Landscape Studies

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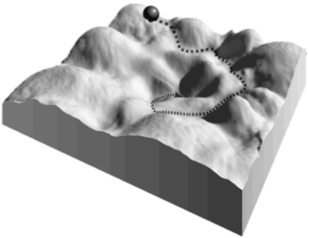
Outline

How to study the landscape?

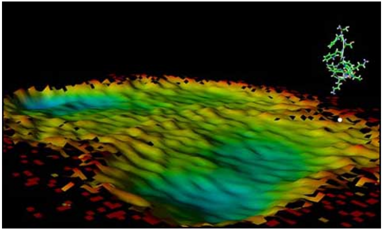
The String Theory Landscape

Vague definition: configuration space of string theory, with some sort of effective potential determining vacua.



One thing is clear: landscape is huge and complex, with huge number of vacua.

The String Theory Landscape



Vaguely similar to energy landscapes in e.g. quantitative biology.

Problems in string theory setting:

- proper degrees of freedom?
- dynamics?
- initial conditions?
- probability distributions?

What can we do?

Apart from trying to solve these deep problems, at least two things:

1. Explicit construction of semi-realistic vacua:
   $\rightarrow$ no massless moduli, cosmological constant $\ll$ KK scale
2. Statistics of vacua: how are vacua distributed over parameter space?
   $\rightarrow$ *number* distributions!

Questions in construction program

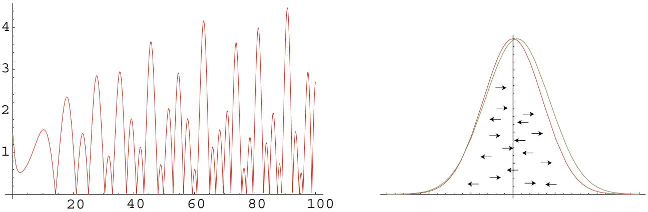
What kinds of vacua exist in string theory?

- Standard model spectrum?
- Susy breaking mechanisms?
- **Mechanism to stabilize moduli?**
  - KKLT scenario most promising for stabilization *in controlled regime*, with cc $\ll$ KK scale
  - but: turns out to impose relatively strong topological constraints
  - but II: computational complexity
  - simple explicit example?

Questions in statistics program

- Guidance for model building? What is possible in given class of models?
- Statistical analysis to exclude properties of vacua (in given ensemble)? e.g. large volume
- How fine is the discretuum of parameters? Sufficiently fine to accommodate small cc? Sufficiently sparse to be distinguishable?
- Notion of naturalness in string theory? For example, given landscape, is it still reasonable to assume low energy susy because otherwise Higgs mass must be fine-tuned? What if low energy susy needs even more fine-tuning?
  $\rightarrow$ Landscape + environmental selection as alternative notion of naturalness?

Power of statistics



- Counting often easier than constructing. Plot: $|\zeta(z)|$ at $\mathrm{Re}\ z = \frac{1}{2}$. Number of nonreal zeros ζ with $|\mathrm{Im}\ z| < L$ is $\sim L \log L$.
- Distributions more robust than individual solutions.

Naive statistics can be misleading

- Naive argument for distribution susy breaking scales:
  - $F_i \equiv D_i W$ uniformly and independently distributed
  - $\Rightarrow \mathcal{N}_{vac}(|F| < F_*) \sim \int_{|F|<F_*} d^{2n}F \sim F_*^{2n}$
  - $\Rightarrow$ Low scale breaking strongly suppressed.
- It sounds equally reasonable that also W and the fermion mass matrix $M_{ij} \equiv D_i D_j W$ are uniformly and independently distributed.
- Cannot be all right: (W, F, M) *not* independent but related by $V' = 0$ condition:

$$M_{ij}\bar{F}^j = 2\bar{W}F_i$$

Explicit construction of vacua

An explicit KKLT example

Problem: find 4d string compactification with large volume, no massless moduli, and $\mathcal{N} = 1$ unbroken susy.

KKLT: IIB with flux + nonperturbative effects:

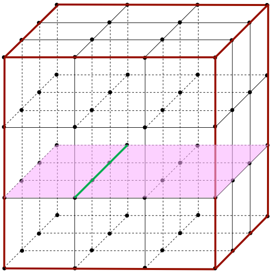
$$W = W_{\rm flux}(\mathrm{dilaton, complex}) + W_{\rm np}(\mathrm{kahler})$$

$\rightarrow$ can fix all moduli in principle.

Examples?? $\rightarrow$ plausibly many, but technically difficult to construct explicitly. [DDF]

Found simple one: $\mathbb{Z}_2$ orientifold of resolved $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$. acting as $(+,-,-),(-,+,-),(-,-,+)$

Geometry and moduli

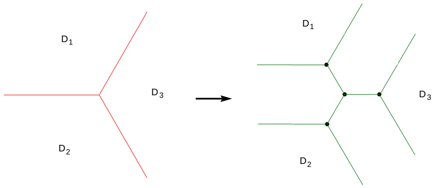


Massless content (classically and without flux):

- 3 complex structure moduli + dilaton
- 3 areas + $3 \times 16 = 48$ blowup modes = 51 Kähler moduli
- $3 \times 4 = 12$ O7 planes, with 4 coincident D7 branes on top $\rightarrow$ $SO(8)^{12}$ gauge theory. No massless brane matter fields.

Absence of massless charged matter

Local model for resolution: $\mathbb{C}^3/\mathbb{Z}_2 \times \mathbb{Z}_2$



$SO(8)$ divisors $D_i : z_i = 0$.

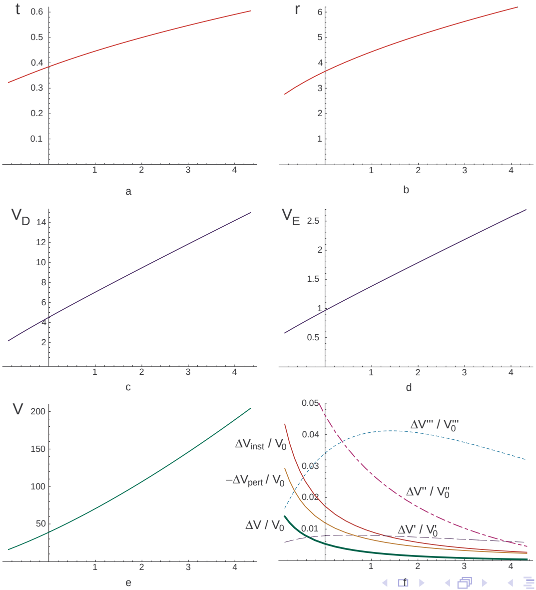
- After blowing up, D_i no longer intersect.
  $\Rightarrow$ No massless bifundamentals
- Global topology D_i before blowup: $T^4/\mathbb{Z}_2 \times \mathbb{Z}_2 = \mathbb{P}^1 \times \mathbb{P}^1$.
  Topology D_i preserved by blowup, so $h^{2,0} = 0$
  $\Rightarrow$ No massless adjoints

Fixing the moduli

(i.e. finding isolated solutions of $DW = 0$)

- Complex structure moduli (3) and dilaton:
  - Fixed in standard way by 3-form fluxes.
  - Saturate D3 tadpole $L = 28$ ($\Rightarrow$ no mobile D3-branes)
- Käher moduli (51)
  - Need nonperturbative effects to produce $W(\text{kahler})$: gaugino condensation in effective SYM theories, and/or Euclidean D3-instantons.
  - No matter $\Rightarrow$ gaugino condensation in each SO(8)
  - Exceptional divisors from blowing up orbifold fixed lines are rigid (more precisely M-theory M5 lifts have $h^{0,i} = 0$) + flux free $\Rightarrow$ 48 contributing D3-instantons
  - Together sufficient to stabilize all moduli, at moderately large radii.
  - Known corrections small (1 % level)

Numerical results



Statistics of supergravity ensembles

Statistics toy model: IIB on rigid Calabi-Yau

- Turning on RR-flux F and NSNS-flux H induces superpotential

$$W(\tau) = (f_1 + if_2) + (h_1 + ih_2)\tau$$

where $\tau = 1/g_s + iC_0$ is dilaton-axion, and $f_i, h_i \in \mathbb{Z}$.

- D3 tadpole cancellation requires

$$L \equiv f_1 h_2 - h_1 f_2 \leq L_*,$$

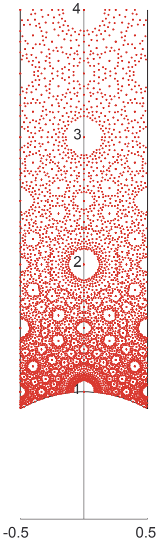
where in more realistic examples L_* is minus D3-charge induced by O3, O7, D7, ...

- Vacua = solutions τ in fundamental domain of

$$D_\tau W = 0 \Leftrightarrow \tau = -\frac{f_1 - if_2}{h_1 - ih_2}$$

- Exact enumeration possible, e.g. $N_{vac}(L_* = 150) = 37208$.

Distribution of dilaton-axion values



Continuous approximation

Number of vacua with $L \equiv f_1 h_2 - h_1 f_2 \leq L_*$ in region $\mathcal{R}$ is

$$N_{vac}(\mathcal{R}) = \sum_{f,h:L\leq L_*} \int_{\mathcal{R}} d^2\tau\, g\, \delta^2(DW)\, |\det D^2 W|$$
$$\approx \int_{\mathcal{R}} d^2\tau\, g \int_{L\leq L_*} d^2 f\, d^2 h\, \delta^2(DW)\, |\det D^2 W|$$

At fixed τ: linear change of variables:

$$(f,h) \in \mathbb{R}^4 \rightarrow (W \equiv W(\tau), F \equiv DW(\tau)) \in \mathbb{C}^2.$$

Then $L = |W|^2 - |F|^2$, Jac $= 4$, and $\det D^2 W = |W|^2$.
So approx. number of vacua is:

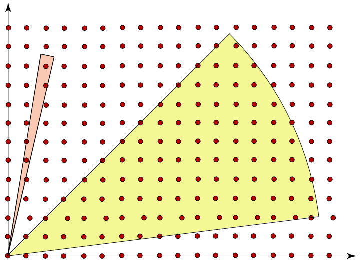
$$\mathcal{N}_{vac}(\mathcal{R}) = 4 \int d^2\tau\, g \int_{|W|^2-|F|^2\leq L_*} d^2 W\, d^2 F\, \delta^2(F) |W|^2$$
$$= 2\pi L_*^2 \int_{\mathcal{R}} d^2\tau\, g$$

Remarks

- Total $\mathcal{N}_{vac} = \frac{\pi^2 L_*^2}{6}$. For our example $L_* = 150$, $\mathcal{N}_{vac} \approx 37011$, within 0.5% of exact result $N_{vac} = 37208$.
- Distribution of string coupling constants is *uniform*:

$$\mathcal{N}_{vac}(g_s < g_*) \sim \int_{\tau_2 = 1/g_*}^{\infty} \frac{d\tau_2}{\tau_2^2} \sim g_*$$

- Nature of approximation: counting lattice points in flux space by computing volume:



More general ensembles

Compactification includes discrete data $\vec{N}$ such as fluxes, brane charges, $\ldots \Rightarrow$ defines ensemble of effective 4d supergravity Lagrangians $\{\mathcal{L}_{\vec{N}}\}$ with potentials

$$V_{\vec{N}}(t) = |DW_{\vec{N}}|^2 - 3|W_{\vec{N}}|^2$$

on moduli space $\mathcal{M}$.

Example
IIB flux compactifications: $W_{\vec{N}}(t) = \vec{N} \cdot \vec{\Pi}(t)$

Number of metastable vacua:

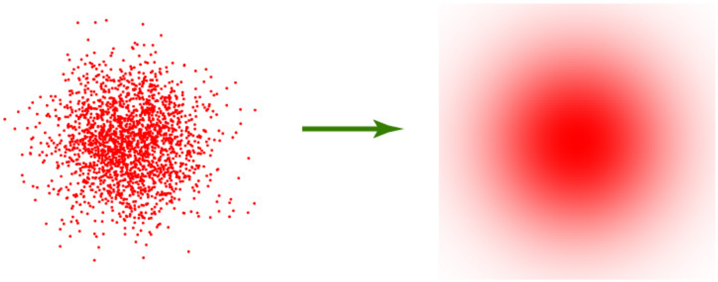
$$N_{vac} = \sum_{\vec{N}} \int_{\mathcal{M}} d^{2m}t \, \delta^{2m}(V') \, |\det V''| \, \Theta(V'')$$

Change of variables

Many physical properties can be expressed in terms of

$$W \equiv W(t), \quad F_A \equiv D_A W, \quad M_{AB} \equiv D_A D_B W, \quad Y_{ABC} \equiv D_A D_B D_C W$$

At any fixed t, varying $\vec{N}$ will define a large discrete set in (W, F, M, Y)-space. The distribution of these points is given by some measure $d\mu_0[W, F, M, Y] \rightarrow$ continuous approximation



IIB flux case

W linear in $\vec{N} \Rightarrow$ distr. uniform and supported on linear subspace:

$$d\mu_0[W, F, M, Y] = 4^m d^2 W \, d^{2m} F_A \, d^{2m-2} M_{0I} \cdot \delta(\cdots).$$

Here 0 = dilaton, I = complex structure indices, and $\delta(\cdots)$ imposes:

- the linear relations

$$M_{IJ} = \mathcal{F}_{IJK}(t) \bar{M}^{0K}, \qquad Y_{ABC} = \cdots$$

  The $\mathcal{F}_{IJK}(t) = \partial_I \partial_J \partial_K \mathcal{F}$ ar the "Yukawa couplings" defining the special geometry of the CY complex structure moduli space.

- the tadpole constraint $L = |W|^2 - |F_A|^2 + |M_{0I}|^2 \leq L_*$.

Vacuum number density

Combining the a priori measure $d\mu_0$ with the constraint $V' = 0$ gives

$$\mathcal{N}_{vac} = \int d\mu[t, W, F, M, Y]$$

where

$$d\mu[t, W, F, M, Y] = d^{2m}t \, d\mu_0[W, F, M, Y] \cdot \delta^{2m}(V') \, |\det V''| \, \Theta(V'').$$

Happily, V' and V'' are easily expressed in terms of W, F, M, Y:

$$\begin{aligned}
\partial_A V &= M_{AB} \bar{F}^B - 2\bar{W} F_A \\
D_A \partial_B V &= -\bar{W} M_{AB} + Y_{ABC} \bar{F}^K \\
\bar{D}_A \partial_B V &= \bar{M}_{AC} M_{BD} - 2\delta_{AB} |W|^2 + R_{\bar{A}B\bar{C}D} F^C \bar{F}^D + \delta_{AB} |F|^2 - \bar{F}_A F_B.
\end{aligned}$$

$\Rightarrow$ problem reduced to finding $d\mu_0$ and doing finite dim. integral!

Some results for supersymmetric IIB flux vacua

- Number of flux vacua in region $\mathcal{R}$ of moduli space

$$\mathcal{N}_{\mathcal{R}}(L \leq L_*) \approx \frac{(2\pi L_*)^{b_3}}{b_3!} \int_{\mathcal{R}} \frac{1}{\pi^m} \det(R + \omega \mathbf{1})$$

- C.c. $\Lambda = -3|W|^2$ uniformly distributed for $|\Lambda| \ll M_p^4$:

$$d\mathcal{N}[\Lambda] \sim d\Lambda$$

- Large volumes strongly suppressed:

$$d\mathcal{N}[V] \sim e^{-cV^{2/3}} d(V^{2/3})$$

- Vacua cluster near conifold degenerations:

$$d\mathcal{N}[|t|] \sim d\frac{1}{\log|t|}$$

$\rightarrow$ dual gauge theory scale hierarchy

Some results for nonsupersymmetric IIB flux vacua

- F-breaking vacua, $F = M_{susy}^2 \ll M_p^2$:
  1. generic negative Λ:

$$d\mathcal{N}[F, \Lambda] \sim dF\, d\Lambda$$

  2. $\Lambda \sim 0$ or $\Lambda > 0$:

$$d\mathcal{N}[F, \Lambda, t] \sim F^5 dF\, d\Lambda$$

$\rightarrow$ low breaking scale disfavored (but much less than naive guess dF^{2n})

- RS-type "D-breaking" vacua (anti-D3 at bottom of warped AdS throat near conifold degeneration), $M_{susy}^2 \sim |t|^\alpha \ll M_p^2$

$$d\mathcal{N}[M_s^2, \Lambda] \sim d\frac{1}{\log M_{susy}^2}\, d\Lambda$$

$\rightarrow$ low breaking scale favored

Some results for M-theory G_2 flux vacua

- Number of flux vacua in region $\mathcal{R}$ of moduli space

$$\mathcal{N}_{\mathcal{R}} \sim c_2^{b_3} \int_{\mathcal{R}} \det g$$

- Large volumes strongly suppressed:

$$d\mathcal{N}[V] \sim (kc_2)^{b_3} dV^{-3b_3/7} \quad \Rightarrow V < (kc_2)^{7/3}$$

- Small cc's strongly suppressed because

$$\Lambda \sim 1/V^3$$

- Small F-breaking susy breaking scales strongly suppressed because

$$M_{susy}^2 \sim 1/V^{3/2}$$

Reason large hierarchies are suppressed: only as many fluxes as moduli $\Rightarrow$ all scales set by V, no further discrete tuning possible once moduli are fixed.

Conclusions

- Distribution of susy / nonsusy vacua over moduli space roughly uniform for all flux compactifications studied thus far, except near singularities.
- Large volume suppression and upper bound on volume is universal feature.
- Once volume (KK scale) fixed, IIB still allows other scales such as Λ and M_{susy} to be (discretely) tuned much smaller, while G_2 does not. Effective scanning of scale hierarchies needs more fluxes than moduli.
- More "fluxes" in G_2 possible? Yes, but correspond to geometric discrete data away from special holonomy.
- Universal for F-breaking by flux potential at small or positive cc: low energy susy numerically disfavored (even when taking into account Higgs mass tuning).

To do

- more general ensembles
- study observable sector
- establish existence of more vacua with stabilized moduli
- hypothesis for probabilities