Solitary-Wave Collisions

by

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with

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Two-Wave Collisions

Head-on Collision

Following Collision

EXPERIMENTS

Mathematical Models

- Linear superposition of KdV Solitons (Yi)
- Higher-order, coupled KdV equations (Wright, Wayne)
- Numerical solutions of Euler's equations (Craig, Guyenne, Sulem)
- N-soliton solution of KdV (N=2)
- Numerical solutions of Euler's equations (CGS)

Experiments

OBJECTIVE: High-Resolution, Spatial Data at Fixed Times.

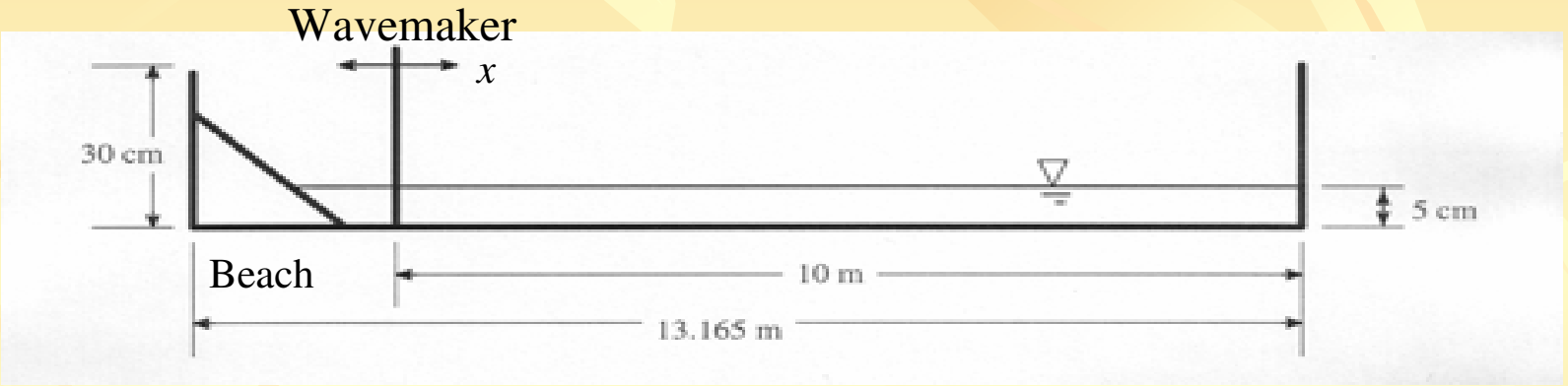
KEY: Repeatable Experiments.

- Wave Channel
- Wave Maker
- Instrumentation Carriage
- Wave Gages
- Pressure Gage
- Data Acquisition
- Control System
- Fluid Domain
- Procedures

Wave Channel & Wave Maker

Channel Width = 25.4cm

QuickTime™ and a
Motion JPEG OpenDML decompressor
are needed to see this picture.



Wavemaker:

- § Linear Motor
- § 20,000 Cnts/cm
- § Zero Backlash
- § Teflon Paddle
- § Negligible Leakage

Instrumentation Carriage, Wave Gages, Pressure Gage

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Motion JPEG OpenDML decompressor
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Instrumentation Carriage

- Programmable, linear belt drive with motor.
- Four capacitance-type wave gages.
- Rides on stainless steel rails.

Wave Gages

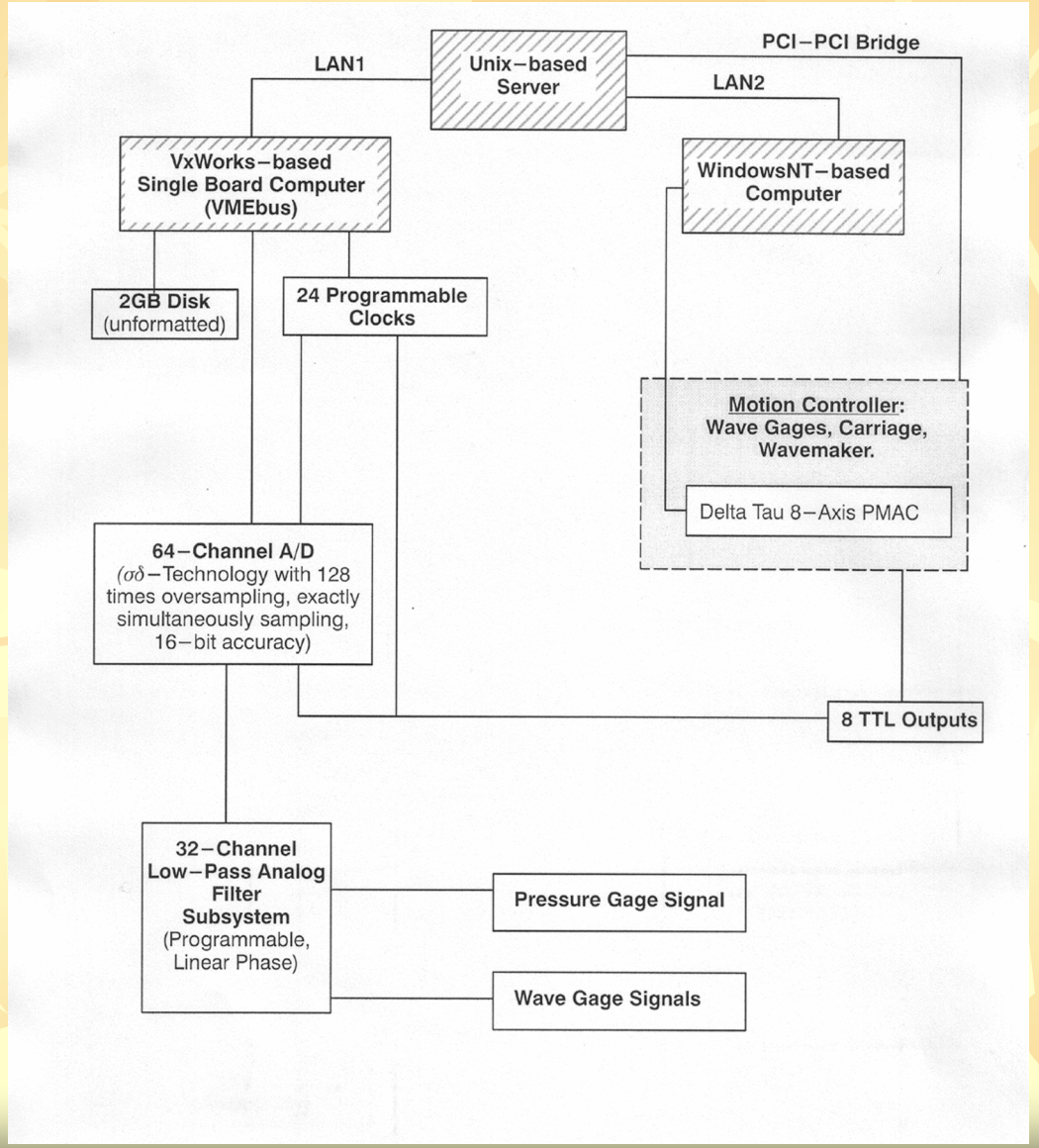
- Non-intrusive.
- Capacitance-type.
- Resolution = 0.04 mm.
- 40 cm apart.
- Linear, repeatable, programmable calibrations.
- Sensing element: 6 mm X 12.7 cm (cross-channel-width), and 2.5 cm above water surface.

Pressure Gage

- Flush-mounted in channel bottom at x=7.155m.
- Linear calibration with 0-5V output for 0-10.16cm water pressure head.
- Depth control: 0.02V ~ 0.3mm depth ~ 1 liter water in the channel.

Data Acquisition & Control Systems

Sampling rate =1627.6 Hz;
Decimated to 325.5 Hz



General Procedures

- Position carriage. Run experiment. Obtain time series at 4 positions under carriage.
- Move carriage downstream 1 cm. Run experiment.
- Repeat above procedure 38 more times.
- Concatenate the 160 time series.

RESULT: High resolution of wave amplitudes at 1cm intervals for 160 cm of x-distance under instrumentation carriage at any time.

IFF EXPERIMENTS ARE *EXACTLY* REPEATABLE AT ALL POSITIONS x AND FOR ALL TIMES t.

- e.g. A difference in wave arrival times at any x between two experiments of 0.01s corresponds to differences in wave positions of 1 cm (fatal!).

Sources of Errors

Sources of small differences between experiments:

- *Latency* in electro-mechanical, servo-systems.
  e.g. servo update period = 0.885 ms.

- *Water Depth* variations.
  e.g. evaporation.

- *Water Surface Contamination.*
  e.g. surfactant accumulation with time.

- *Residual boundary layers.*
  e.g. wave propagation through wave-wakes.

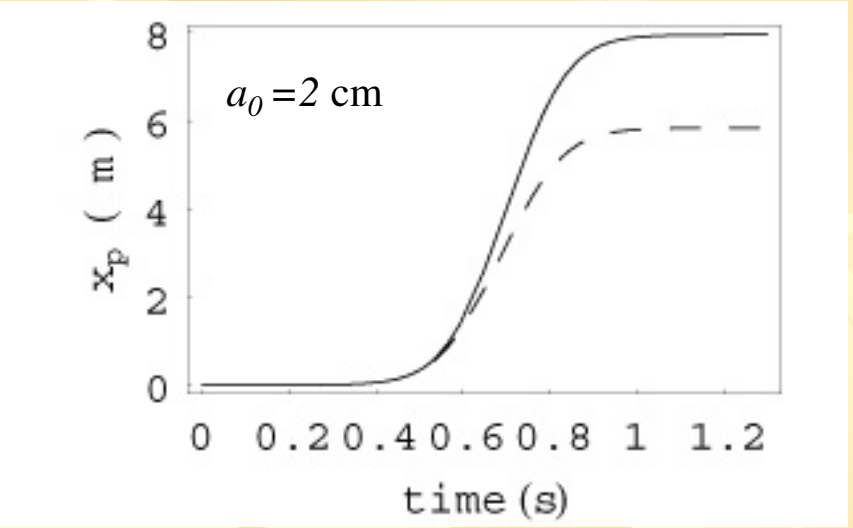
- *Pilot Error.*
  e.g. deterministic, real-time programming.

Wave Generation

KdV solitary-wave, horizontal velocity field:

a_0 := Wave Height

h_0=5 cm := Water Depth



Wavemaker displacement, x_p, found numerically from

Wave Generation

Carriage fixed during experiments: $a_0 = 2.0$ cm.

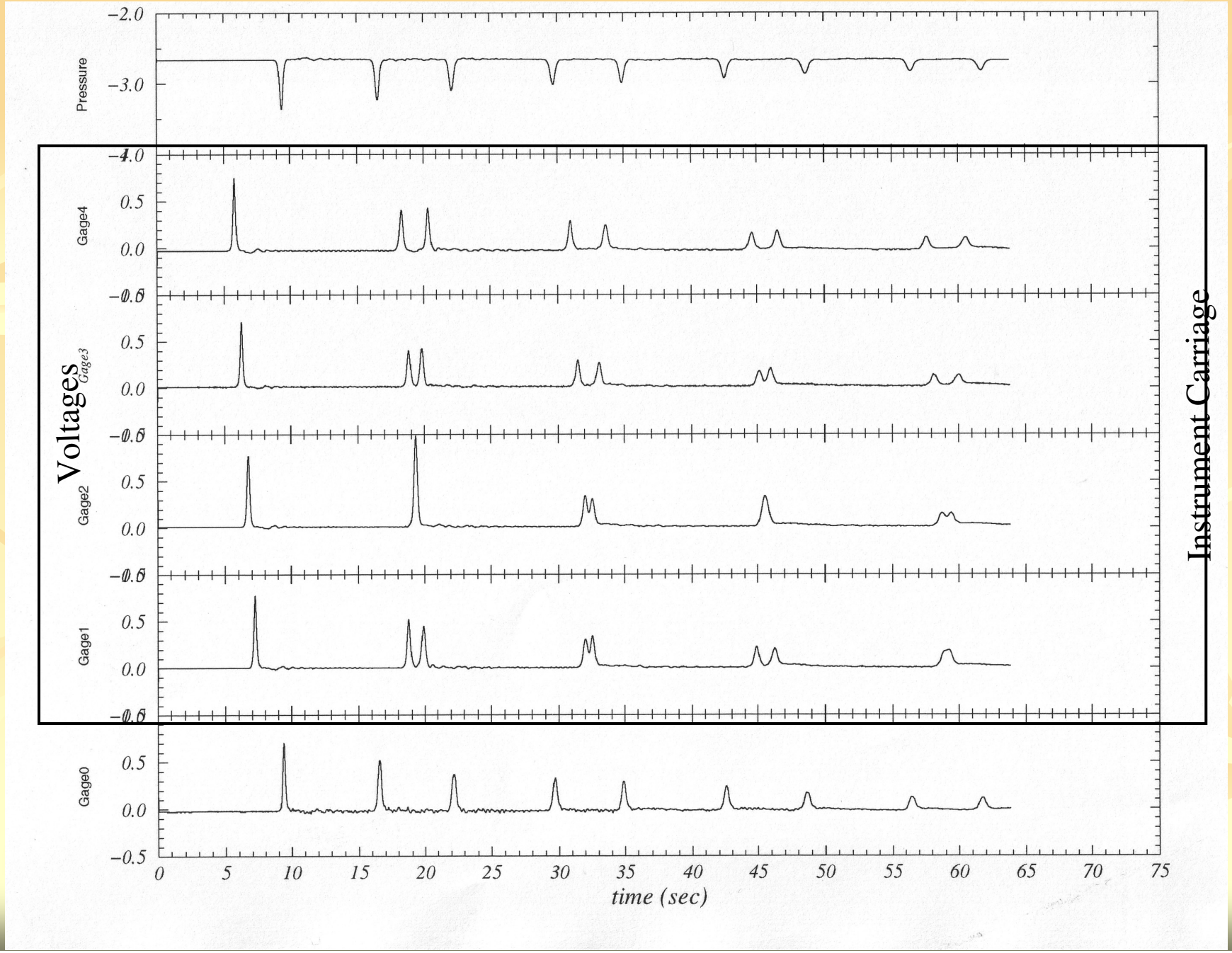
QuickTime™ and a
Video decompressor
are needed to see this picture.

Head-on Collision

PROCEDURES:

- Carriage measurement window:
- Generate 1st solitary wave with a_1=2.00 cm.
- Wait 10.8 sec, generate 2nd solitary wave with a_2=1.25 cm.
- First wave reflects from channel endwall and then collides with 2nd wave under instrumentation carriage and near center of channel. Data collected.
- Wait 12 mins. Move carriage 1cm downstream. Repeat experiment. (40 times total.)

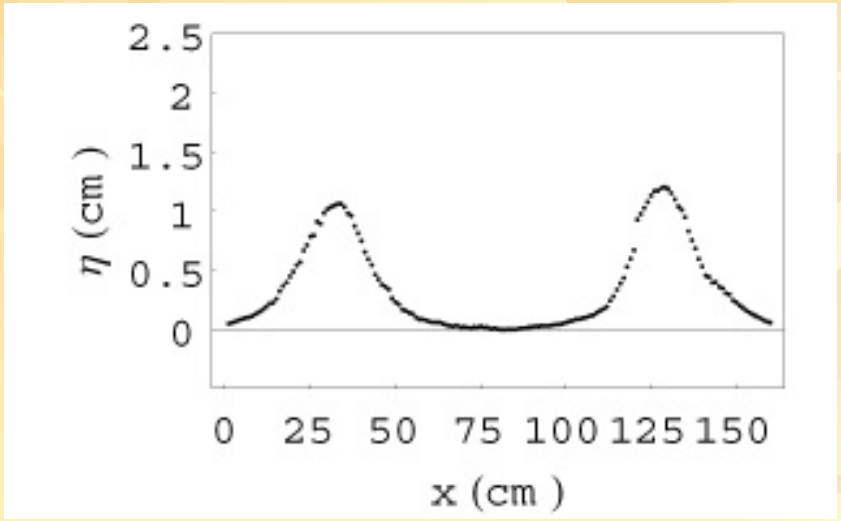
Head-on Collision: Raw Time Series



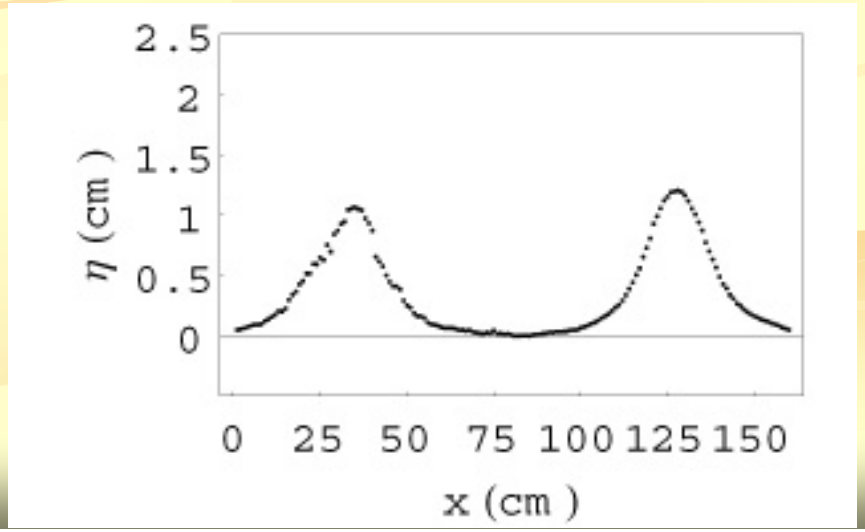
Procedure: Shifting individual data sets in time using pressure data.

Maximum shift: 1st day = 0.0154 sec; 2nd day = 0.0584 sec.

Shift based on the incident wave passage over the pressure gage.



Shift based on the reflected wave passage over the pressure gage.



Use this one.

Head-on Collision

QuickTime™ and a
Video decompressor
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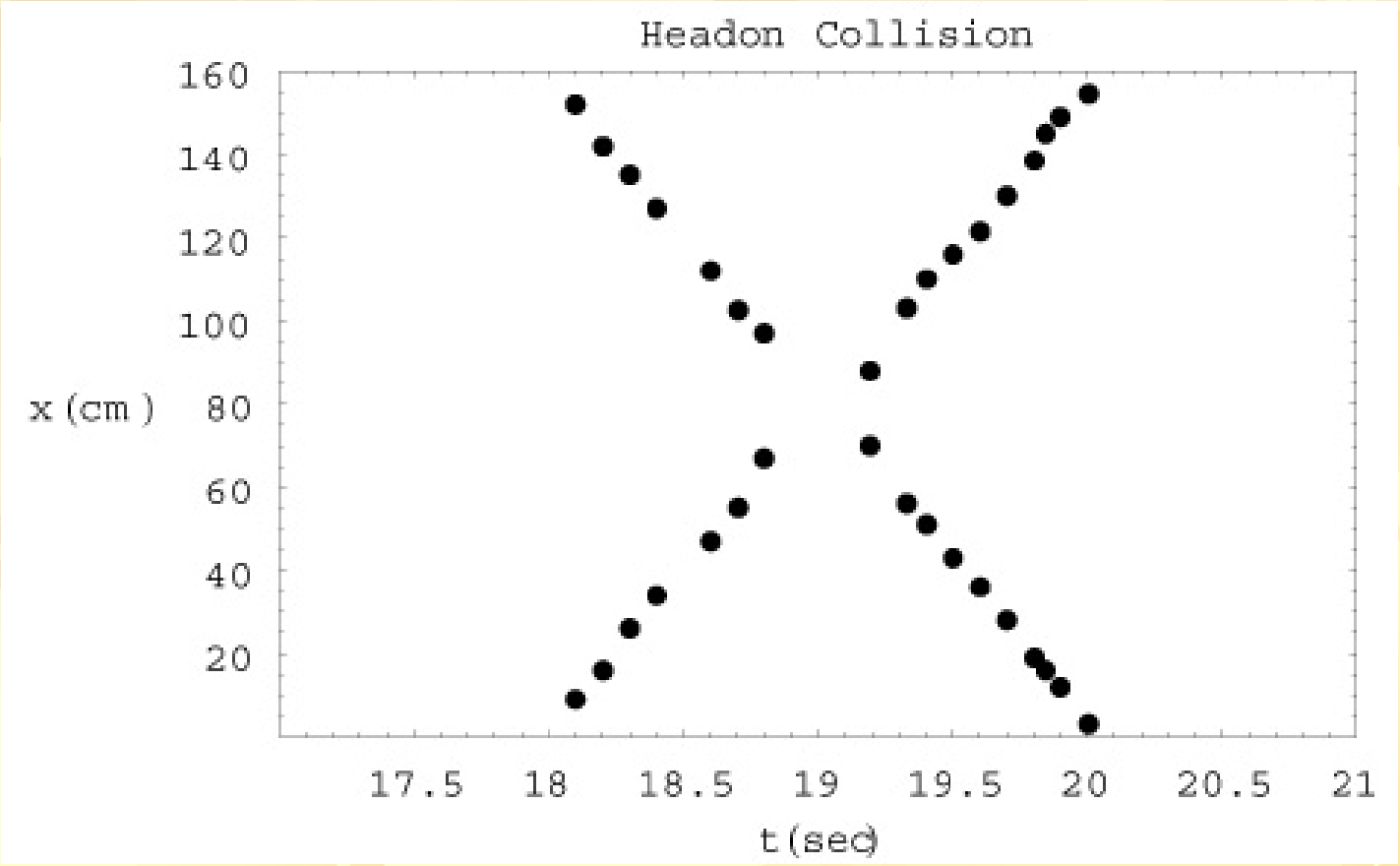
Head-on Collision



Head-on Collision

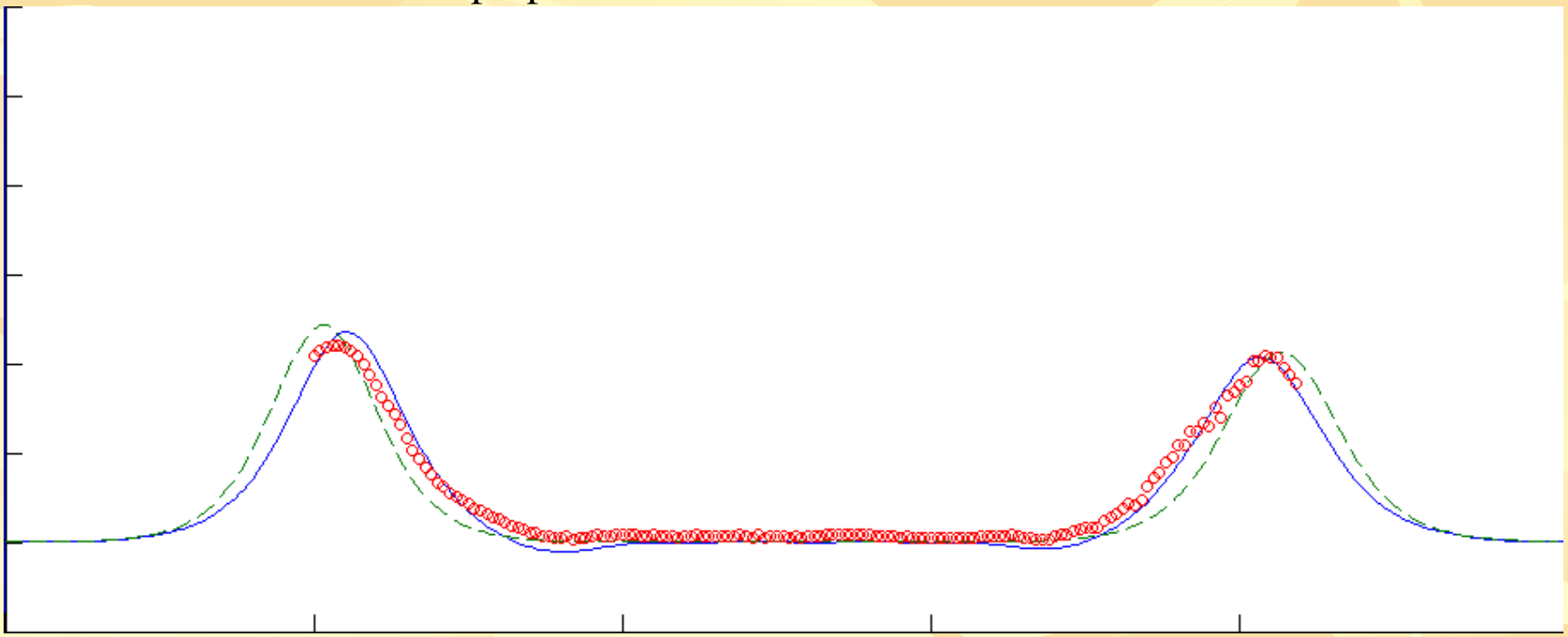
Solid blue: Euler
Dashed green: Linear superposition of KdV Solitons

a_1 =1.052 cm
a_2 =1.220 cm

QuickTime™ and a
decompressor
are needed to see this picture.

Head-on Collision

Solid Blue: Euler
Dashed Green: Linear superposition of KdV Solitons



Maximum measured height: 2.658 cm
Maximum predicted height: 2.763 cm
Linear sum of heights: 2.272 cm

Following Collision

PROCEDURES:

- Carriage initial measurement window:
- Run experiment with wave amplitudes set to zero to obtain water level record.
- Generate 1st solitary wave with a_1=0.75 cm, immediately followed by 2nd solitary wave with a_2=2.0 cm.
- Wait until waves are under carriage, and then begin carriage motion.
- Waves collide under the carriage, which travels 6.35m. Data collected.
- Wait 12 min. Move carriage 1 cm downstream. Repeat experiment (40 times total).

Following Collision

QuickTime™ and a
Motion JPEG OpenDML decompressor
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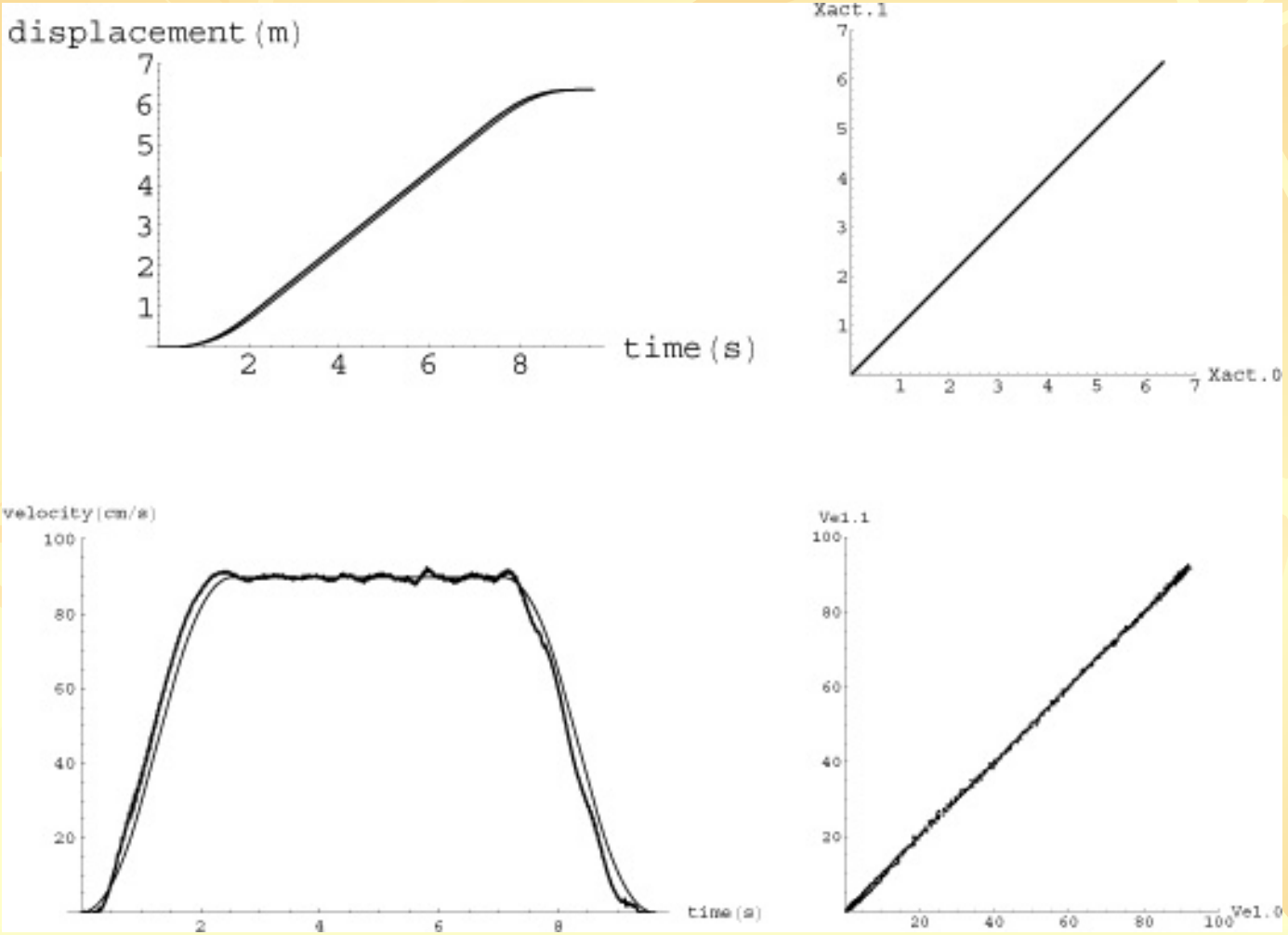
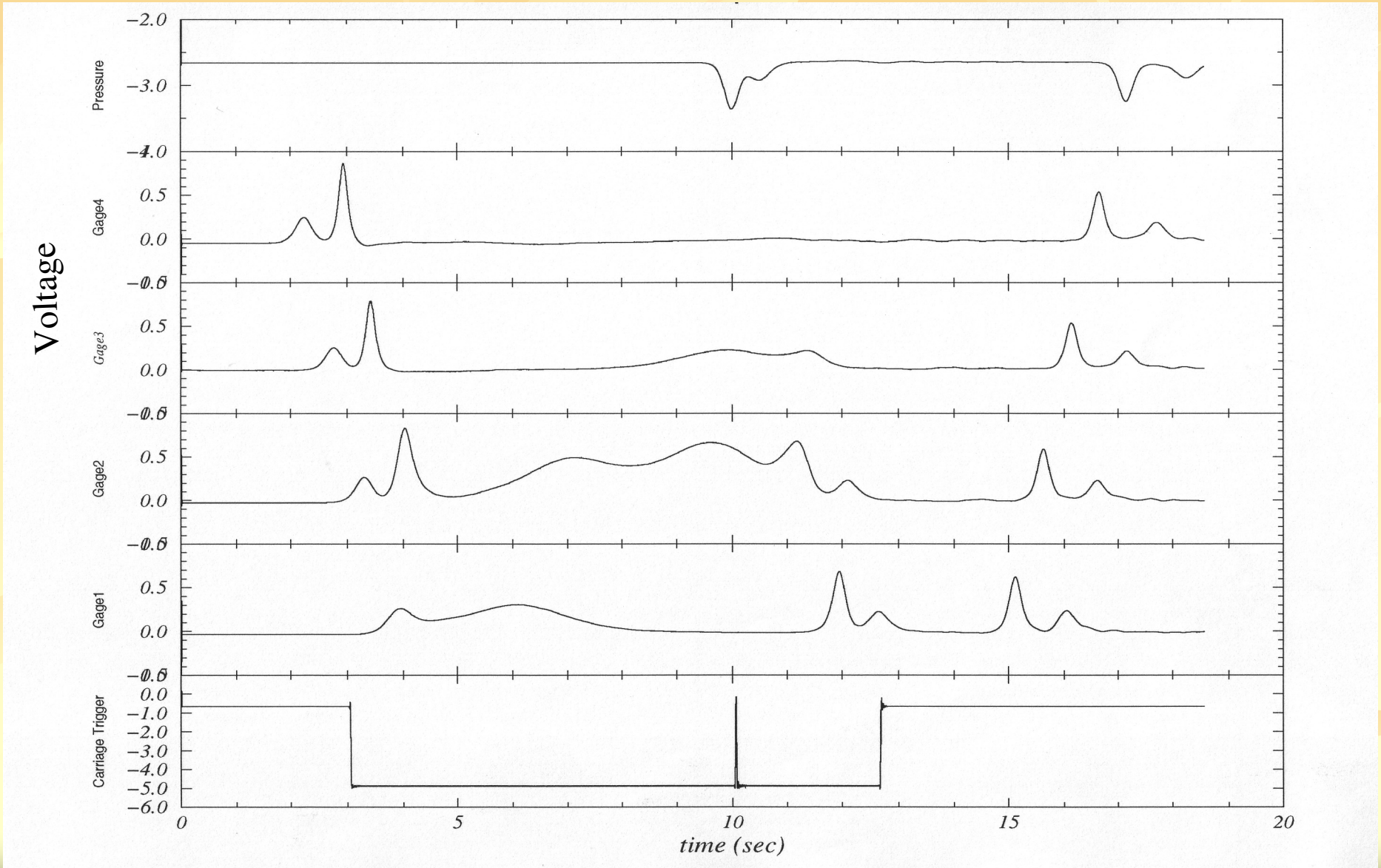
Following Collision

Programmed and Actual Carriage Motions



Following Collision: Raw Time Series



Following Collision

Dots: Experimental data

Line: 2-soliton solution of KdV using a_1 =2.23 cm and a_2 =0.73 cm.

QuickTime™ and a
Video decompressor
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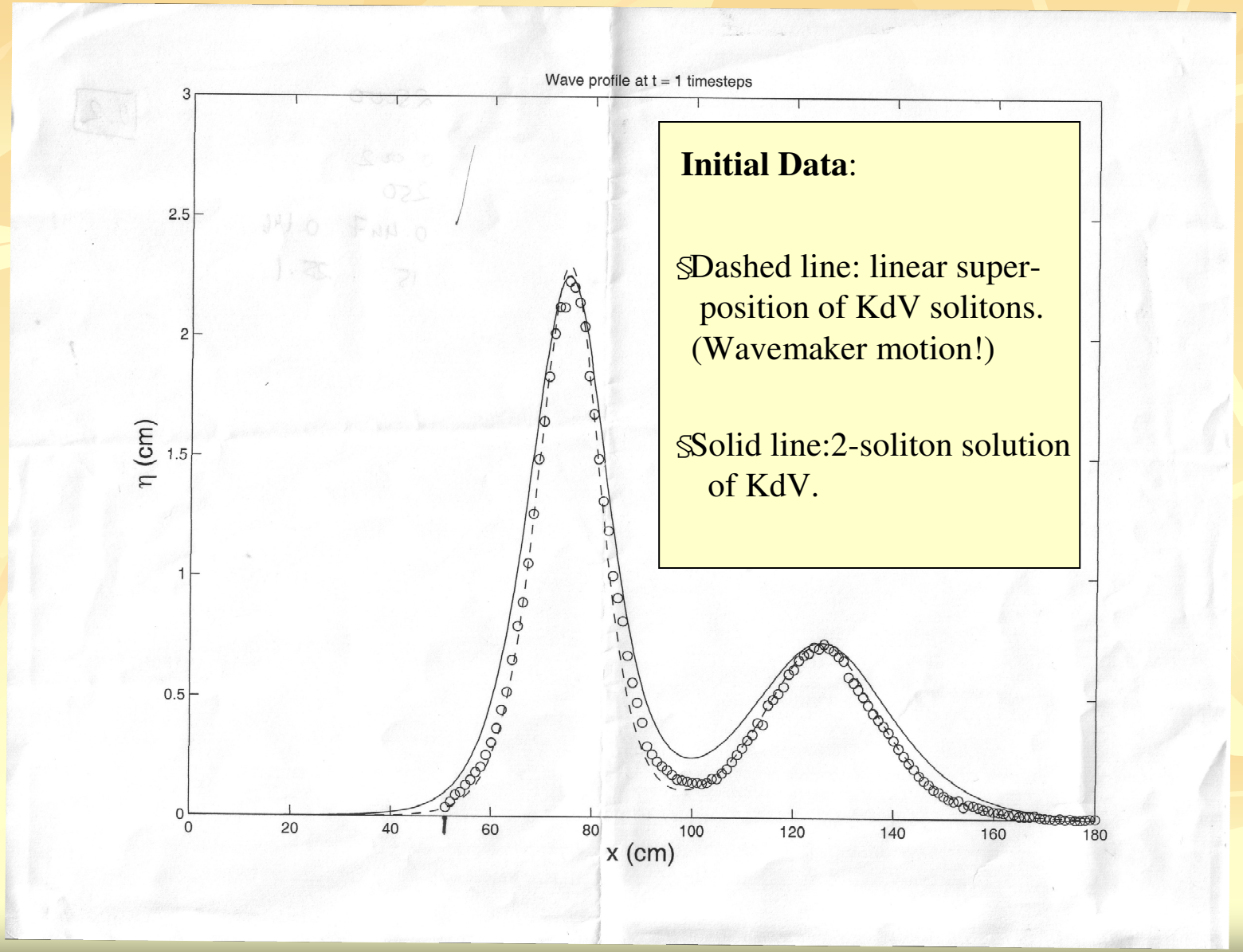
Following Collision



Initial Data:

- Dashed line: linear super-position of KdV solitons. (Wavemaker motion!)
- Solid line:2-soliton solution of KdV.

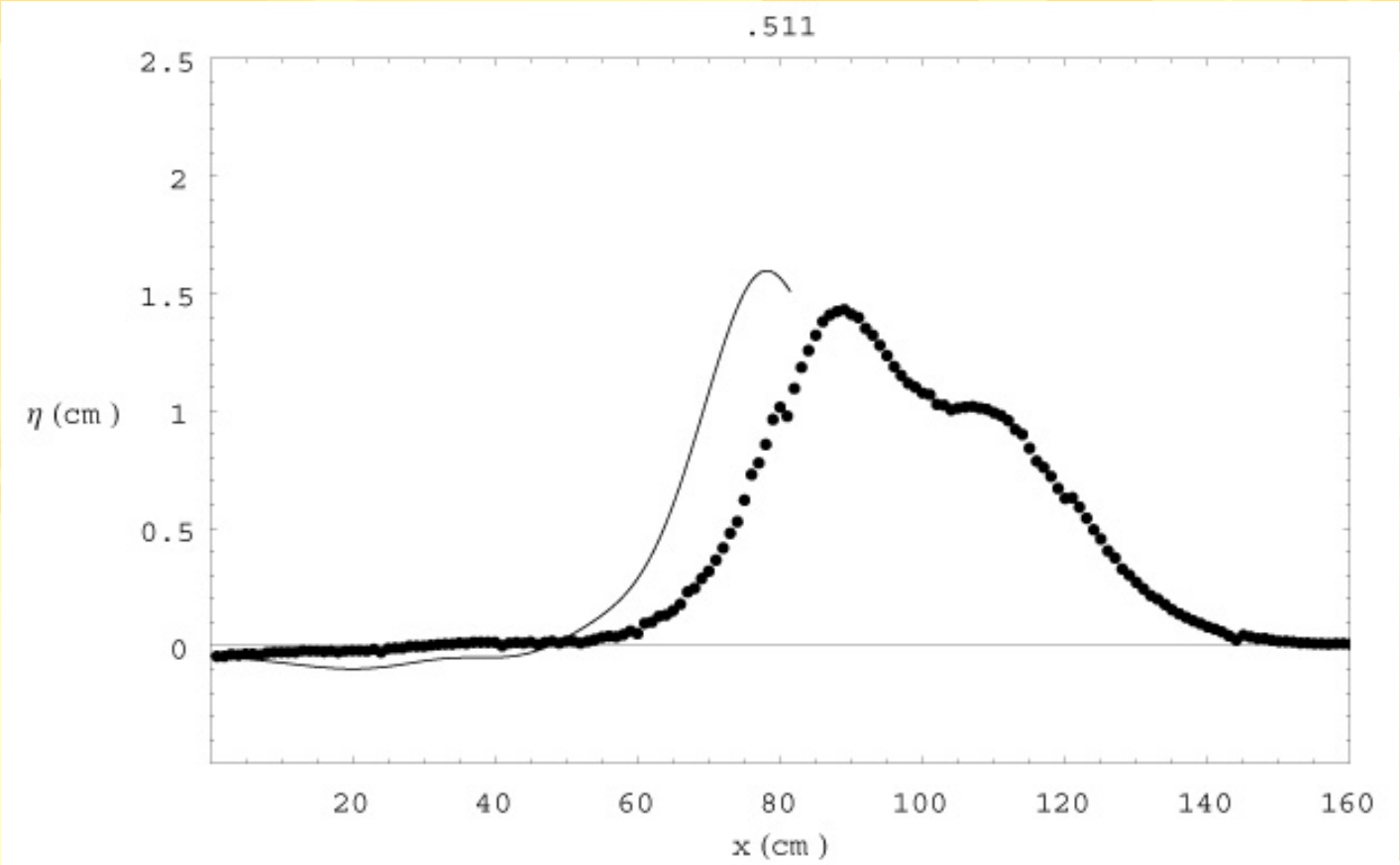
Following Collision- Movie

Dots: experimental data; Solid line: Euler

QuickTime™ and a
Video decompressor
are needed to see this picture.

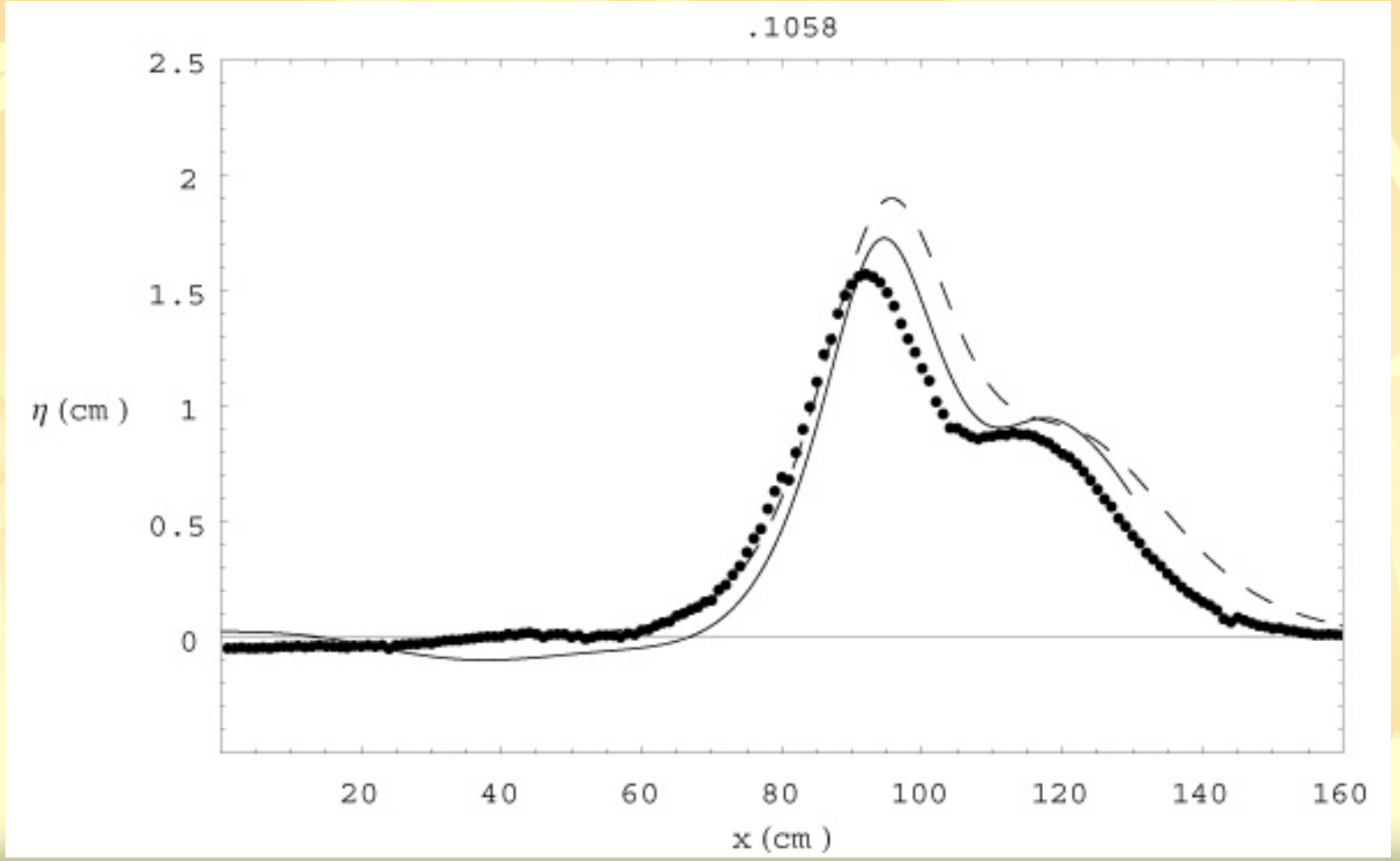
Following Collision - Frames

Dots: experimental data; Solid line: Euler



Following Collision - Comparison

Dots: experiments. Solid line: Euler. Dashed line: KdV; 2-soliton.



Wave Damping

$a_0 = 2.0$ cm