

subdiffusion in granular segregation experiments



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systems

Consist of discrete macroscopic particles with distinctive

A micrograph showing a dense network of green and yellow lines (stress chains) against a blue background. A large black circle is superimposed in the center, containing text. The lines are oriented diagonally, indicating the direction of shear stress.

systems

Stress chains in a 2D shear experiment

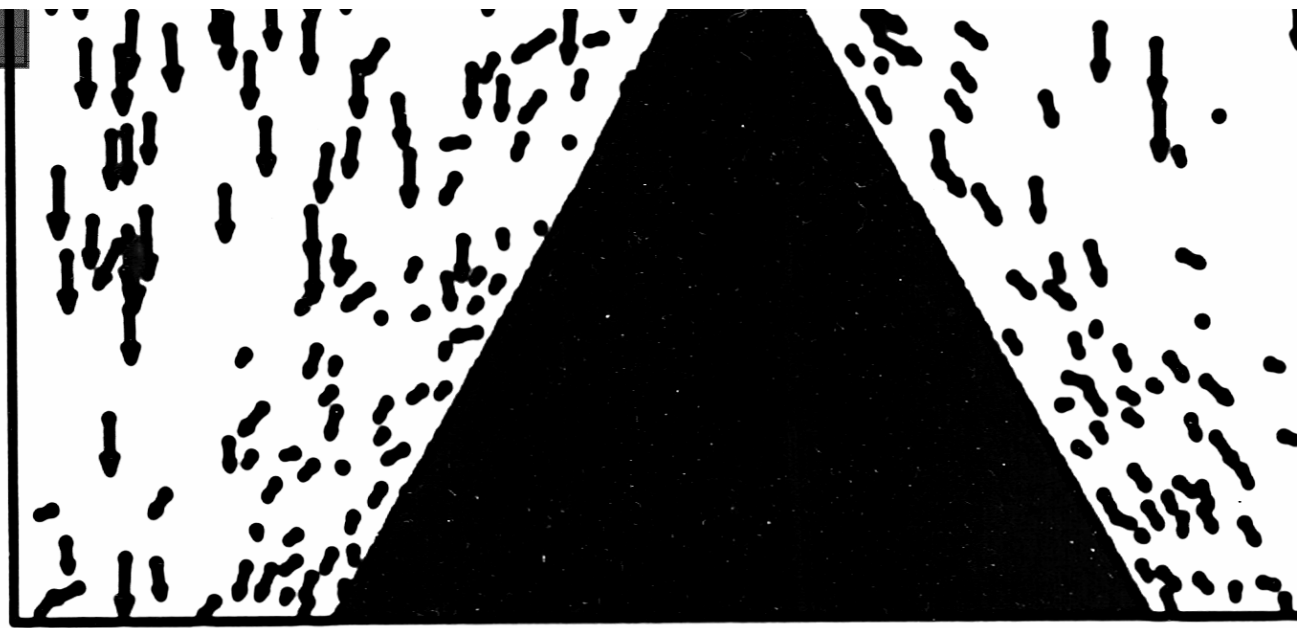
D. Howell and R. B. Behringer
Phys. Rev. Lett. 82, 5241 (1999).

Collective continuum behaviour is not captured by
any single well-agreed-upon differential equation.

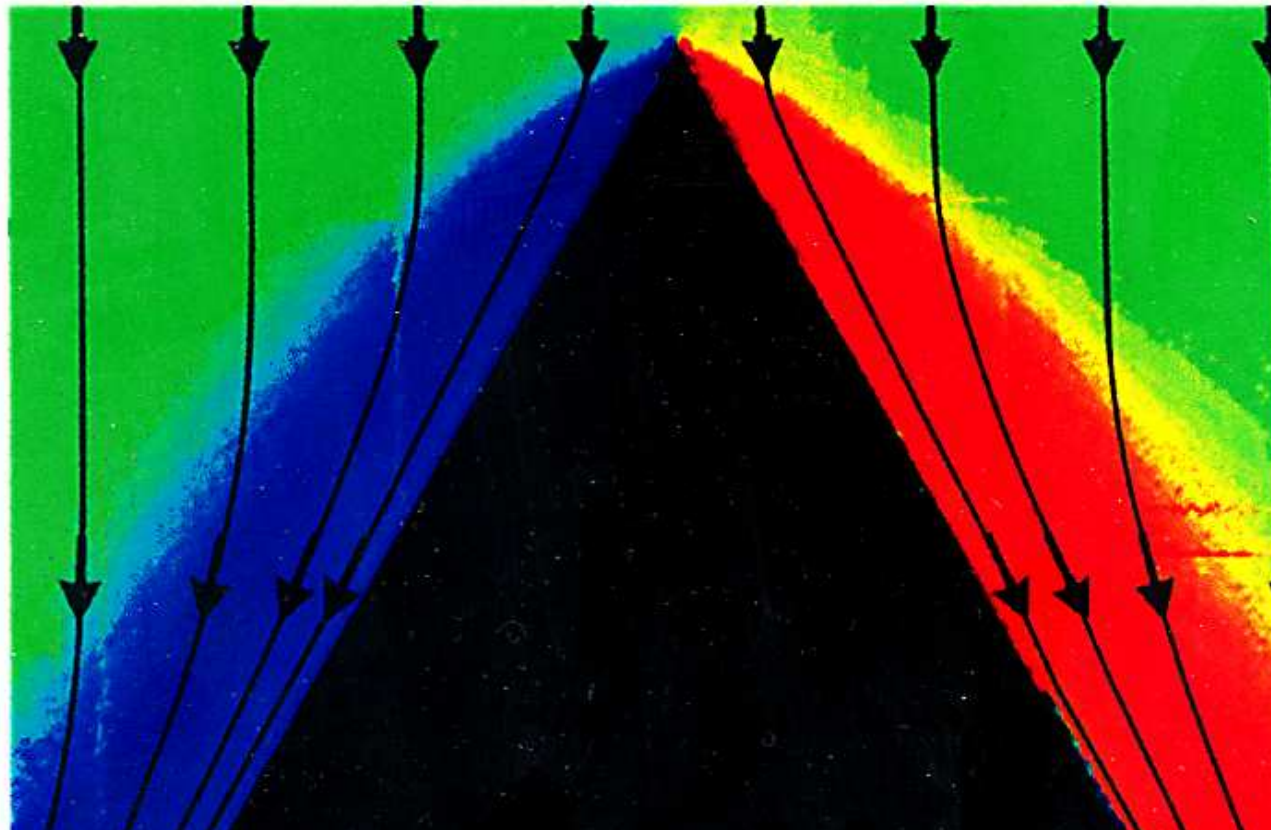
systems

highly
ed" flows are
y
equilibrium,
dense gas
, shocks etc.

richa, C. Bizon, M.
k and H. L. Swinney,
ev. Lett. 88, 014302
(2002).



Time Averaged Horizontal Velocity Field



Granular systems inelastic collapse and clustering in granular gases

1000 inelastic
, coefficient of
restitution 0.6, 500
collisions per particle

Heinrich and G.
Phys. Rev. Lett.
1610 (1992)

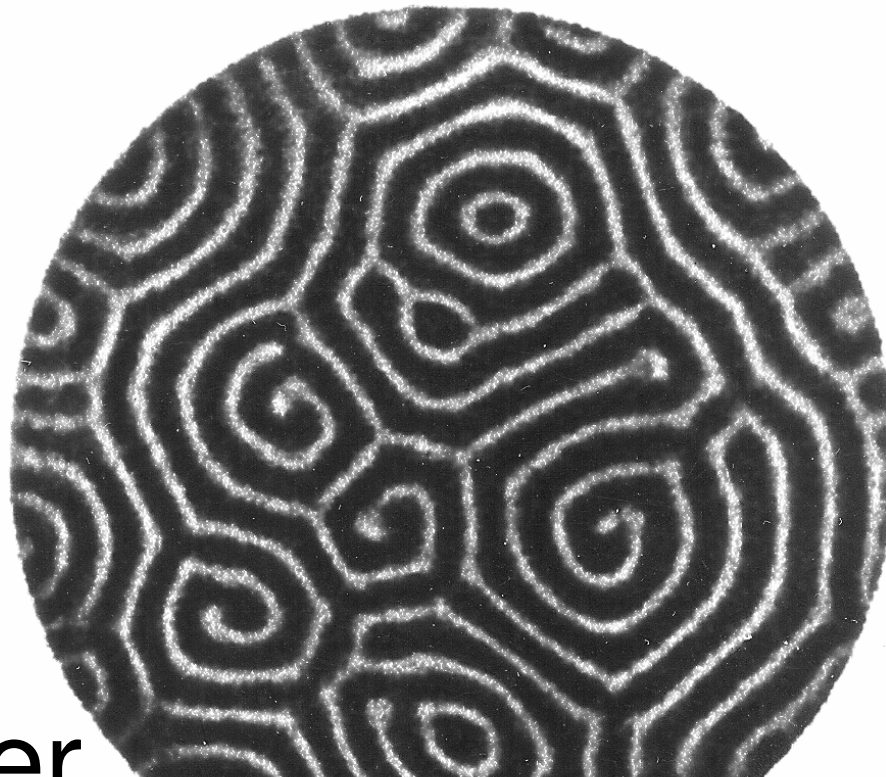
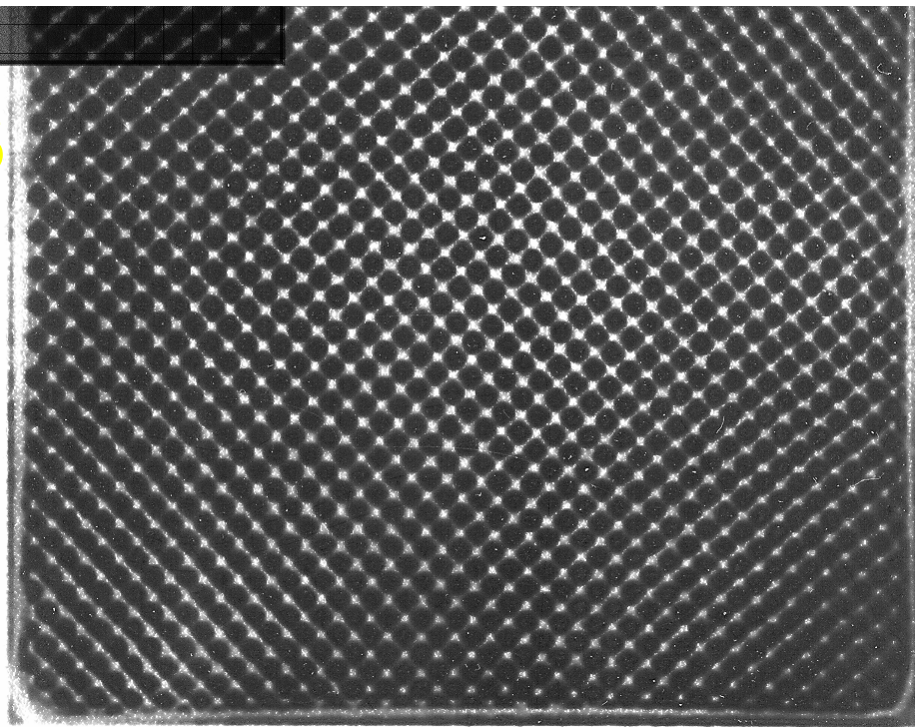


Granular systems

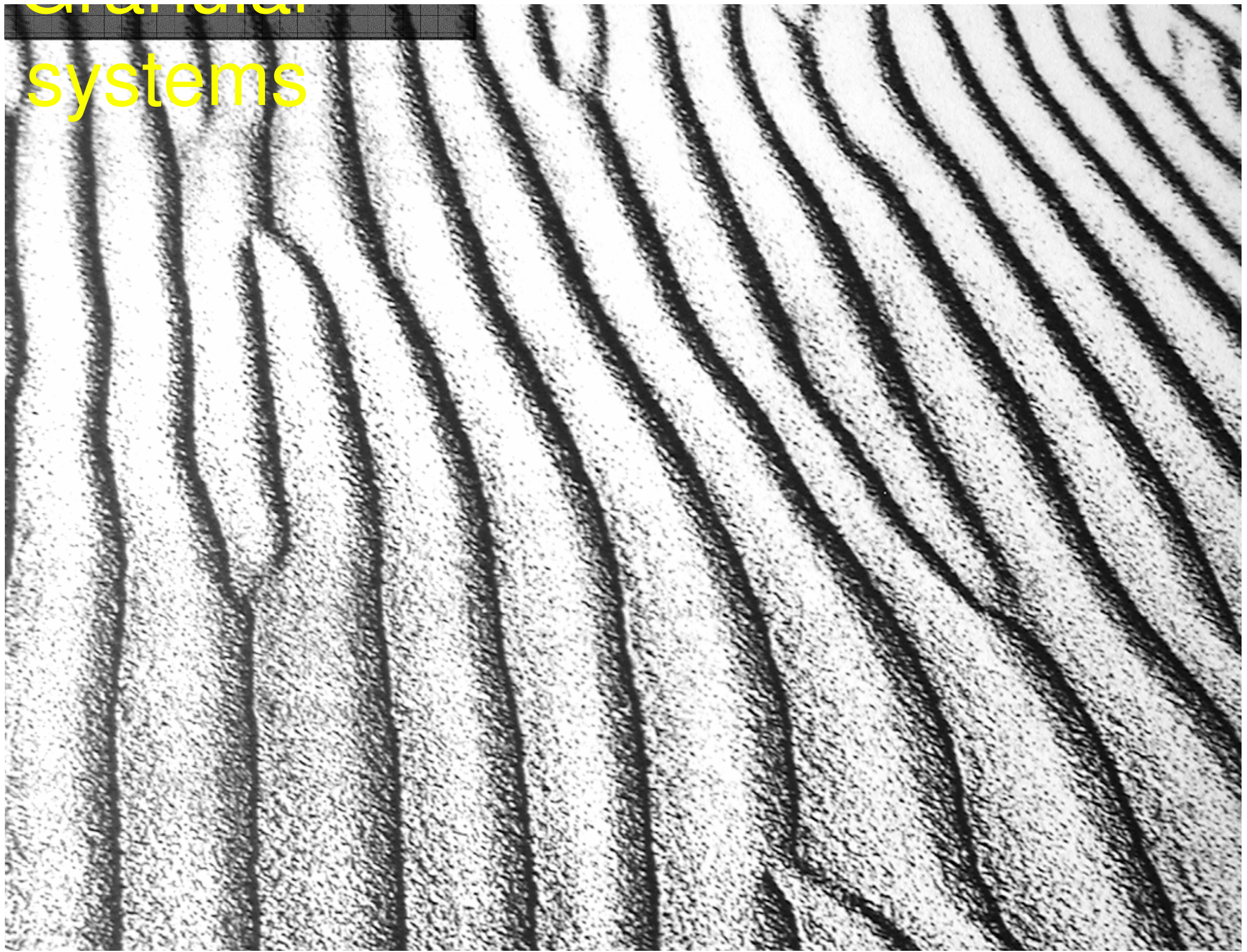
Granular layers
subjected to
vertical shaking



shaker



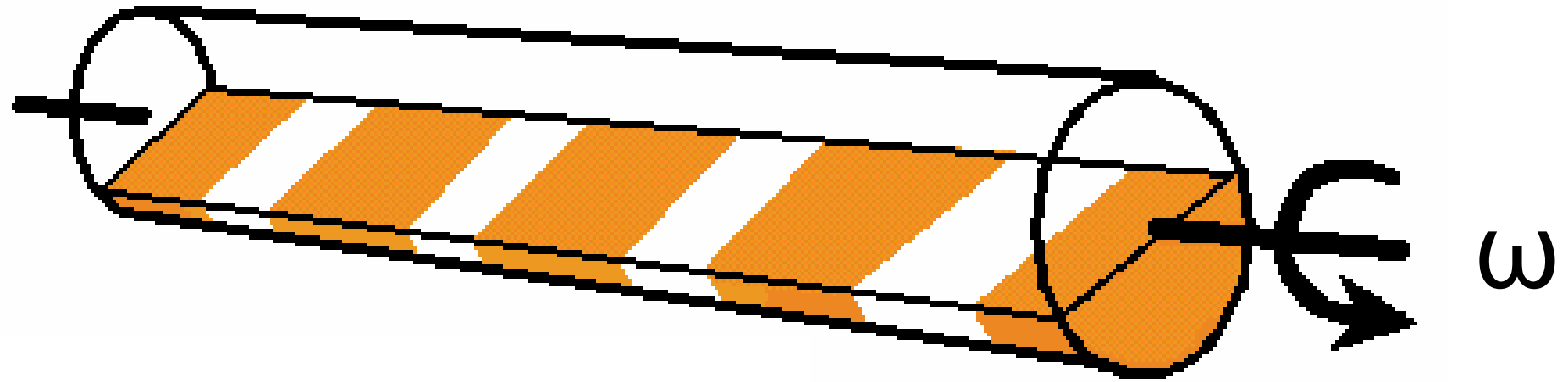
channel systems



Channel systems



size segregation in a long drum mixer



grains = white (*table salt or glass spheres*)
 $\sim 0.1\text{mm}$

.If grains = **black** (*hobby sand*) $\sim 0.2\text{mm}$
 ω = rotation frequency $\sim 2\pi/15$

η = filled volume fraction $\sim 1/3$

Φ = composition fraction

Movie of axial segregation with random premixed initial conditions

QuickTime™ and a
None decompressor
are needed to see this picture.

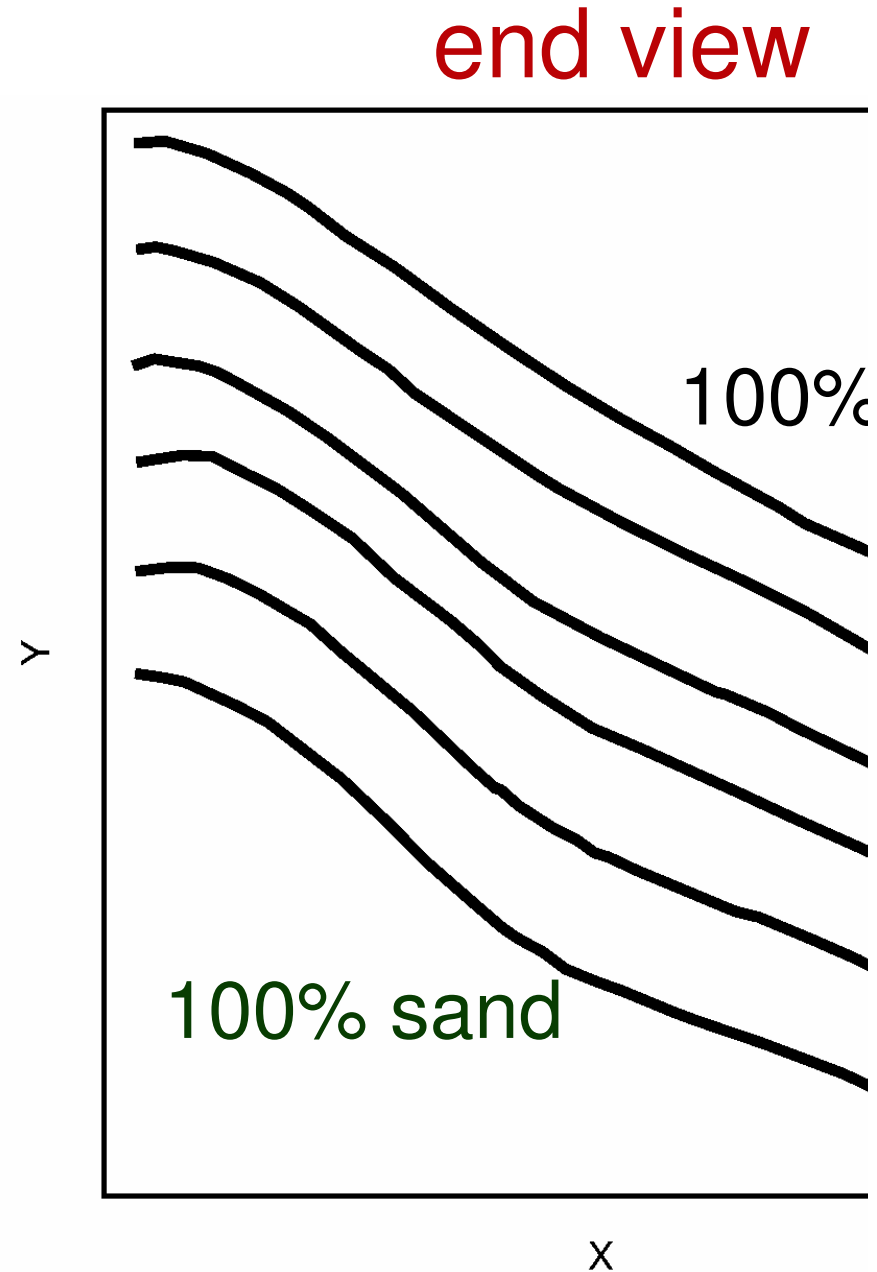
Speeded up 60 X real time

Perfectly premixed initial conditions are difficult to produce. We use a long U channel which is rotated in the tube to gently deposit premixed

ial segregation
des axial. Smaller
les move toward axis of
on.

aming surface shapes
erent concentrations
fferent. S-P. Breton.

y ideas tried to explain
segregation with a
ive effective axial



with random premixed initial

conditions

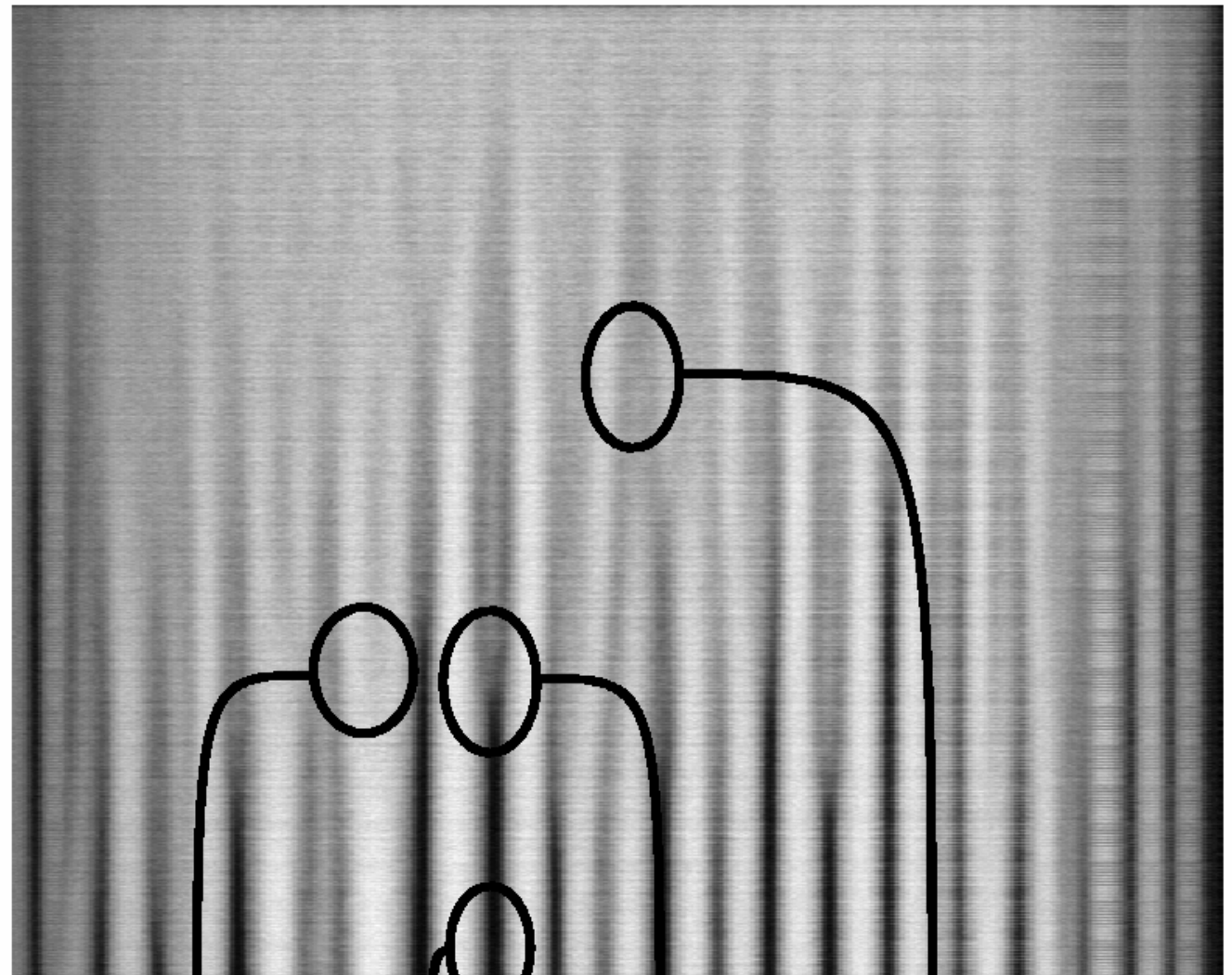
0 cm Space → 109 cm

0 s

Time



900 s



Band dis-
appearance

Band
merging

Band
splitting

ve rather
ex
rics
g to

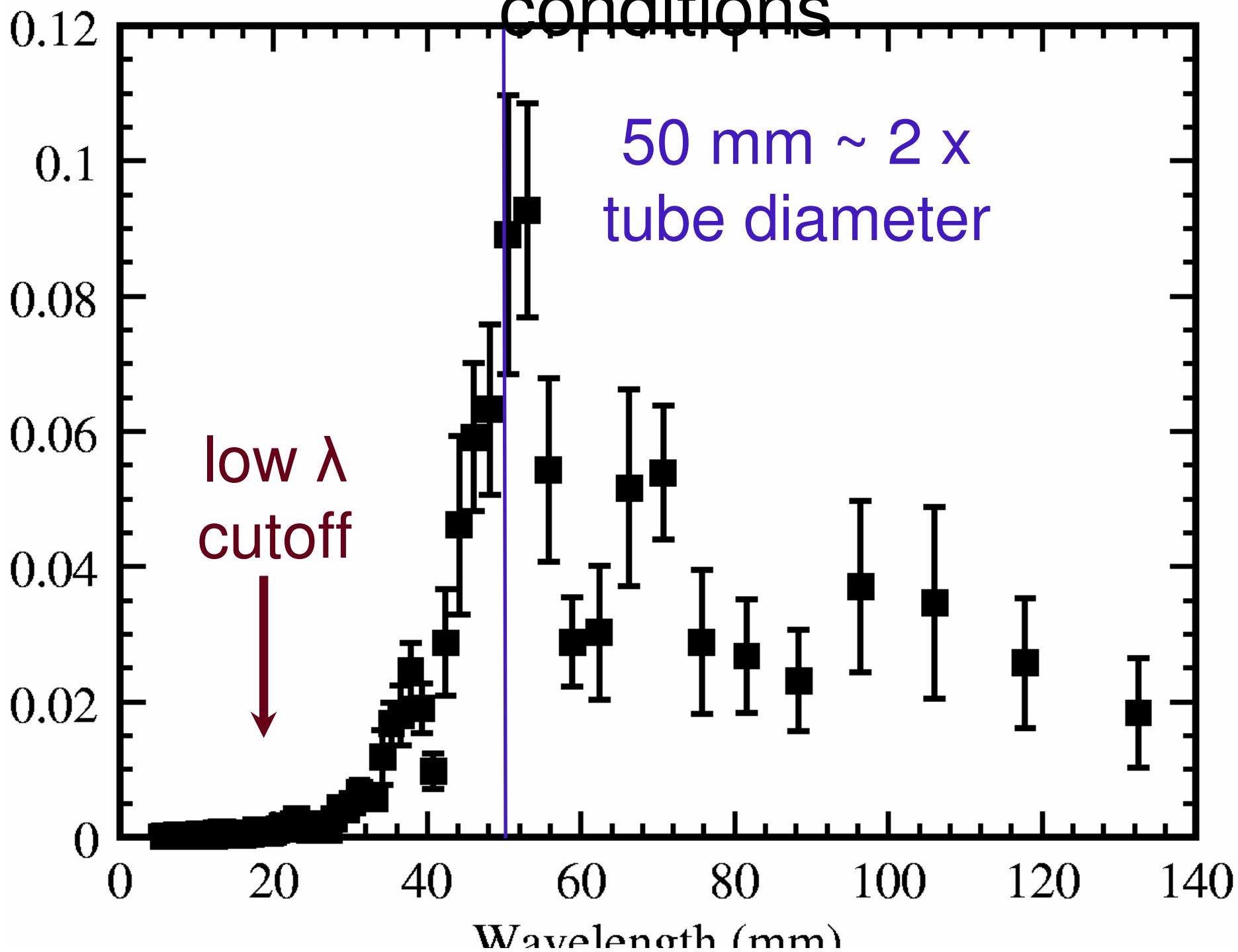
gated

.

= 0.5

bands with random premixed initial

conditions



with random premixed initial

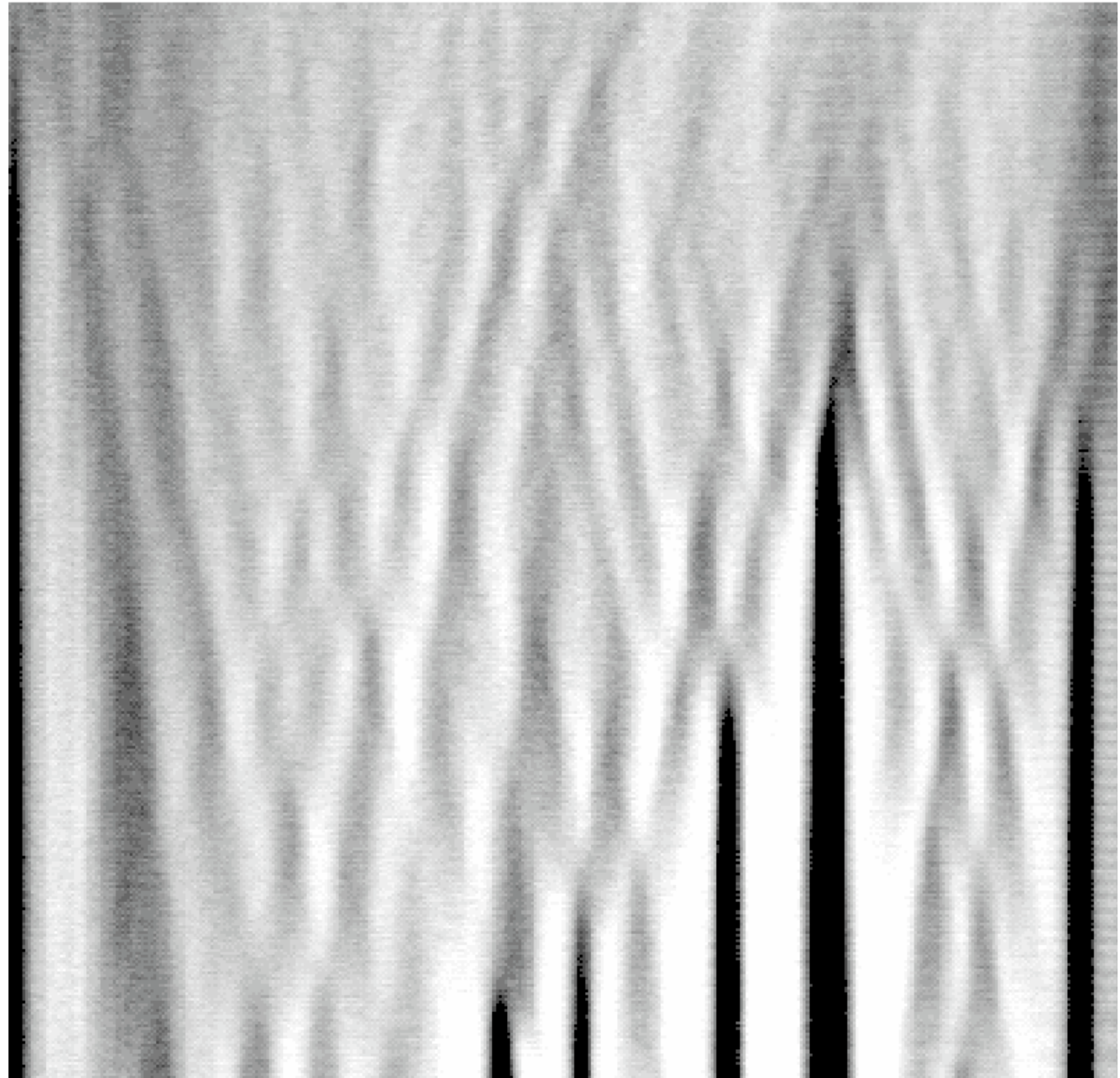
0 cm Space → conditions

0 s

= 0.66

some
ations of
velling
ctures.

Time



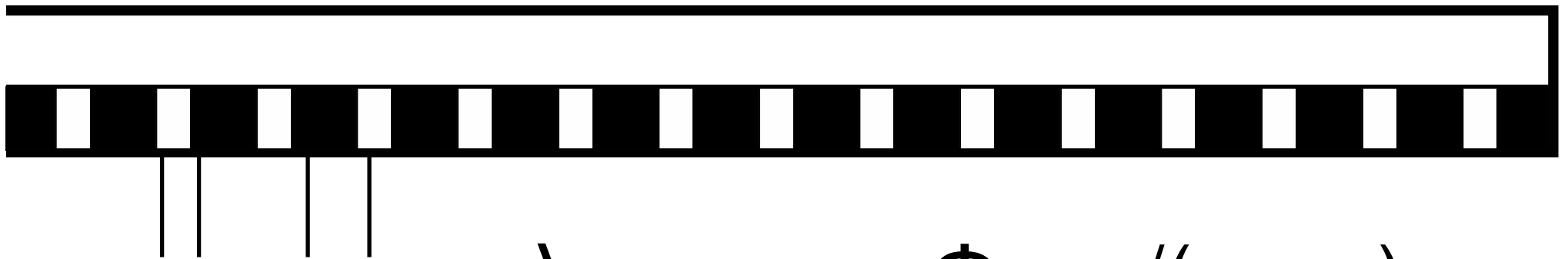
premixed initial conditions

instability amplifies fluctuations in the initial concentration distribution so the transient dynamics are reproducible. Solution: use **presegregated** conditions.

premixed with fluctuations



presegregated with fixed Φ and λ



with *presegregated* initial conditions

0 cm

Space →

10

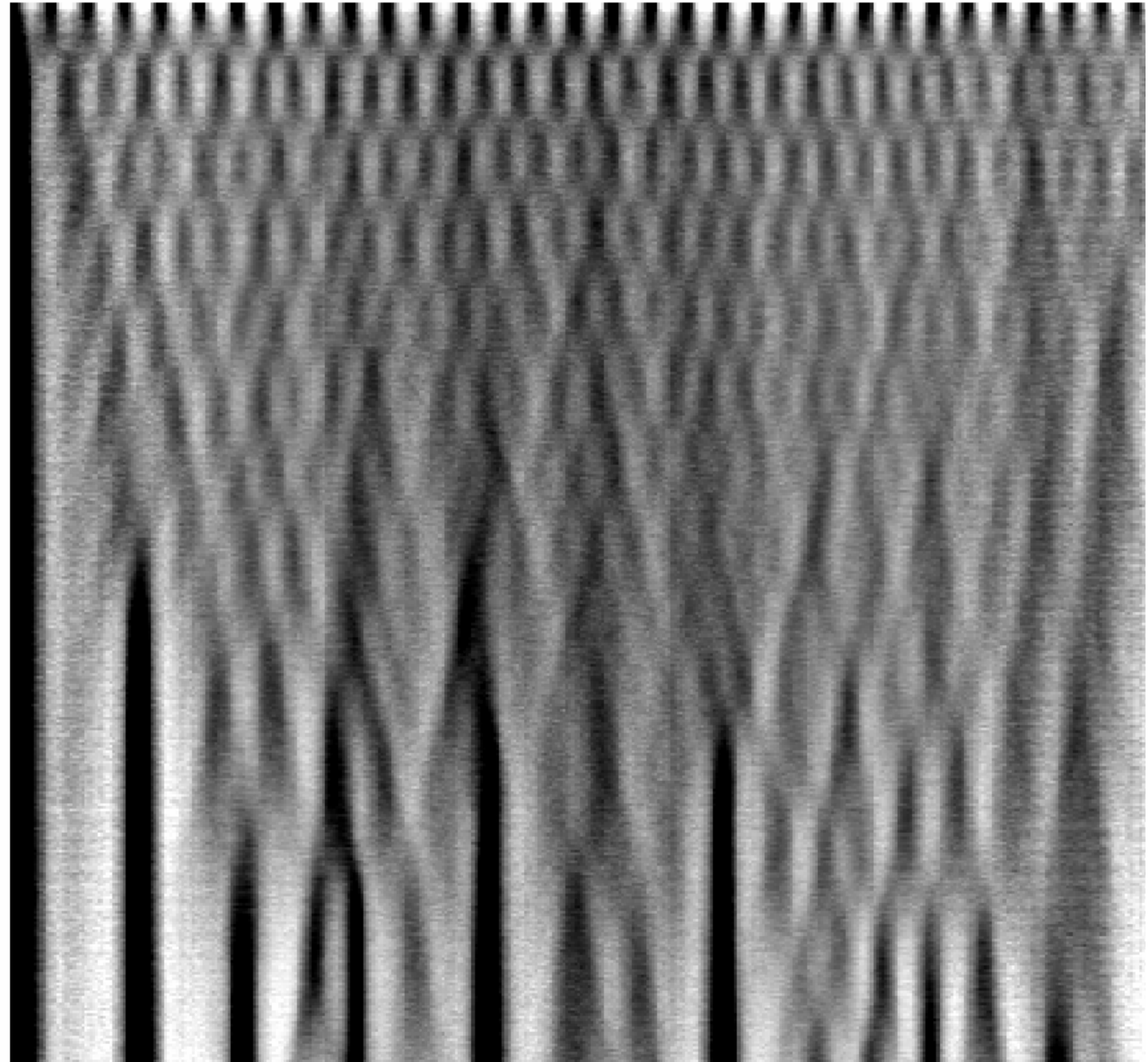
0 s

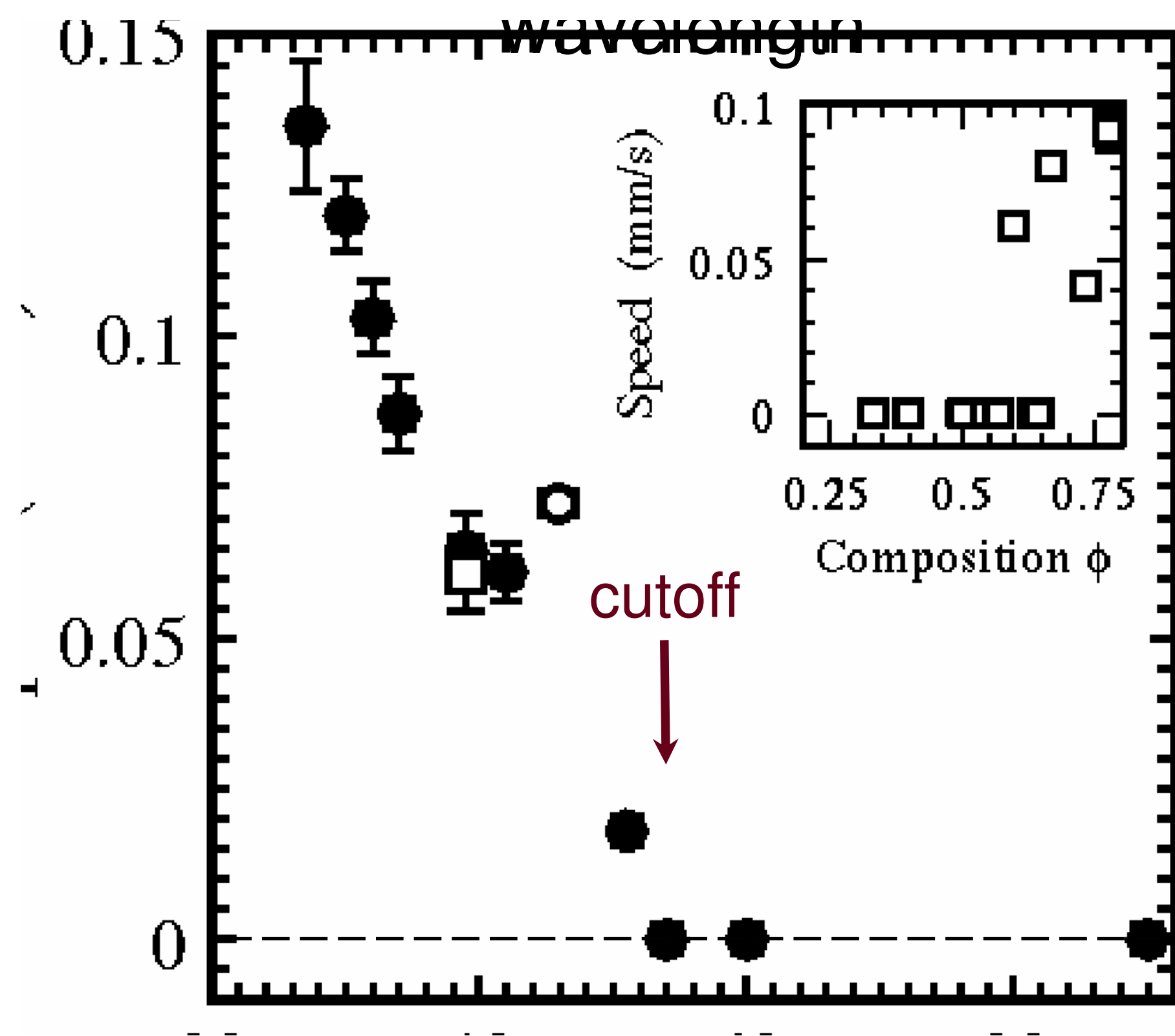
see a clear
ing wave as
leaving
transient

Time



= 0.66





ve a coupled diffusion process between
 concentration and the local streaming

$$\frac{\partial c}{\partial t} = - \frac{\partial}{\partial z} \left[- D \frac{\partial c}{\partial z} + g(c) \frac{\partial \theta}{\partial z} \right]$$

Fick diffusion

axial slope
 gradient flux

$$g(c) \sim 1 - c^2$$

$$\alpha [\Omega - \theta + f(c)] + D_\theta \frac{\partial^2 \theta}{\partial z^2} + \gamma \frac{\partial \theta}{\partial z}$$

rotation rate and
 concentration dependent
 source of slope

slope diffusion

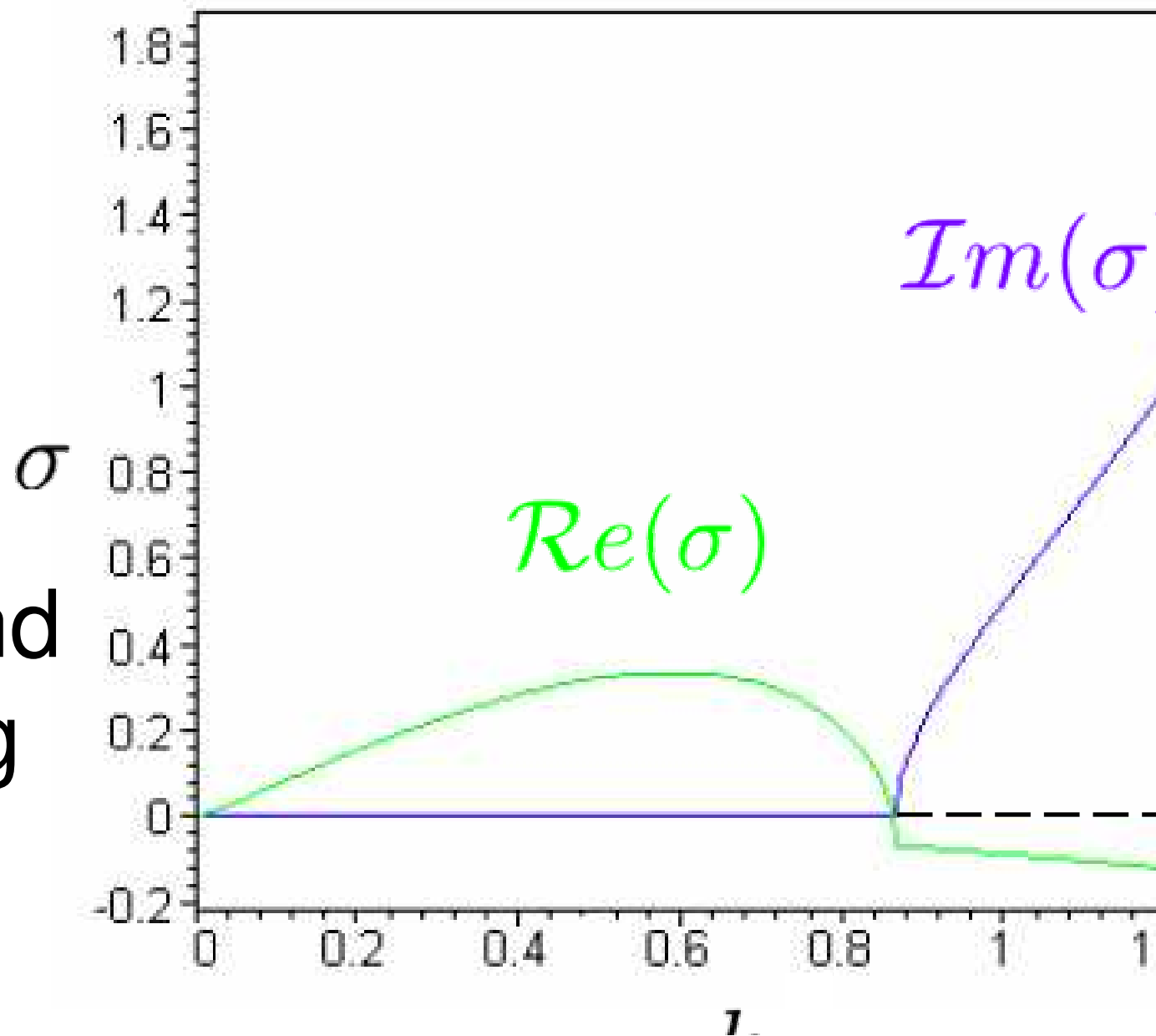
col
 allow

Uniform state: $c = c_o$, $\theta_o = \Omega + f(c_o)$

$$c_o + Ae^{\sigma t + ikz}$$

$$\theta_o + Be^{\sigma t + ikz}$$

growing non-
stationary modes
with a cutoff k , and
stationary decaying
modes above.



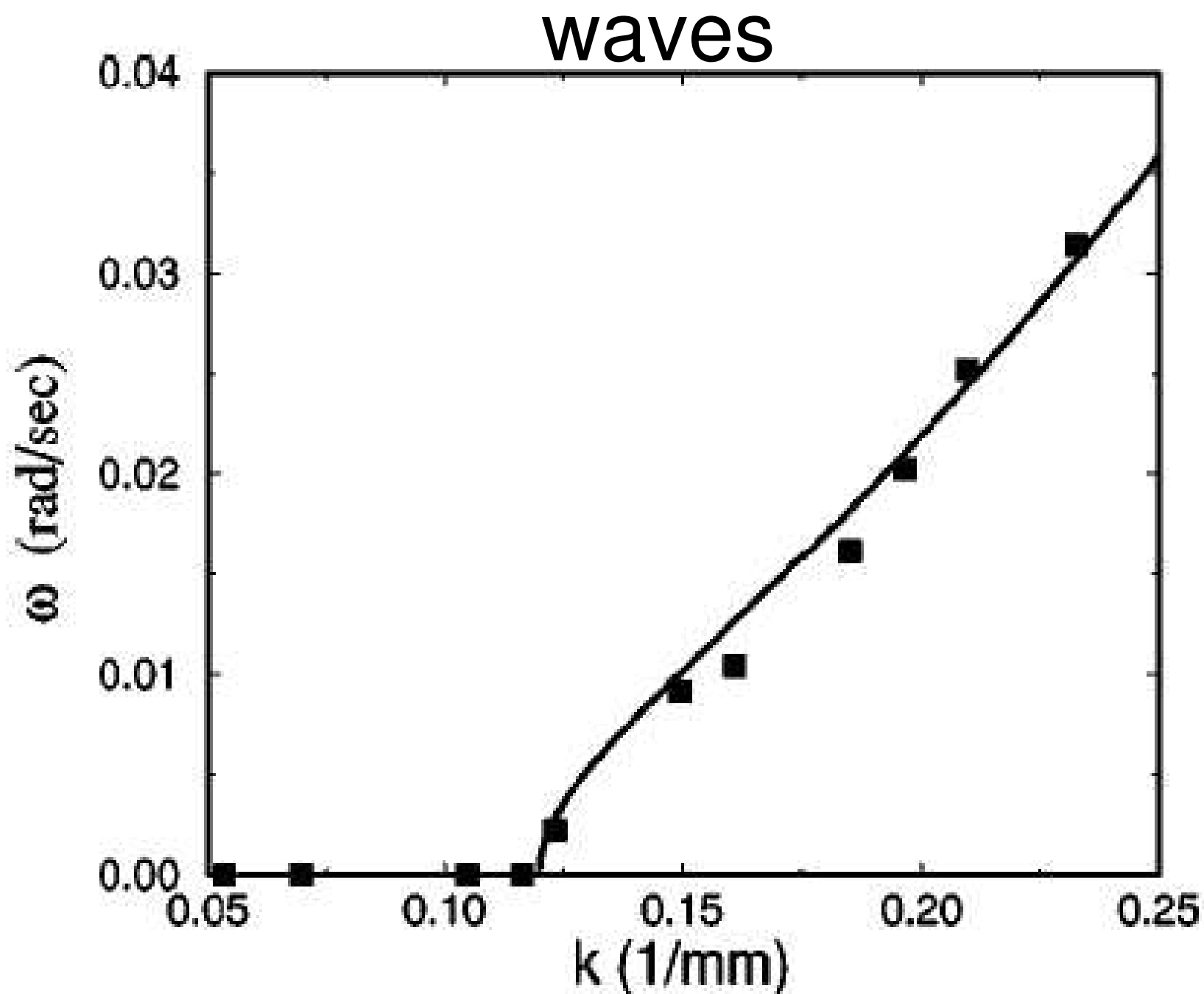
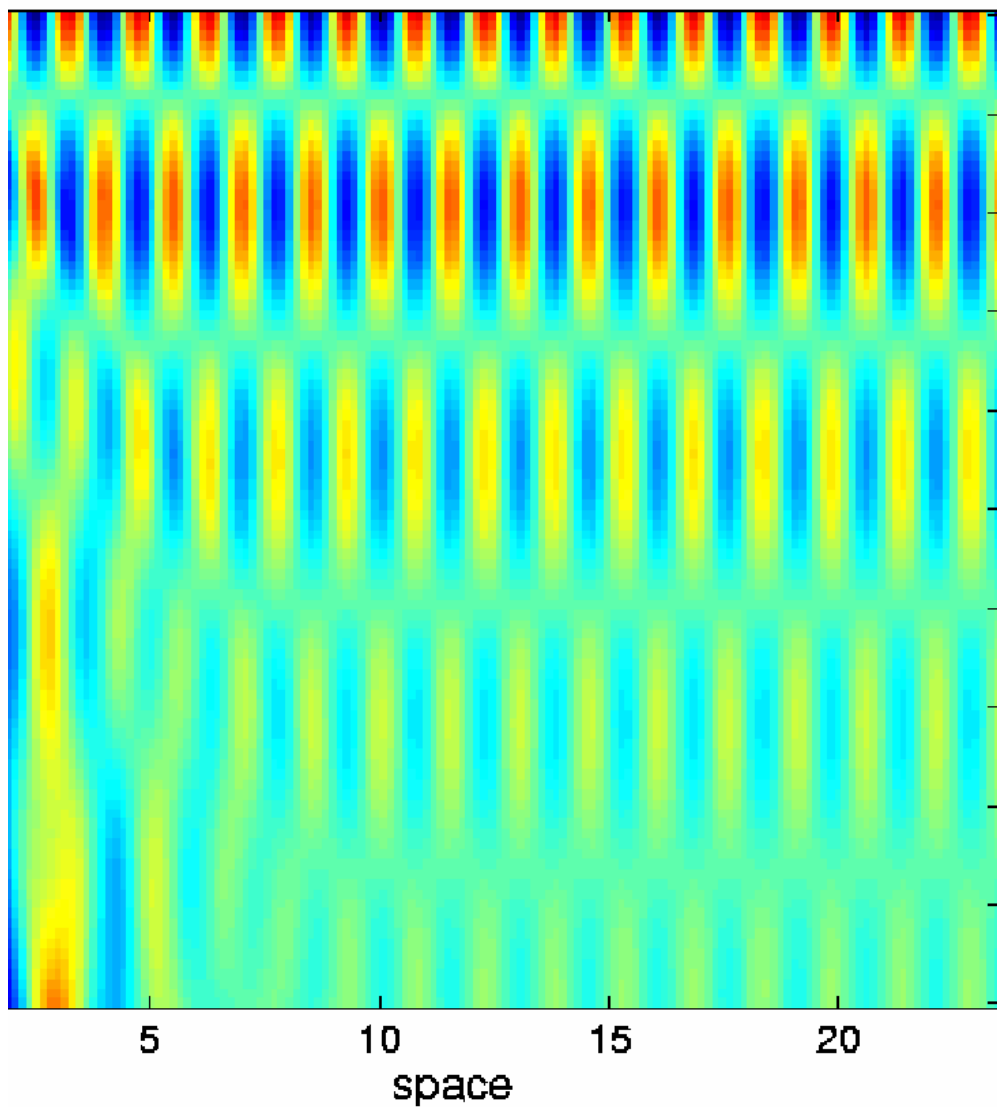


FIG. 3. Comparison of theoretical results ω vs k (solid line) with experimental data of Ref. [6] (squares). The values of parameters

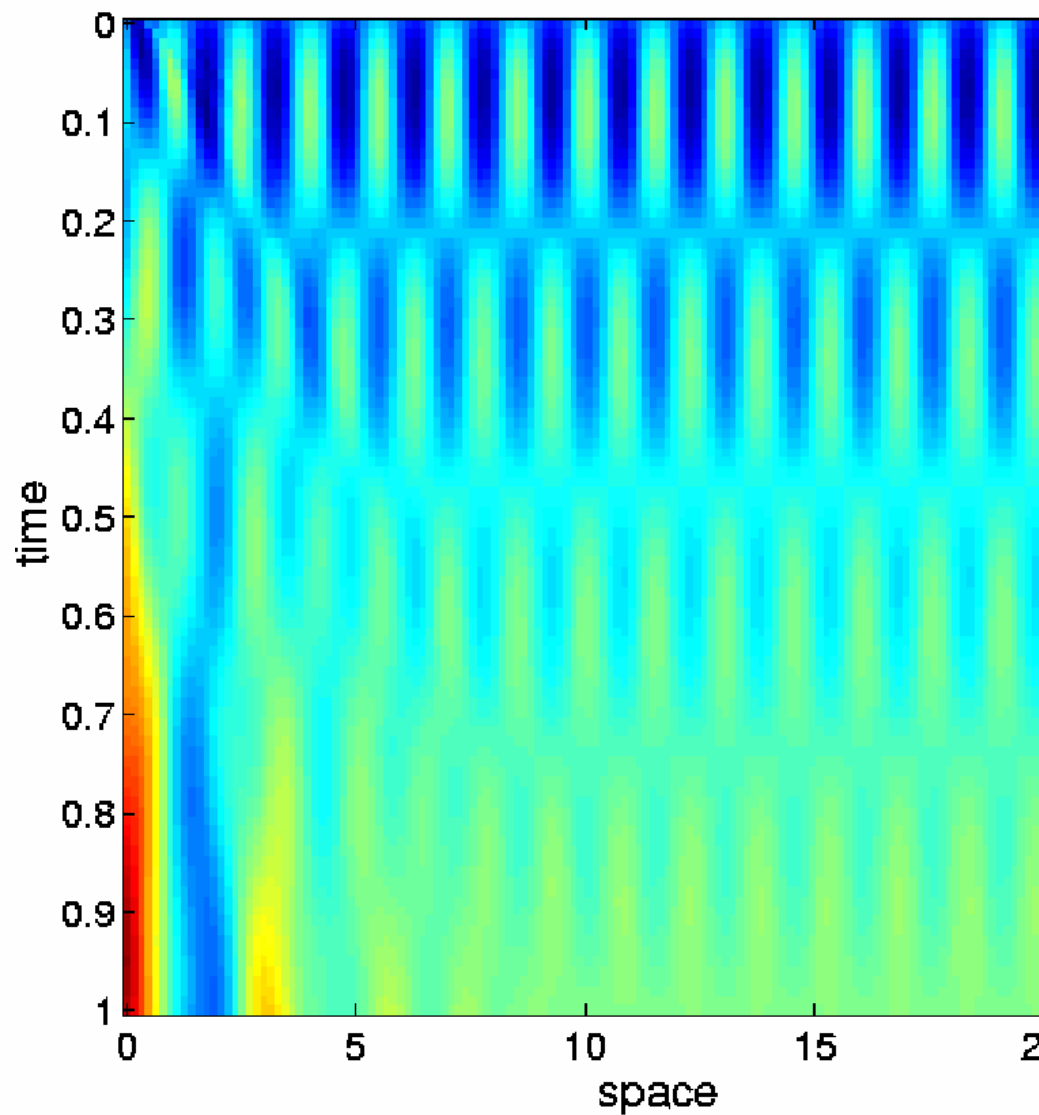
C

Concentration



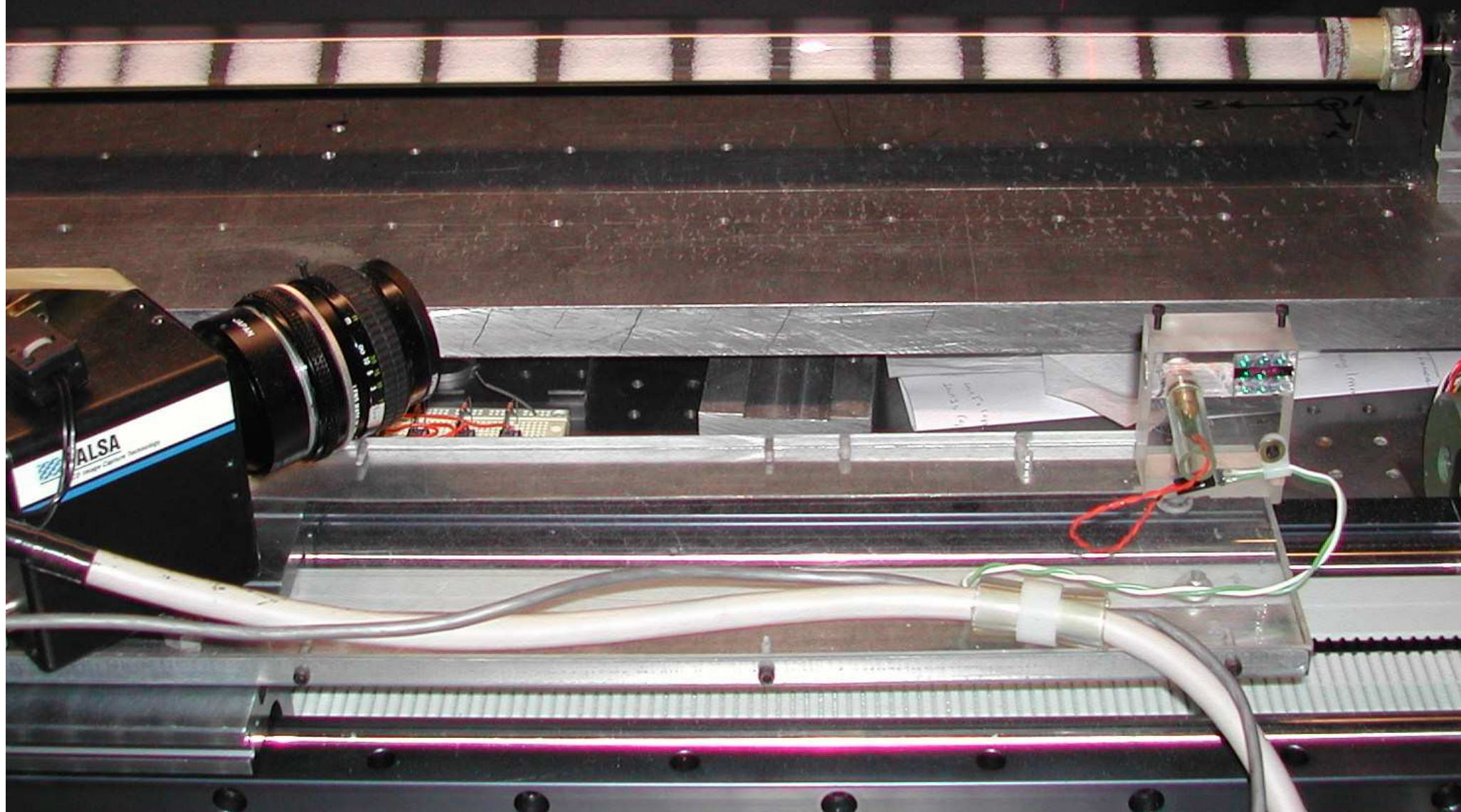
θ

Dynamic Angle



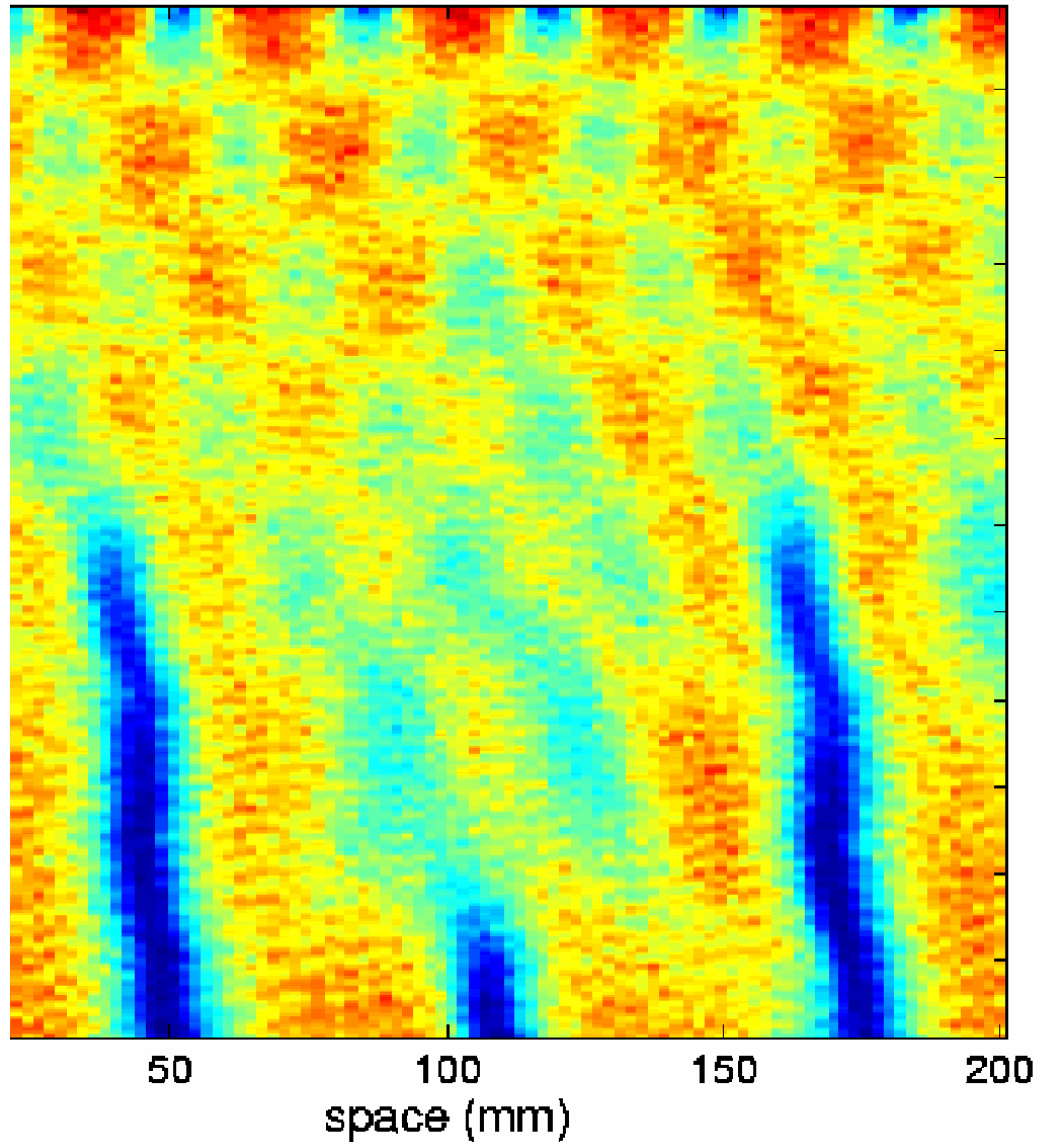
c and θ are $\pi/2$ out of phase at early

profilometer

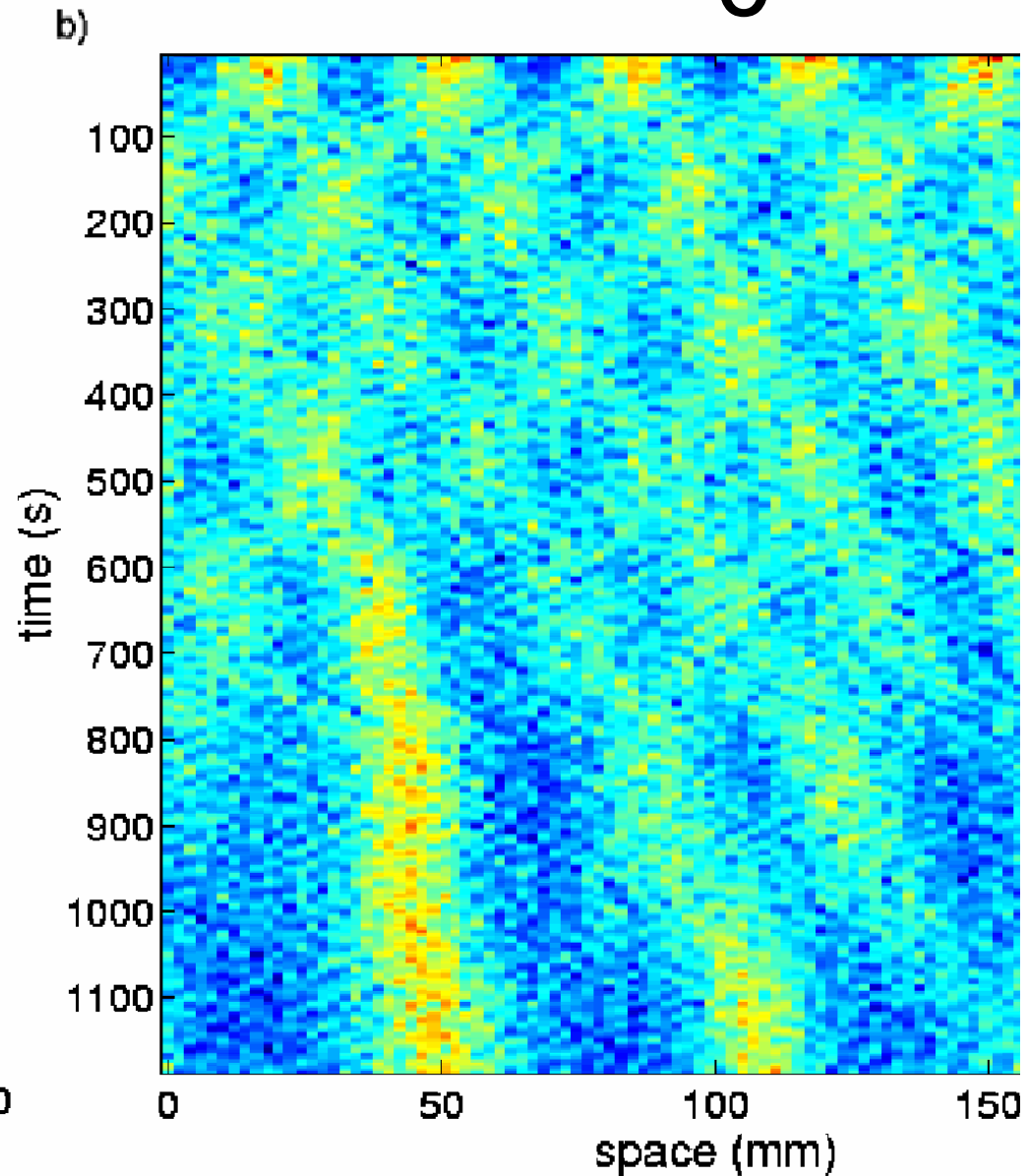


measurements contradict ATV theory

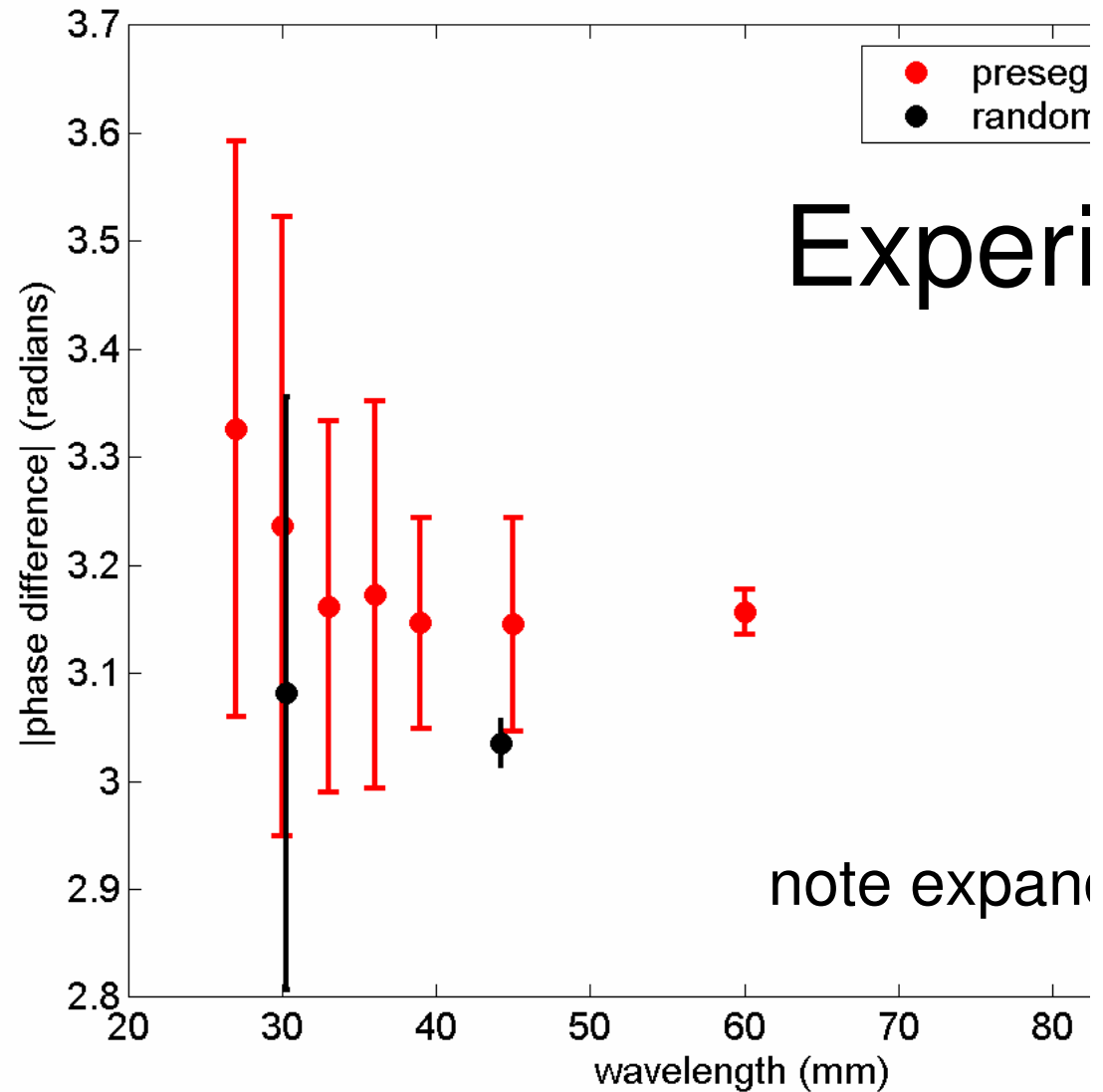
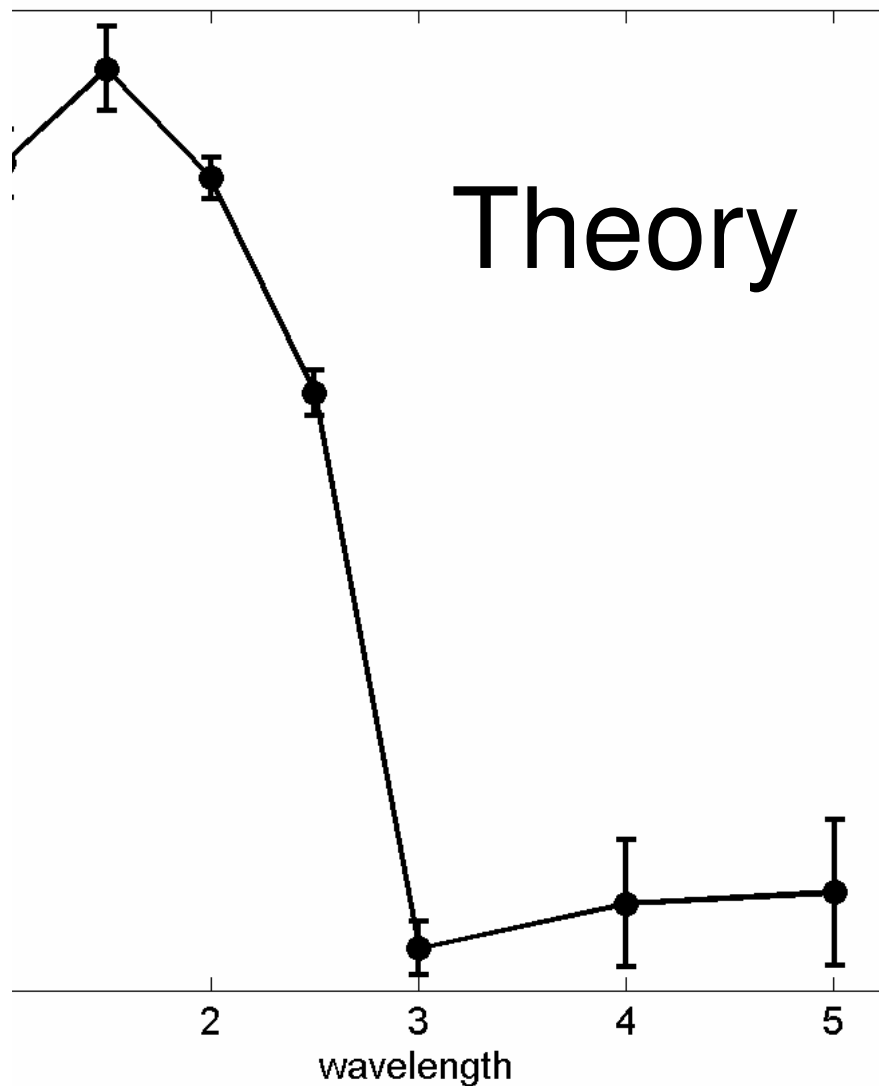
C



θ



measurements contradict ATV theory



This shows that surface slope is *not*

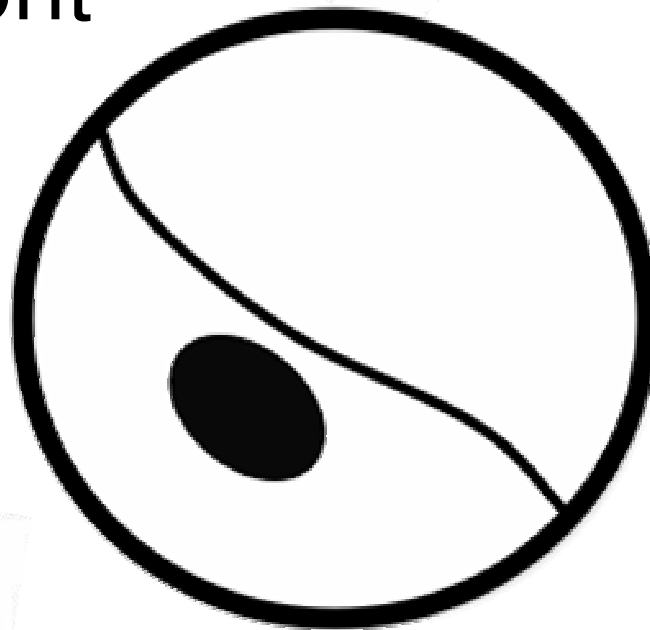
segregated core?

Is the buried core of small particles dynamically mixing material with the surface and playing the role of a catalyst?

Contrast between front and back lighting



camera



See the shadow of the dark subsurface



projection movies

front view

QuickTime™ and a
None decompressor
are needed to see this picture.

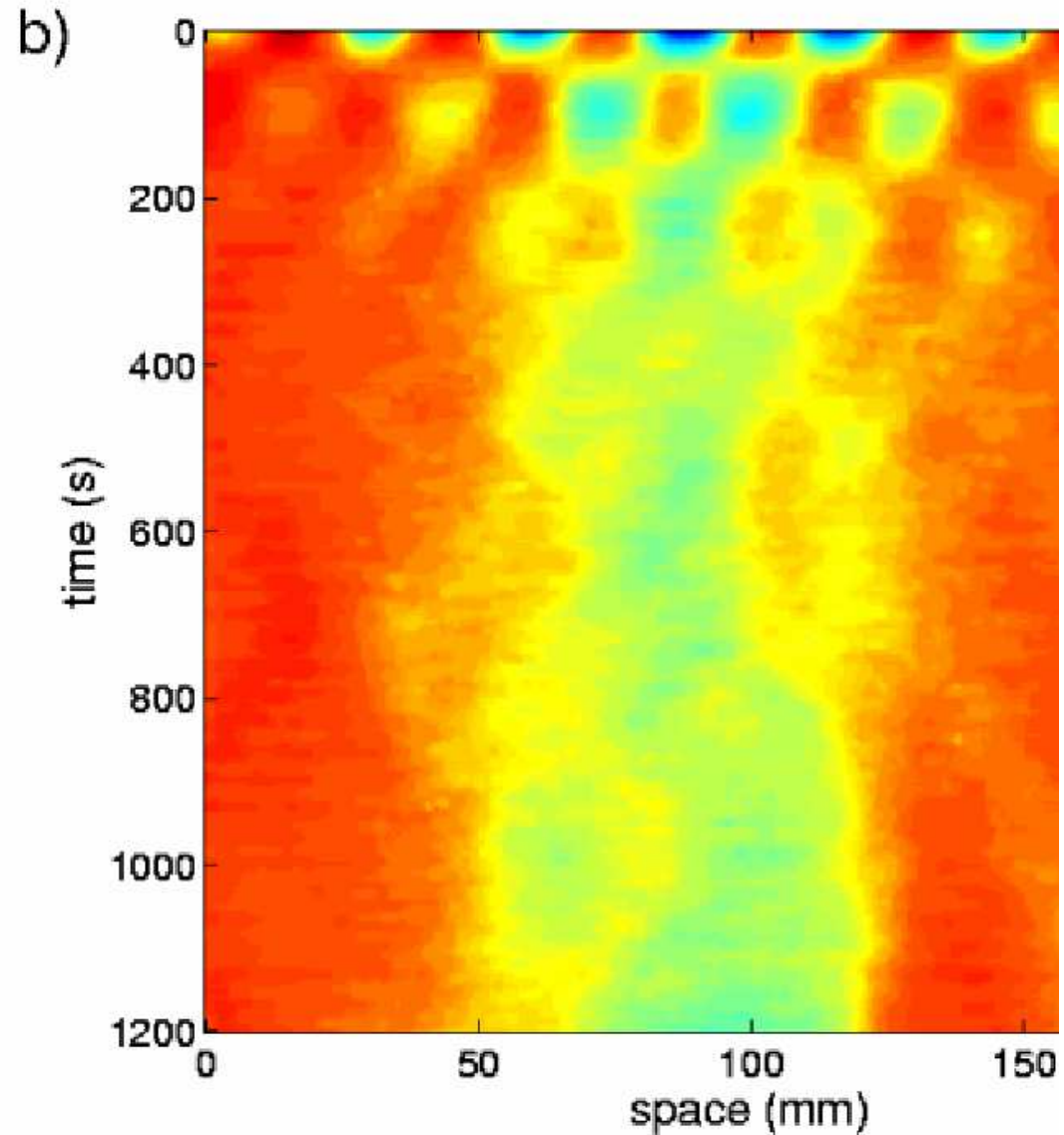
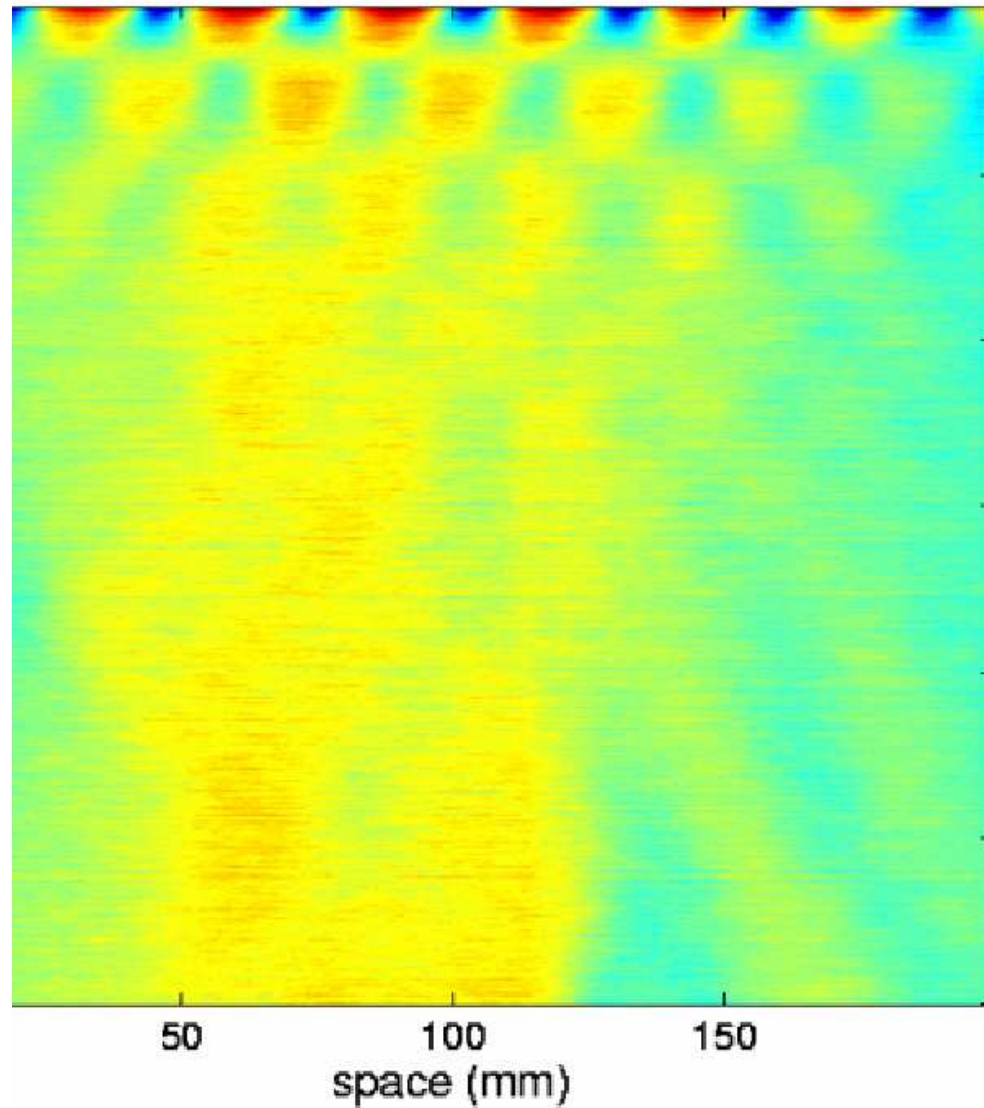
projection view

QuickTime™ and a
None decompressor
are needed to see this picture.

spacetime view of surface and core projections

C

core width



Oscillatory axial segregation remains a mystery!

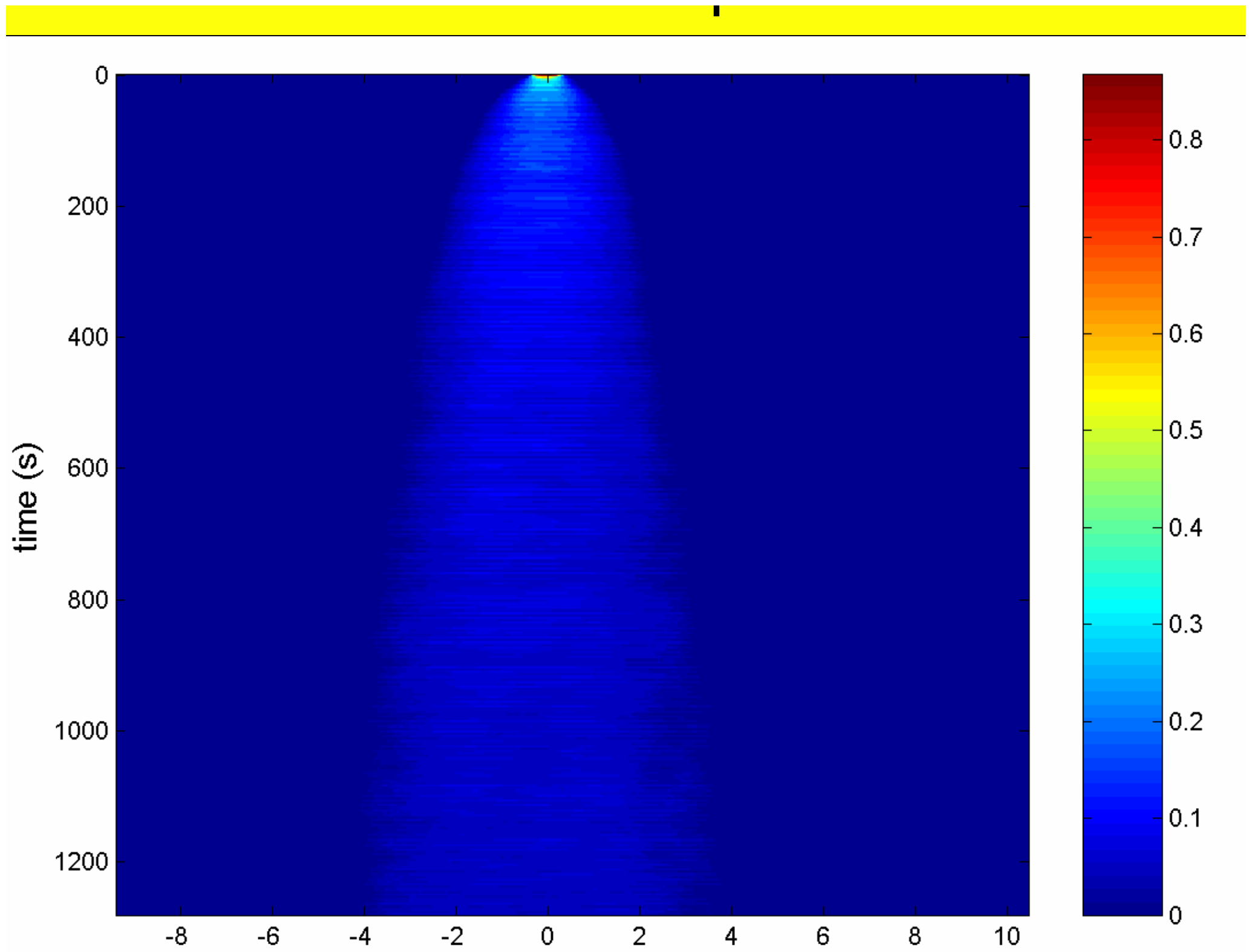
- Segregation looks like an oscillatory instability of the radial core.
- We have not identified an out-of-phase field analogous to θ in ATV theory.
- Subtle charge effects *etc.* seem to have been eliminated.
- Return to simplest questions about axial transport in the tube.

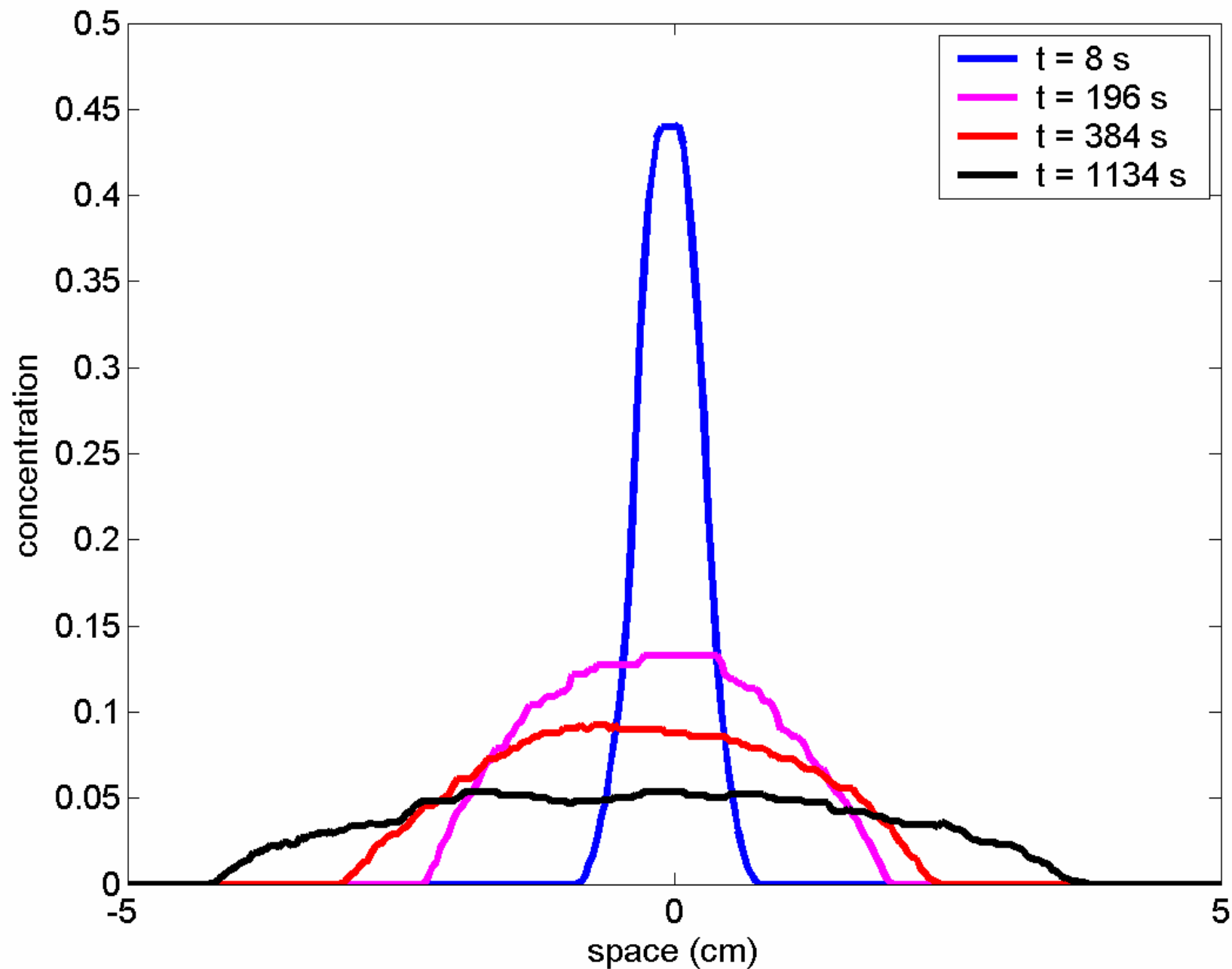
condition projection view

QuickTime™ and a
decompressor
are needed to see this picture.

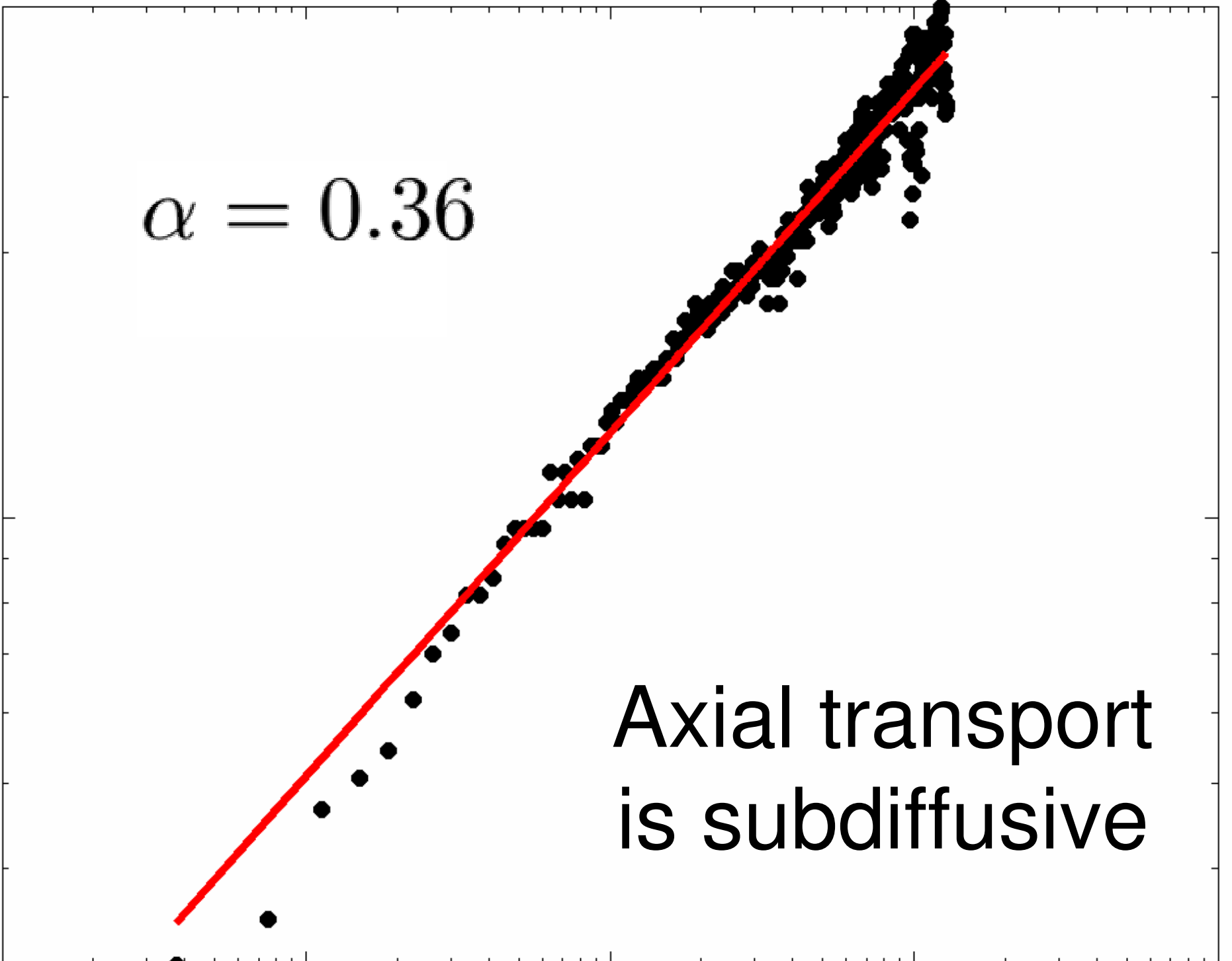
else quickly moves to axis of rotation and becomes
surface. It then expands slowly along the axis of
tube.

Is this regular diffusion?





studied the growth of the width of this distribution
time For normal diffusion we expect



A log-log plot showing a series of black data points that follow a linear trend on the logarithmic scale. A solid red line represents a power-law fit to the data. The plot is enclosed in a black frame with tick marks on all four sides. A yellow horizontal bar is located at the top of the image.

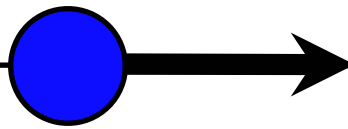
$$\alpha = 0.36$$

Axial transport
is subdiffusive

Fractional diffusion equation

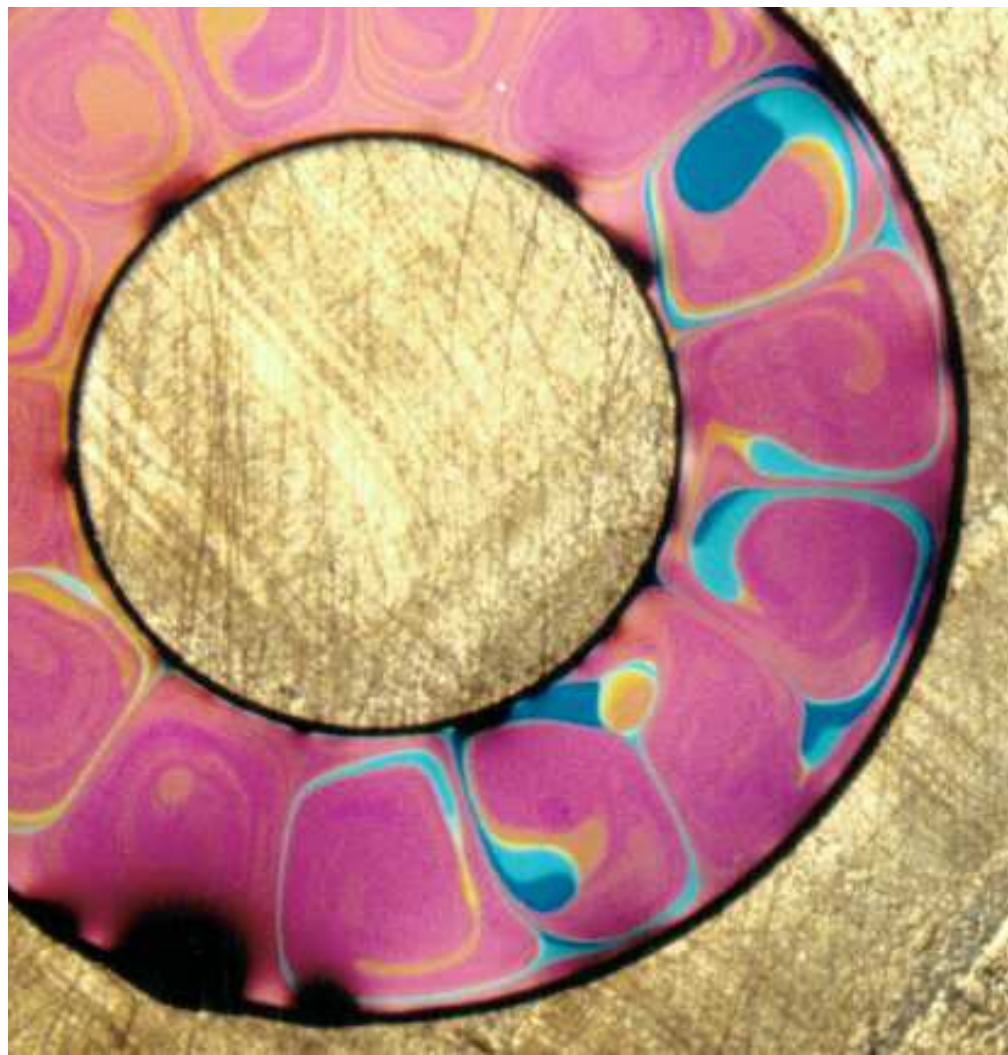
$$\frac{\partial^\beta C}{\partial t^\beta} = D \frac{\partial^2 C}{\partial z^2}$$

$\beta = 2\alpha, \quad \alpha = 1/2$ for normal diffusion



A 1D random walker with equal spatial steps taken at time intervals Δt such that $\langle \Delta t \rangle \rightarrow \infty$

find $\alpha \sim$ independent of tube rotation frequency.



You are invited to
the nonlinear physics
group lab!

Sunday at 1:00pm
Monday at 1:00pm

or anytime by appointment
smorris@physics.utoronto.ca

St. George St., Room 090 (north
sement). Go out the back door of Fields
and past the big smokestack...