# Betting on Death and Capital Markets in Retirement:

A Shortfall Risk Analysis of Life Annuities versus Phased Withdrawal Plans

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Three Uncertainties in Retirement:
A Financial Perspective

Investment
Returns ?

Bequest ?

Source: Die Zeit
„Rente"=Retirement / „Ziel"=Goal
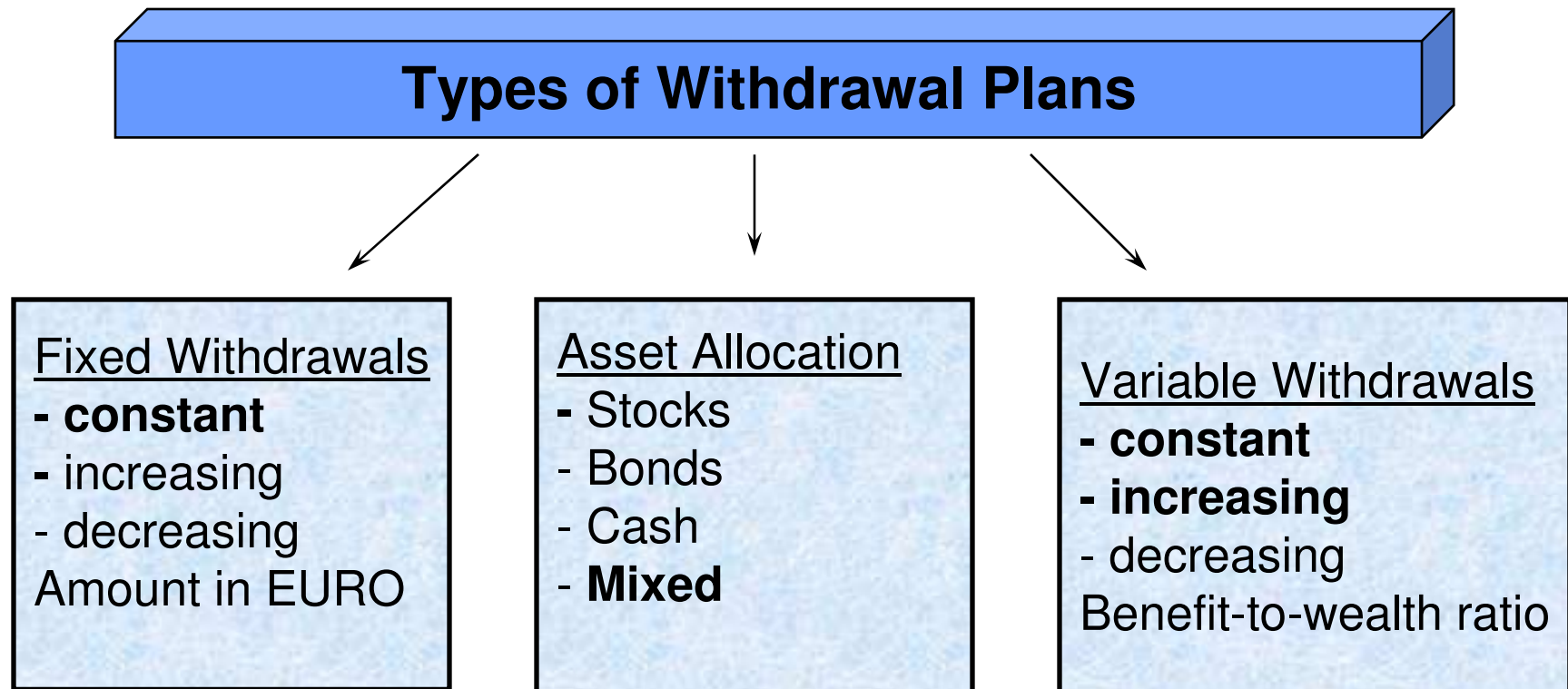
Investment horizon ?
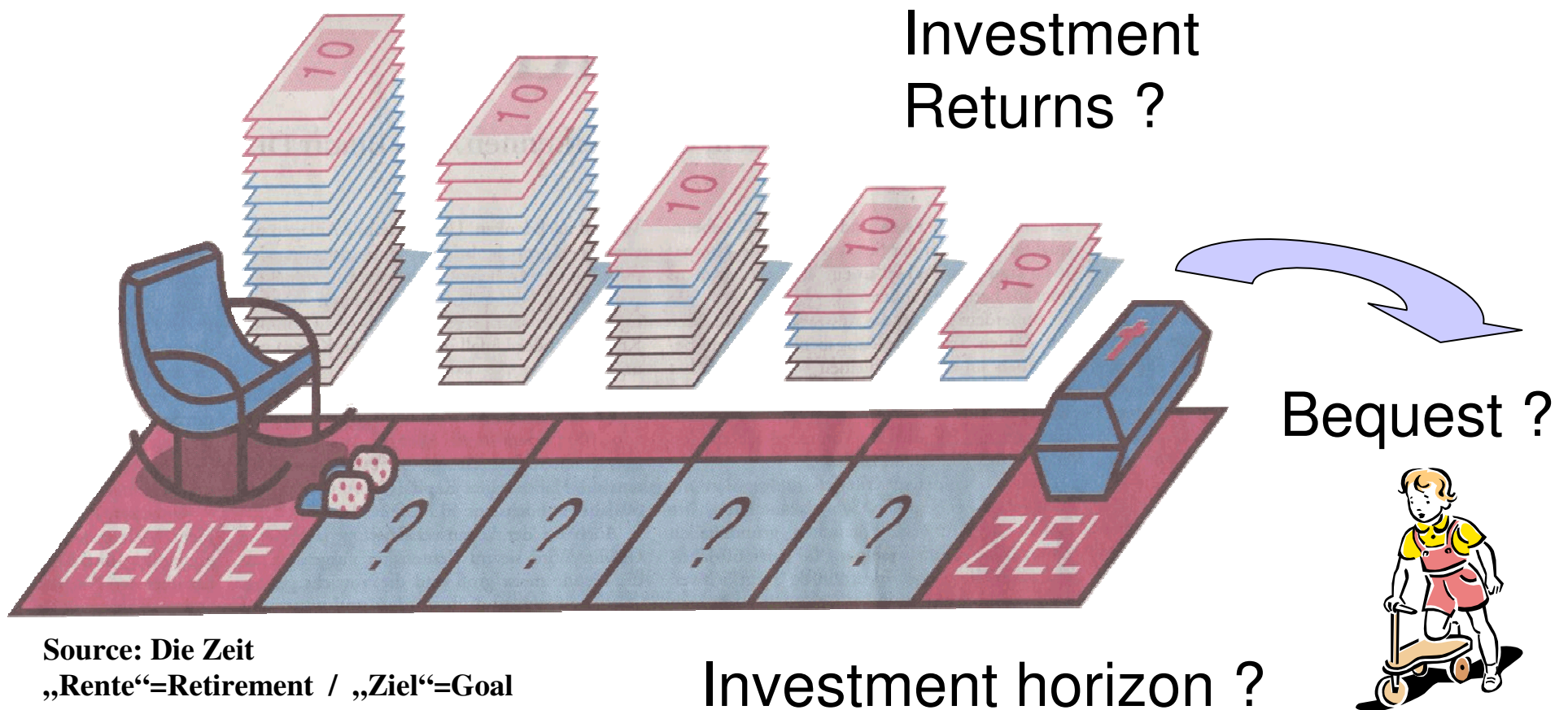
Motivation

- Compared to accumulation phase:
  - Ø Uncertainty about capital markets
  - Ø Uncertainty about investment horizon

- Interest in alternative payout designs:
  - Ø Risk-return tradeoffs: Benefits, shortfalls, and bequests
  - Ø Incorporate asset allocation and withdrawal rules

- Importance:
  - Ø 1st pillar state pensions in decline, more DC plans
  - Ø Retirees responsible for decumulation phase
  - Ø Some countries (UK, Germany) require mandatory annuitization (75/85)

Phased Withdrawal Plans

- Retirement assets invested in **I**ndividual **P**ension **A**ccount
  - Ø Asset Allocation ?

- Retiree consumes from the IPA periodically
  - Ø Withdrawal Rule ?

- Advantages compared to Life Annuity
  - Ø High flexibility, liquidity
  - Ø Bequest potential
  - Ø Higher benefits

- Risks of Phased Withdrawal Plans
  - Ø Lower benefits than Life Annuity   à   Consumption Shortfall
  - Ø Longevity risk (No risk pooling)   à   "Betting on Death"
  - Ø Capital market risk   à   "Betting on Capital Markets"

Phased Withdrawal Plans

Types of Withdrawal Plans

Fixed Withdrawals
- **constant**
- increasing
- decreasing
Amount in EURO

Asset Allocation
- Stocks
- Bonds
- Cash
- **Mixed**

Variable Withdrawals
- **constant**
- **increasing**
- decreasing
Benefit-to-wealth ratio

Fixed Withdrawal Plan

Retiree has sum of money V_0

- invested in financial assets earning returns R_t.
- Each period, he consumes B equal to the life annuity as long as possible:

$$B_t = \min(B, V_t).$$

- <u>Non-linear</u> Intertemporal budget constraint:

$$V_{t+1} = (V_t - B_t)(1 + R_t) = \begin{cases} (V_t - B)(1 + R_t) & V_t > B \\ 0 & V_t \leq B. \end{cases}$$

⚡ Consumption risk = fund exhaustion while still alive

Variable Withdrawal Plans

- Plan pays an *ex ante* specified fraction ω_t of remaining retirement funds [e.g. 5%].

$$B_t = \omega_t \cdot V_t$$

- Linear Intertemporal budget constraint:

$$V_{t+1} = (V_t - B_t) \cdot (1 + R_t) = (1 - \omega_t) \cdot V_t \cdot (1 + R_t)$$

Consumption risk = lower benefits than benchmark while still alive

Specific Variable Withdrawal Rules

"Fixed Percentage" withdrawal rule :

  – Constant and fixed fraction $\qquad \omega = \omega_0 = \omega_1 = ...\omega_t.$

"1/T Rule" withdrawal rule:

  – Withdrawal fraction set to maximum possible plan duration T

$$\omega_t = \frac{1}{T-t}.$$

"1/E[$T(x)$]" withdrawal rule:

  – Withdrawal fraction determined by retiree's remaining life expectancy

$$\omega_t = \frac{1}{E[T(x+t)]}.$$

The Benchmark Life Annuity

- **Characteristics**
  - ØConstant (real) annuity payments until death
  - ØOffered by commercial insurance companies
  - ØPro: Pooling of longevity risk / mortality "spread"
  - ØCon: No bequest potential, low flexibility

- **Present Relevance**
  - ØThin private annuity markets around the world
  - ØAlso countries with substantial DC-pension plans

Life Annuity Benefits:
Using German / US data

| Mortality Table	Male	Female
Retirement Age	Life Annuity $ (€) p.a.	
65	5.83 (5.82)	5.22 (5.02)
70	7.00 (7.03)	6.22 (5.99)

Parenthesis: Results for German Annuity

Immediate Annual Life-long Real Annuity Benefits per EUR 100 Single Premium: Total Expense Loadings 2.785% for Germany; 1% for US; (Real) Discount Factor **1.5%;** German DAV R 94 annuitant mortality table (max. age 110); US 2000 basic annuitant mortality table (max age 115)

Ø **Mortality "drag" at the cost of no bequest potential**

Historical Analysis: Retire in 1957 (German-Case)

Historical Benefits of Withdrawal Plans Conditional on Survival
(60% Equities / 40% Bonds): Life Annuity Benchmark
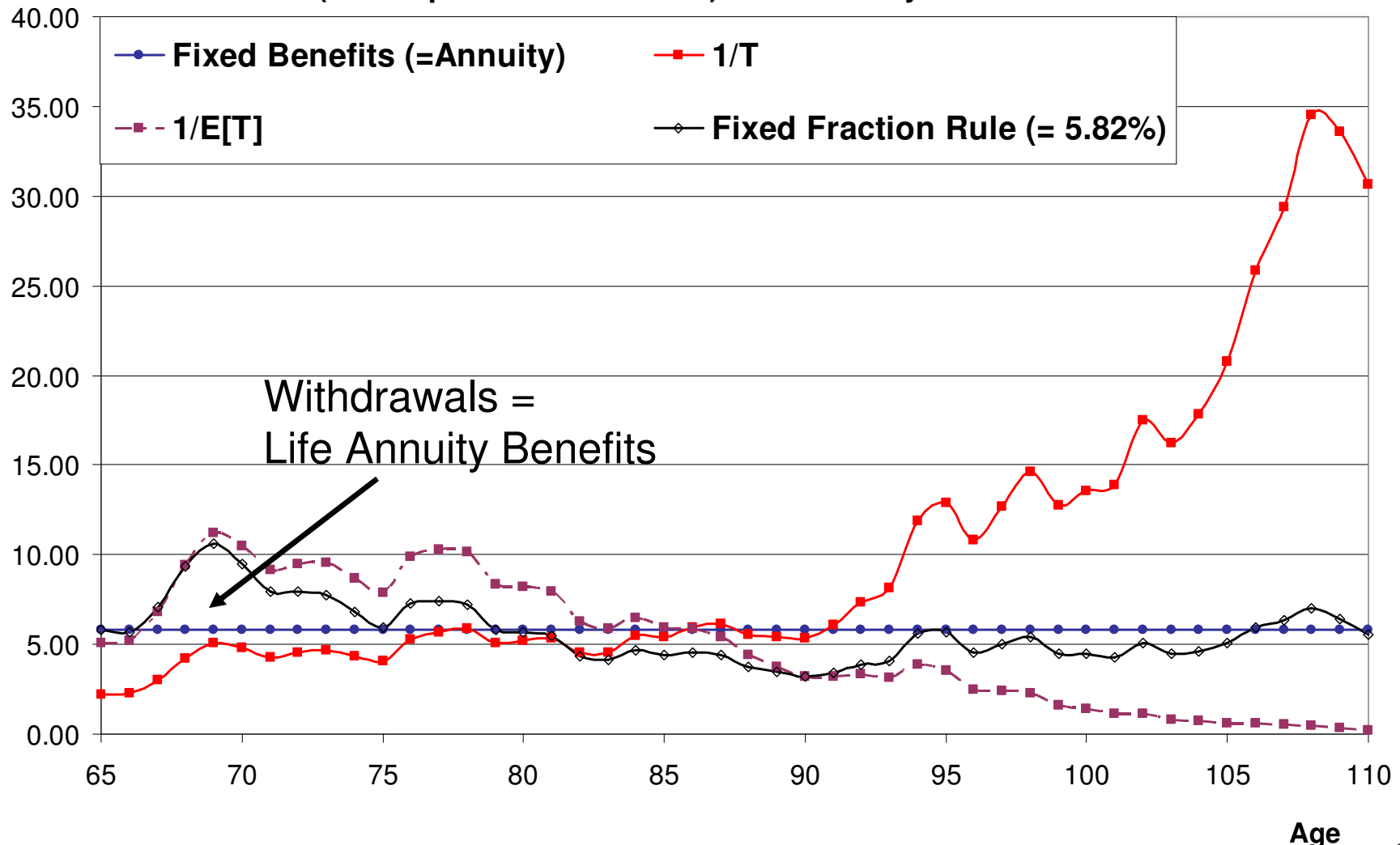
Historical Analysis: Retire in 1957 (US- Case)

Historical Benefits of Withdrawal Plans Conditional on Survival (60% Equities / 40% Bonds): Life Annuity Benchmark

Legend:
- Fixed Benefits (=Annuity)
- 1/T
- 1/E[T]
- Fixed Fraction Rule (= 5.83%)

Withdrawals = Life Annuity Benefits

Age
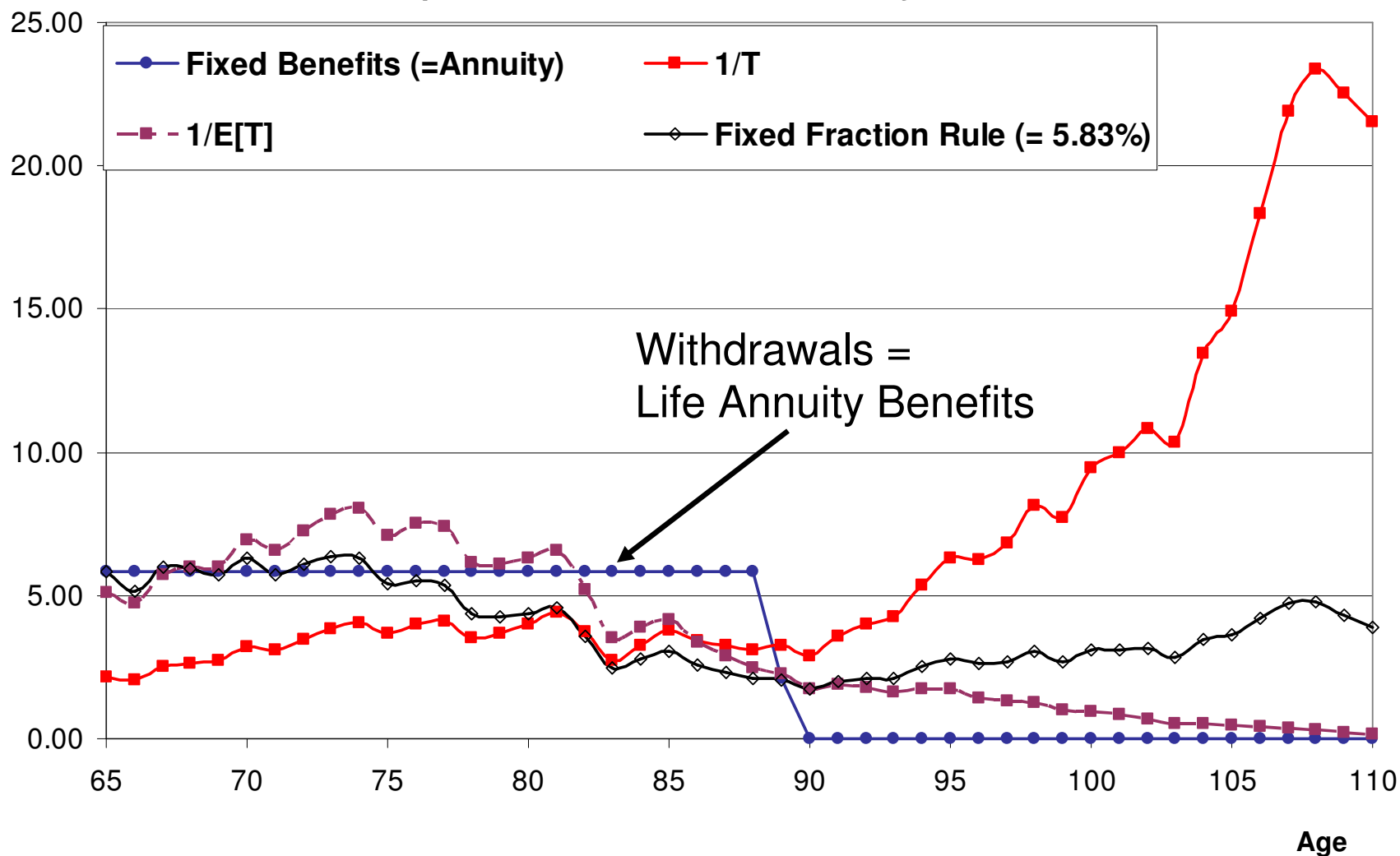
Research Approach

- Evaluate these different strategies against <u>life annuity benchmark</u>

- Stochastic Model (mortality / investments)

- Possible objective functions

  Ø Risk value models (Milevsky et al. 1994, 1998, 2000, 2001 Albrecht/Maurer 2002)

  – Only look at shortfall probability

  – Only examine withdrawal plans with fixed benefits

  Ø Specific utility functions (Blake, Campbell/Viciera)

  – Must assume exact risk preferences, but…

Our Contributions

Ł **Using risk value models:**

Ø Our risk measure incorporates both probability and size of loss

Ø Compare fixed with different variable withdrawal rules

Ø Optimize asset allocation

Ø Optimize design parameters of variable payment schedule

Ø Study portfolios of withdrawal plans and annuities

Shortfall Risk and "Return" Measures:

Risk

- Shortfall Probability
  $$SP = P(B_t < z)$$

- Mean Excess Loss
  $$MEL = E(z - B_t \mid B_t < z)$$

- Expected Shortfall
  $$SE = E[max(z - B_t, 0)]$$
  $$= SP * MEL$$

where
B_t = benefit of the withdrawal plan
z = benefit of the benchmark life annuity

Return

- Expected Benefit
  $$E[B_t]$$

- Expected Bequest
  $$E[V_t]$$

Withdrawal plans:
Risk-Minimizing Investment Allocation

- Objective function:

$$EPVShortfall = \sum_{t=1}^{T} \frac{{}_t p_x \, \mathrm{E}[\max(z - B_t, 0)]}{(1+r)^t}$$

- This risk measure accounts for:
  - ü Mortality risk
  - ü Time preferences
  - ü Risk preferences for investment uncertainty

- Vary investment mix and withdrawal fraction to minimize Expected PV of Shortfall

Optimized Withdrawal Rules in Risk-Return Context

ü EPV_Benefits reflects expected present value of benefit payments conditional on survival:

$$EPVBenefits = \sum_{t=1}^{T} \frac{{}_t p_x E(B_t)}{(1+r)^t}$$

ü EPV_Bequest measures expected present value of inheritance the retiree passes to heirs in the event of death:

$$EPVBequest = \sum_{t=1}^{T} \frac{{}_{t-1} p_x q_{x+t} E(V_t)}{(1+r)^t}$$

Methodology

§ We model withdrawal plans: age 65 to 110 (115)

§ Benchmark Annuity
- Ø **US** / German Mortality Tables
- Ø Assumptions about loadings

§ Stochastic Model
- Ø Price dynamics: GBM
- Ø 1967-2002 yearly real returns
  - § German Data
  - § **US-Data from Ibbotson**
- Ø 100,000 alternative paths for fixed withdrawal plans
  - § (Alternative: IG-Approximation accord. Milevski et al.)
- Ø Analytical closed form solution for variable withdrawal plans
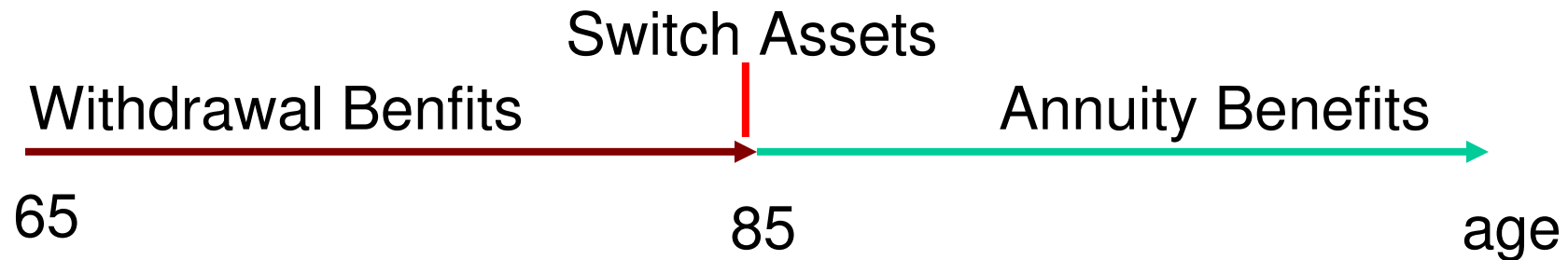
Optimization Results:
"Stand Alone Withdrawal Rules" (German case)

Benefits from Withdrawal Plan →

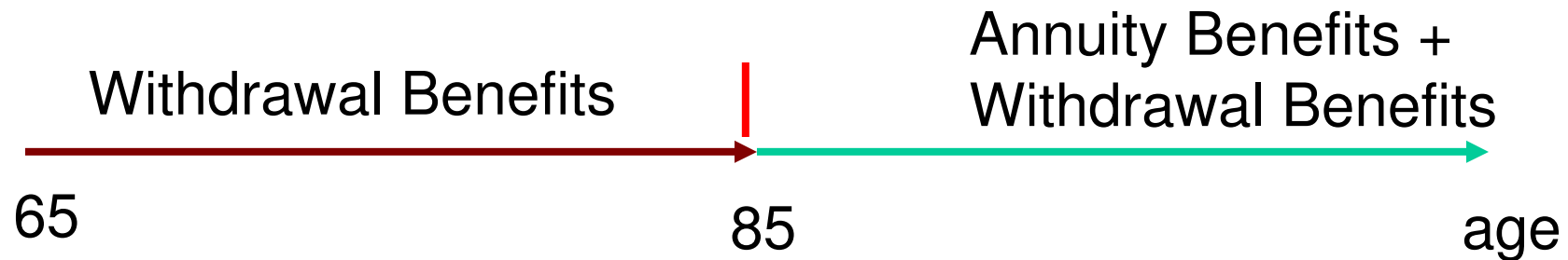65   age

Results for Male (Retirement Age 65): Benchmark Real Life Annuity €5.82 p.a./ €100						
Strategy	EPV Shortfall	EPV Benefits	EPV Bequest	Investment Weights (in %)		
				Equity	Bonds	Cash
Real Annuity €5.82	0	97.29	0			
Fixed Benefit = €5.82	3.58	93.41	53.19	20	80	0
Fixed Pct. = 5.82%	12.58	92.53	66.06	30	70	0
1/T Rule Age 110	34.95	82.68	134.41	50	50	0
1/E(T) Rule	8.27	103.08	39.80	20	80	0

Impact of Mandatory Switching into a Life Annuity at Age 85 (German Case)

Switch Assets

Withdrawal Benfits | Annuity Benefits

65

85

age

Results for Male (Retirement Age 65 Switching Age 75): Benchmark Real Life Annuity €5.82 p.a./ €100						
Strategy	EPV Shortfall	EPV Benefits	EPV Bequest	Investment Weights (in %)		
				Equity	Bonds	Cash
Real Annuity €5.82	0	97.3	0			
Fixed Benefit until 85	2.8	103.4	33.5	15	80	5
Fixed Pct. Opt ω=7.4%	7.4	108.8	32.3	25	75	0
1/T Rule Opt Age 88	9.5	108.3	35.1	20	80	0
1/E(T) Rule	5.4	104.1	31.2	15	75	10

Portfolio of Phased Withdrawal Plan and Deferred Life Annuity starting at Age 85

Withdrawal Benefits

Annuity Benefits +
Withdrawal Benefits

65

85

age

Results for Male (Retirement Age 65 Switching Age 75): Benchmark Real Life Annuity €5.82 p.a./ €100						
Strategy	EPV Shortfall	EPV Benefits	EPV Bequest	Investment Weights (in %)		
				Equity	Bonds	Cash
Real Annuity 5.828	0	99.0	0			
Fixed Payment until 85	5.3	100.0	34.4	50	40	10
Fixed Perct. opt. 9.1%	13.4	110.1	33.7	79	21	0
1/T-Rule (T=84)	10.0	110.2	21.2	50	36	14
1/E(T)-Rule	14.6	111.9	37.7	68	32	0

Comparison US vs. German Data

Rule	Risk	Benefits	Bequest	Equity Exposure	Withdrawal Fraction
Fixed Benefits					
- Stand Alone	++	-	++	++	
- Switching (85)	++	++	+-	++	
- Deferring (85)	++	-+	++	++	
Fixed Fraction					
- Stand Alone	+	++	-	++	+-
- Switching (85)	+	+	0	++	-+
- Deferring (85)	+	+	-+	++	+-
1/T-Rule					
- Stand Alone	+	+-	-+	++	0
- Switching (85)	+	++	-+	++	0
- Deferring (85)	+	+	0	++	0
1/E(T)-Rule					
- Stand Alone	+	++	0	++	
- Switching (85)	+	+	+-	++	
- Deferring (85)	+	+	+-	++	

++ (--)   Substantial Higher (Lower) compared with German Data
+ (-)   Higher (Lower) compared with German Data
+- (-+)   slightly higher (Lower) compared with German Data
0   no change compared with German Data

Conclusions

- Phased withdrawal plans offer many advantages: flexibility, bequests, and possibly higher consumption than life annuities.

- Yet a phased withdrawal plan also requires that attention be devoted to asset allocation and withdrawal rules.

- To minimize the shortfall-risk of consuming less than a real annuity benchmark, retirees should invest their assets more in fixed income than in equities.

- For a fixed withdrawal rule compared to no annuity:
  – Mandatory deferred annuitization and/or a switching rule can enhance expected payouts & cut expected shortfall risk
  – But at cost of reduced bequests.

"The secret to living well is to die without a cent in your pocket"
# "But I seem to have miscalculated"

Jorge Guinle, 88, a Playboy Who Outlived His Millions

By LARRY ROHTER

BUENOS AIRES, March 5 — Jorge Guinle, one of the last of the millionaire playboys whose free-spending ways and romantic exploits made them global celebrities from the 1930's onward, died early Friday in Rio de Janeiro. He was 88 years old and died in a suite at the luxurious Copacabana Palace Hotel, which his family built and owned for more than half a century.

Mr. Guinle had been at a hospital awaiting surgery for an aortic aneurysm. But on Thursday he insisted on being released, and checked in at the hotel where he had spent much of his gilded childhood, choosing to dine on chicken stroganoff, raspberry sherbet and tea for what turned out to be his final meal.

"He died as he lived, in grand style and with his eyes shining," said Clau-
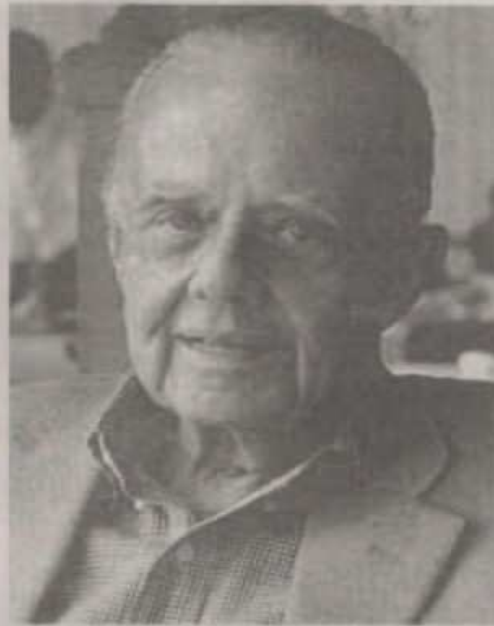
John Maier/Image Works, for The New York Times
Jorge Guinle in 2003.

Source: Financial Times

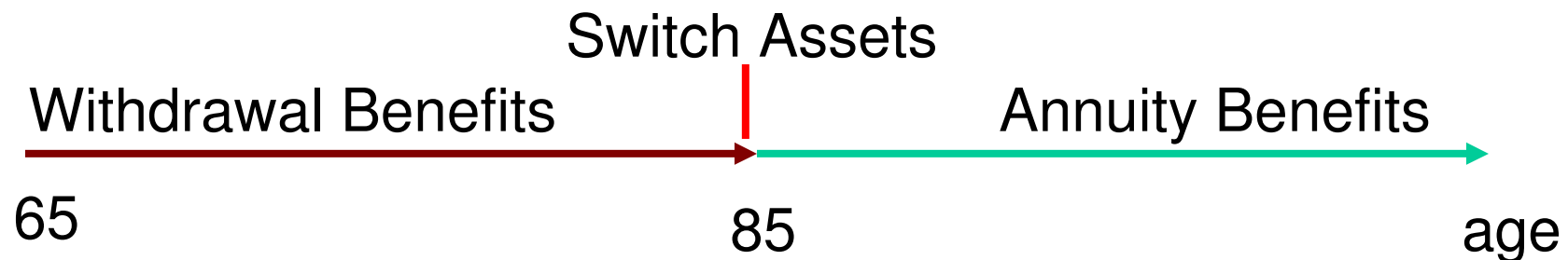# BACKUP

Optimization Results:
"Stand Alone Withdrawal Rules" (US case)
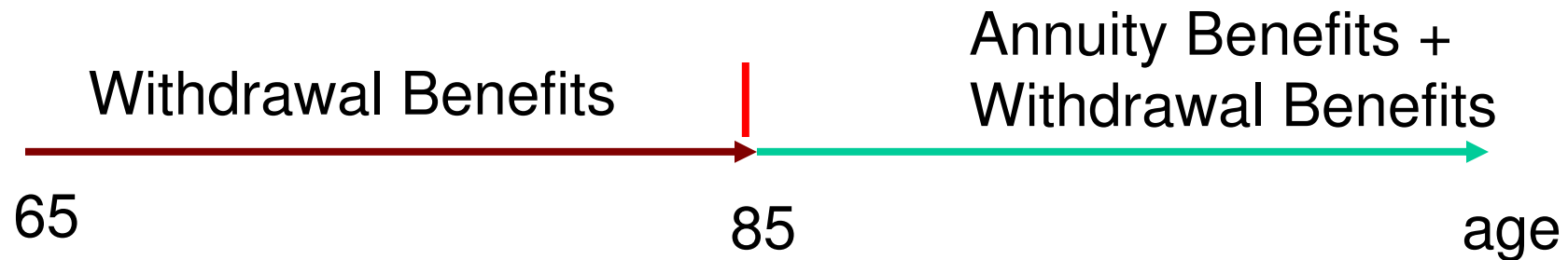
Benefits from Withdrawal Plan

65
age

Results for Male (Retirement Age 65): Benchmark Real Life Annuity €5.83 p.a./ US$ 100						
Strategy	EPV Shortfall	EPV Benefits	EPV Bequest	Investment Weights (in %)		
				Equity	Bonds	Cash
Real Annuity €5.83	0	99.0	0			
Fixed Benefit = €5.83	7.0	91.6	72.4	60	40	0
Fixed Pct. = 5.83%	14.7	106.9	46.1	75	25	0
1/T Rule Age 87	18.6	105.1	30.0	45	31	24
1/E(T) Rule	12.4	112.6	39.0	63	37	0

Impact of Switching into a Life Annuity at Age 85 (US case)

Switch Assets

Withdrawal Benefits | Annuity Benefits

65

85

age

Results for Male (Retirement Age 65 Switching Age 75): Benchmark Real Life Annuity €5.82 p.a./ €100						
Strategy	EPV Shortfall	EPV Benefits	EPV Bequest	Investment Weights (in %)		
				Equity	Bonds	Cash
Real Annuity €5.82	0	99.000	0			
Fixed Benefit until 85	6.7	112.5	37.6	55	45	0
Fixed Pct. Opt ω=7.4%	10.9	116.7	32.6	64	36	0
1/T Rule Opt Age 88	13.0	119.4	34.2	63	37	0
1/E(T) Rule	10.2	114.1	32.3	55	40	5

Portfolio of Phased Withdrawal Plan and Deferred Life Annuity starting at Age 85 (US-Case)

Withdrawal Benefits

Annuity Benefits +
Withdrawal Benefits

65

85

age

Results for Male (Retirement Age 65 Switching Age 75): Benchmark Real Life Annuity €5.82 p.a./ €100						
Strategy	EPV Shortfall	EPV Benefits	EPV Bequest	Investment Weights (in %)		
				Equity	Bonds	Cash
Real Annuity 5.828	0	99.0	0			
Fixed Payment until 85	5.3	100.0	34.4	50	40	10
Fixed Perct. opt. 9.1%	13.4	110.1	33.7	79	21	0
1/T-Rule (T=84)	10.0	110.2	21.2	50	36	14
1/E(T)-Rule	14.6	111.9	37.7	68	32	0

Summary Statistics for Annual Real Log-Returns on Stocks / Corp. Bonds / Cash 1967-2002

Asset Class	Mean (% p.a)	Volatility (% p.a.)	Correlations		
			Stocks	Bonds	Cash
Stocks	5.31 (5.53)	17.22 (25.36)	1	(0.235)	(-0.174)
Bonds	3.31 (3.98)	11.78 (5.21)	0.432	1	(0.326)
Cash	1.41 (2.84)	2.35 (1.69)	0.446	0.591	1

Parenthesis: Results for German Capital Market

Literature

Dus/Maurer/Mitchell (2004): „Betting on Death and Capital Markets in Retirement:A Shortfall Risk Analysis of Life Annuities versus Phased Withdrawal Plans", Working Paper, Pension Research Council, 1-2004.