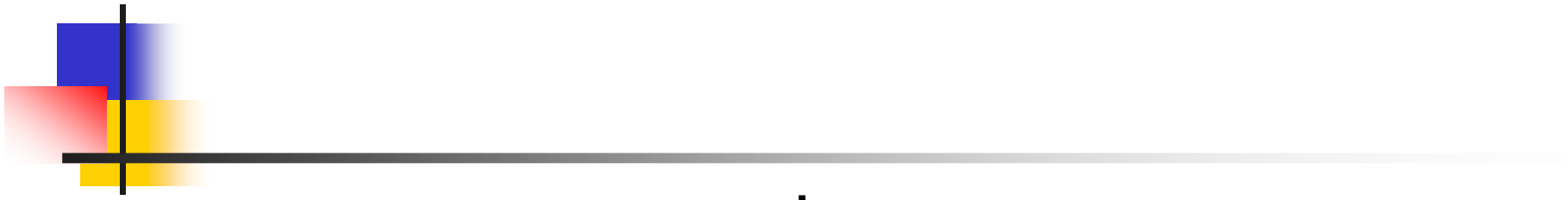
Asset Location and Allocation with Multiple Risky Assets



by

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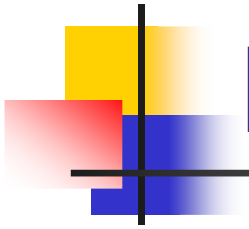
at

The IFID Centre at Fields



Plan of the Presentation

- Introduction and motivation
- Related works
- The model
- Results from numerical analysis
- Conclusion

Introduction and Motivation

- Consider average investors
- Taxable brokerage account (TBA)
- Tax-deferred retirement account (TDRA)
- Asset Allocation: Risky Vs. Risk Free Assets
- Asset Location: TBA Vs. TDRA

Basic Questions in this Literature

- How should investors allocate their asset holdings in both taxable and tax-deferred accounts?
- Tax-timing options in taxable account Vs. pre-tax returns in tax-deferred account
- What are the determinants of the optimal holdings in these two accounts?

Related Works

- Constantinedes (1983) [Econometrica]
  -Tax-timing option
- Dammon and Spatt (1996) [RFS]
  -Valuing tax timing option
- Shoven (1999) [NBER]
  -Asset location and allocation

Related Works Contd.

- Dammon, Spatt, and Zhang (2004) [JF]

Optimal Location

*Taxable bonds: TDRA

*Equities: TBA

Optimal Location (with borrowing constraint)

*Mixed holding possible but not simultaneously in both of the accounts

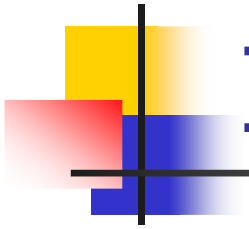
- Bergestresser and Poterba (2001) [Conf.]

*Mixed holdings are observed in both accounts

*Some investors allocates all of their TDRA wealth to equities

The Model

- Preference: Constant relative risk-aversion (Power Utility)
- The investor starts working at age 20. Works for 50 years, and lives in retirement for 30 years.
- Bequest motive (H): Leaves bequest in the form of H period annuity (e.g. H=20)
- A constant fraction of the total wealth comes from non-financial or labor income (15%)
- A constant fraction of the non-financial income is contributed toward the retirement account (20%)



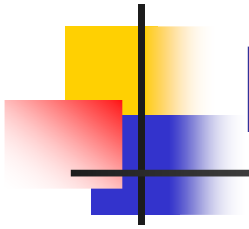
Investment Opportunities

- Two risky assets (e.g. stocks)
- A risk-free bond
- The risky asset prices follow binomial process, and the price processes are independent of each other
- A constant fraction of each of the asset prices is paid as dividend

Tax Environment

- Taxable Account
  * Dividends and interests taxed as received
  * Capital gains and losses taxed only at sale
  * Investor's tax basis is reset to the market value at death (taxes are forgiven)
- Tax-Deferred Account
  * Investment returns are not taxed
  * Withdrawals taxed as ordinary income
  * Retirement account wealth is taxed as ordinary income at death

Limitations & Potential Extensions



- Average cost basis
- Stochastic labor income
- Shocks to labor income
- Fixed date for death
- Behavioral issues

Parameters	Base portfolio
Basis-price ratio of asset 1 (p^*_{1t-1})	1.0
Basis-price ratio of asset 2 (p^*_{2t-1})	1.0
Prior holdings in equity 1 (s_{1t})	0.4
Prior holdings in equity 2 (s_{2t})	0.4
Mean return of equity 1 (μ_1)	9%
Standard deviation of equity 1 (σ_1)	20%
Mean return of equity 2 (μ_2)	13%
Standard deviation of equity 2 (σ_2)	30%
Correlation coefficient(ρ)	0
Bequest horizon (H)	20
Retirement wealth ratio (y)	0.4
Interest rate (r)	6%
Inflation rate (i)	3.5%
Ordinary income tax rate (τ_d)	36%
Capital gains tax rate (τ_g)	20%
Risk aversion parameter (γ)	3
Fraction of total wealth earned as labor income (l)	15%
Fraction of labor income contributed to the TDRA (α)	20%

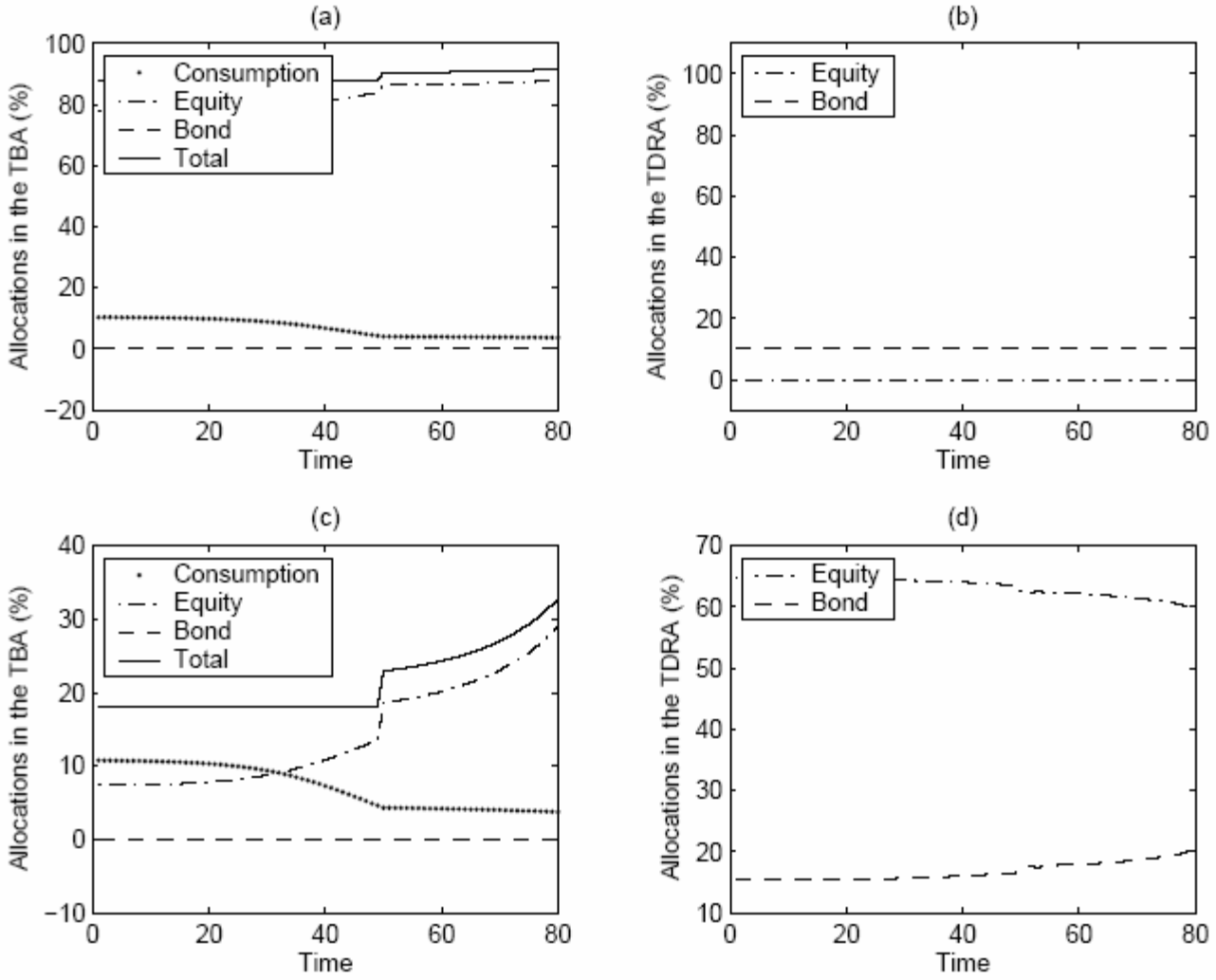


Fig. 5.1. Allocations within the TBA and the TDRA

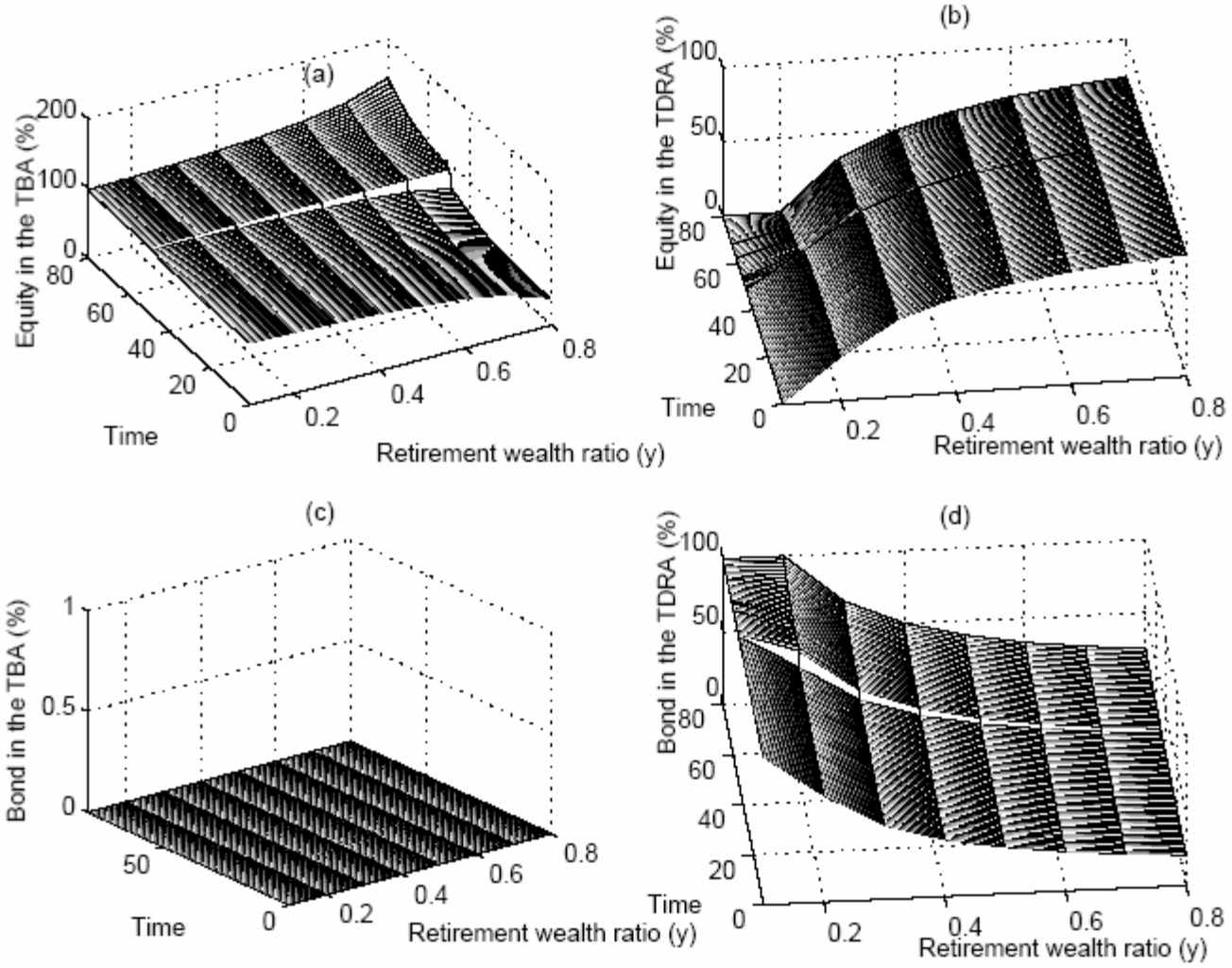


Fig. 5.2. Asset Allocation and Location Under Borrowing and Short Sale Constraints

Table 5.2
Asset Holding Frequency and Wealth Allocations for an Investor with Low Retirement Wealth Ratio ($y = .1$)

	TBA			TDRA	
Magnitude	Consumption	Equity	Bond	Equity	Bond
$< -90\%$	0.00	0.00	0.00	0.00	0.00
$< -40\%$	0.00	0.00	0.00	0.00	0.00
$< -10\%$	0.00	0.00	98.75	0.00	0.00
$< 0\%$	0.00	0.00	100	0.00	0.00
$> 0\%$	100	100	0.00	0.00	100
$> 5\%$	57.50	100	0.00	0.00	100
$> 10\%$	22.50	100	0.00	0.00	0.00
$> 50\%$	0.00	100	0.00	0.00	0.00
Mean wealth allocations (Working age)	8.75%	92.73%	−13.40%	0.00%	10%
Mean wealth allocations (Retirement age)	3.92%	97.56%	−10.70%	0.00%	10%

Table 5.3
Asset Holding Frequency and Wealth Allocations for an Investor with High Retirement Wealth Ratio ($y = .8$)

	TBA			TDRA	
Magnitude	Consumption	Equity	Bond	Equity	Bond
$< -90\%$	0.00	0.00	0.00	0.00	0.00
$< -40\%$	0.00	0.00	0.00	0.00	0.00
$< -10\%$	0.00	0.00	100	0.00	0.00
$< 0\%$	0.00	0.00	100	0.00	0.00
$> 0\%$	100	100	0.00	100	100
$> 5\%$	60	100	0.00	100	100
$> 10\%$	32.50	100	0.00	100	100
$> 50\%$	0.00	0.00	0.00	0.00	0.00
Mean wealth allocations (Working age)	9.30%	28.77%	-20%	45.18%	34.82%
Mean wealth allocations (Retirement age)	4.14%	42.09%	-20%	40.31%	39.69%

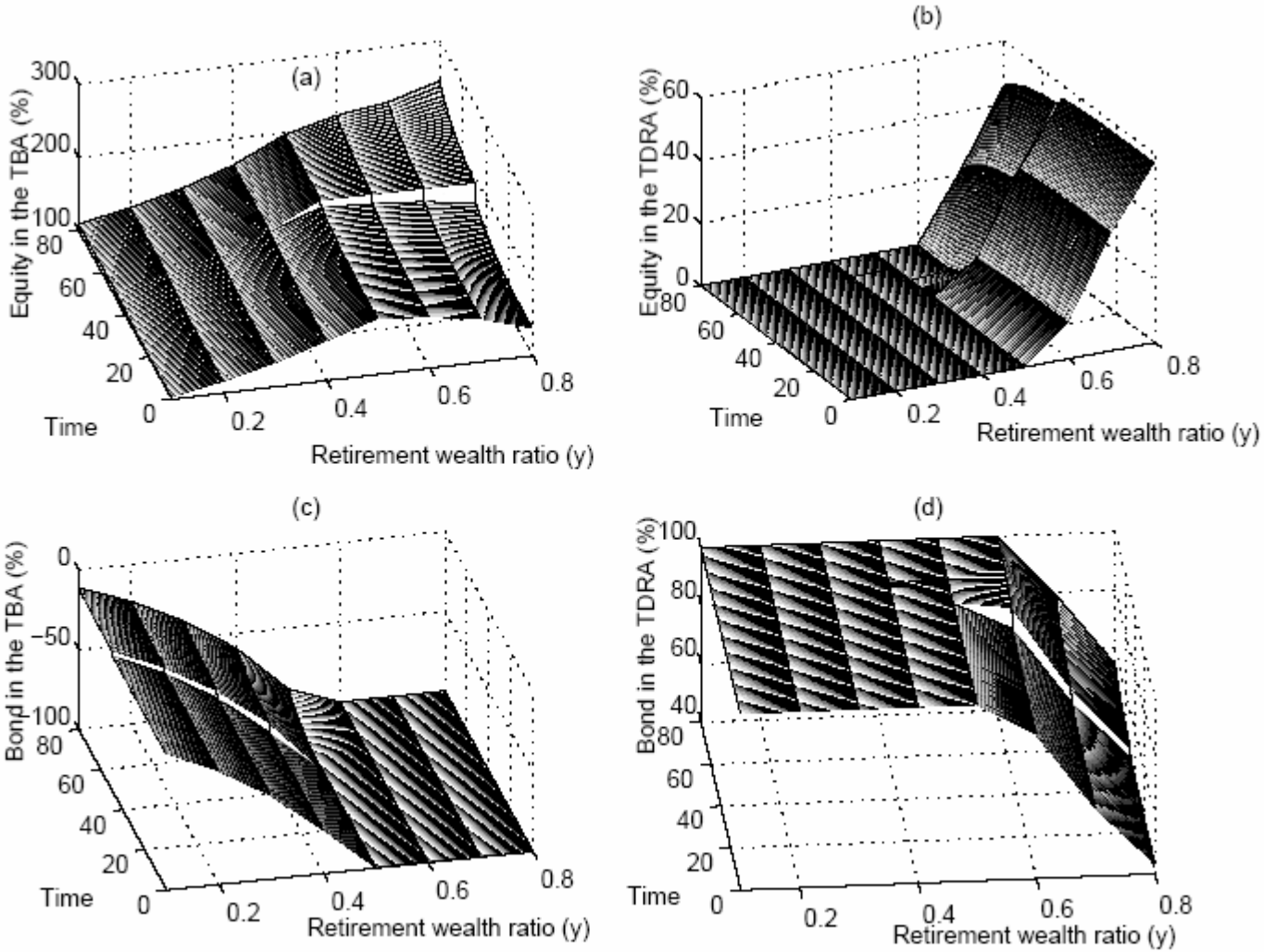


Fig. 5.3. Asset Allocation and Location when Borrowing Constraint is Relaxed

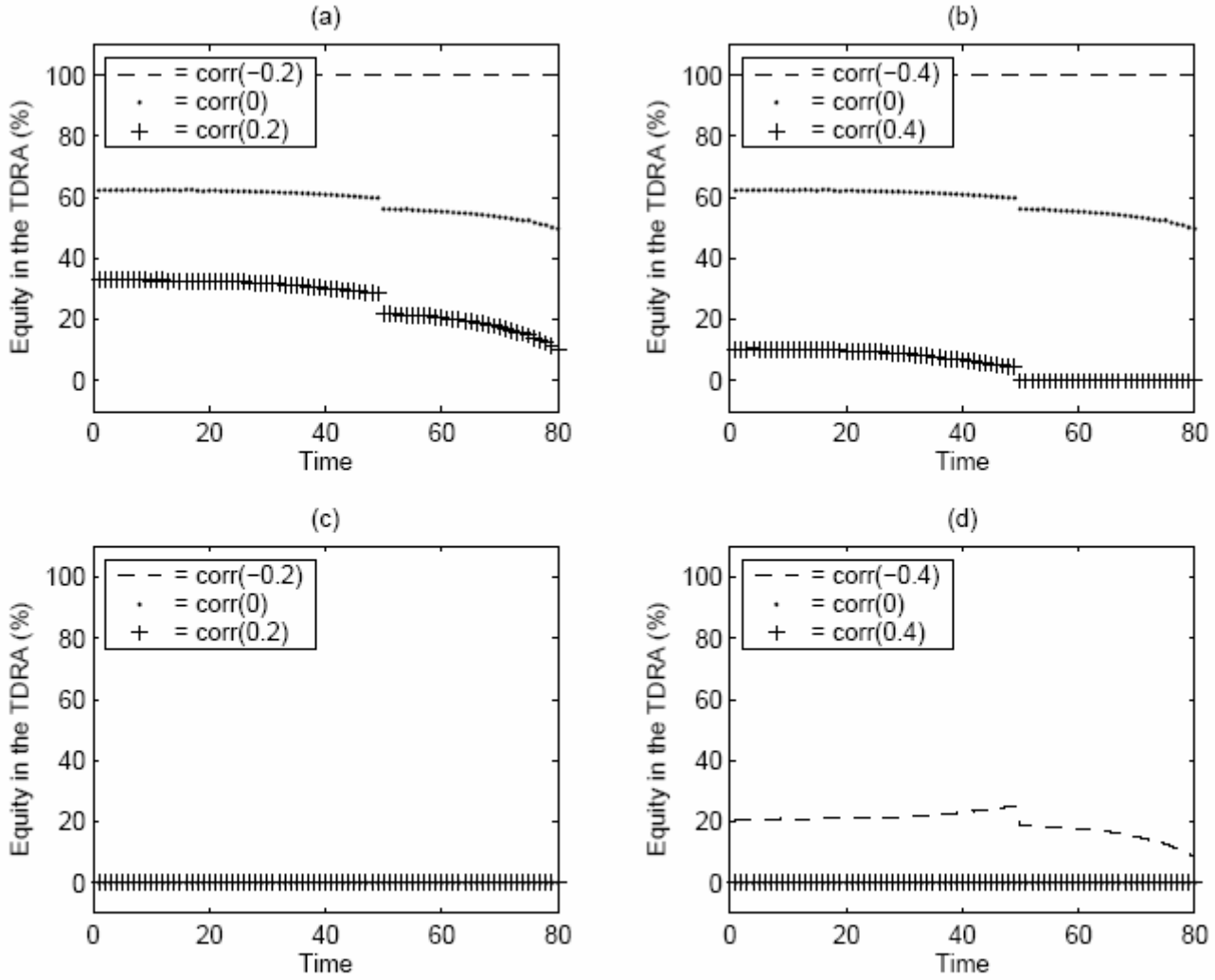


Fig. 5.4. Correlation Structure and Equity Holdings in the TDRA

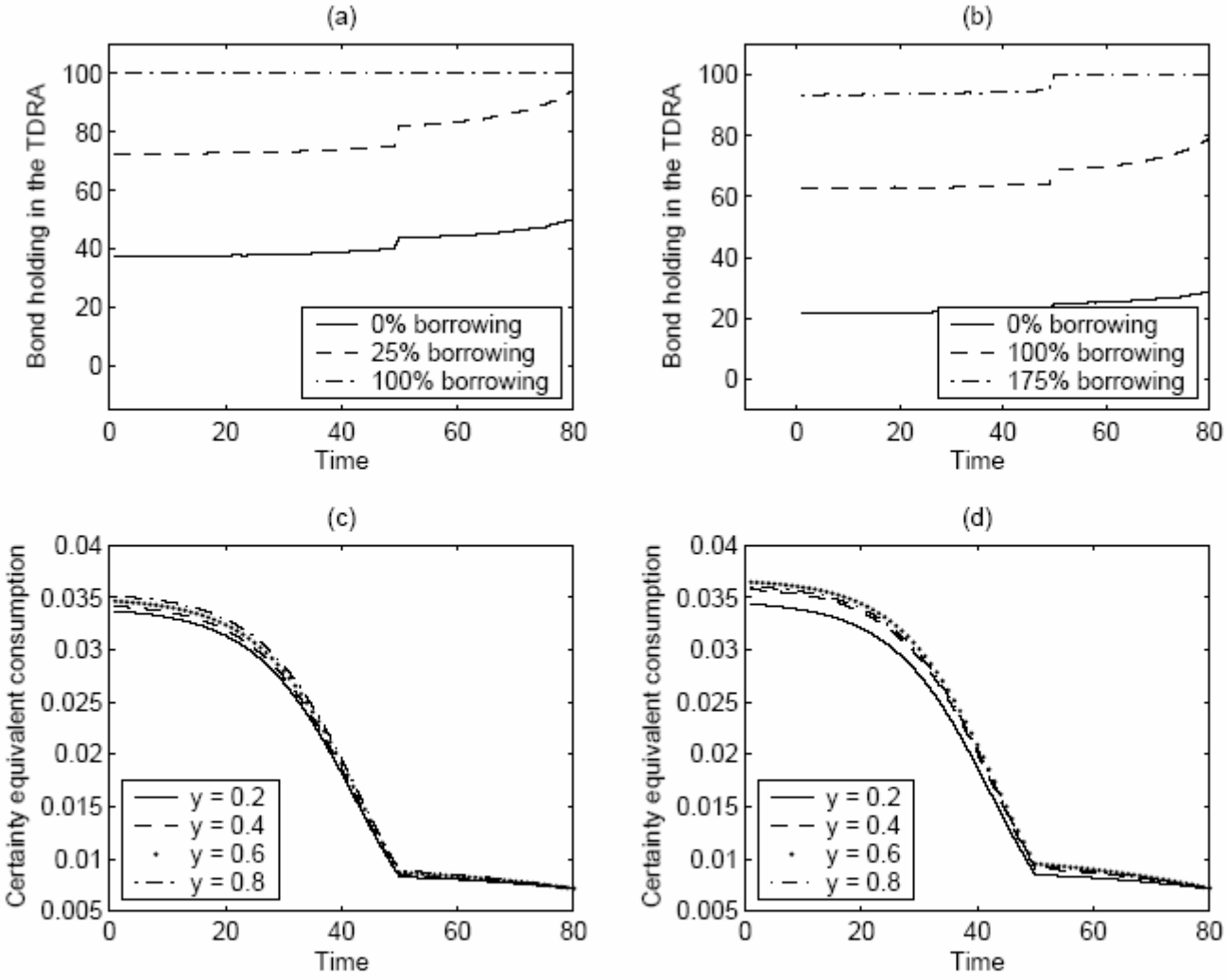


Fig. 5.6. Interaction between Borrowing Restrictions and Retirement Wealth Ratio, and Optimal Size of the TDRA

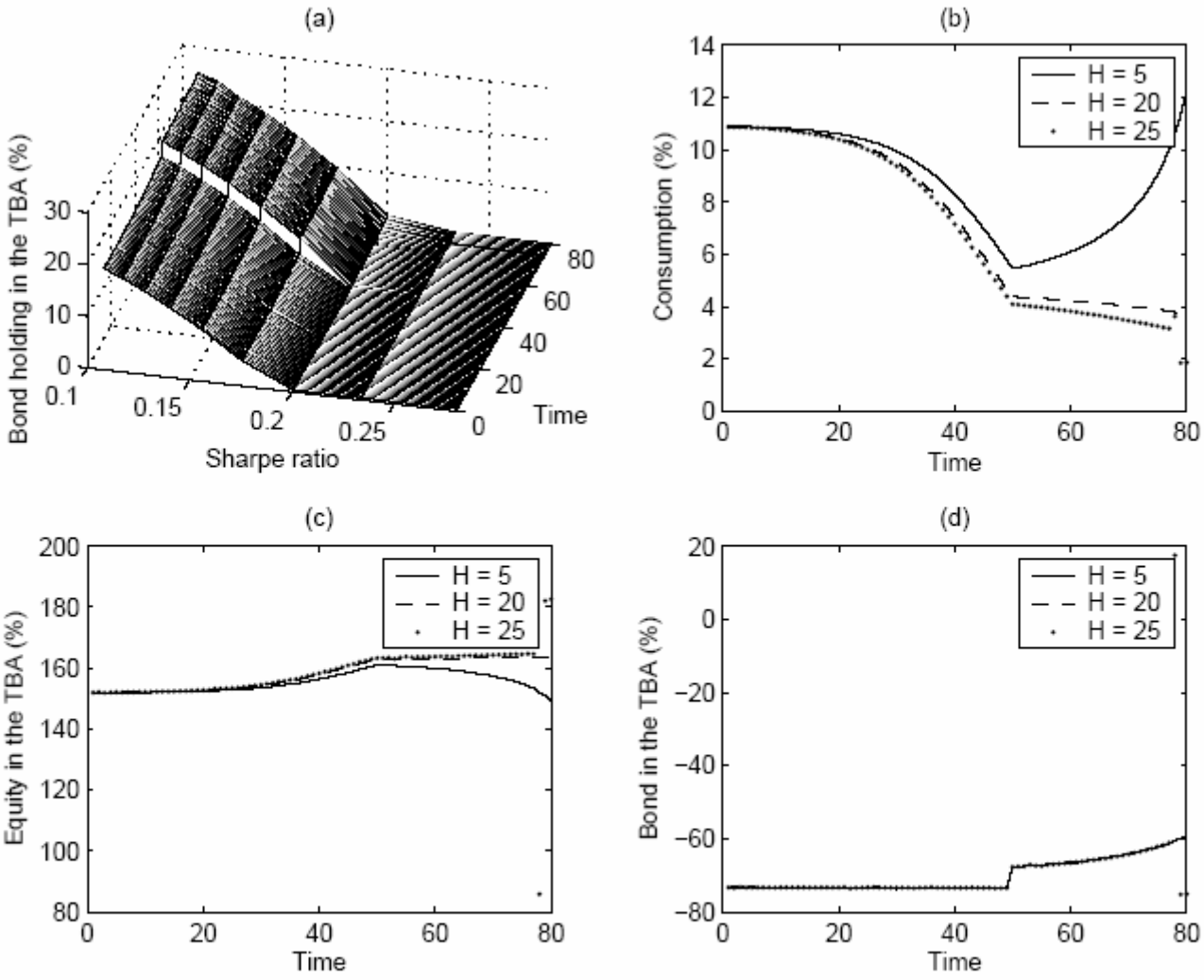


Fig. 5.5. Sharpe Ratio and Bequest Motive

Table 5.4
Changes in Utility Due to the Variations in the Retirement Contribution Level

y	$\Delta v_{5-20\%}$ with borrowing constraint	$\Delta v_{5-20\%}$ without borrowing constraint	$\Delta v_{20-30\%}$ with borrowing constraint	$\Delta v_{20-30\%}$ without borrowing constraint
.1	0.141%	0.265%	0.086%	0.176%
.4	0.090%	0.254%	0.060%	0.169%
.7	0.088%	0.086%	0.059%	0.057%

Conclusions

- Correlation structure of the risky assets is a key determinant of asset location decision
- Size of the retirement account (retirement wealth ratio) along with level of borrowing constraint affect location decision
- Borrowing constraint (or borrowing ability) is a key determinant of location decision
- There is a trade off between diversification and tax-timing
- Location decisions are robust to bequest motives and retirement contribution level
- With reasonable relaxation of borrowing constraint, mixed holdings of equities and bonds in TDRA is optimal
- Investors are not necessarily making suboptimal investment decisions
- There may be a possible link between the optimal retirement account size and borrowing constraint