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Kinetic Theory and Simulation of Nonlinear Magnetic Structures

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Outline

- Introduction – Nonlinearity and Relaxation in collisionless plasmas
- Kinetic Model – Vlasov and Gyrokinetic Vlasov
- Discrete Formalism – Particle-in-Cell Approach
- Low Noise Method – Delta-F Technique
- Kinetic Simulations I – Linear and Nonlinear Landau Damping
- Kinetic Simulations II – Nonlinear Magnetic Structures
- Summary and Work in Progress

Introduction

-plasmas are a unique medium in that it has a nonlinearity associated with particle trapping

-this has important consequences on the dynamics and transport properties as well as the relaxation toward a new equilibrium

-In this talk, an initial value nonlinear kinetic simulation approach based on the particle-in-cell method, is described to address questions regarding relaxation processes in collisionless plasmas

Kinetic Model – Vlasov Maxwell

Vlasov equation in continuity form

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial}{\partial z} \cdot (\dot{z} f) = 0$$

where $z = (\vec{x}, \vec{v})$ and $\dot{z} = (\vec{v}, (q/m)[\vec{E} + \frac{\vec{v} \times \vec{B}}{c}])$

Maxwell equations

$$\nabla \times E^T = -\frac{1}{c}\frac{\partial B^T}{\partial t}$$

$$\nabla \times B^T = \frac{1}{c}\frac{\partial E^T}{\partial t} + \frac{4\pi}{c} J^T$$

$$\nabla \cdot E^L = 4\pi\rho$$

where $\rho = \int q f d^3 v$ and $J = \int q f v d^3 v$

Finite-Sized Particle-in-Cell

Equations of motion

$$\dot{z}_i = (\vec{v}, (q/m) \int d^n x S(x - x_i)[\vec{E} + \frac{\vec{v}\times\vec{B}}{c}])$$

Maxwell equations

$$\nabla \times E^T = -\frac{1}{c}\frac{\partial B^T}{\partial t}$$

$$\nabla \times B^T = \frac{1}{c}\frac{\partial E^T}{\partial t} + \frac{4\pi}{c} J^T$$

$$\nabla \cdot E^L = 4\pi\rho$$

where

$$\rho = \sum_{i=1}^{N} q_i S(x - x_i) \text{ and } J = \sum_{i=1}^{N} q_i v_i S(x - x_i)$$

References:

Birdsall, Langdon,'85
Hockney, Eastwood, '88
Dawson, '83

Gyrokinetic Vlasov: Ordering and Equations

$$\frac{\omega}{\Omega_i} \sim \frac{\rho_i}{L} \sim \frac{k_\parallel}{k_\perp} \sim \frac{e\phi}{T_e} \sim \frac{\delta B}{B} \sim O(\epsilon)$$
$$k_\perp \rho_i \sim O(1)$$

Electrons – drift kinetic

$$\boxed{\frac{\partial f_e}{\partial t} + (v_\parallel \hat{b}^* + \frac{\hat{b}\times\nabla\phi}{B_o}) \cdot \nabla f_e - \frac{e}{m_e}(-\hat{b}^* \cdot \nabla\phi - \frac{\partial A_z}{\partial t})\frac{\partial f_e}{\partial v_\parallel} = 0}$$

Ions - gyrokinetic

$$\boxed{\begin{aligned}\frac{\partial f_i}{\partial t} + (v_\parallel \hat{b}^* + \frac{\hat{b}\times\nabla(J_o\phi)}{B_o}) \cdot \nabla f_i& \\ + \frac{e}{m_i}(-\hat{b}^* \cdot \nabla(J_o\phi) - \frac{\partial(J_o A_z)}{\partial t})\frac{\partial f_i}{\partial v_\parallel} &= 0\end{aligned}}$$

$$\hat{b}^* = \hat{b} + \frac{\nabla A_z \times \hat{b}}{B_o}$$

$$J_o\phi = < \int \phi(r)\delta(r - R - \rho)dr >$$

Gyrokinetic Particle Simulation

Time integrate using characteristics of gyrokinetic-Vlasov equation

$$F = \Sigma_i^N \delta(\mathbf{R} - \mathbf{R_i})\delta(\mathbf{v}_{||} - \mathbf{v}_{||\mathbf{i}})\delta(\mu - \mu_\mathbf{i})$$

$$\mu_i = \frac{v_{i\perp}}{2B}$$

Gyrokinetic Poisson Equation

$$\boxed{\frac{T_e}{T_i \lambda_{De}^2}(1 - \Gamma_o)\phi = -4\pi e(n_e - < n_i >)}$$

$$< n_i > = \int 2\pi v_\perp dv_\perp dv_{||} J_o F_i$$

$$\Gamma_o = I_o(k_\perp^2 \rho_i^2) e^{-k_\perp^2 \rho_i^2}$$

Ampere's Equation

$$\boxed{\nabla_\perp^2 A_z = -\frac{4\pi}{c} \int v_{||} F_e dv_{||}}$$

Normal Mode – Kinetic Shear Alfven Wave

$$\omega^2 = \frac{k_{||}^2 V_A^2}{1 + k_\perp^2 \frac{c^2}{\omega_{pe}{}^2}} \left[\frac{k_\perp^2 \rho_i^2}{1 - \Gamma_o} + k_\perp^2 \rho_s^2\right]$$

Gyrokinetic Magnetostatic Model

Introducing a canonical momentum: $p_z = v_z + \frac{q}{m} A_z(\mathbf{R}, t)$

and generalized potential: $\Psi(\mathbf{R}, t) = \phi(\mathbf{R}, t) - v_z A_z(\mathbf{R}, t)$

Equations of motion

$$\frac{d\mathbf{R}_j}{dt} = v_{z_j}\hat{b} - \frac{c}{B}\left(\frac{\partial \Psi}{\partial \mathbf{R}} \times \hat{b}\right)_j$$

$$\frac{dp_{z_j}}{dt} = -\hat{b} \cdot \left(\frac{q}{m}\frac{\partial \Psi}{\partial \mathbf{R}}\right)_j$$

Ampere's Equation

$$\left(\nabla_\perp^2 - \frac{\omega_{pe}^2}{c^2}\right) A_z = -4\pi|e| \sum_j p_{z_j} S(\mathbf{R} - \mathbf{R_j}) + \frac{\omega_{pe}^2}{c^2} A_z\left(\frac{n_e}{n_o} - 1\right)$$

Gyrokinetic Poisson Equation

$$\frac{T_e}{T_i \lambda_{De}^2}(1 - \Gamma_o)\phi = -4\pi e(n_e - < n_i >)$$

(Refs. Hahm et al, '88, Naito et al. '95, Sydora, '01, Phys. Plasmas)

Low Noise Method

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial}{\partial z} \cdot (\dot{z} f) = 0$$

Splitting the distribution

$$f(z,t) = f_o(z) + \delta f(z,t) \text{ gives}$$

$$\frac{d\delta f}{dt} = -\frac{df_o}{dt}$$

Equilibrium condition

$$\dot{z}_o \cdot \frac{\partial f_o(z)}{\partial z} = 0$$

Evolution equation for δf

$$\frac{\partial \delta f}{\partial t} + \frac{\partial}{\partial z} \cdot (\dot{z} \delta f) = -\dot{z}_1 \cdot \frac{\partial f_o}{\partial z}$$

(Refs. Kotschenreuther, ’88, Dimits,Lee, ’93, Sydora, ’93, Parker,Lee,’93, Hu, ’94)

Delta-F Method

Representation of δf

$$\delta f(z,t) = \sum_i w_i \delta(z - z_i)$$

where the particle weight is

$$w_i = \frac{\delta f}{g}$$

and g is an arbitrary 'marker' distribution

$$g(z,t) = \sum_i \delta(z - z_i)$$

Delta-F Evolution Equation

δf evolution equation

$$\frac{\partial \delta f}{\partial t} + \frac{\partial}{\partial z} \cdot (\dot{z} \delta f) = -\dot{z}_1 \cdot \frac{\partial f_o}{\partial z}$$

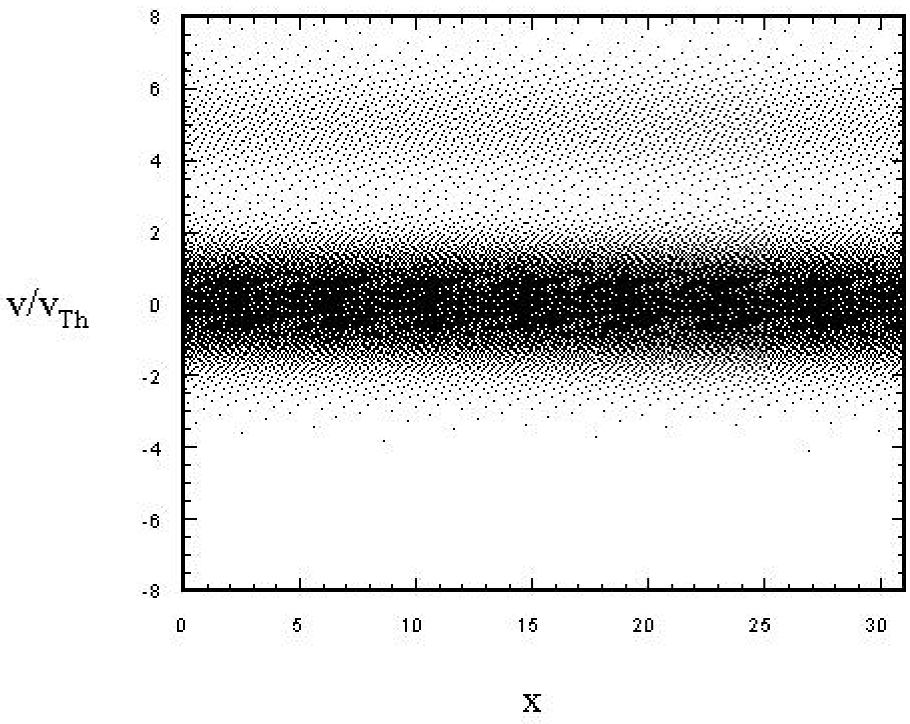
becomes

$$\frac{dw_i}{dt} = -\dot{z}_1 \cdot \frac{1}{g(z,t)} \frac{\partial f_o}{\partial z}|_z$$

and since both f and g satisfy df/dt=0 and dg/dt=0

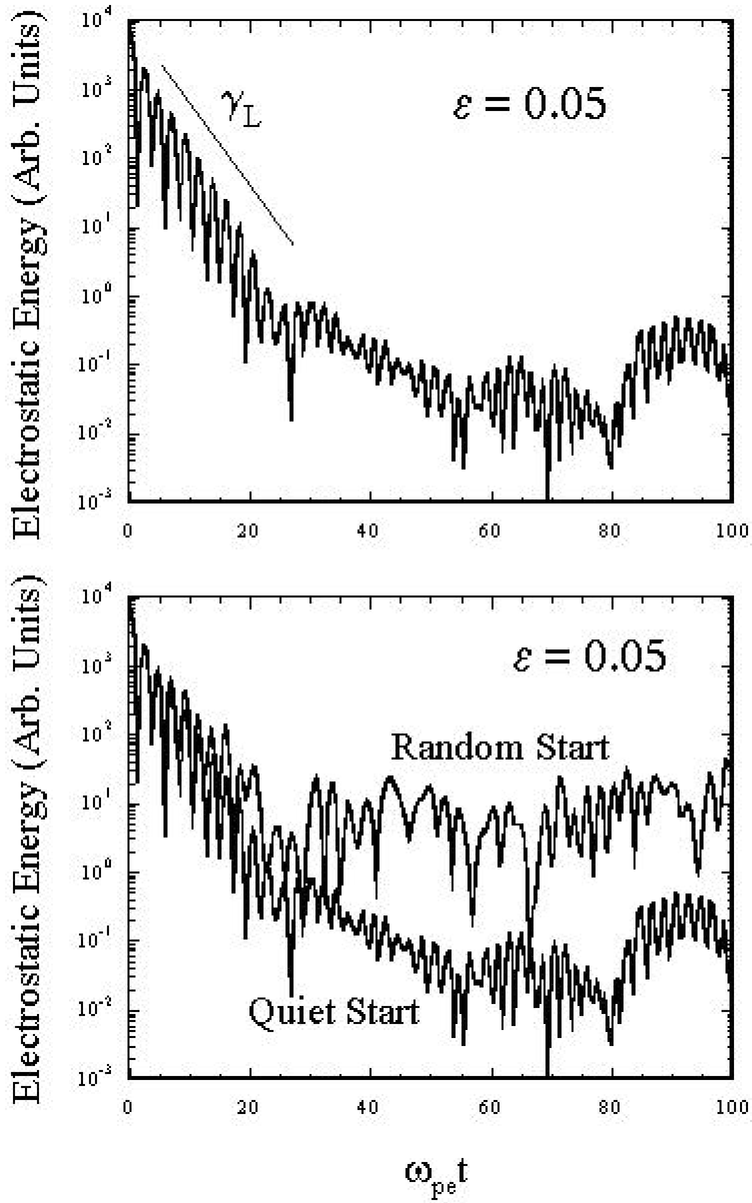
$$\frac{dw_i}{dt} = -(\frac{f(0)}{g(0)} - w_i)\dot{z}_1 \cdot \frac{1}{f_o(z,t)} \frac{\partial f_o}{\partial z}|_z$$

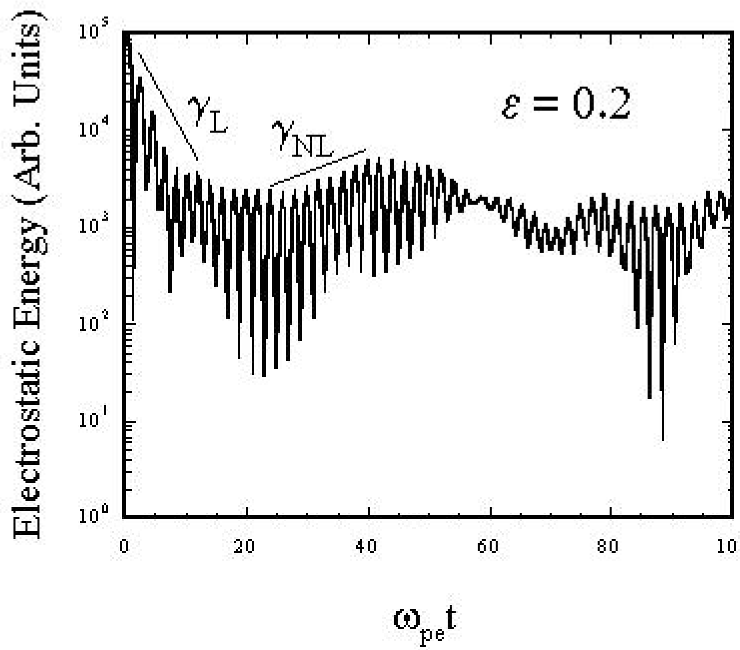
Kinetic Simulations I – Linear and Nonlinear Landau Damping



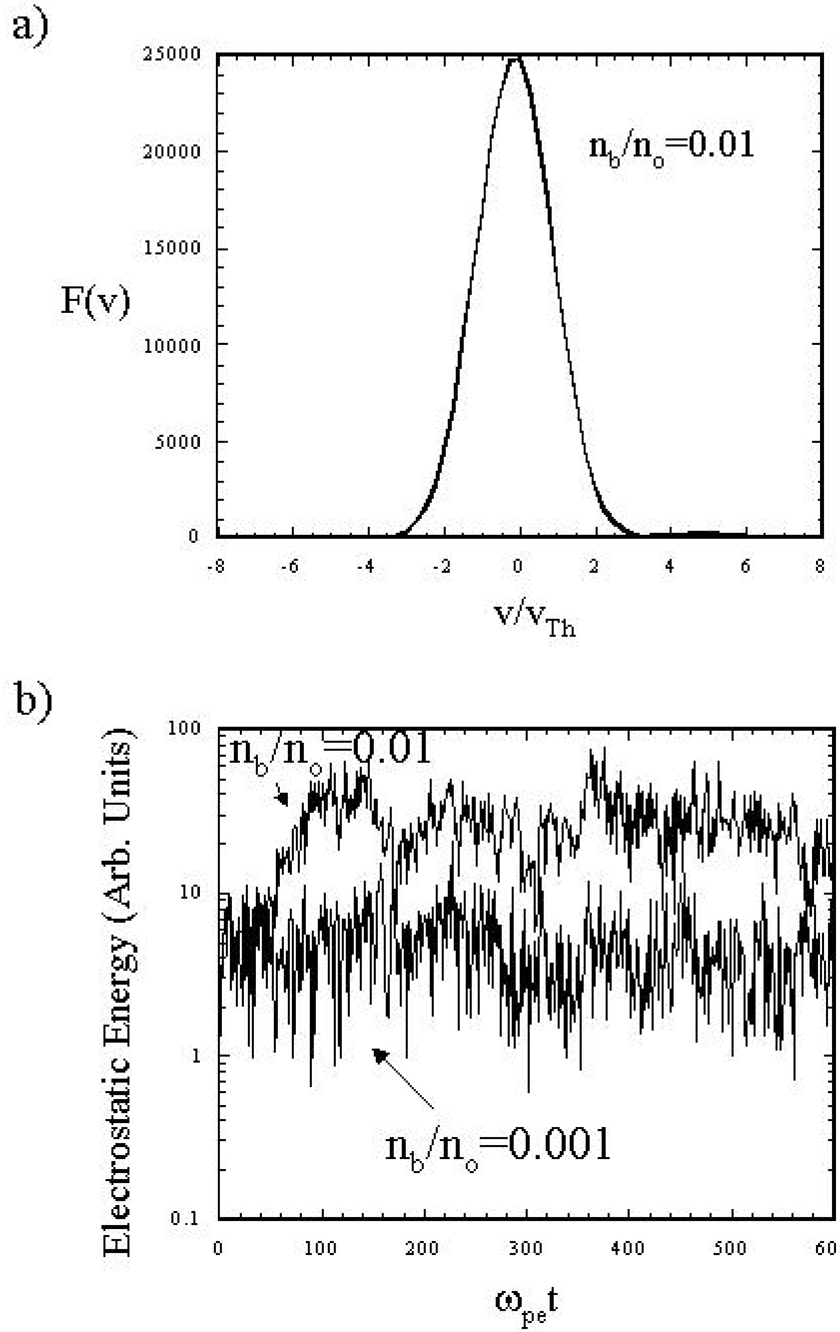
Bit-reversed quasi-random sequence phase space loading

$$\delta f = (1 + \epsilon cos(kx))f_o$$

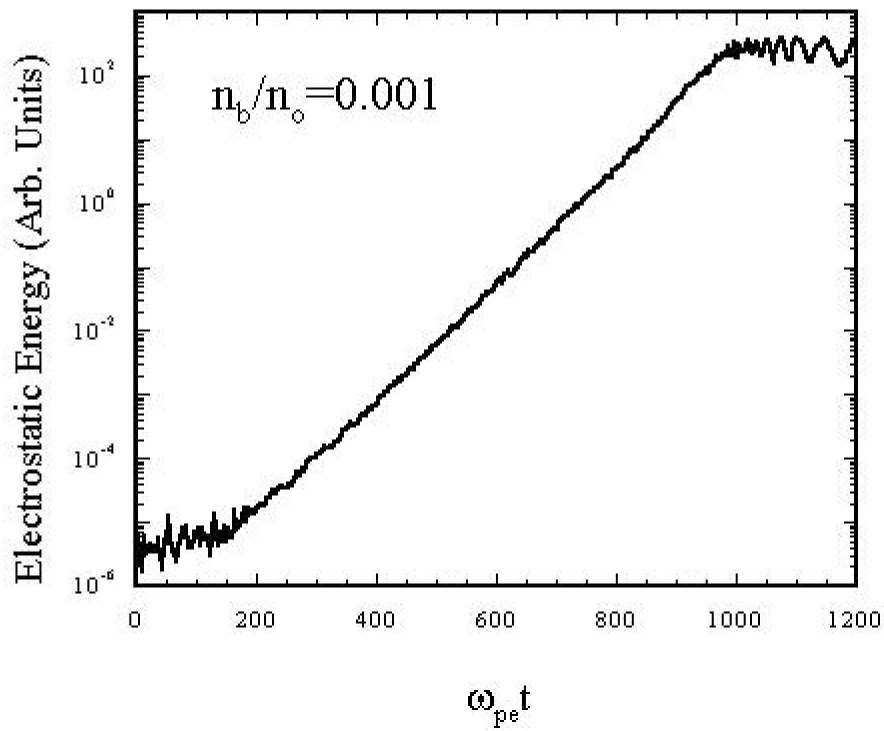



Electrostatic Energy (Arb. Units)
$\varepsilon = 0.2$
γ_L
γ_{NL}
$\omega_{pe}t$
0
20
40
60
80
100
10^0
10^1
10^2
10^3
10^4
10^5

a)

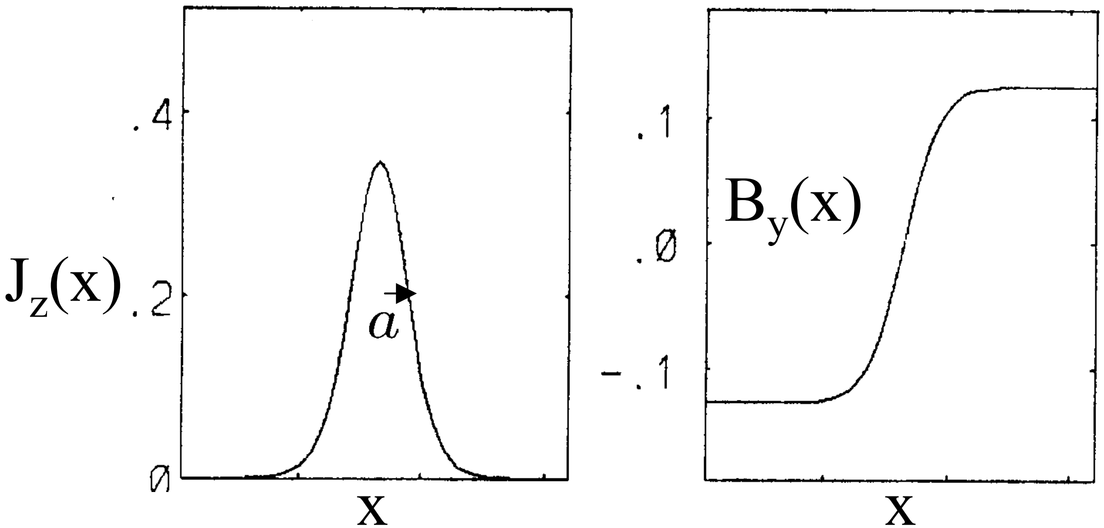


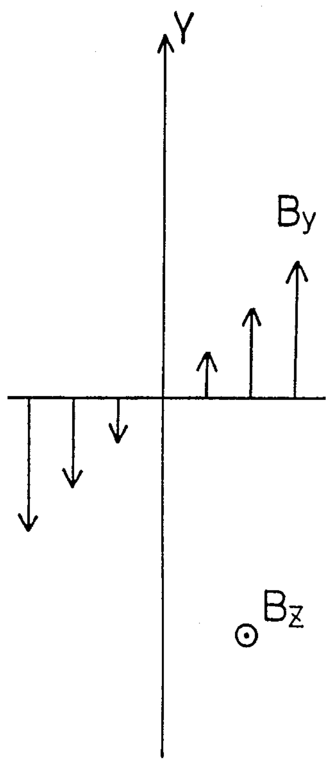
F(v)
25000
20000
15000
10000
5000
0
-8
-6
-4
-2
0
2
4
6
8
v/v_{Th}
n_b/n_o=0.01
b)
Electrostatic Energy (Arb. Units)
100
10
1
0.1
0
100
200
300
400
500
600
$\omega_{pe}t$
n_b/n_o=0.01
n_b/n_o=0.001



n_b/n_o=0.001
Electrostatic Energy (Arb. Units)
10^{2}
10^{0}
10^{-2}
10^{-4}
10^{-6}
0
200
400
600
800
1000
1200
$\omega_{pe}t$

Kinetic Simulations II – Nonlinear Magnetic Structures



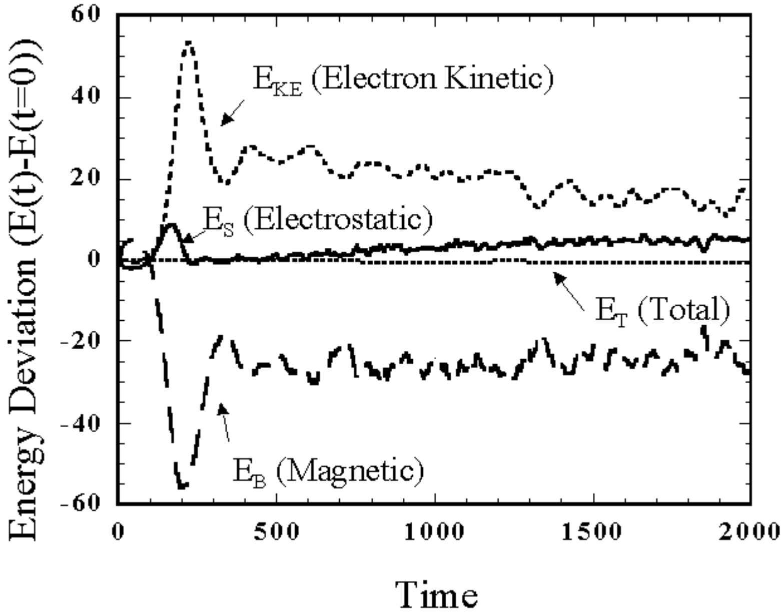


Parameters:

$$T_e/T_i = 1, m_i/m_e = 1837, \rho_i = 1 - 4c/\omega_{pe}$$
$$L_x = 30 - 60c/\omega_{pe}, N_o = 100 - 200 particles/cell$$

$$\Delta' = -2k_y + \frac{2}{k_y a^2}$$

Energy Time Evolution

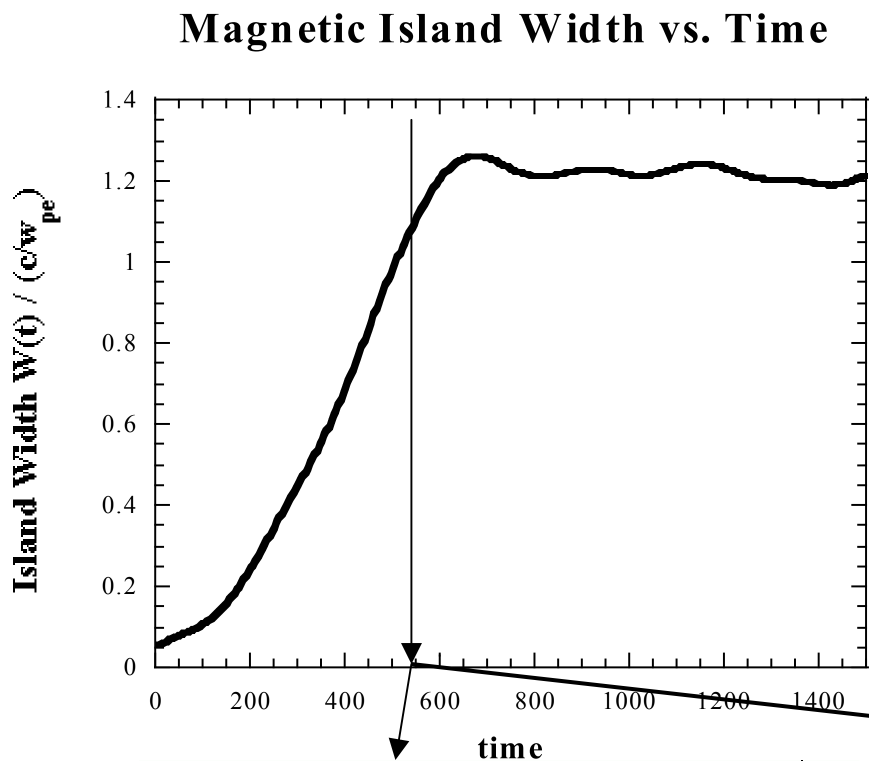


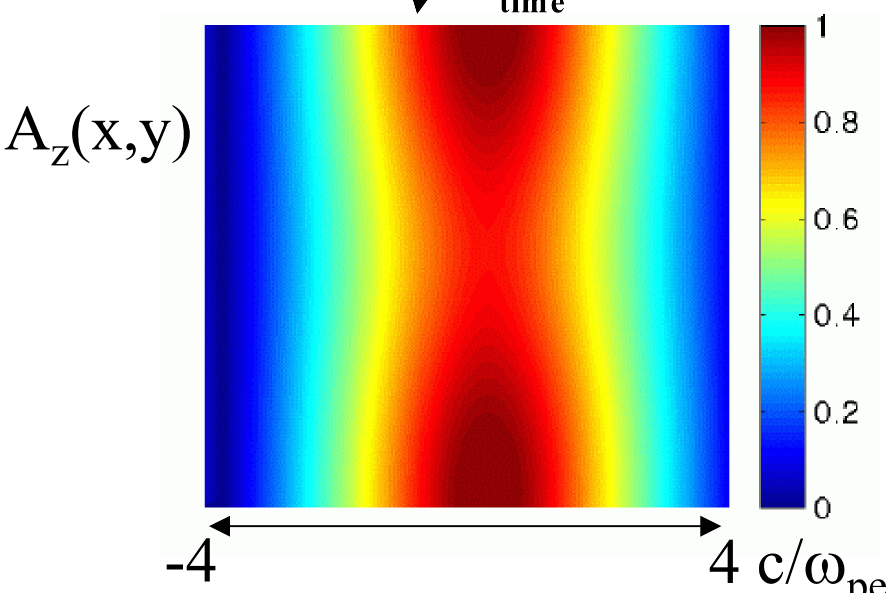
$$\gamma \simeq (\frac{kv_{te}}{L_s})(\frac{1}{\sqrt{\pi}})(\frac{c}{\omega_{pe}})^2\Delta'$$

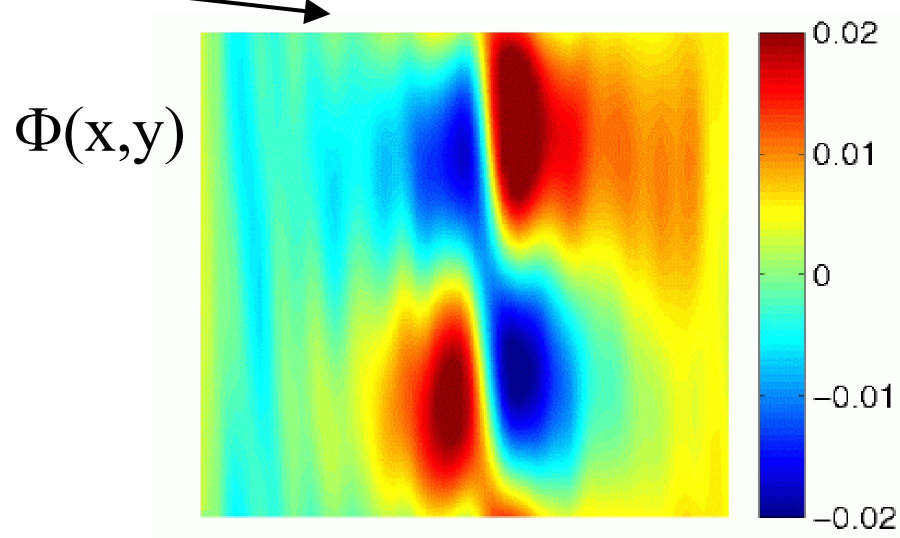
Magnetic Island Evolution

$$W = 2(\tilde{A}_z L_s / B_o)^{1/2}$$

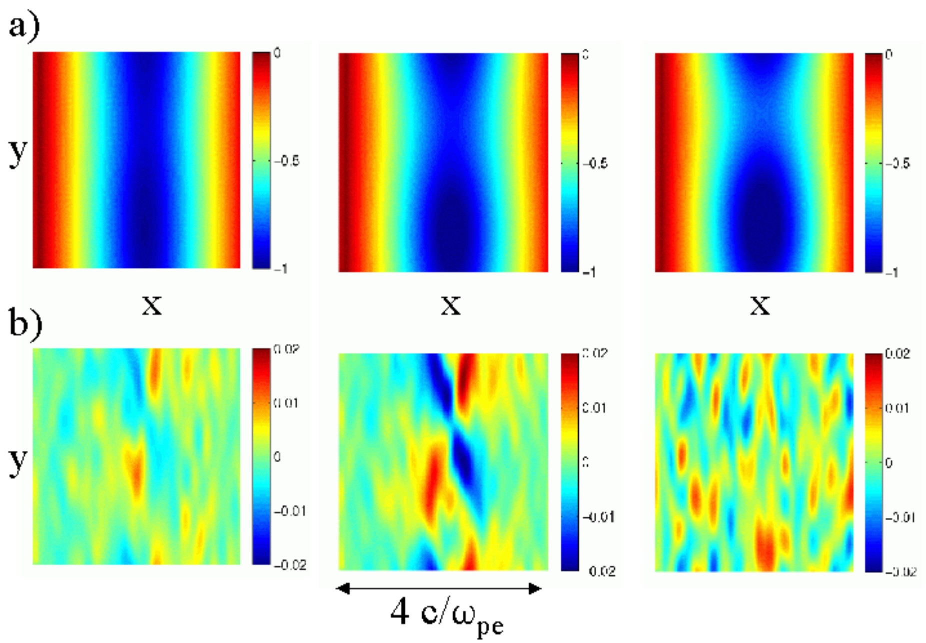
$$L_s^{-1} = (\partial B_y / \partial x)/B_y$$

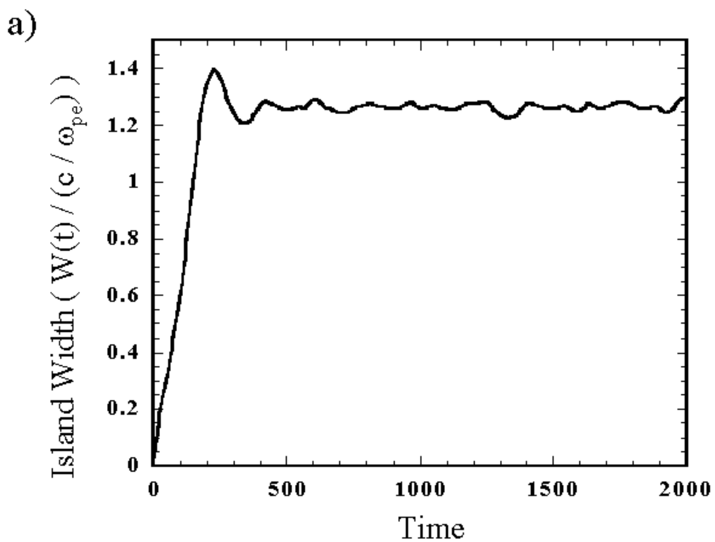




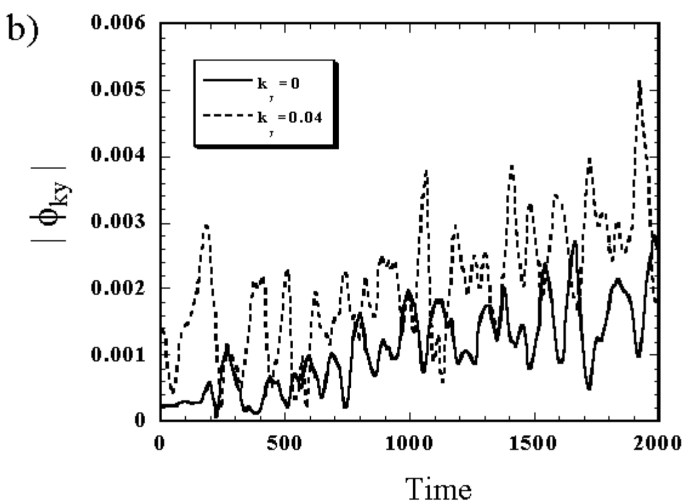


Magnetic Flux and Electrostatic Potential





Magnetic Island Width Evolution



Electrostatic Mode Evolution

$$\phi(x,y) = \Sigma_{k_y}\phi_{k_y}(x)e^{ik_y y} = \phi_{k_y=0}(x) + \Sigma_{k_y\neq 0}\phi_{k_y}(x)e^{ik_y y}$$

$$\updownarrow$$

$$<\phi>(x)$$

Shear Flow Development

-Radial (x-direction) electrostatic fields build up due to:

-ion polarization currents

-magnetic island

-Pure E_x gives rise to a y-direction flow

$$E_x = -\frac{\partial <\phi>}{\partial x}$$

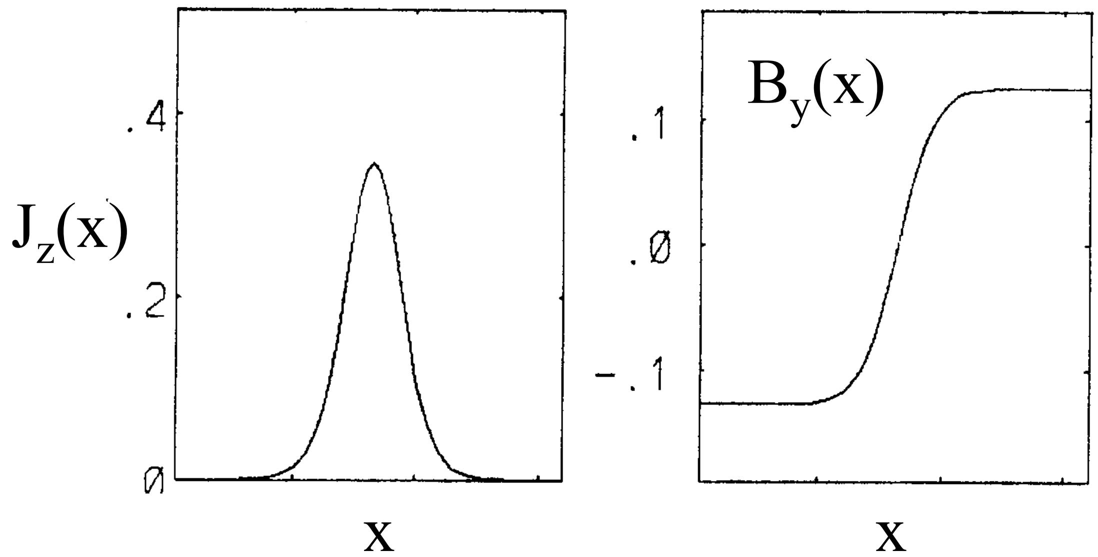
$$V_{E\times B} \sim E_x / B_o$$

-shear flow when E_x non-uniform

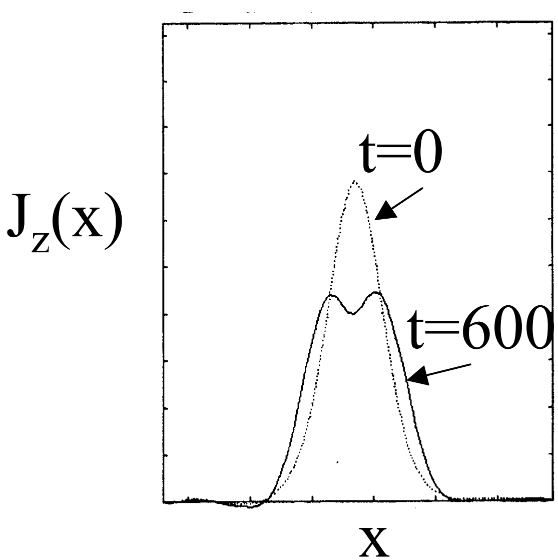
$$\frac{dV_{E\times B}}{dx}$$

which can be unstable (Kelvin-Helmholtz)

Initial Current and B-field Profiles



Current Profile Evolution



Summary and Work in Progress

-Particle-in-cell method has been described for both the Vlasov and gyrokinetic Vlasov system

-Issues related to the implementation of a numerical algorithm for this method discussed for the magnetostatic, low-beta collisionless plasmas

-Results presented for a low noise formulation based on the discrete representation of the perturbed distribution function

-Work in progress related to

→long time dynamics of marker distribution

→Connection with quasilinear theory