Fluctuations near Bifurcations in spatially extended Non-Equilibrium Systems

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Rayleigh-Benard convection: Jaechul Oh (now at NRL), Nathan Becker
Electroconvection: Michael Scherer, Xinliang Qiu, Sheng-Qi Zhou

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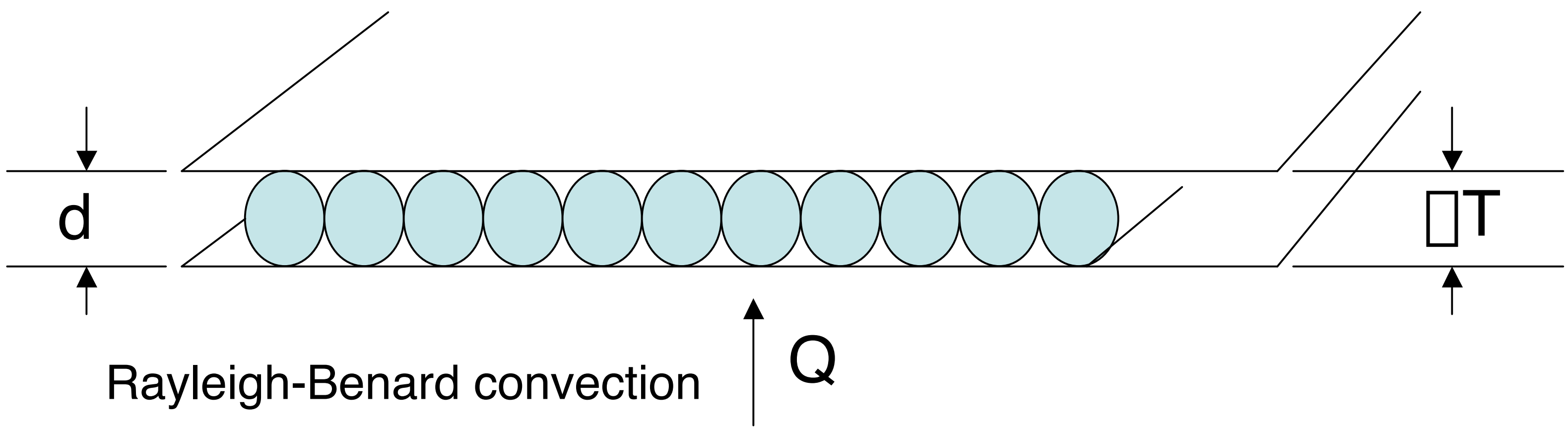

Rayleigh-Benard convection

Prandtl number

□ = □ / □

□ = kinematic viscosity

□ = thermal diffusivity

□ = □T/□T_c - 1

Supported by the US National Science Foundation Grant DMR0243336

1.) Fluctuations below the bifurcation

2.) Effect of fluctuations on the bifurcation

3.) Effect of fluctuations on the “ordered” state
above the bifurcation

1. Stochastic Boussinesq equations, 3-dimens. (Landau and Lifshitz 1959)
   (NS with temperature independent fluid properties)

2. Stochastic Swift-Hohenberg equation, 2-dimens.

   J. Swift, P. C. Hohenberg, Phys. Rev. A **15**, 319 (1977).
   P. C. Hohenberg, J. Swift, Phys. Rev. A **46**, 4 773 (1992)

$$\dot{\psi} = \epsilon\psi + \xi_0^2(k_c^2 + \nabla^2)^2\psi - \psi^3 + \eta(\vec{r}, t)$$

$\eta =$ White noise of intensity F_{th}

3. Linear approximation:

$$\langle\psi^2\rangle = F_{th}/(2|\epsilon|^{1/2}), \qquad \epsilon < 0$$

$$F_{th} = 0.19\frac{k_B T}{d^3\rho(\nu/d)^2}(\nu/\kappa)$$

ν = kinematic viscosity

κ = thermal diffusivity

4. Nonlinear: Fluctuation-induced subcritical bifurcation
   (first-order phase transition, Brazovskii universality class $\square_c = F^{2/3}$

$$F_{th} = \frac{k_B\, T}{\square\, \square^2\, d}\; 0.19(\,\square / \square\,)$$

Water $F_{th} = 10^{-9}$ $\square_c = 10^{-6}$

CO_2 40 bars $F_{th} = 3\text{x}10^{-7}$ $\square_c = 5\text{x}10^{-5}$

Linear Boussinesq eqs., H. van Beijeren, E. G. D. Cohen, J. Stat. Phys. **53**, 77 (1988)

$$F_{th} = \frac{k_B\, T}{\square\, \square^2\, d}\; 0.19(\, \square / \square\,)$$

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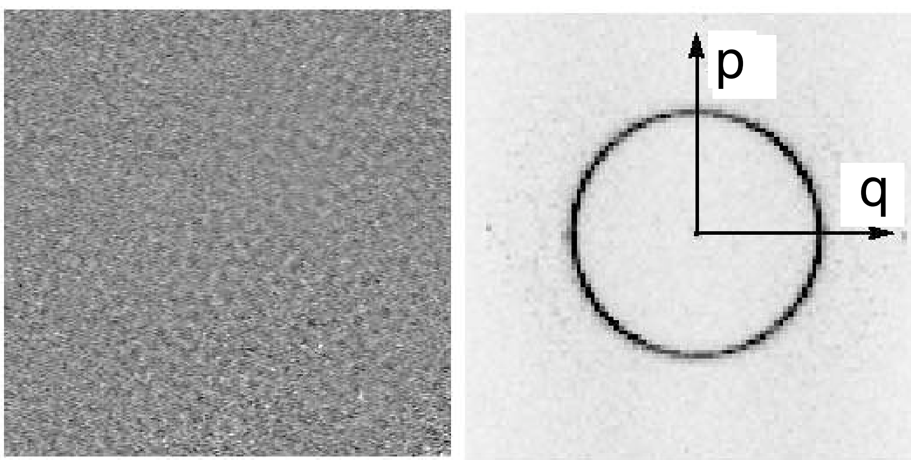
Linear Boussinesq eqs., H. van Beijeren, E. G. D. Cohen, J. Stat. Phys. **53**, 77 (1988)

P = 42 bars, $\square = 2 \text{ x } 10^{-4}$



Wu, A. + Cannell, PRL **75**, 1743 (1995).

$$F_{th} = \frac{k_B\, T}{\square\, \square^2\, d}\; 0.19(\,\square / \square\,)$$

Linear Boussinesq eqs., H. van Beijeren, E. G. D. Cohen, J. Stat. Phys. **53**, 77 (1988)

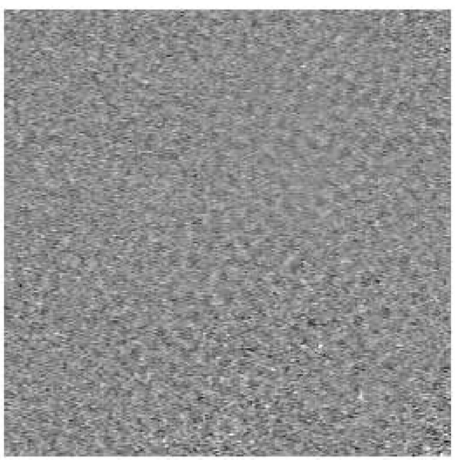
Water $F_{th} = 10^{-9}$ $\square_c = 10^{-6}$

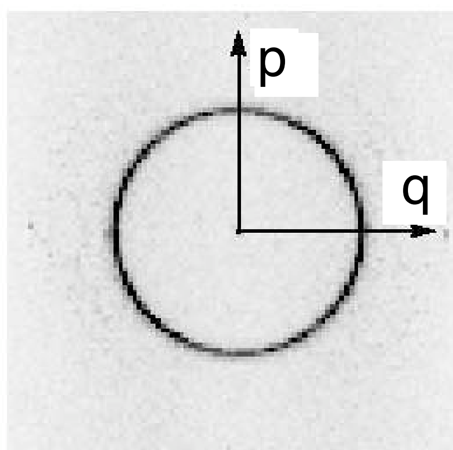
CO_2 40 bars $F_{th} = 3\mathrm{x}10^{-7}$ $\square_c = 5\mathrm{x}10^{-5}$

Linear theory :

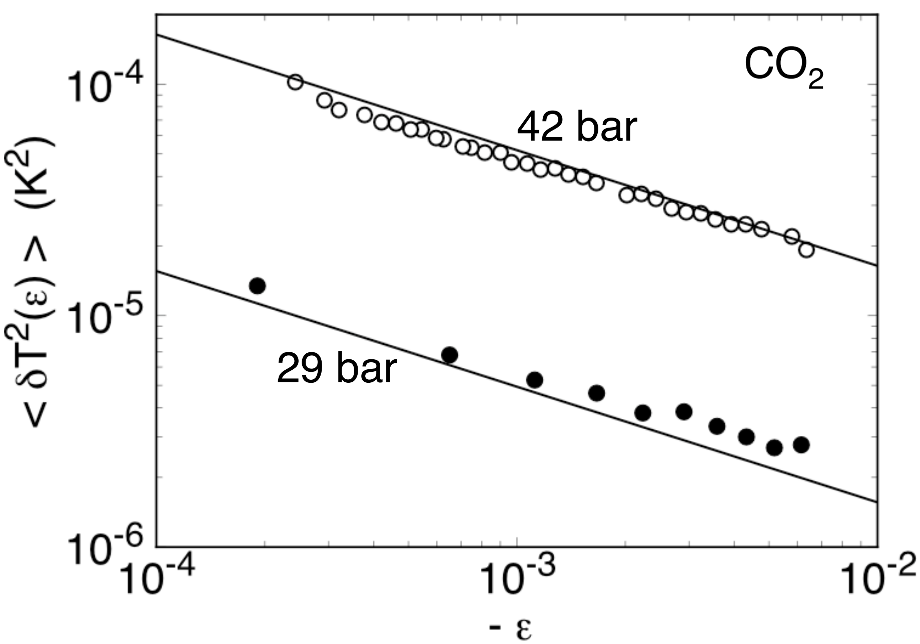
$$< \square T^2 > \;\sim\; F_{th} / |\square|^{1/2}$$

P = 42 bars, $\square = 2 \times 10^{-4}$





Wu, A. + Cannell, PRL **75**, 1743 (1995).



CP of SF_6

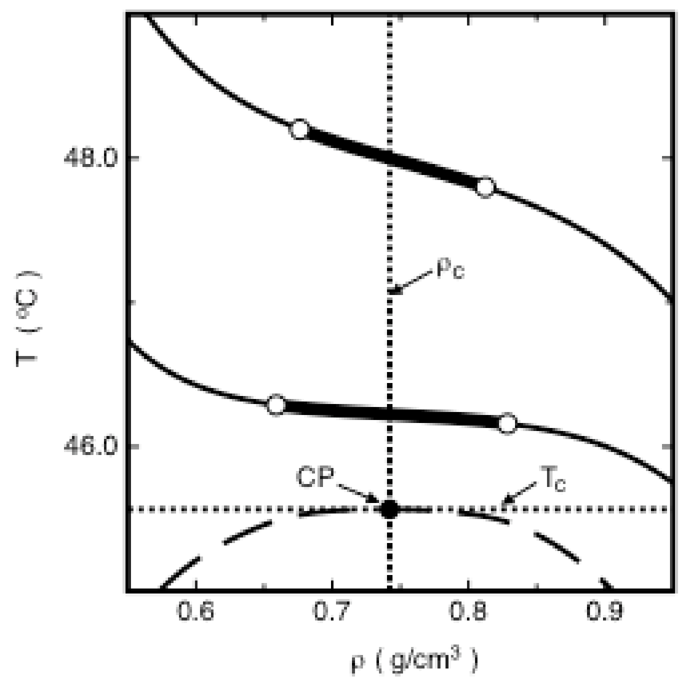
$$F_{th} = \frac{k_B\, T}{\square\, \square^2\, d}\; 0.19(\square / \square)$$

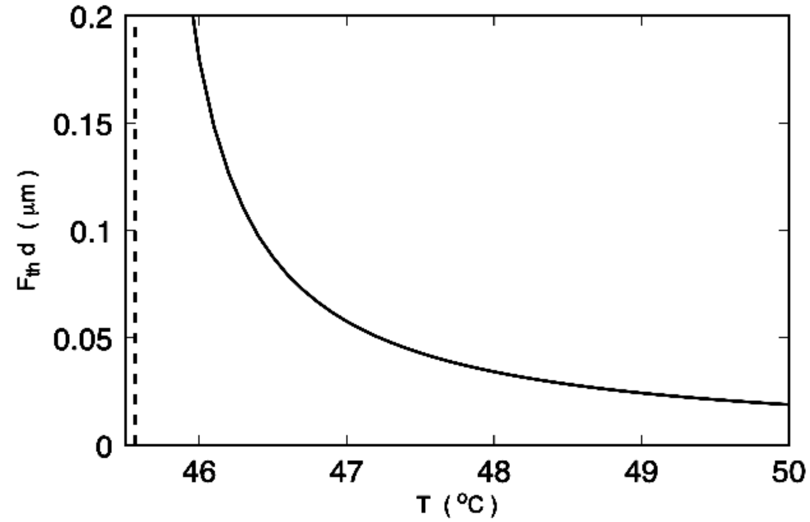
On critical isochore:

46.5 ^{o}C, d = 34.3 μm, $F_{th} = 5.1 \times 10^{-4}$ $\square_c = 6 \times 10^{-3}$

48.0 ^{o}C, d = 59.0 μm, $F_{th} = 0.8 \times 10^{-4}$ $\square_c = 2 \times 10^{-3}$

dn / dT ~ 0.1 K^{-1}

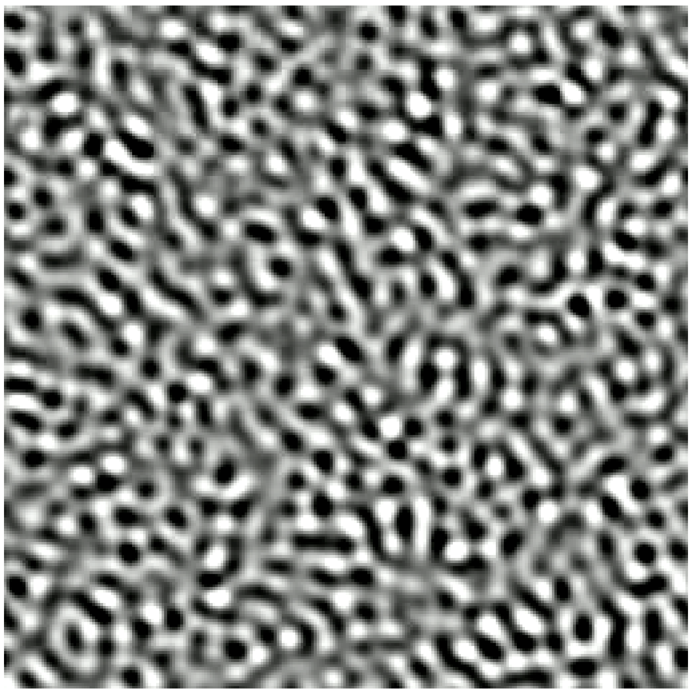




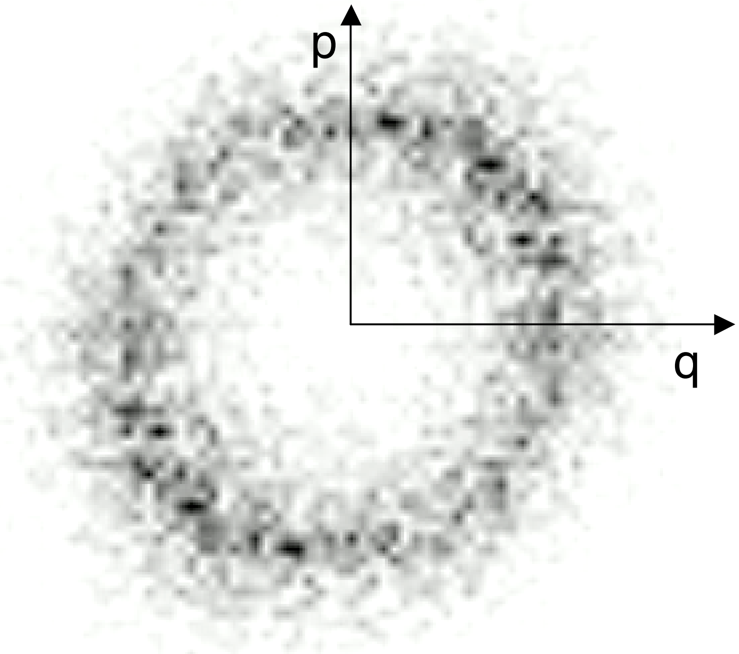
J. Oh and G.A., Phys. Rev. Lett. **91**, 094501 (2003).

Fluctuations well below the onset of convection

Snapshot in real space



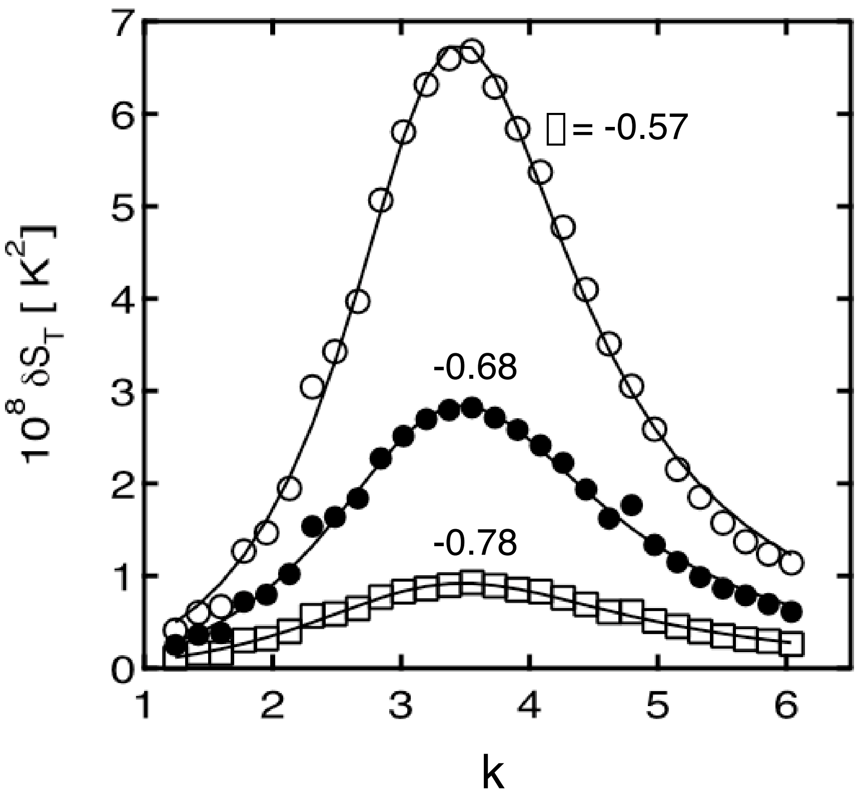
Structure factor =
square of the modulus
of the Fourier transform
of the snapshot

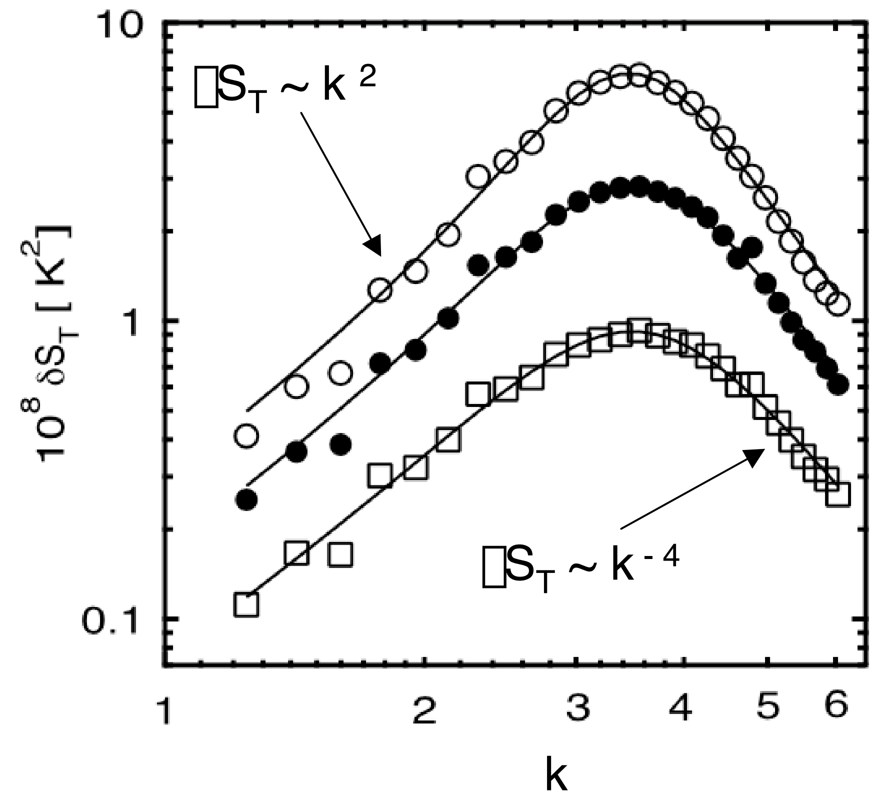


□= -0.06

Movie by Jaechul Oh

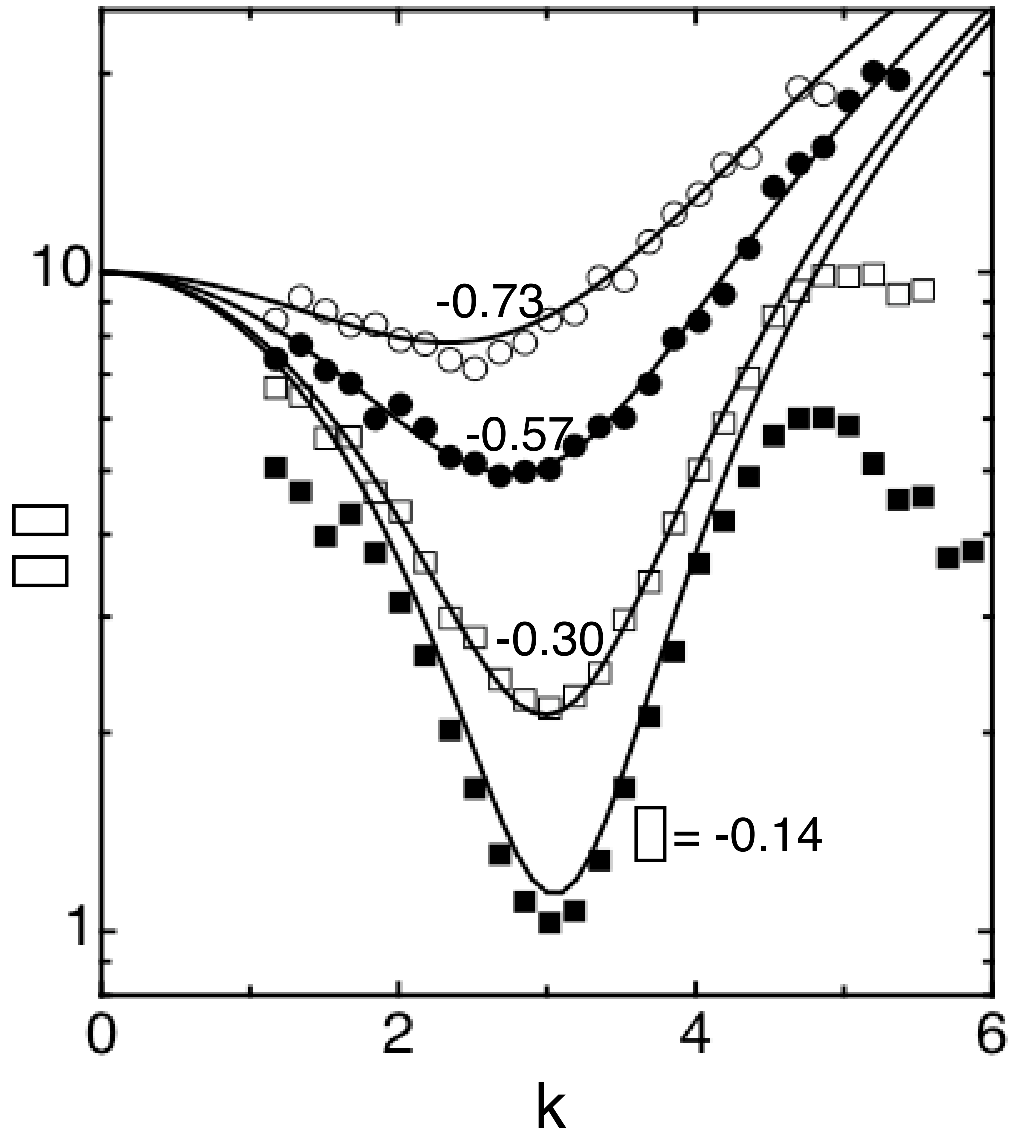
46.5 ^{o}C, d = 34.3□m, F_{th} = 5.1x10 $^{-4}$



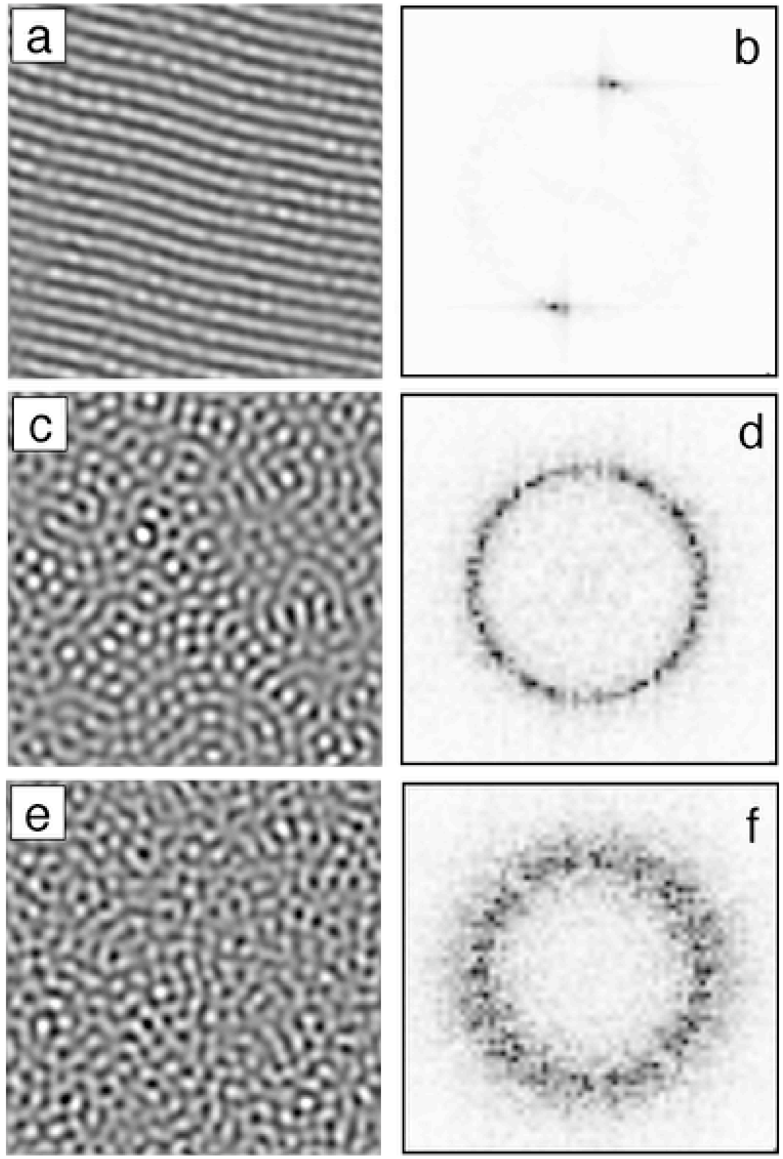


Experiment: J. Oh and G.A., unpublished.

Linear Theory: J. Ortiz de Zarate and J. Sengers, Phys. Rev. E **66**, 036305 (2002).



J. Oh, J. Ortiz de Zarate, J. Sengers, and G.A., Phys. Rev. E, in print.

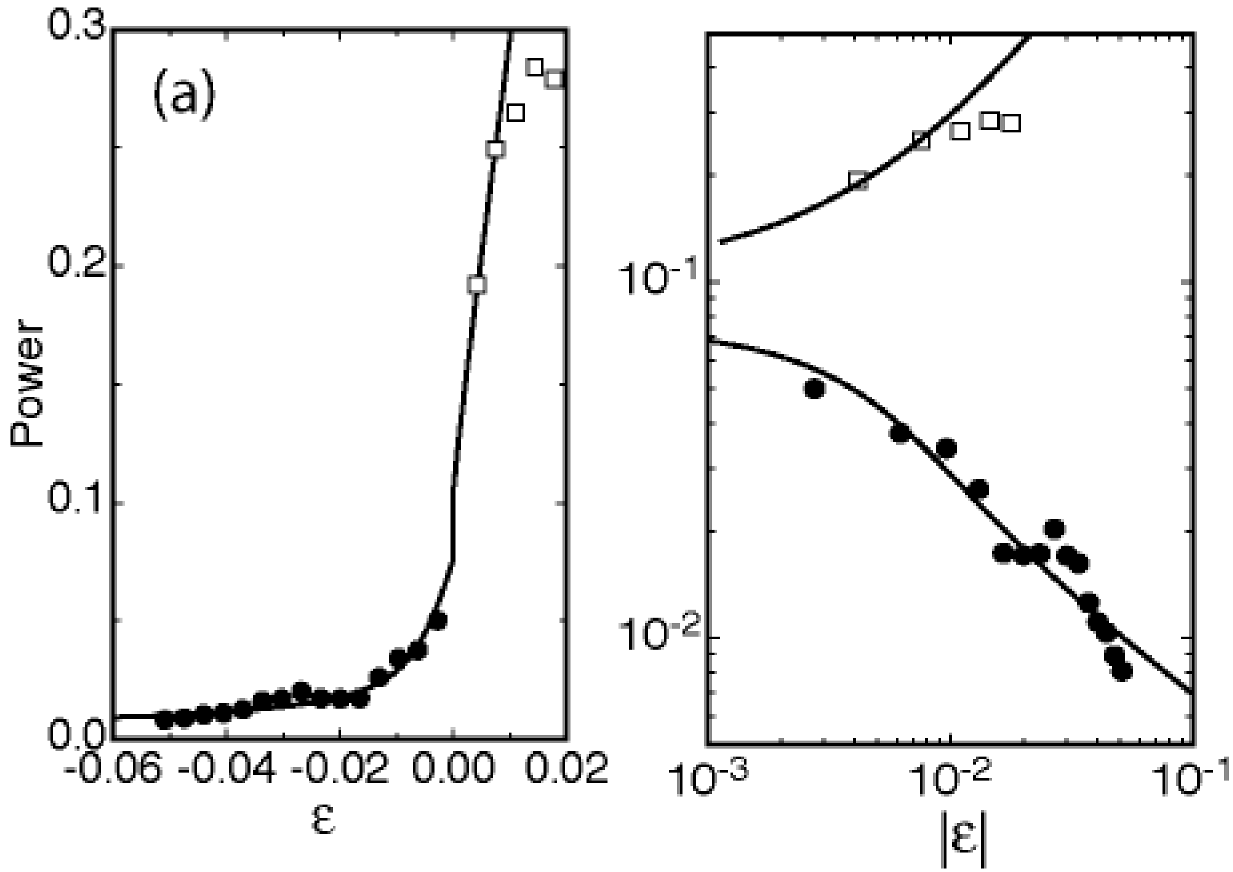


□T = 0.1320 K
□= 0.007

0.1311 K
-0.007

0.1248 K
-0.054

1.28x1.28x0.0343 mm^3



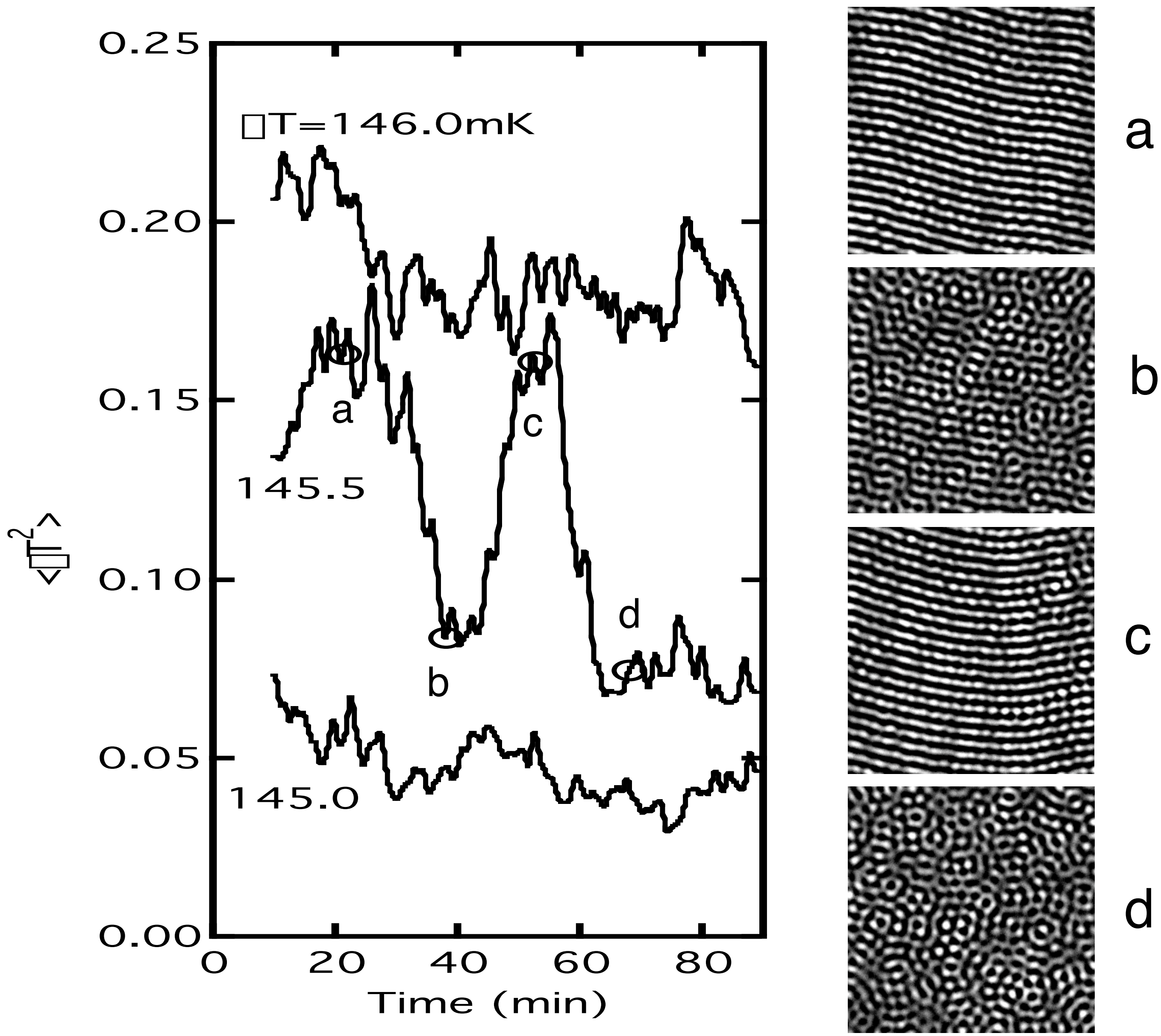
d = 34.3□m

$F_{exp} = 7.1x10^{-4}$

$F_{th} \ \ = 5.1x10^{-4}$

SH prediction is quantitative only for □ < F $^{-2\,/\,5}$

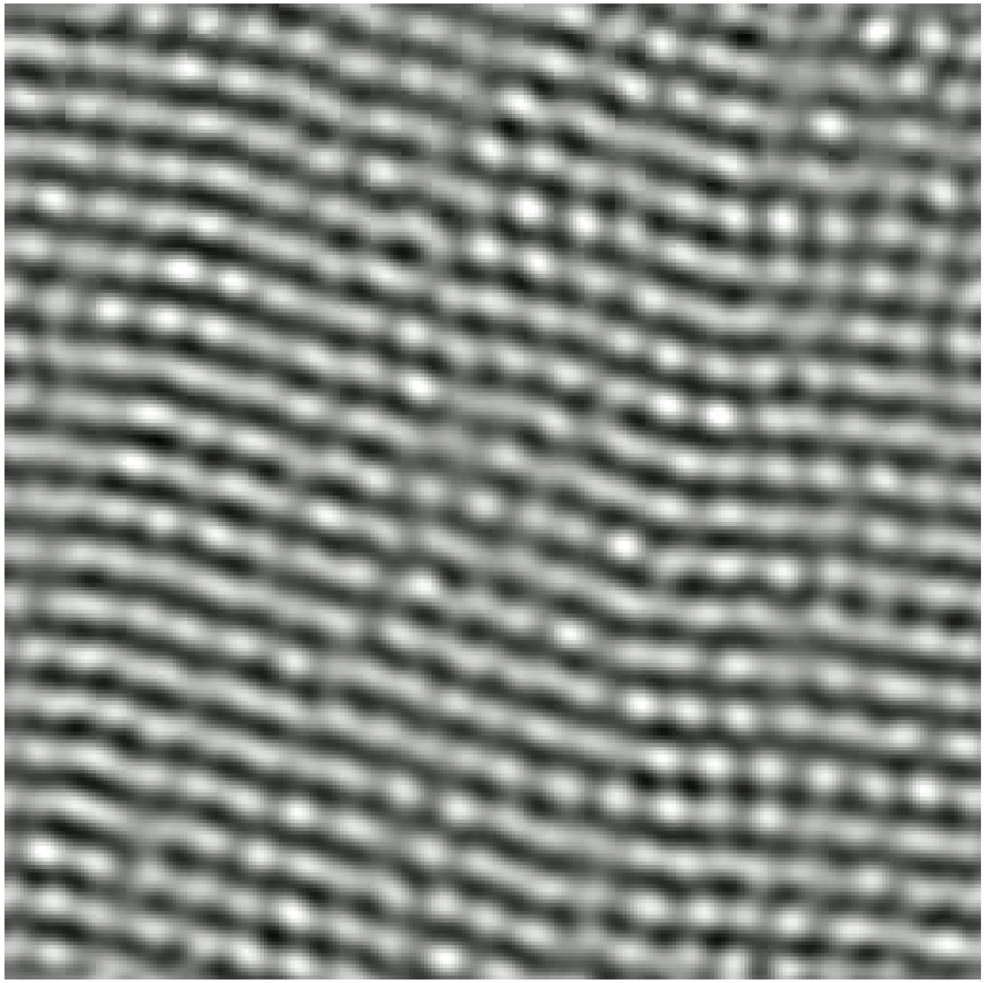
We have □ = 145 > F $^{-2\,/\,5}$ = 14

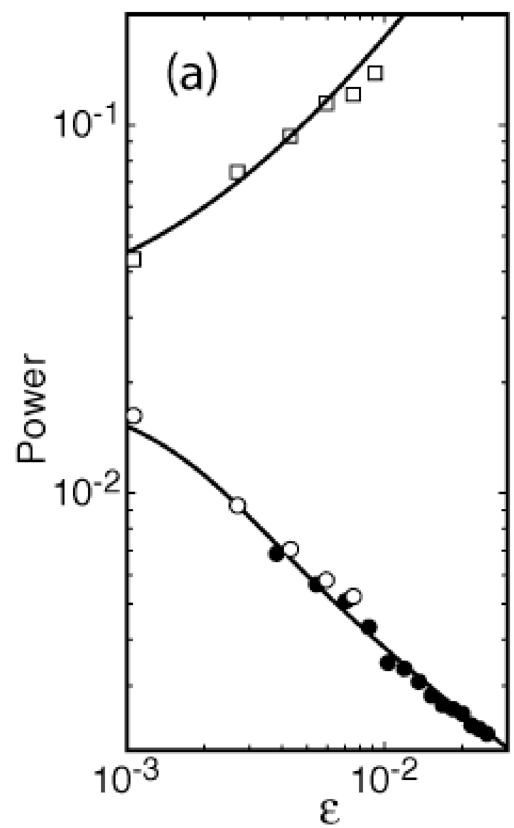


0.25
0.20
0.15
0.10
0.05
0.00
0
20
40
60
80
Time (min)
T=146.0mK
145.5
145.0
a
b
c
d
a
b
c
d

<T> = 46.2ºC

□= 0





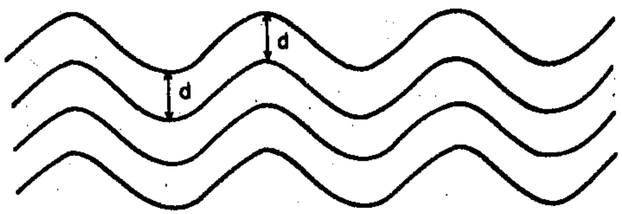
$F_{exp} = 1.4\text{x}10^{-4}$

$F_{th} = 0.8\text{x}10^{-4}$

d = 0.059 mm

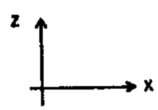
two types of
“phonons” :

(a)

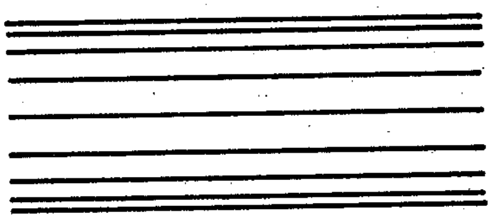


$q_x > 0$

$q_z = 0$



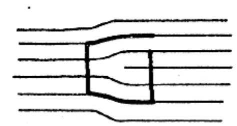
(b)



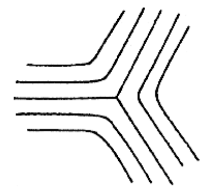
$q_x = 0$

$q_z > 0$

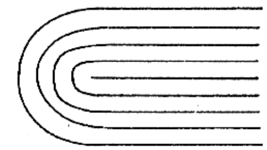
dislocations



disclinations

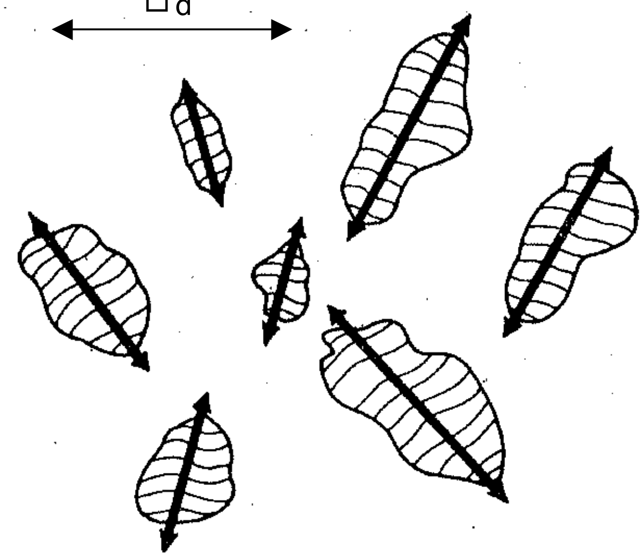


concave

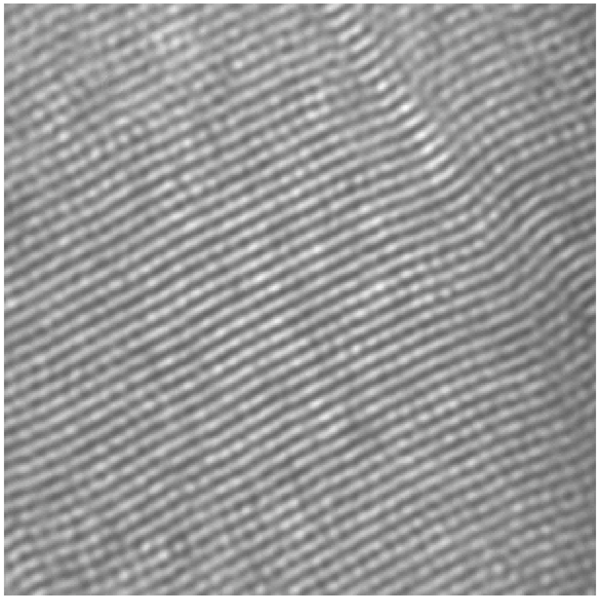


convex

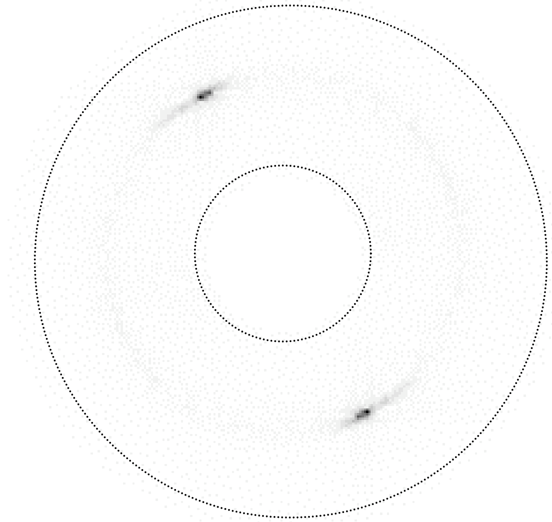
□d



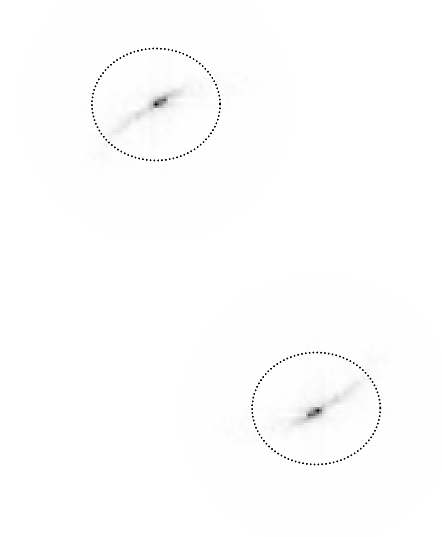
J. Toner and D.R. Nelson, Phys. Rev. B **23**, 316 (1981).

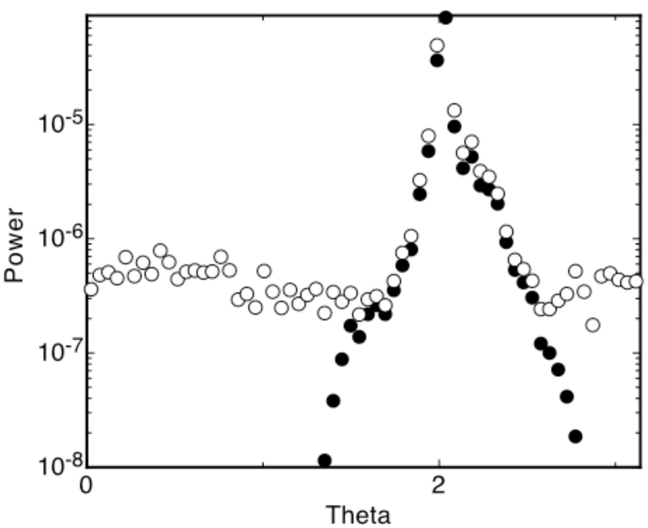


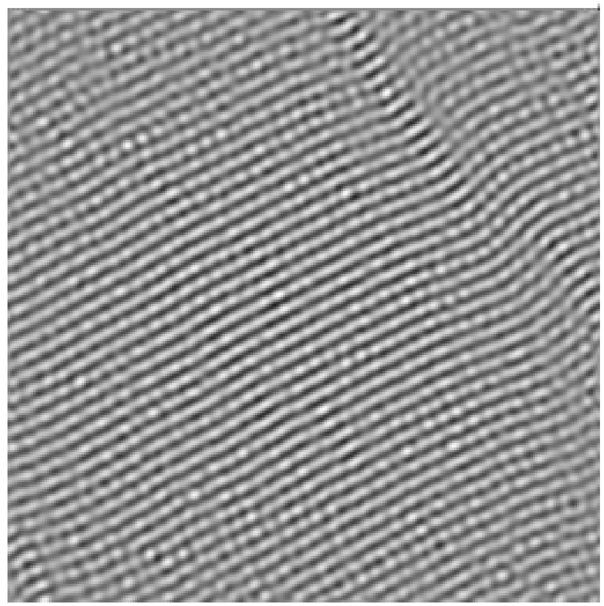
Radial Bandpass filter

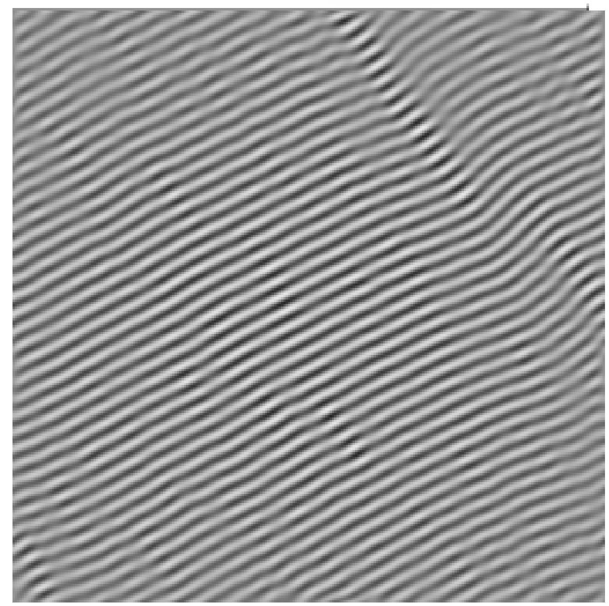


Peak filter



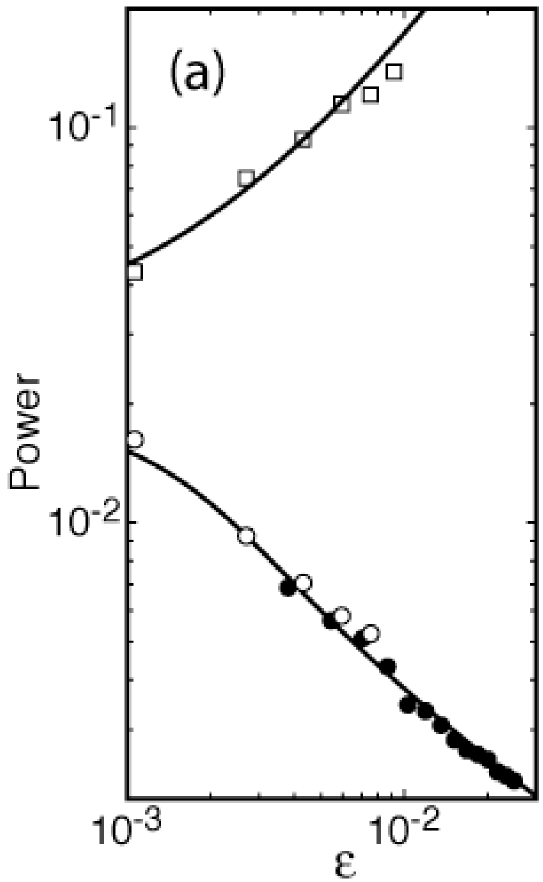




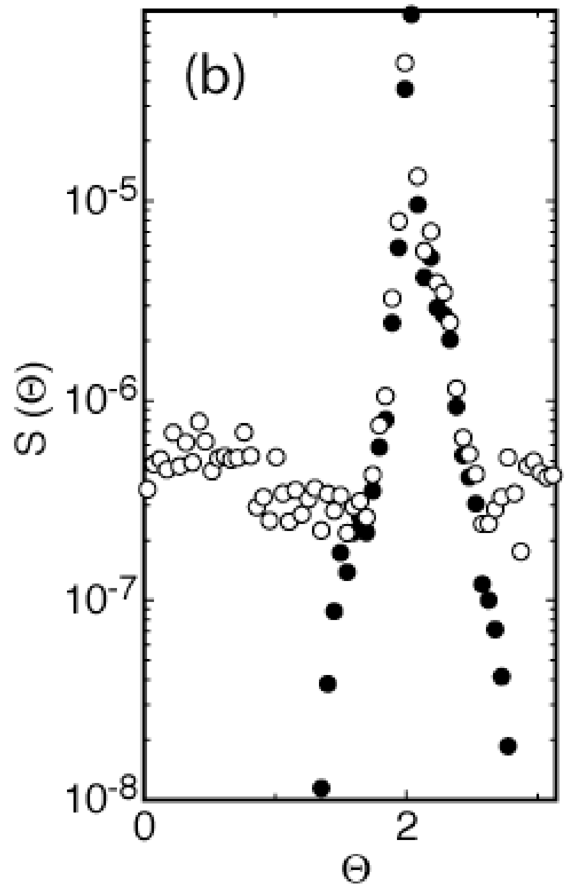


021212a/621/*.999*

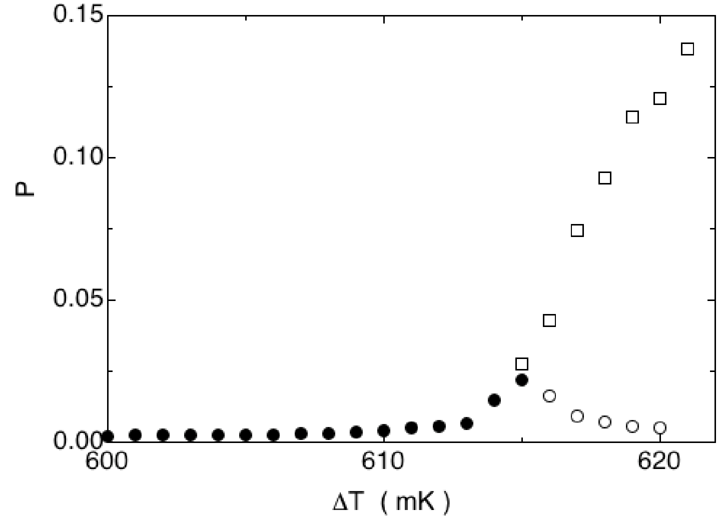
□= 0.009



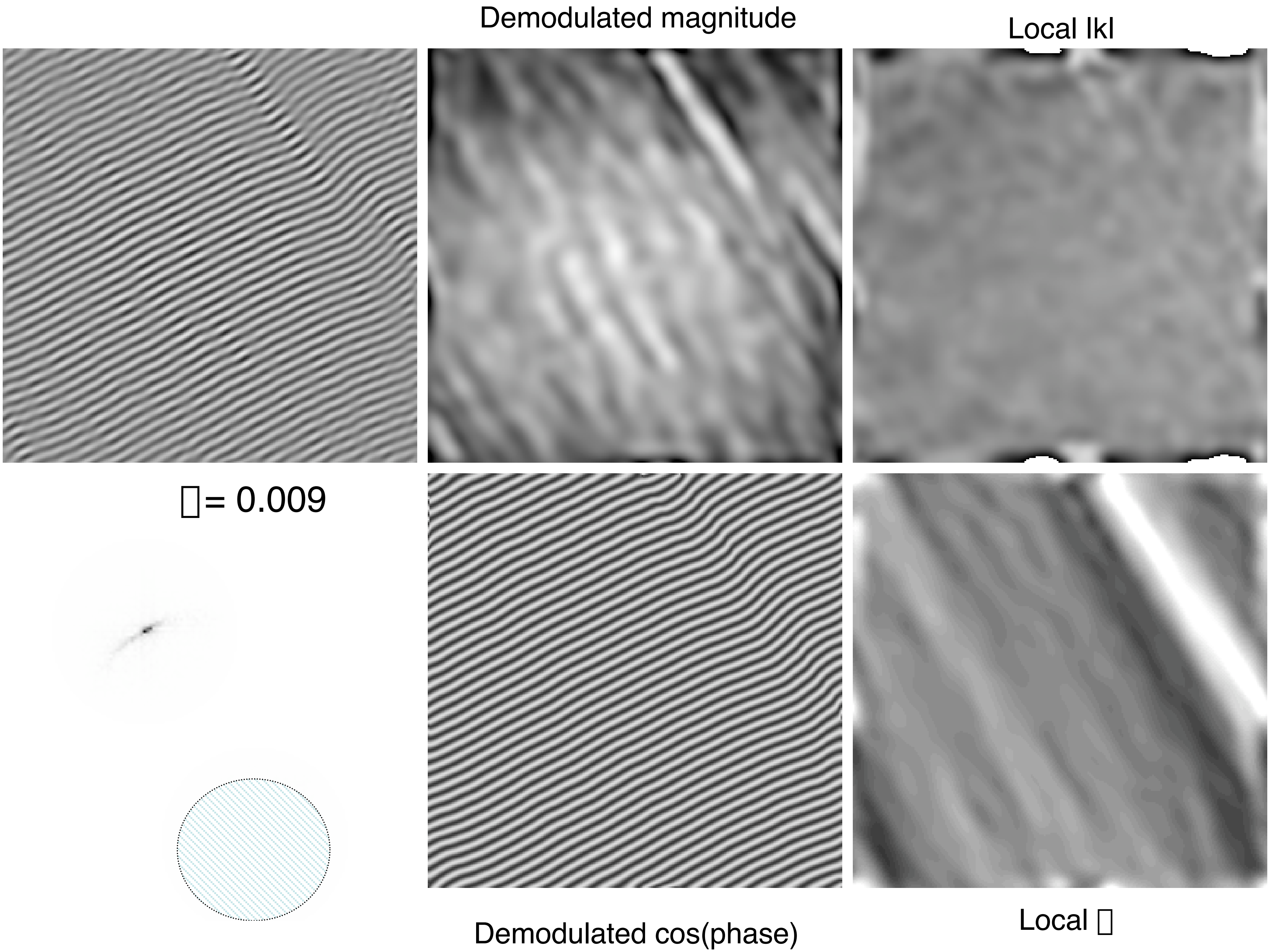
(a)
Power
10^{-1}
10^{-2}
10^{-3}
10^{-2}
ε

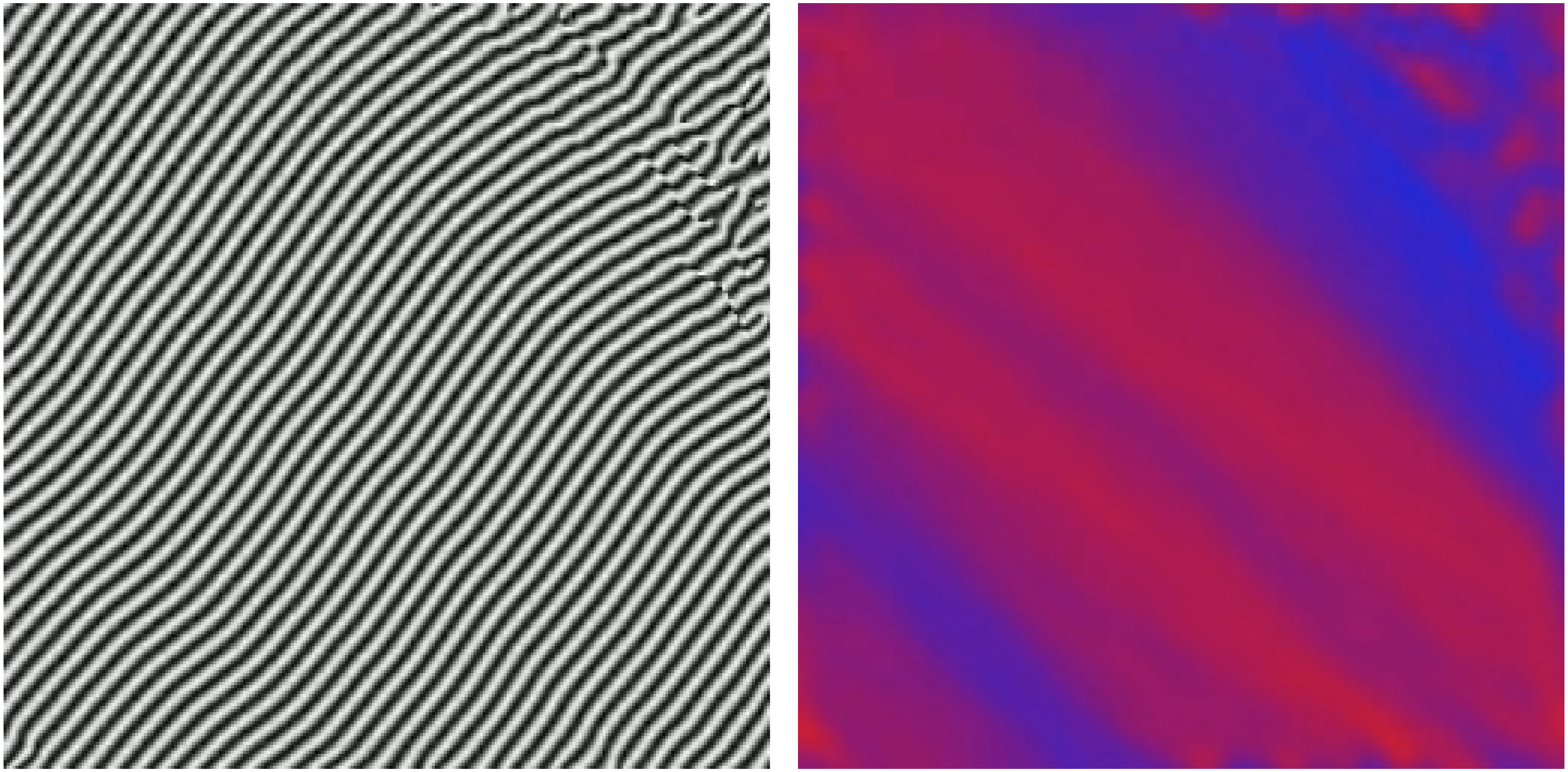


(b)
S (Θ)
10^{-5}
10^{-6}
10^{-7}
10^{-8}
0
2
Θ



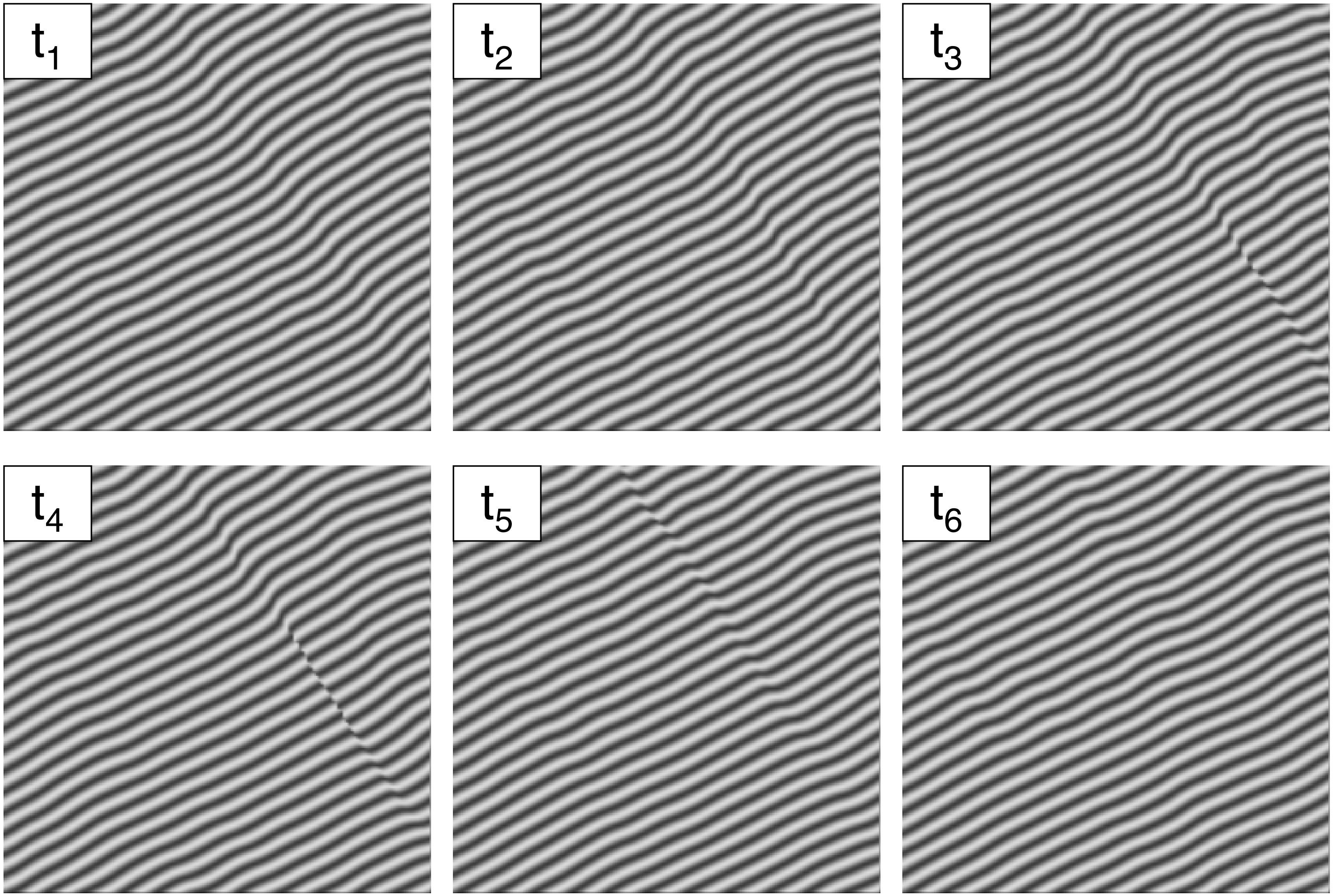
P
0.15
0.10
0.05
0.00
600
610
620
ΔT (mK)


Demodulated magnitude
Local |k|
□= 0.009
Demodulated cos(phase)
Local □

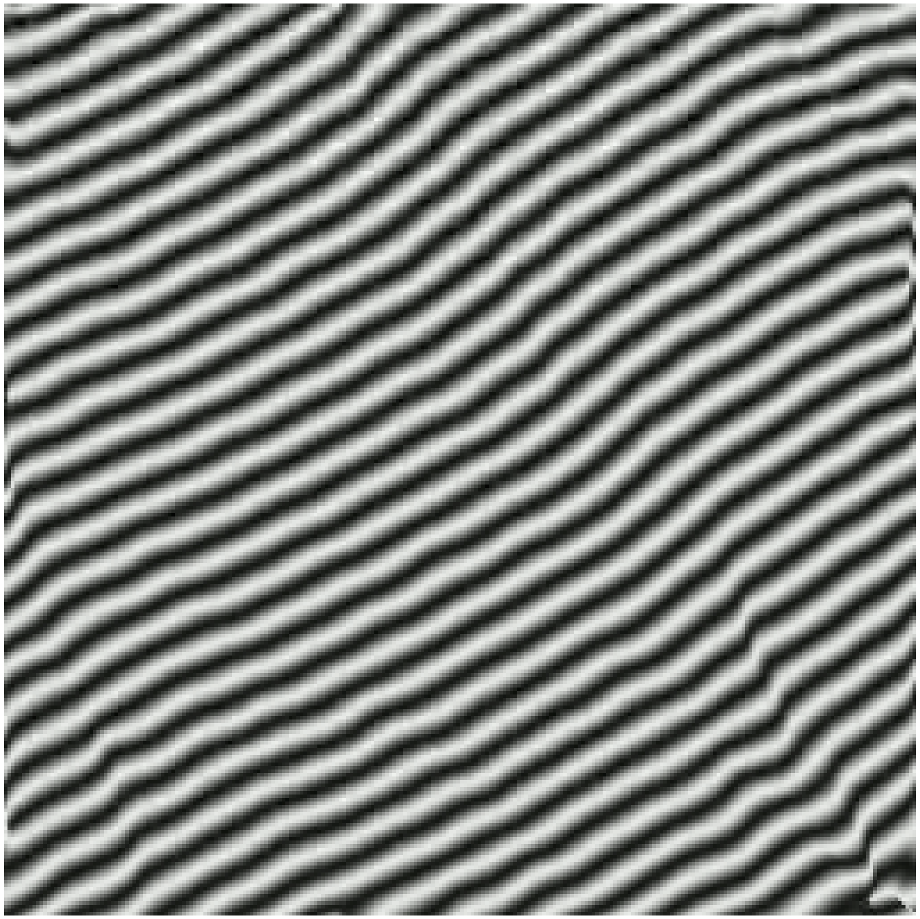


□= 0.004

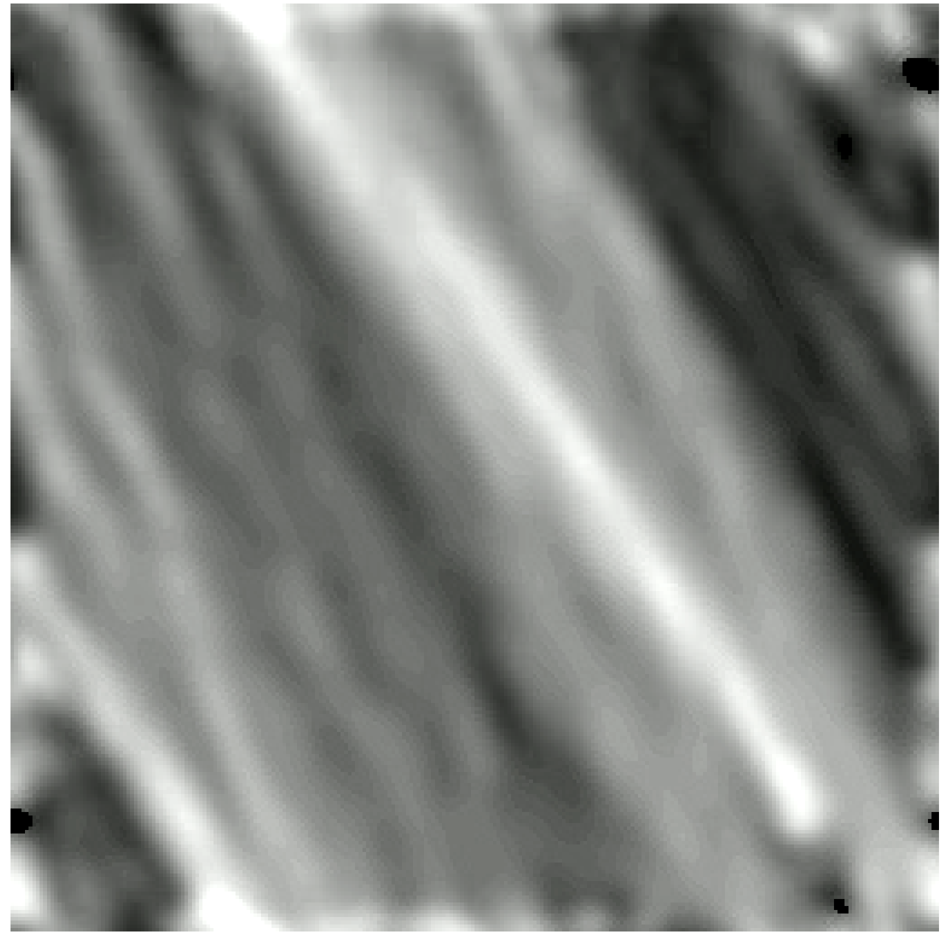
620/617.5



021212a/619 138, 140, 142, 144, 146, 148 0.0024

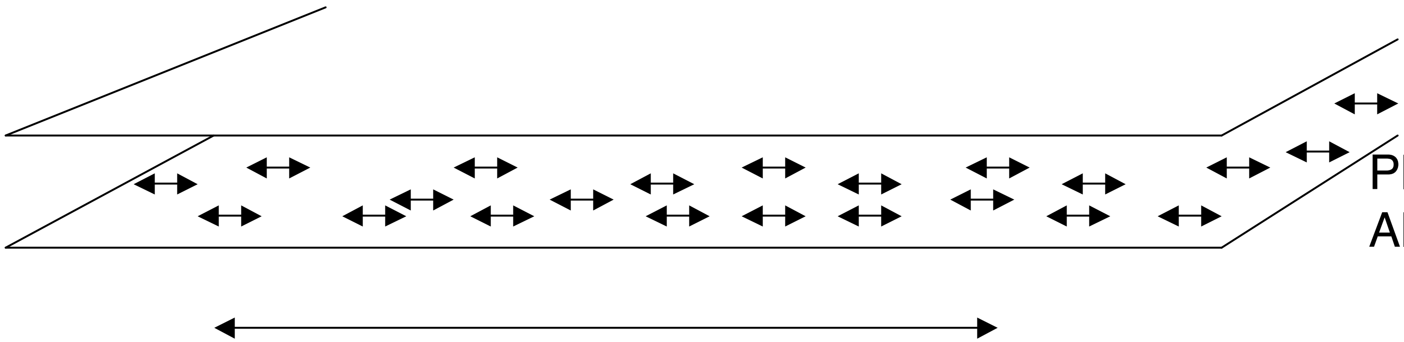


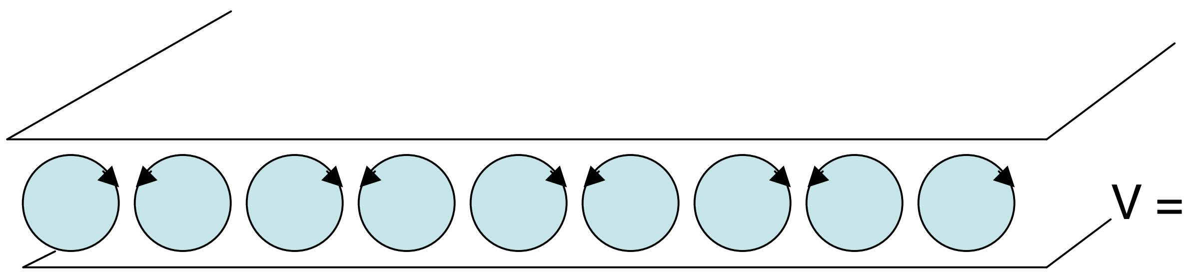
cos(phase)



roll angle

Electroconvection in a nematic liquid crystal

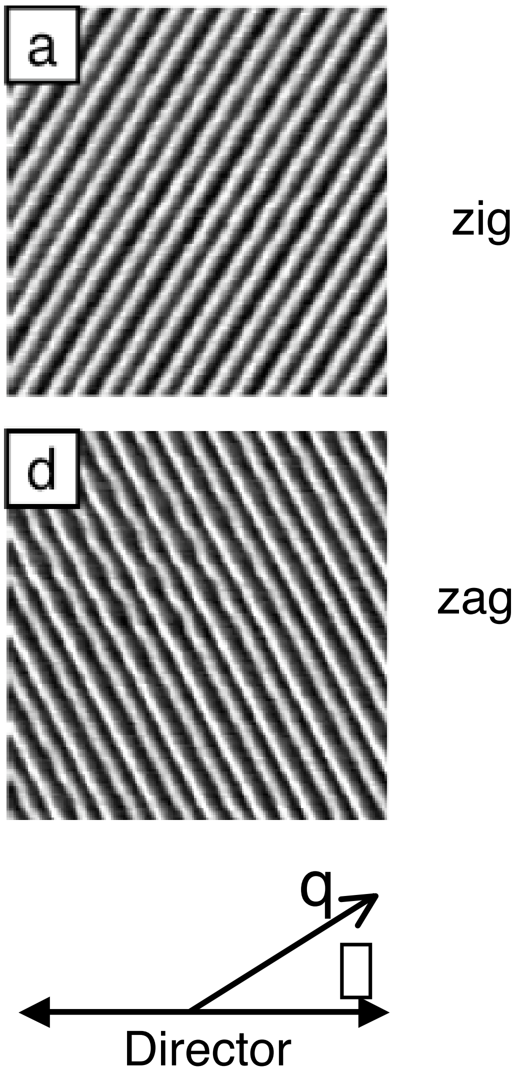


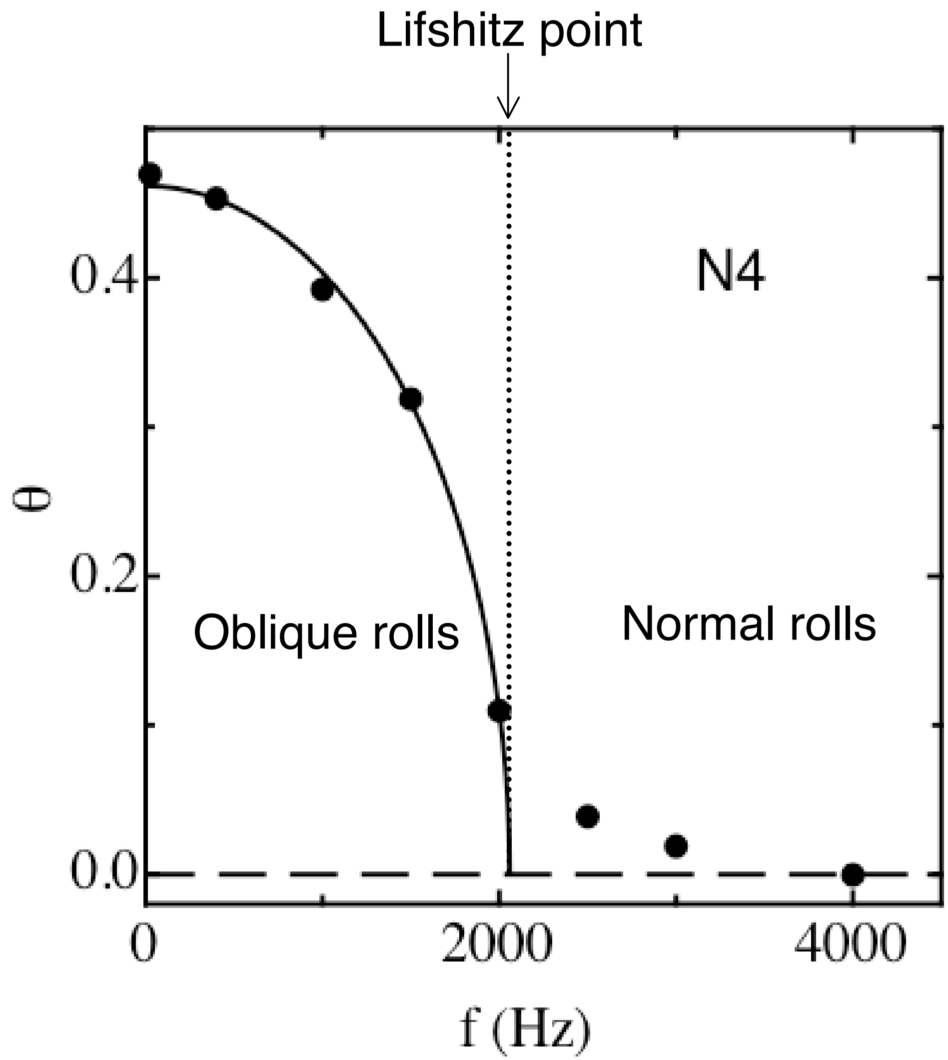


Convection for $V_0 > V_c$ $\varepsilon = (V_0 / V_c)^2 - 1$

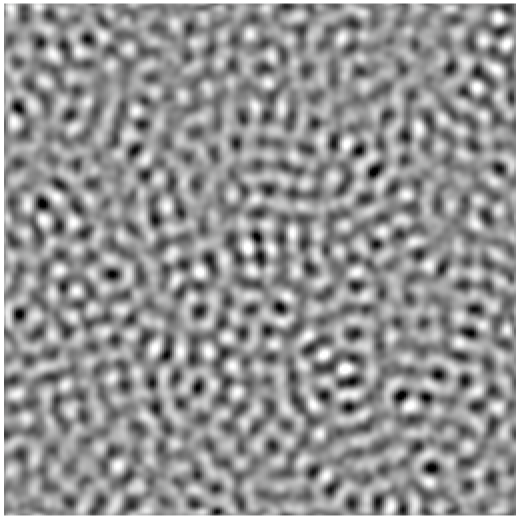
Anisotropic !

Stationary Bifurcation





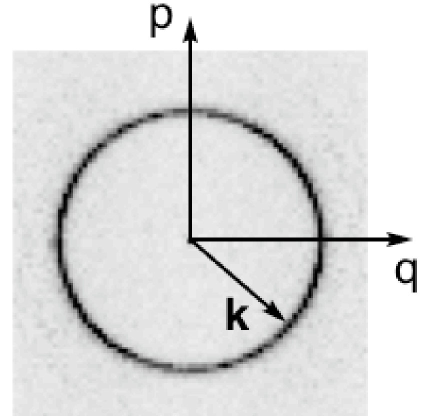
$f = \omega / 2\pi$



Rayleigh-Benard :

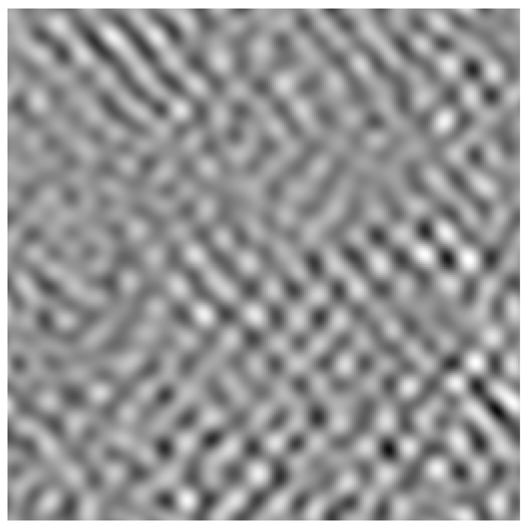
$$S(k) = \frac{S_0}{\tilde{\xi}^2(k^2 - k_0^2)^2 + \epsilon}$$

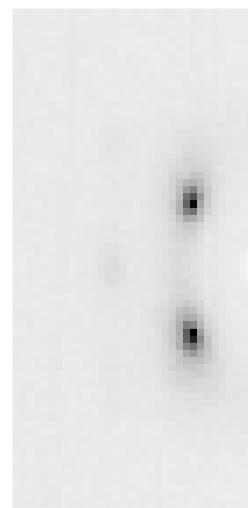
$$k^2 = q^2 + p^2$$

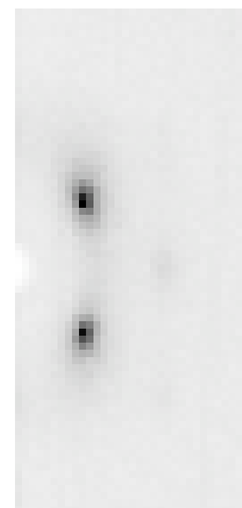


Electroconvection :

$$S(\mathbf{k}) = \frac{S_0}{\xi_q{}^2(q - q_0)^2 + \xi_p{}^2\,(p - p_0)^2 + \xi_{qp}{}^2\,(q - q_0)\,(p - p_0) + \varepsilon}$$







Different types of EC patterns:

Planar alignment:

Stationary Bifurcations

normal rolls
oblique rolls
Lifshitz point

Hopf Bifurcations

normal traveling rolls
oblique traveling rolls
Lifshitz point

Codimension-two points

Homeotropic alignment:

Preceded by Fredericz
transition

Directly to EC

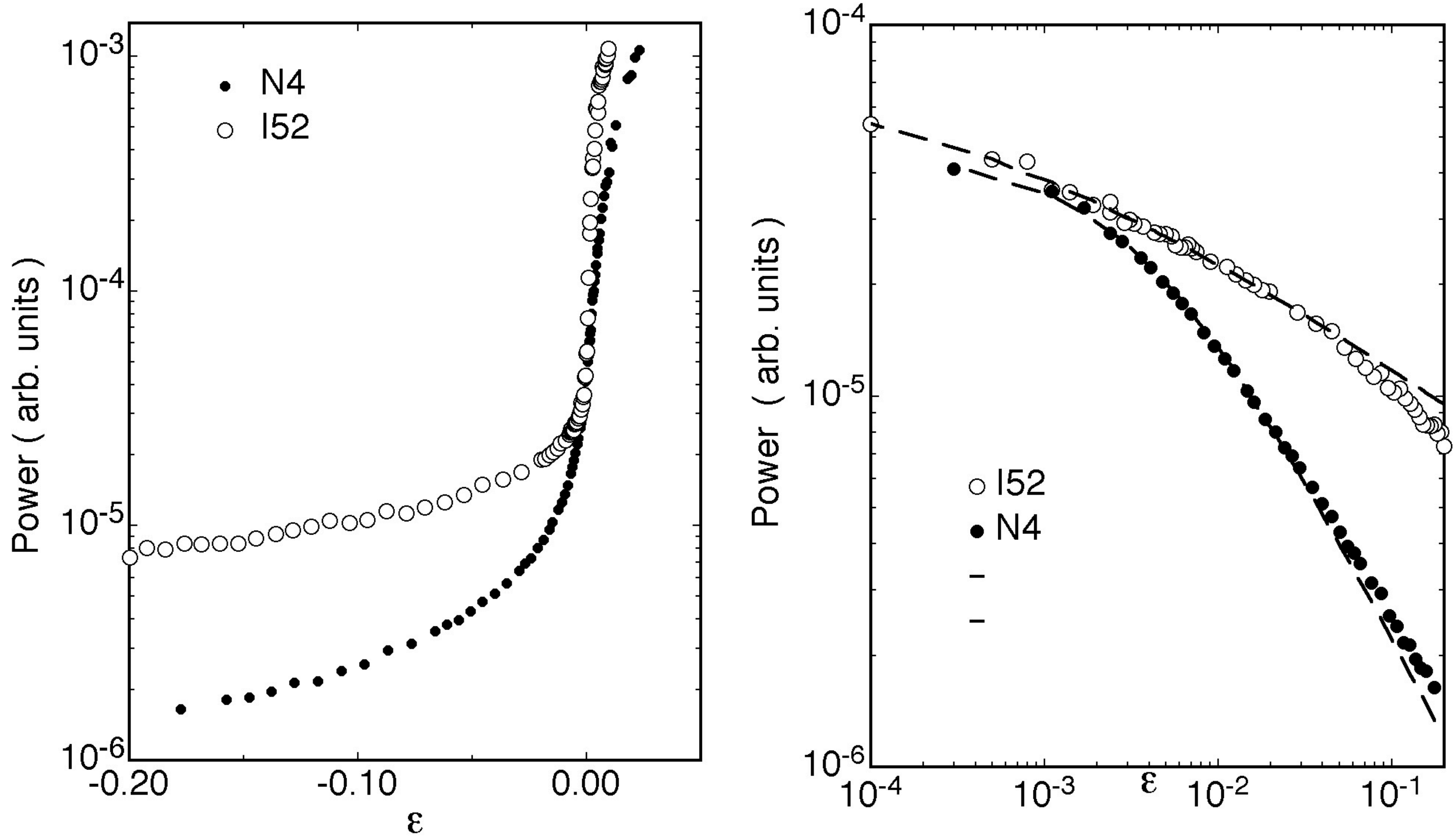
$$F = \frac{k_B T}{d < k_{el} >} \simeq 10^{-5}$$

Critical region probably not accessible

For oblique or normal stationary rolls:

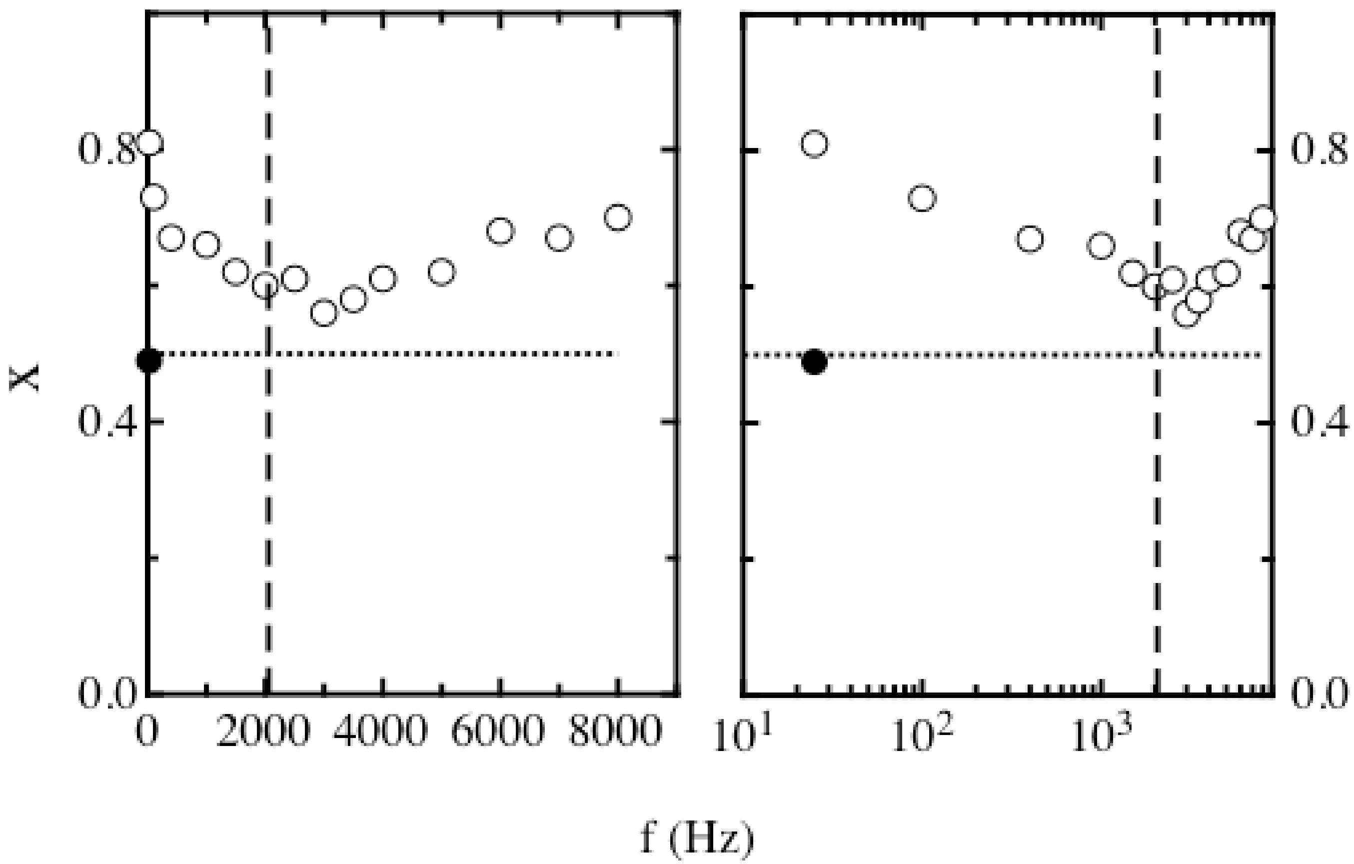
$$\dot{A} = (\epsilon + \xi_{||}^2 \partial_x^2 + \xi_{\perp}^2 \partial_y^2 + \xi_{cross}^2 \partial_x \partial_y - |A|^2)A + \eta(\vec{r}, t)$$

Anisotropic gradient terms, not invariant under rotation
Not Brazovskii
Probably 2-d XY (P.C. Hohenberg, private comm.)



○ I52: Hopf to oblique traveling waves

● N4: Stationary oblique rolls



$P = (1 / P_1 + 1 / P_0)^{-1}$, $P_1 = P_{10} | \square^{-x}$, $P_0 = \text{const}$

X. Qiu + GA, unpub.

Summary :

- Fluctuations near RB instability (Brazovskii)
  - fluctuation induced 1st order transition to rolls (or stripes)
  - fluctuations in striped phase
  - “phonons” in striped phase
    - amplitude modulation
    - roll angle modulation
  - dislocations in striped phase
- Fluctuations near oblique traveling wave bif. In EC (I52)
- Fluctuations near oblique and normal stationary bif. in EC (N4)
- Fluctuations near Lifshitz point of stationary rolls (N4)

Possible universality classes:

- **RBC: Brazovskii**
- **EC Normal and oblique stationary rolls: Probably 2-d XY**
- EC Normal and **Oblique travelling rolls: ????**
- EC codimension-two points
- **EC Lifshitz stationary rolls**
- EC Lifshitz traveling rolls
- EC Homeotropic after Fredericsz transition
- EC Homeotropic w/out Fred.: Brazovskii

Other systems:

micro-crystallization of diblock co-polymers (Fredrickson et al.), equilibrium phase transition of the Brazovskii universality class

stripe phases in 2-d Coulomb gases

high Tc superconductors

2-d high Landau levels.

Brazovskii? Probably not because of coupling to lattice.