A Heterotic Standard Model

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Burt's talk next week
hep-th/0410055: Elliptic Calabi-Yau Threefolds with $\mathbb{Z}_3 \times \mathbb{Z}_3$ Wilson Lines
hep-th/0501070: A Heterotic Standard Model
hep-th/0502155: A Standard Model from the $E_8 \times E_8$ Heterotic Superstring
hep-th/0504xxx: Standard-Model Vacua in Heterotic String Theory

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Introduction

- Geometric compactification of the $E_8 \times E_8$ heterotic string.

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- $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$
  $\Rightarrow$ proton decay suppressed.
- No exotic matter.
- All of the ordinary matter fields (including right-handed Neutrino).

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An Organizational Principle

Ancient Lore: $\mathrm{Spin}(10)$ GUT with $\mathbb{Z}_3 \times \mathbb{Z}_3$ Wilson lines "works":

$\underline{\mathbf{16}}$ of $\mathrm{Spin}(10)$: Breaks into one family of quarks and leptons including a right-handed Neutrino.

$\overline{\underline{\mathbf{16}}}$ of $\mathrm{Spin}(10)$: Anti-family.

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However, we do not care about GUTs:

Compactification scale $\sim$ GUT scale

... but nice way to package representations.

Wilson Line Breaking

$$\mathrm{Spin}(10) \supset SU(3) \times SU(2) \times U(1) \times U(1) \times \mathbb{Z}_3 \times \mathbb{Z}_3$$

$$\left\{ \begin{array}{c} \text{Standard Model} \\ \text{gauge group} \end{array} \right\} \times U(1)_{B-L} \times \{\text{Wilson lines}\}$$

$\mathbb{Z}_3 \times \mathbb{Z}_3$ is smallest Wilson line possible.

Wilson Line Breaking

$$\text{Spin}(10) \supset SU(3) \times SU(2) \times U(1) \times U(1) \times \mathbb{Z}_3 \times \mathbb{Z}_3$$

$$\begin{aligned}\underline{\mathbf{16}} = &\chi_1^2\chi_2(\underline{\mathbf{3}}, \underline{\mathbf{2}}, 1, 1) \oplus \chi_1^2(\underline{\mathbf{1}}, \underline{\mathbf{1}}, 6, 3) \oplus \\ &\oplus \chi_1^2\chi_2^2(\overline{\underline{\mathbf{3}}}, \underline{\mathbf{1}}, -4, -1) \oplus \chi_2^2(\overline{\underline{\mathbf{3}}}, \underline{\mathbf{1}}, 2, -1) \oplus \\ &\oplus (\underline{\mathbf{1}}, \overline{\underline{\mathbf{2}}}, -3, -3) \oplus \chi_1(\underline{\mathbf{1}}, \underline{\mathbf{1}}, 0, 3)\end{aligned}$$

$$\begin{aligned}\underline{\mathbf{10}} = &\chi_1(\underline{\mathbf{1}}, \underline{\mathbf{2}}, 3, 0) \oplus \chi_1\chi_2(\underline{\mathbf{3}}, \underline{\mathbf{1}}, -2, -2) \oplus \\ &\oplus \chi_1^2(\underline{\mathbf{1}}, \overline{\underline{\mathbf{2}}}, -3, 0) \oplus \chi_1^2\chi_2^2(\overline{\underline{\mathbf{3}}}, \underline{\mathbf{1}}, 2, 2)\end{aligned}$$

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\end{aligned}$$

Right-handed Neutrino

More Group Theory

$$G = \mathbb{Z}_3 \times \mathbb{Z}_3 = G_1 \times G_2$$

Fix generators g_1 and g_2.

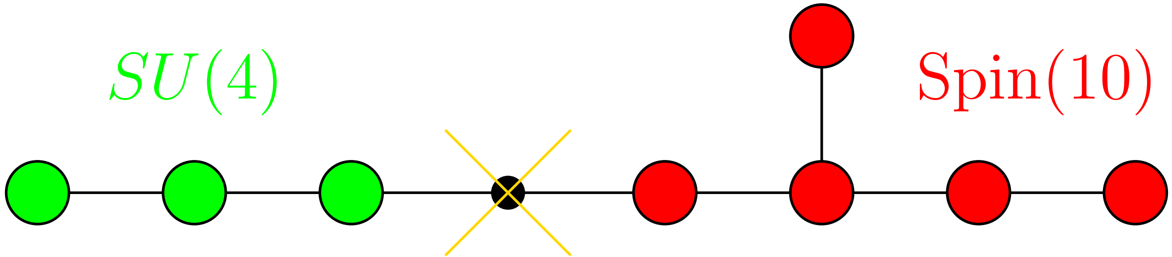
Characters (=1-d representations): Denote generators by χ_1 and χ_2, where ($\omega = e^{\frac{2\pi i}{3}}$)

$$\chi_1(g_1) = \omega \qquad \chi_1(g_2) = 1$$
$$\chi_2(g_1) = 1 \qquad \chi_2(g_2) = \omega\,.$$

All other characters are products of χ_1 and χ_2.

Yet More Group Theory

Maximal regular subgroup $SU(4) \times \text{Spin}(10) \subset E_8$:



The adjoint of E_8 (fermions in the $E_8 \times E_8$ heterotic string) decomposes as

$$\underline{\mathbf{248}} = (\underline{\mathbf{1}}, \underline{\mathbf{45}}) \oplus (\underline{\mathbf{15}}, \underline{\mathbf{1}}) \oplus (\underline{\mathbf{4}}, \underline{\mathbf{16}}) \oplus (\overline{\underline{\mathbf{4}}}, \overline{\underline{\mathbf{16}}}) \oplus (\underline{\mathbf{6}}, \underline{\mathbf{10}})$$

Wish List

To make use of this group theory, we would like

- A Calabi-Yau threefold X with $\mathbb{Z}_3 \times \mathbb{Z}_3$ fundamental group.

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- A Calabi-Yau threefold X with $\mathbb{Z}_3 \times \mathbb{Z}_3$ fundamental group.
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- With the "right" cohomology groups (low energy spectrum).

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CY Introduction

Calabi-Yau threefold
X with
$\pi_1(X) = \mathbb{Z}_3 \times \mathbb{Z}_3$

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elliptically fibered		torus fibered
(torus fibered with section)		(assuming $\mathbb{Z}_3 \times \mathbb{Z}_3$ preserves fibration)

Calabi-Yau Construction

Start with two dP_9 surfaces B_1 and B_2.

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Note: dP_9 are elliptically fibered; Fibers over a generic point $x \in \mathbb{P}^1$ are

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- $\widetilde{X} \stackrel{\text{def}}{=} B_1 \times_{\mathbb{P}^1} B_2$ is a simply connected Calabi-Yau threefold, $c_1(\widetilde{X}) = 0$.

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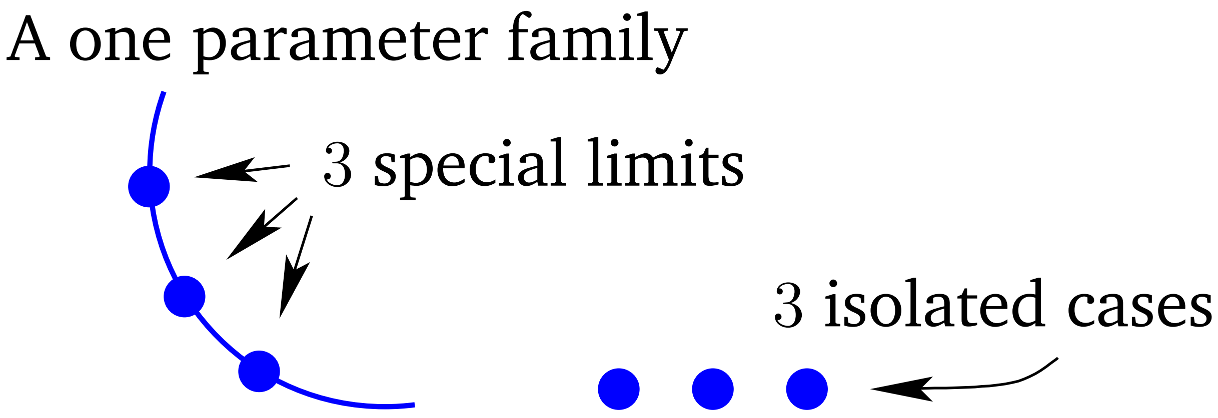
- $h^{1,1}(\widetilde{X}) = 19 = h^{2,1}(\widetilde{X})$
- Group actions on B_1, B_2 lift to $\widetilde{X}$ if their action on the common base $\mathbb{P}^1$ is identical.

Group Actions on the del Pezzo I

We classified all $\mathbb{Z}_3 \times \mathbb{Z}_3$ actions on dP_9 surfaces.
The moduli space looks like this:



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Group Actions on the del Pezzo II

All such dP_9 surfaces with $G = \mathbb{Z}_3 \times \mathbb{Z}_3$ action give rise to a G action on $\widetilde{X}$.

- The 3 isolated cases never yield a free $\mathbb{Z}_3 \times \mathbb{Z}_3$ action.
- The one-parameter family and its limits can give a free $\mathbb{Z}_3 \times \mathbb{Z}_3$ action on $\widetilde{X}$.

We only consider this one-parameter family in the following.

Group Action on Cohomology

$G = \mathbb{Z}_3 \times \mathbb{Z}_3$ acts on the cohomology groups of B_1, B_2, and $\widetilde{X}$.

On $H^2(B_i, \mathbb{Z}) = \mathbb{Z}^{10}$ the group generators g_1, g_2 act as

$$\begin{pmatrix} 0&0&0&0&0&0&3&-1&-1&-1\\ 0&1&0&1&0&0&3&-1&-1&-1\\ 0&0&0&0&1&0&-2&0&1&0\\ 0&0&0&0&0&0&-2&0&2&1\\ 0&0&0&-1&0&1&-2&1&1&1\\ 0&0&1&0&0&0&-1&0&1&0\\ 0&0&0&-1&0&0&-1&1&0&1\\ 1&0&0&0&0&0&0&1&0&1\\ 0&0&0&0&0&0&0&1&-1&0\\ 0&0&0&0&0&0&-3&0&3&1 \end{pmatrix}, \begin{pmatrix} 0&0&3&0&0&0&0&-1&2&-1\\ 0&1&3&0&0&1&1&-1&2&-1\\ 0&0&-2&0&0&0&0&0&-1&1\\ 0&0&-2&0&0&-1&0&1&-2&1\\ 0&0&-2&0&0&0&-1&1&-1&1\\ 0&0&-1&1&0&-1&0&0&-1&0\\ 0&0&-1&0&1&0&-1&1&-1&0\\ 0&0&0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&0&0&1&0\\ 1&0&-3&0&0&0&0&1&-2&2 \end{pmatrix}$$

Similarly, 19×19 matrices for g_1, g_2 on $H^{1,1}(\widetilde{X}) = \mathbb{C}^{19}$.

Invariant Cohomology

$G = \mathbb{Z}_3 \times \mathbb{Z}_3$ action free $\Rightarrow H^{p,q}(X) = H^{p,q}(\widetilde{X})^G$

$$\Rightarrow \text{Hodge diamond } h^{p,q}(X) = \begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & 3 & & 0 & \\ 1 & & 3 & & 3 & & 1 \\ & 0 & & 3 & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array}$$

$h^{1,1}(X) = 3$ dimensional space of divisor classes.

Divisors on the Calabi-Yau I

$$\dim_{\mathbb{C}} = 3 : \qquad \widetilde{X}$$

$$\dim_{\mathbb{C}} = 2 : \qquad B_1 \xleftarrow{\pi_1} \widetilde{X} \xrightarrow{\pi_2} B_2$$

$$\dim_{\mathbb{C}} = 1 : \qquad B_1 \xrightarrow{\beta_1} \mathbb{P}^1 \xleftarrow{\beta_2} B_2$$

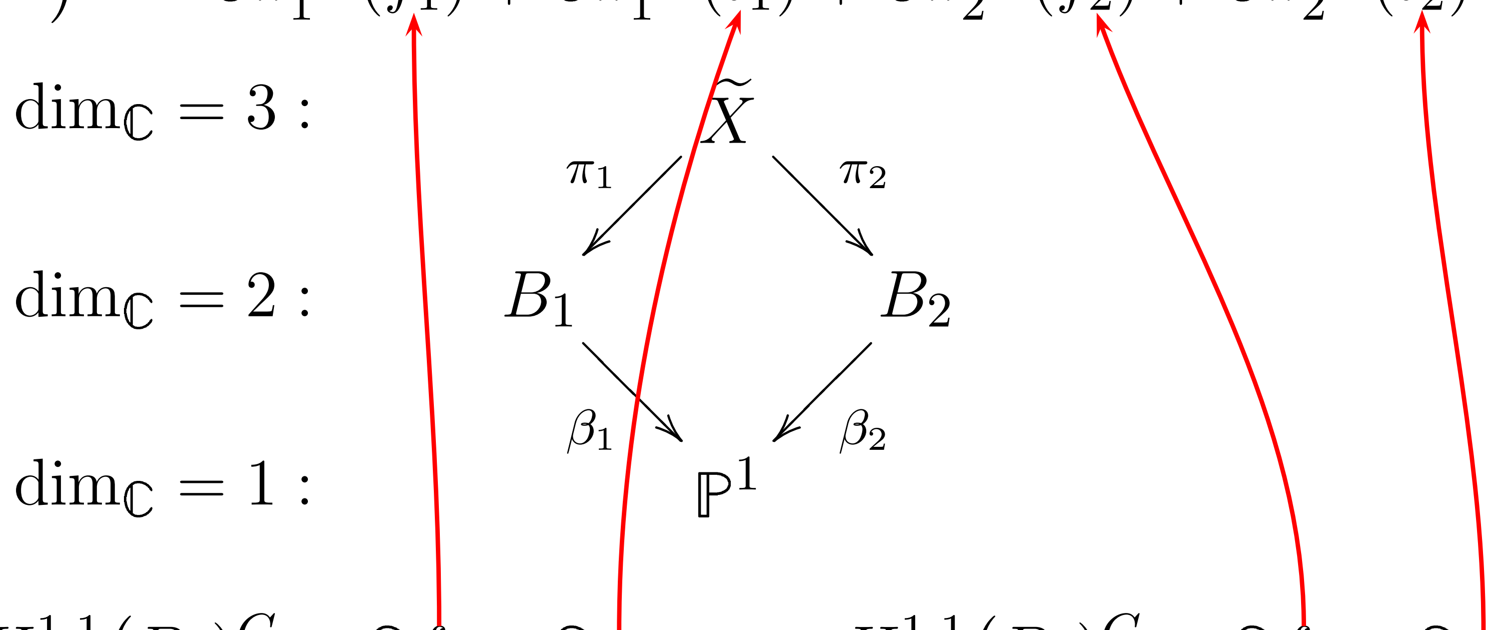
Divisors on the Calabi-Yau I

$$
\begin{array}{lccccc}
\dim_{\mathbb{C}} = 3: & & & \widetilde{X} & & \\
 & & \swarrow^{\pi_1} & & \searrow^{\pi_2} & \\
\dim_{\mathbb{C}} = 2: & B_1 & & & & B_2 \\
 & & \searrow_{\beta_1} & & \swarrow_{\beta_2} & \\
\dim_{\mathbb{C}} = 1: & & & \mathbb{P}^1 & &
\end{array}
$$

$$H^{1,1}\big(B_1\big)^G = \mathbb{C}f_1 \oplus \mathbb{C}t_1 \qquad H^{1,1}\big(B_2\big)^G = \mathbb{C}f_2 \oplus \mathbb{C}t_2$$

Divisors on the Calabi-Yau I

$$H^{1,1}\big(\widetilde{X}\big)^G = \mathbb{C}\pi_1^{-1}(f_1) + \mathbb{C}\pi_1^{-1}(t_1) + \mathbb{C}\pi_2^{-1}(f_2) + \mathbb{C}\pi_2^{-1}(t_2)$$



$$H^{1,1}(B_1)^G = \mathbb{C}f_1 \oplus \mathbb{C}t_1 \qquad H^{1,1}(B_2)^G = \mathbb{C}f_2 \oplus \mathbb{C}t_2$$

Divisors on the Calabi-Yau II

$$H^{1,1}\big(\widetilde{X}\big)^G = \mathbb{C}\pi_1^{-1}(f_1) + \mathbb{C}\pi_1^{-1}(t_1) + \mathbb{C}\pi_2^{-1}(f_2) + \mathbb{C}\pi_2^{-1}(t_2)$$
$$= \mathbb{C}\phi \oplus \mathbb{C}\tau_1 \oplus \mathbb{C}\tau_2$$

There is one relation:

$$\pi_1^{-1}(f_1) = \left\{ T^4 \text{ fiber of } \begin{array}{c} \widetilde{X} \\ \big\downarrow \\ \mathbb{P}^1 \end{array} \right\} = \pi_2^{-1}(f_2) \stackrel{\text{def}}{=} \phi$$

$$\pi_1^{-1}(t_1) \stackrel{\text{def}}{=} \tau_1 \qquad \pi_2^{-1}(t_2) \stackrel{\text{def}}{=} \tau_2$$

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Line Bundles

On any variety Y, we have

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Linear equivalence

For B_1, B_2, $\widetilde{X}$ that is just cohomology class of the divisor in $H^{1,1}$.

Every line bundle is of the form

- $\mathcal{O}_{\widetilde{X}}(x_1\tau_1 + x_2\tau_2 + x_3\phi)$, $x_1, x_2, x_3 \in \mathbb{Z}$.
- $\mathcal{O}_{B_i}(y_1 t_i + y_2 f_i)$, $y_1, y_2 \in \mathbb{Z}$.

Equivariant Line Bundles I

Line bundles on
$X = \widetilde{X}/G$

Equivariant Line Bundles I

Work with | Have in mind

G-equivariant line bundles on $\widetilde{X}$	$=$	Line bundles on $X = \widetilde{X}/G$

Equivariant Line Bundles I

Work with

G-equivariant line bundles on $\widetilde{X}$

$=$

Have in mind

Line bundles on $X = \widetilde{X}/G$

An equivariant line bundle is a line bundle $\mathcal{L}$ together with a group action $\gamma : G \times \mathcal{L} \to \mathcal{L}$:

$$\begin{array}{ccc} \mathcal{L} & \xrightarrow{\gamma_g} & \mathcal{L} \\ \downarrow & & \downarrow \\ \widetilde{X} & \xrightarrow{g} & \widetilde{X} \end{array}$$

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- In these cases, there is always more than one G action

$\Rightarrow$ Different equivariant line bundles!

Equivariant Structures

Consider the trivial line bundle $\mathcal{O}_{\widetilde{X}} = \widetilde{X} \times \mathbb{C}$.

- Obvious equivariant action
  $\gamma_g : \widetilde{X} \times \mathbb{C} \to \widetilde{X} \times \mathbb{C}, \ (p, v) \mapsto \big(g(p), v\big)$

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  $\chi\gamma_g : \widetilde{X} \times \mathbb{C} \to \widetilde{X} \times \mathbb{C},\ (p, v) \mapsto \big(g(p), \chi(g)v\big)$
- We write $\chi\mathcal{O}_{\widetilde{X}}$ for this different equivariant line bundle.

The Serre Construction

A way to construct may stable rank 2 vector bundles on a surface (here: B_1 and B_2).

- Take two line bundles $\mathcal{L}_1$, $\mathcal{L}_2$.
- An ideal sheaf I (sheaf of functions vanishing at some fixed points).
- Define $\mathcal{S}$ as an extension

$$0 \longrightarrow \mathcal{L}_1 \longrightarrow \mathcal{S} \longrightarrow \mathcal{L}_2 \otimes I \longrightarrow 0$$

- Cayley-Bacharach property $\Rightarrow$ generic extension is a rank 2 vector bundle.

Serre Construction Example

$$0 \longrightarrow \mathcal{O}_{B_2}(-2f_2) \longrightarrow \mathcal{W} \longrightarrow \mathcal{O}_{B_2}(2f_2) \otimes I_9 \longrightarrow 0$$

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- I_9 is the ideal sheaf of (certain) nine points.
- Has the Cayley-Bacharach property.
- $\mathrm{Ext}^1\Big(\mathcal{O}_{B_2}(2f_2) \otimes I_9,\ \mathcal{O}_{B_2}(-2f_2)\Big) = \mathbb{C}^9$, so there exist extensions.

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G-equivariant vector bundles on $\widetilde{X}$	$=$	Vector bundles on $X = \widetilde{X}/G$

Problem: Even if $\mathcal{E}$, $\mathcal{F}$ are equivariant,

$$0 \longrightarrow \mathcal{E} \longrightarrow \mathcal{V} \longrightarrow \mathcal{F} \longrightarrow 0$$

Extension is not necessarily equivariant!

Only extensions in $\mathrm{Ext}^1\left(\mathcal{F}, \mathcal{E}\right)^G$ are equivariant.

Equivariant Example

$$0 \longrightarrow \mathcal{O}_{B_2}(-2f_2) \longrightarrow \mathcal{W} \longrightarrow \chi_2 \mathcal{O}_{B_2}(2f_2) \otimes I_9 \longrightarrow 0$$

- $\mathcal{O}_{B_2}(-2f_2)$, $\chi_2 \mathcal{O}_{B_2}(2f_2)$ are equivariant.

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  so there exist extensions.

Constructing Vector Bundles

Building blocks:

- Line bundles on $\widetilde{X}$.
- Rank 2 bundles pulled back from B_1, B_2.

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- Line bundles on $\widetilde{X}$.
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Operations:

- Tensor product of bundles.
- ~~Sums of bundles.~~ Never (slope-) stable!
- Extensions of bundles.

Our Solution

Define these two rank 2 vector bundles

$$
\begin{aligned}
\mathcal{V}_1 &\stackrel{\text{def}}{=} \chi_2 \mathcal{O}_{\widetilde{X}}(-\tau_1 + \tau_2) \oplus \chi_2 \mathcal{O}_{\widetilde{X}}(-\tau_1 + \tau_2) = \\
&= 2\chi_2 \mathcal{O}_{\widetilde{X}}(-\tau_1 + \tau_2) \\
\mathcal{V}_2 &\stackrel{\text{def}}{=} \mathcal{O}_{\widetilde{X}}(\tau_1 - \tau_2) \otimes \pi_2^*(\mathcal{W})
\end{aligned}
$$

We define the rank 4 bundle $\mathcal{V}$ finally as a generic extension

$$
0 \longrightarrow \mathcal{V}_1 \longrightarrow \mathcal{V} \longrightarrow \mathcal{V}_2 \longrightarrow 0
$$

Table of Contents

Low Energy Spectrum I

The massless spectrum
$= $ zero modes of $\not{D}_{E_8}$
$= H^1$ cohomology of the adjoint bundle $\mathcal{E}_8^{\mathcal{V}/G}$.

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Low Energy Spectrum I

The massless spectrum
= zero modes of $\not{D}_{E_8}$
= H^1 cohomology of the adjoint bundle $\mathcal{E}_8^{\mathcal{V}/G}$.

Work with

$$H^1\left(\widetilde{X},\ \mathcal{E}_8^{\mathcal{V}}\right)^G$$

=

Have in mind

$$H^1\left(X,\ \mathcal{E}_8^{\mathcal{V}/G}\right) = \\ = H^1\left(X,\ \mathcal{E}_8^{\mathcal{V}}/G\right)$$

Low Energy Spectrum II

$$\underline{\mathbf{248}} = (\underline{\mathbf{1}}, \underline{\mathbf{45}}) \oplus (\underline{\mathbf{15}}, \underline{\mathbf{1}}) \oplus (\underline{\mathbf{4}}, \underline{\mathbf{16}}) \oplus (\overline{\underline{\mathbf{4}}}, \overline{\underline{\mathbf{16}}}) \oplus (\underline{\mathbf{6}}, \underline{\mathbf{10}})$$

$$\underline{\mathbf{10}} = \chi_1 (\underline{\mathbf{1}}, \underline{\mathbf{2}}, 3, 0) \oplus \chi_1\chi_2 (\underline{\mathbf{3}}, \underline{\mathbf{1}}, -2, -2) \oplus$$
$$\oplus \chi_1^2 (\underline{\mathbf{1}}, \overline{\underline{\mathbf{2}}}, -3, 0) \oplus \chi_1^2\chi_2^2 (\overline{\underline{\mathbf{3}}}, \underline{\mathbf{1}}, 2, 2)$$

Correspondingly, the fermions split as...

Low Energy Spectrum II

$$\mathcal{E}_8^V = \left(\mathcal{O}_{\widetilde{X}} \otimes \theta(\underline{\mathbf{45}})\right) \oplus \left(\mathrm{ad}(\mathcal{V}) \otimes \theta(\underline{\mathbf{1}})\right) \oplus$$

$$\oplus \left(\mathcal{V} \otimes \theta(\underline{\mathbf{16}})\right) \oplus \left(\mathcal{V}^\vee \otimes \theta(\underline{\overline{\mathbf{16}}})\right) \oplus \left(\wedge^2 \mathcal{V} \otimes \theta(\underline{\mathbf{10}})\right)$$

where $\theta(\cdots)$ is the trivial bundle.

$$\theta(\underline{\mathbf{10}}) = \left[\chi_1 \theta\left(\underline{\mathbf{1}}, \underline{\mathbf{2}}, 3, 0\right)\right] \oplus \left[\chi_1 \chi_2 \theta\left(\underline{\mathbf{3}}, \underline{\mathbf{1}}, -2, -2\right)\right] \oplus$$

$$\oplus \left[\chi_1^2 \theta\left(\underline{\mathbf{1}}, \underline{\overline{\mathbf{2}}}, -3, 0\right)\right] \oplus \left[\chi_1^2 \chi_2^2 \theta\left(\underline{\overline{\mathbf{3}}}, \underline{\mathbf{1}}, 2, 2\right)\right]$$

Low Energy Spectrum III

For example, focus on the fields in the $\underline{\mathbf{10}}$:

$$H^1\Big(\widetilde{X},\, \mathcal{E}_8^V\Big)^G = (\text{lots of other fields}) \oplus$$

$$\oplus \left[\chi_1 \otimes H^1\Big(\widetilde{X},\, \wedge^2\mathcal{V}\Big)\right]^G \otimes (\underline{\mathbf{1}}, \underline{\mathbf{2}}, 3, 0) \oplus$$

$$\oplus \left[\chi_1\chi_2 \otimes H^1\Big(\widetilde{X},\, \wedge^2\mathcal{V}\Big)\right]^G \otimes (\underline{\mathbf{3}}, \underline{\mathbf{1}}, -2, -2) \oplus$$

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$$\oplus \left[\chi_1^2\chi_2^2 \otimes H^1\Big(\widetilde{X},\, \wedge^2\mathcal{V}\Big)\right]^G \otimes (\underline{\overline{\mathbf{3}}}, \underline{\mathbf{1}}, 2, 2)\ .$$

Cohomology I

The necessary cohomology groups for $\mathcal{V}$ are

$$\begin{aligned} H^1\Big(\widetilde{X}, \mathcal{V}\Big) &= 3\,\mathrm{Reg}(G) \\ H^1\Big(\widetilde{X}, \mathcal{V}^\vee\Big) &= 0 \\ H^1\Big(\widetilde{X}, \wedge^2\mathcal{V}\Big) &= H^1\Big(\widetilde{X}, \mathcal{V}_1 \otimes \mathcal{V}_2\Big) = \\ &= 2 \oplus 2\chi_1 \oplus 2\chi_2 \oplus 2\chi_1^2 \oplus 2\chi_2^2 \oplus \\ &\qquad \oplus 2\chi_1\chi_2^2 \oplus 2\chi_1^2\chi_2 \end{aligned}$$

Cohomology II

$$H^1\left(\widetilde{X}, \wedge^2\mathcal{V}\right) =$$

$$2 \oplus 2\chi_1 \oplus 2\chi_2 \oplus 2\chi_1^2 \oplus 2\chi_2^2 \oplus 2\chi_1\chi_2^2 \oplus 2\chi_1^2\chi_2$$

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$$2 = \left[\chi_1 \otimes H^1\left(\widetilde{X}, \wedge^2 \mathcal{V}\right)\right]^G$$ up Higgs

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$$0 = \left[\chi_1\chi_2 \otimes H^1\left(\widetilde{X}, \wedge^2\mathcal{V}\right)\right]^G$$ **3**

Cohomology II

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Heterotic Anomaly I

So far, we only considered the visible E_8 bundle!

It turns out that we need a instanton $\mathcal{H}$ in the hidden E_8 for anomaly cancellation.

$$c_2\big(T\widetilde{X}\big) - c_2\big(\mathcal{V}\big) - c_2\big(\mathcal{H}\big) = PD(C) \in H^4\big(\widetilde{X}, \mathbb{Z}\big)$$

where C is the curve wrapped by five-branes.

$$\begin{aligned} c_2\big(T\widetilde{X}\big) &= 12\left(\tau_1^2 + \tau_2^2\right) \\ c_2\big(\mathcal{V}\big) &= -2\tau_1^2 + 7\tau_2^2 + 4\tau_1\tau_2 \end{aligned}$$

Heterotic Anomaly II

We found a rank 2 bundle with

- no hidden matter whatsoever.
- which requires five branes.

$$c_2\big(T\widetilde{X}\big) - c_2\big(\mathcal{V}\big) - c_2\big(\mathcal{H}_{(1)}\big) = 6\tau_1^2 \,.$$

Note: the curve τ_1^2 is just an elliptic fiber,

$$\tau_1^2 = \pi_1^{-1}(\{\text{pt.}\})$$

Heterotic Anomaly III

We found another (reducible) rank 4 bundle with

- two hidden $\mathrm{Spin}(12)$ multiplets.
- which requires no five branes.

$$c_2(T\widetilde{X}) - c_2(\mathcal{V}) - c_2(\mathcal{H}_{(2)}) = 0\,.$$

Spectral Sequences I

How did we compute all these cohomology groups?

Leray spectral sequence for any sheaf $\mathcal{F}$ on $\widetilde{X} \to B_2$:

$$E_2^{p,q} = H^p\Big(B_2,\ R^q\pi_{2*}\mathcal{F}\Big) \quad \Rightarrow \quad H^{p+q}\Big(\widetilde{X},\ \mathcal{F}\Big)$$

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Think of $E_2^{p,q}$ as the "forms with p legs along the base and q legs along the fiber".

Spectral Sequences II

Example: $H^1\Big(\widetilde{X}, \wedge^2 \mathcal{V}\Big) = H^1\Big(\widetilde{X}, 2\chi_2 \pi_2^*(\mathcal{W})\Big)$

$$\pi_{2*}\Big(2\chi_2\pi_2^*(\mathcal{W})\Big) = 2\chi_2\mathcal{W}$$

$$R^1\pi_{2*}\Big(2\chi_2\pi_2^*(\mathcal{W})\Big) = 2\chi_1\chi_2\mathcal{W} \otimes \mathcal{O}_{B_2}(-f_2)$$

Compute $H^p(B_1, \cdots)$ by two more Leray SS...

$$\Rightarrow E_2^{p,q} = \begin{array}{c|ccc} q{=}1 & 0 & 2\oplus 2\chi_1 \oplus 2\chi_2 \oplus 2\chi_1^2 \oplus 2\chi_2^2 \oplus 2\chi_1\chi_2^2 \oplus 2\chi_1^2\chi_2 & 0 \\ q{=}0 & 0 & 2\oplus 2\chi_1 \oplus 2\chi_2 \oplus 2\chi_1^2 \oplus 2\chi_2^2 \oplus 2\chi_1\chi_2^2 \oplus 2\chi_1^2\chi_2 & 0 \\ \hline & p{=}0 & p{=}1 & p{=}2 \end{array}$$

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  $\Rightarrow$ no enhanced spectrum.
- Unique solution?
- Equivariant actions are the key.

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Equivariant Structures I

Recall our solution: $0 \longrightarrow \mathcal{V}_1 \longrightarrow \mathcal{V} \longrightarrow \mathcal{V}_2 \longrightarrow 0$

$$\mathcal{V}_1 \stackrel{\text{def}}{=} \chi_2\ \mathcal{O}_{\widetilde{X}}(-\tau_1 + \tau_2) \oplus \chi_2\ \mathcal{O}_{\widetilde{X}}(-\tau_1 + \tau_2)$$

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Without this, $\mathcal{W}$ is not a bundle!

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Constraint on characters:
$SU(4)$ bundle, not $U(4)$
$\Leftrightarrow \quad \wedge^4 \mathcal{V} = \mathcal{O}_{\widetilde{X}}$.

Without this, $\mathcal{W}$ is not a bundle!

Equivariant Structures II

Different G-action: $$0 \longrightarrow \mathcal{V}'_1 \longrightarrow \mathcal{V}' \longrightarrow \mathcal{V}'_2 \longrightarrow 0$$

$$\mathcal{V}'_1 \stackrel{\text{def}}{=} \quad \mathcal{O}_{\widetilde{X}}(-\tau_1 + \tau_2) \oplus \chi_2^2 \mathcal{O}_{\widetilde{X}}(-\tau_1 + \tau_2)$$

$$\mathcal{V}'_2 \stackrel{\text{def}}{=} \mathcal{O}_{\widetilde{X}}(\tau_1 - \tau_2) \otimes \pi_2^*(\mathcal{W}')$$

$$0 \longrightarrow \mathcal{O}_{B_2}(-2f_2) \longrightarrow \mathcal{W}' \longrightarrow \chi_2 \mathcal{O}_{B_2}(2f_2) \otimes I_9 \longrightarrow 0$$

This is a different equivariant bundle on $\widetilde{X}$

$\Rightarrow$ a different quotient bundle on X!

Equivariant Structures III

$$H^1\left(\widetilde{X}, \wedge^2\mathcal{V}\right) = 2 \oplus 2\chi_1 \oplus 2\chi_2 \oplus 2\chi_1^2 \oplus 2\chi_2^2 \oplus \oplus 2\chi_1\chi_2^2 \oplus 2\chi_1^2\chi_2$$

$$H^1\left(\widetilde{X}, \wedge^2\mathcal{V}'\right) = 2 \oplus \chi_1 \oplus \chi_1^2 \oplus 2\chi_2 \oplus 2\chi_1\chi_2 \oplus \oplus \chi_1^2\chi_2 \oplus 2\chi_2^2 \oplus \chi_1\chi_2^2 \oplus 2\chi_1^2\chi_2^2$$

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Equivariant Structures III

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Different spectrum:

One up Higgs, one down Higgs,

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$$\oplus \chi_1^2\chi_2 \oplus 2\chi_2^2 \oplus \chi_1\chi_2^2 \oplus 2\chi_1^2\chi_2^2$$

Different spectrum:

One up Higgs, one down Higgs, two $\underline{\mathbf{3}}$, and two $\underline{\overline{\mathbf{3}}}$.

Spectrum Summary

- 3 families of quarks and leptons.
- Zero anti-families.
- 4 Higgs (twice MSSM).
- Doublets and triplets are completely split, all triplets are projected out.
- Hidden pure E_7 or $\mathrm{Spin}(12)$ with 2 matter fields.

Within our Ansatz, there is only one solution that is even close to a realistic spectrum.

Future Directions

- Supersymmetry breaking.
- $U(1)_{B-L}$ breaking.
- Yukawa couplings.
- Moduli stabilization.
- Revisit $SU(5)$ with $\mathbb{Z}_2$ Wilson line: no $U(1)_{B-L}$.