fibers degenerate to T^2/\mathbb{Z}_2 over the fixed point set of α and $\alpha \circ \beta$
- the fixed point set in the base consists of 4 copies of the line parameterized by z_1 and 4 copies of the line parameterized by z_2
- the fixed lines intersect in 16 points

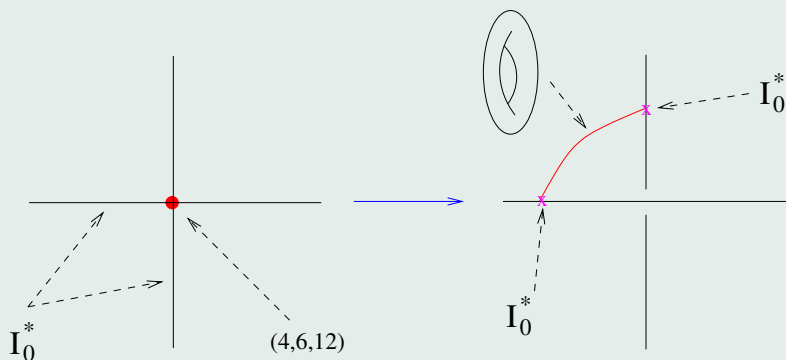
$$T^2 \longrightarrow T^6 / \mathbb{Z}_2 \times \mathbb{Z}_2$$

$$\downarrow \pi$$

$$T^4 / \mathbb{Z}_2 \times \mathbb{Z}_2$$



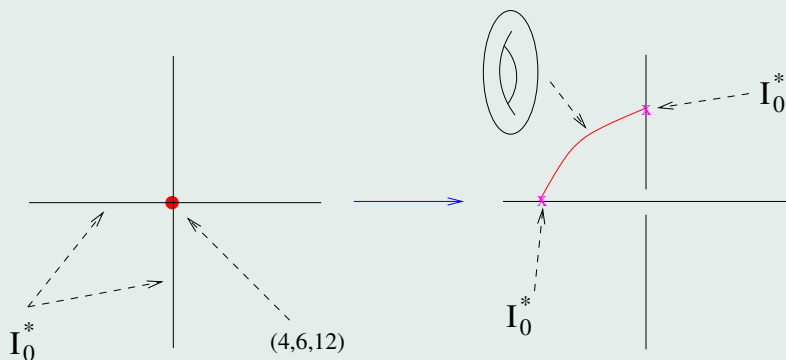
- IIB theory: singularities in the base correspond to locations of $O7$ -planes; on top of each $O7$ -plane there are 4 $D7$ -branes $\rightsquigarrow SO(8)$ worldvolume gauge theory
- Smooth 3fold \rightsquigarrow need to separate the $D7$ -branes that intersect transversely



- This modification of geometry is possible, but comes with a price: blow-up \rightsquigarrow introduce 16 new curves
- Therefore, introduce 16 new Kähler parameters; the $D7$ -branes do not intersect any longer
- Last step: resolve the elliptic fibration \rightsquigarrow the surfaces S which are the 3fold lifts of the new curves become **rigid**:

$$h^{1,0}(S) = h^{2,0}(S) = 0$$

- We considered an orientifold of T^4/\mathbb{Z}_2 and its **resolution**.



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- We considered an orientifold of T^4/\mathbb{Z}_2 and its **resolution**.
- Go one dimension higher!

5. Main Example

- Consider a higher-dimensional Borcea-Voisin example: take $Y = T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ and construct $Y \times T^2/(\sigma, -\mathbb{1}) \simeq T^8/\mathbb{Z}_2^3$, where σ acts as $-\mathbb{1}$ on $H^{3,0}(Y)$;
- The \mathbb{Z}_2 actions are summarized by the following table:

| | z_1 | z_2 | z_3 | z_4 |
|----------|-------|-------|-------|-------|
| α | + | - | - | + |
| β | - | + | - | + |
| Ω | - | - | - | - |

by definition, this is an orientifold of $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$;

- Again, think of T^8/\mathbb{Z}_2^3 as an elliptic fibration over $B = T^6/\mathbb{Z}_2^3$,

$$\begin{array}{ccc}
 T^2 & \longrightarrow & T^8/\mathbb{Z}_2^3 \\
 & & \downarrow \pi \\
 & & T^6/\mathbb{Z}_2^3
 \end{array}$$

with I_0^* fibers along the fixed point locus of each of $\alpha\Omega$, $\beta\Omega$ and $\alpha\beta\Omega$ in B .

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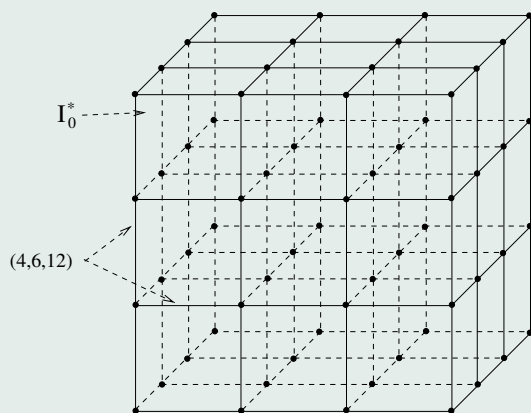
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- 12 fixed planes w/ I_0^* fibers;
- 48 lines where 2 such planes intersect and fiber degenerates to Kodaira type $(4,6,12)$;
- 64 points where three $(4,6,12)$ lines meet; these points are fixed by Ω .

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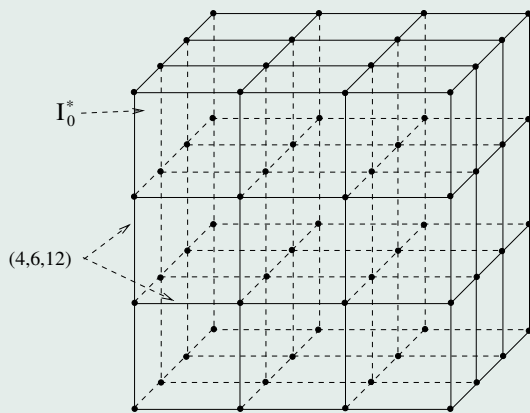
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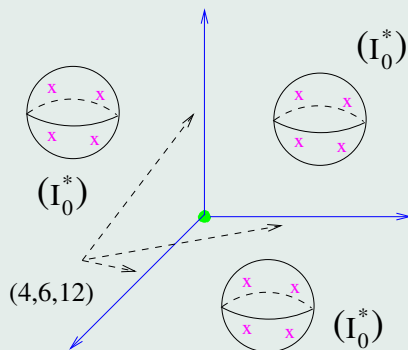
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- Locally, the geometry looks as follows:



- **Blow-up!** 48 fixed lines \rightsquigarrow 48 Kähler moduli;
- $4 \times 12 = 48$ Kähler moduli from resolution of the fiber singularities;
- 3 Kähler moduli from the base and 1 from the section of the elliptic fibration;

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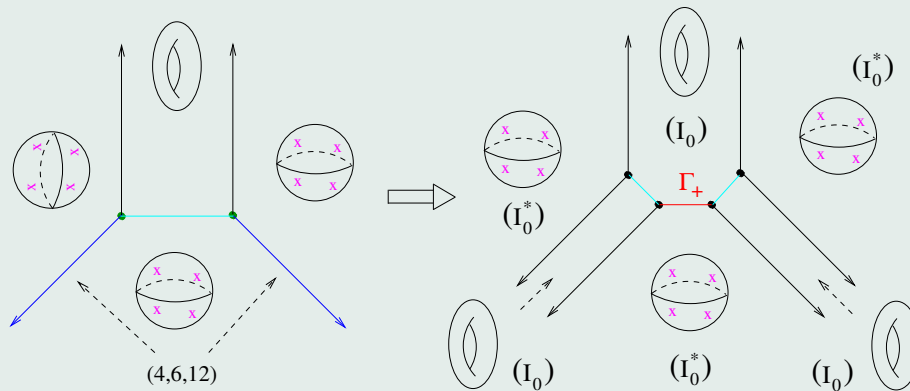
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- Calabi-Yau 4fold X has $h^{1,1}(X) = 100$ and $h^{3,1}(X) = 4$; we need to stabilize the 51 Kähler moduli of the base ($h^{1,1}(B) = 51$) and the 4 complex structure moduli.
- In order to show that we can accomplish that, we need to understand better the resolution of the singularities; below we present an 'asymmetric resolution';



- Perform sequence of blow-ups in a given order in each local patch and glue \rightsquigarrow 4fold asymmetric resolution; we will call the corresponding base B_+ ;
 - in the base $B_+ \rightsquigarrow$ 16 exceptional divisors which are del Pezzo surfaces dP_9 and 32 \mathbb{F}_0 exceptional divisors;

- the surfaces $D_{\bullet,+}$ wrapped by $D7$ -branes do not intersect any longer; they are either \mathbb{F}_0 's or \mathbb{F}_0 's blown-up at 16 points.
- What is the topology of the exceptional divisors in the Calabi-Yau 4fold X_+ ?
 - $E_{1+} \simeq E_{3+} \simeq \mathbb{P}^1 \times dP_9$ (this is $\mathbb{P}^1 \times$ the story in $6D$);
 - E_{2+} is the blow-up along 8 rational curves of $\mathbb{P}^1 \times dP_9$;
 - all exceptional divisors have holomorphic Euler characteristic 1, $\chi(E_{\bullet,+}) = 1$; moreover, $h^{0,i}(E_{\bullet,+}) = 0$, $i = 1, 2, 3$, therefore these divisors have the right topological properties to give a superpotential contribution;
 - **The exceptional divisors take care of themselves!**; blowing-up, we introduced 48 additional Kähler parameters, but (\forall) Kähler parameters (\exists) a corresponding divisor of holomorphic Euler characteristic 1;
- What about the remaining 3 Kähler parameters of the base?
 - after blowing-up, the I_0^* surfaces do not intersect any longer \rightsquigarrow no fundamental matter in the gauge theory on the $D7$ -brane worldvolume;
 - $D7$ -branes wrap **rigid** surfaces $D_{\bullet,+}$ (they have $h^{0,1}(D_{\bullet,+}) = h^{0,2}(D_{\bullet,+}) = 0$) \rightsquigarrow no adjoint matter either;

- $D7$ -branes worldvolume theory: **pure** $SO(8)$ gauge theory \rightsquigarrow (\exists) superpotential term generated by strong infrared dynamics (gaugino condensation):

$$W \sim e^{-\frac{8\pi^2}{g_{YM}^2}}, \quad \frac{1}{g_{YM}^2} \sim Vol(D_{\bullet+}).$$

- (\exists) sufficient contributions to the superpotential to stabilize the remaining Kähler moduli!

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- $D7$ -branes worldvolume theory: **pure** $SO(8)$ gauge theory $\rightsquigarrow (\exists)$ superpotential term generated by strong infrared dynamics (gaugino condensation):

$$W \sim e^{-\frac{8\pi^2}{g_{YM}^2}}, \quad \frac{1}{g_{YM}^2} \sim Vol(D_{\bullet+}).$$

- (\exists) sufficient contributions to the superpotential to stabilize the remaining Kähler moduli!
- Where do we stand?
 - **Flux vacua**: possible to find explicit vacua, due to the simplicity of the problem: only 3 complex structure moduli and the axion-dilaton. In fact, in the present case, it is perfectly fine to work in the orientifold limit, since moving away from the limit does not affect the complex structure;
 - Example: search on the symmetric locus (same complex structure for all three $T^2 \subset T^6$), yields an isolated vacuum with

$$W_0 = e^{\mathcal{K}/2} |W| = 10^{-1}, \quad (2\pi\sqrt{\alpha'} = 1).$$

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- Supersymmetric vacua are given by Kähler covariant critical points of

$$W = W_0 + \sum_{i=1}^{51} B_i e^{-2\pi\tau_{E_i}} + \sum_{j=1}^{12} C_j e^{-2\pi\tau_{D_j}/6}$$

that is

$$D_i W = \partial_i W + (\partial_i \mathcal{K}) W = 0.$$

- Prefactors B_i : holomorphic functions of the complex structure moduli coming from one-loop determinants of massive modes; we will take them to be of order 1;
- Find critical point $\text{vol}(D_i) \sim 10$, $\text{vol}(E_i) \sim 2$, $\text{vol}(B) \sim 60$ (in units of $l_s = 2\pi\sqrt{\alpha'}$); $W \sim -10^{-9}$ (AdS solution); supergravity potential is at a local minimum at this critical point in all directions;
- Solution is stable under reasonable variations of W_0 and the coefficients;
- For $g_s > 0.01$, the $\zeta(3)$ correction to the Kähler potential is negligible.

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- Everything looks good; but the isolated critical point is **not** in the Kähler cone!
 - The curves in the class Γ_+ have negative area! :(

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- Everything looks good; but the isolated critical point is **not** in the Kähler cone!

– The curves in the class Γ_+ have negative area! :(

- What to do next?

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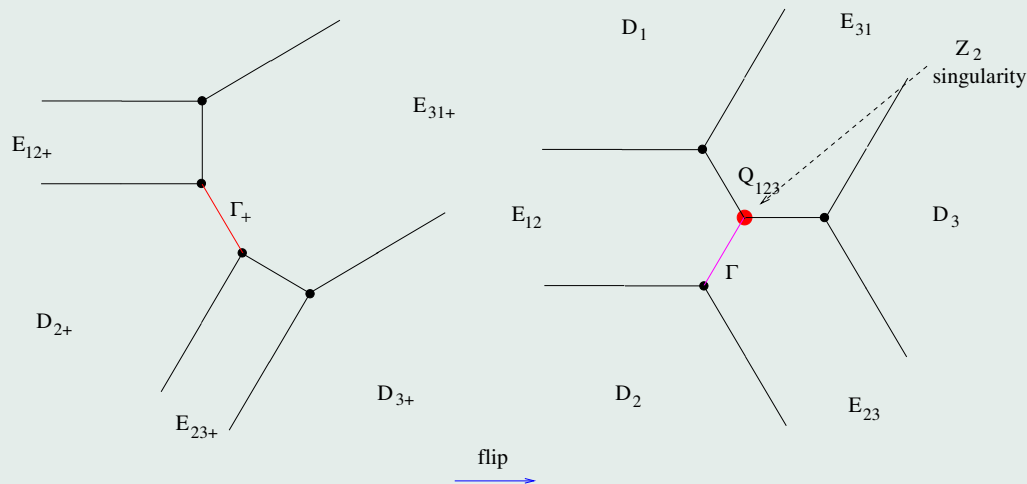
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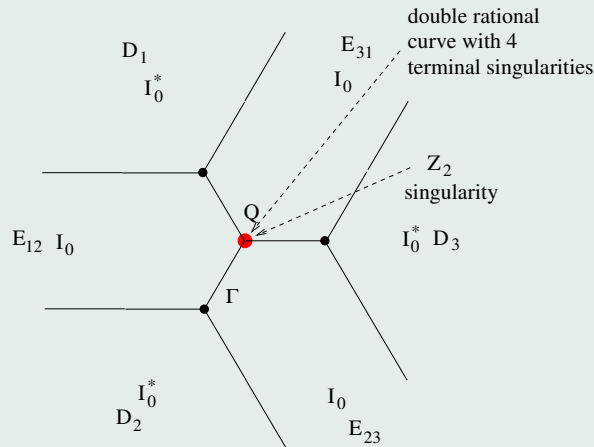
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- There (\exists) another 4fold resolution X ('symmetric resolution'):
 - X and X^+ are related by a (4fold) flop;
 - the bases B and B^+ are related by a flip;



- to see it is a flip, check intersection with the canonical classes: $K_{B^+} \cdot \Gamma_+ = 1$ and $K_B \cdot \Gamma = -\frac{1}{2}$;
- B has a \mathbb{Z}_2 singularity at Q_{123} ; (IIB string: the singular points correspond to $O3$ -planes transverse to the base);

- Study the birational factorization of the 4fold flop: sequence of blow-ups and blow-downs connecting X_+ and X_-



- Exceptional divisors $E_{\bullet\bullet} \subset X$ have $\chi = 1$; they are rigid and **do** contribute to the superpotential;
 - $D7$ -branes wrap \mathbb{F}_0 surfaces; they do not intersect and the worldvolume theory is pure $SO(8)$ gauge theory \rightsquigarrow gaugino condensation superpotential;
- Find 'good' critical point of superpotential; the supergravity potential is at a local minimum in all possible directions.

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- What could go wrong in our model?
 - **P1**: Prefactor of the instanton contribution to the superpotential might vanish if transverse $D3$ -brane hits the $D3$ -brane instanton (**Ganor**).
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 - **P2**: M-theory picture, we need to sum in principle over self-dual worldvolume fluxes $h \rightsquigarrow \Theta$ -function in the instanton prefactor; Θ -functions have the nasty habit of having zeroes. Moreover, if the divisor wrapped by the instanton is not spin, the Θ -function vanishes identically (**Moore** - private communication).
 - **A2**: The 4fold instanton divisors have vanishing third homology \rightsquigarrow no Θ -function in the instanton prefactor.

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 - **A2**: The 4fold instanton divisors have vanishing third homology \rightsquigarrow no Θ -function in the instanton prefactor.
 - **P3**: **Freed-Witten** anomaly: if a D -brane wraps a non-spin divisor, the worldsheet theory is anomalous \rightsquigarrow need to turn on some abelian flux on the D -brane worldvolume to cancel the anomaly; the flux needs to satisfy the DUY equations \rightsquigarrow possible constraints on the Kähler class. Moreover, this introduces additional bundle moduli that need to be stabilized.

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- **A3**: $D7$ -branes wrap spin divisors; the $M5$ torsion anomaly vanishes due to the fact that the third cohomology of the divisor is trivial. The four-form flux is trivial when reduced to the $M5$ worldvolume.

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- **P5**: Are there superpotential contributions from $\chi > 1$ as in **Görllich, Kachru, Tripathy, Trivedi**?
- **A5**: If they do (\exists), their contributions will be subleading.

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6. Conclusions

- Presented a simple example of an F-theory compactification on a Calabi-Yau 4fold where background fluxes, together with nonperturbative effects from Euclidean $D3$ -instantons and gauge dynamics on $D7$ -branes, allowed us to fix all closed and open string moduli.

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- Establish existence of de Sitter vacua?

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- Establish existence of de Sitter vacua?
- CFT duals of AdS flux vacua?

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