A Landau fluid model for dispersive MHD

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Workshop on Kinetic Theory, The Fields Institute, March 2004

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I. Introduction

In natural and fusion plasmas, collisions usually negligible.

The description of the intermediate-scale dynamics in terms of usual magnetohydrodynamics is thus questionable.

In most situations, direct numerical integration of the Vlasov-Maxwell equations are beyond the capabilities of the present day computers.

This suggests the development of a reduced description that retains most of the aspects of a fluid description but includes realistic approximations of the pressure tensor and wave-particle resonances.

This model should be simple enough to allow numerical simulation of 3D dispersive MHD turbulence with realistic dissipation. The approximations must however remain controlled to allow a precise prediction of the nonlinear dynamics leading to coherent structures such as magnetic holes or shocklets.

The media under consideration are the magnetosheath, the solar wind as well as the warm phase of the ISM.

II. Previous works:

- Closure can only be rigorously justified in presence of collisions, with an expansion in the Knudsen number.

- "N-moments methods", that derive from Grad's work (1958), assume that the distribution function (d.f.) remains close to an equilibrium d.f., the deviation being expanded in a polynomial in terms of particle velocities. Good for the weakly collisional case but limited to small deviations.

- In the collisionless case, fluid behavior can only result from collective constraints. In presence of a strong magnetic field, Chew et al (1956) proposed the "double adiabatic laws" or CGL equations for the parallel and perpendicular gyrotropic pressure components. The equations are usually closed in the adiabatic regime ($\nabla \cdot \mathbf{Q} = 0$). Conditions of validity analyzed by Belmont and Rezeau (1987).

- Closures that reproduce linear results from kinetic theory: depend on ground state and often presented in Fourier space. Effective polytropic indices [Belmont and Mazelle (1992)].

- Gyrofluids (used for fusion plasmas): equations for hydrodynamic moments obtained from gyrokinetic equations. Able to describe FLR effects (i.e. valid in a range of scales extending up to the ion gyroradius and beyond), but written in a local coordinate system and rather complicated.

- Landau fluids [Hammett and co-authors (1990s)]: built to account for wave-particle resonance effects. Full electromagnetic case presented by Snyder, Hammett and Dorland (1997). In its original form, limited to the very large scales (Hall effect and finite Larmor radius corrections neglected).

- P. and Sulem (2003) have revisited Landau fluids and benchmarked these equations for parallel Alfvén waves and magneto-sonic waves in the case where the Hall term and finite Larmor radius corrections are relevant.

III. Outline of the method:

- Goal: Simple monofluid model able to reproduce the weakly nonlinear dynamics of MHD (magnetosonic and Alfvén) waves, whatever their propagation direction and thus in particular the dynamics of kinetic Alfvén waves (KAW) with $k\rho_L \leq 1$, with the most relevant kinetic effects, i.e. Landau damping and FLR corrections, and a generalized Ohm's law.

- Starting point: Vlasov-Maxwell (VM) equations

- Small parameter: ratio between the ion Larmor radius and the typical (smallest) wavelength. The fields amplitudes are also supposed to be small.

- Main problem: Exact hydrodynamic equations are obtained by taking moments of the VM equations. The hierarchy must however be closed and the main work resides in a proper determination of the pressure tensor.

- Assumptions: Simple geometry (no curvature drift), homogeneous basic state with bi-Maxwellian distribution functions (could be relaxed for some aspects of the problem).

IV. The equations:

• From Vlasov-Maxwell equations, derive a hierarchy of moment equations for each particle species r:

density $\rho_r = m_r n_r \int f_r d^3v$

hydrodynamic velocity $u_r = \dfrac{\int v f_r d^3v}{\int f_r d^3v}$

pressure tensor $P_r = m_r n_r \int (v - u_r) \otimes (v - u_r) f_r d^3v$

heat flux tensor $Q_r = m_r n_r \int (v - u_r) \otimes (v - u_r) \otimes (v - u_r) f_r d^3v$.

$$\partial_t \rho_r + \nabla \cdot (u_r \rho_r) = 0$$

$$\partial_t u_r + u_r \cdot \nabla u_r + \frac{1}{\rho_r} \nabla \cdot P_r - \frac{q_r}{m_r}(e + \frac{1}{c} u_r \times b) = 0$$

$$\partial_t P_r + \nabla \cdot (u_r P_r + Q_r) + [P_r \cdot \nabla u_r + \frac{q_r}{m_r c} b \times P_r]^{\mathcal{S}} = 0$$

where $[A]^{\mathcal{S}} = A + A^{\mathrm{Tr}}$

A monofluid approximation is possible when the dynamics is weakly nonlinear:

$$\partial_t \rho + \nabla \cdot (u\rho) = 0$$

$$\partial_t(\rho u) + \nabla \cdot (\rho u \otimes u) + \nabla \cdot \mathbf{p} - \frac{1}{c} j \times b = 0$$

where $u = \frac{1}{\rho}\sum_r \rho_r u_r$, $\rho = \sum_r \rho_r$ and $j = \frac{c}{4\pi}\nabla \times b$.
On has $\mathbf{p} = \sum_r \mathbf{p}_r$ where the pressure tensor $\mathbf{p}_r$ (and also $\mathbf{q_r}$) is defined in terms of the deviation from the barycentric velocity. $[\mathbf{p}_r = \mathbf{P}_r - \rho_r(u - u_r) \otimes (u - u_r)]$.
Up to subdominant terms one has:

$$\partial_t \mathbf{p}_r + \nabla \cdot (u\mathbf{p}_r + \mathbf{q}_r) + [\mathbf{p}_r \cdot \nabla u + \frac{q_r}{m_r c} b \times \mathbf{p}_r]^{\mathcal{S}} = 0$$

Induction equation with Hall-effect and electron pressure:

$$\partial_t b - \nabla \times (u \times b) = -\frac{m_i c}{q_i} \nabla \times [\frac{1}{4\pi\rho}(\nabla \times b) \times b - \frac{1}{\rho}\nabla \cdot \mathbf{p}_e].$$

Two problems:

(a) Heat fluxes require a CLOSURE APPROXIMATION.

(b) Small timescales in the equation for $\mathbf{P}$. $\longrightarrow$ separation between CGL equations with flux and FLR corrections

- (a) Closure

Reductive perturbative expansion on the Vlasov-Maxwell equations associated with the various types of MHD waves provides ASYMPTOTICALLY EXACT (possibly nonlocal) RELATIONS BETWEEN THE HEAT FLUXES AND LOWER ORDER MOMENTS, FROM WHICH WE INFER GENERAL CLOSURE ASSUMPTIONS. (*Rogister (Phys. Fluids* **14**, *2733 (1971))*)

Advantages

Relative simplicity: allows to neglect non relevant terms. In contrast with usual linearization it isolates the dynamics of individual waves (small amplitude with a typical wave length much larger than the ion inertial length).

Rigor: expansion in terms of a single small parameter.

Bonus: allows to test the equations in the weakly nonlinear regime.

This method allows to obtain relations between heat fluxes and lower order moments.

Which wave does one have to consider?

It turns out that oblique Alfvén waves are the worst case scenario. This is because in this case the distinguished limit imposes a scaling where the Hall term and non-gyrotropic heat flux components enter at dominant order.

The equations obtained in this case are also valid for magnetosonic waves and parallel Alfvén waves. Some terms become subdominant in these situations.

The reductive perturbative expansion allows to only keep those subdominant terms that play a role in the context of oblique Alfvén waves.

In the following we shall present the ideas on the simpler case of parallel Alfvén waves and briefly discuss the additional terms arising for oblique AW.

Scaling argument (within a reductive perturbative expansion):

When the propagation coordinate is rescaled by $\epsilon^{1/2}$, the distinguished limit associated with each type of wave is obtained as follows:

$\star$ Parallel Alfvén waves: $b_x \sim b_y = O(\epsilon^{1/4})$ and $b_z - B_0 = O(\epsilon^{1/2})$

$\star$ Oblique magnetosonic waves: $b_x \sim b_z - B_0 = O(\epsilon)$ and $b_y = O(\epsilon^{3/2})$,

$\star$ Oblique Alfvén waves: $b_x \sim b_z - B_0 = O(\epsilon)$ and $b_y = O(\epsilon^{1/2})$.

- From CGL eqs., gyrotropic heat fluxes comparable to pressure perturbations:
  - $\star$ of order $\epsilon^{1/2}$ for parallel Alfvén waves
  - $\star$ of order ϵ for oblique Alfvén and magnetosonic waves.

- Non-gyrotropic heat fluxes involve a space derivative arising with the $1/\Omega_r$ factor, and thus typically behave not like $[p_r^{(0)} v]$ but rather like $[p_r^{(0)} \frac{v_A}{\Omega_r} \nabla v]$.

Modeling the heat fluxes:

The gyrotropic and non-gyrotropic contributions to the heat fluxes $\mathbf{q}_r$ are separated by writing $\mathbf{q}_r = \mathbf{q}_r^G + \mathbf{q}_r^{NG}$ with

$$q^G_{ijk,r} = q_{\parallel r}\widehat{b}_i\widehat{b}_j\widehat{b}_k + q_{\perp r}(\delta_{ij}\widehat{b}_k + \delta_{ik}\widehat{b}_j + \delta_{jk}\widehat{b}_i - 3\widehat{b}_i\widehat{b}_j\widehat{b}_k),$$

In the long-wave asymptotics

$$\frac{q^{(1)}_{\parallel r}}{v_{th,r}p^{(0)}_{\parallel r}} = \frac{c_r(c_r^2 - 3 + \mathcal{W}_r^{-1})}{c_r^2 - 1 + \mathcal{W}_r^{-1}}\frac{T^{(1)}_{\parallel r}}{T^{(0)}_{\parallel r}}$$

$$\frac{q^{(1)}_{\perp r}}{v_{th,r}p^{(0)}_{\perp r}} = -\frac{T^{(0)}_{\perp r}}{T^{(0)}_{\parallel r}}\frac{c_r\mathcal{W}_r}{1 - \frac{T^{(0)}_{\perp r}}{T^{(0)}_{\parallel r}}\mathcal{W}_r}\frac{T^{(1)}_{\perp r}}{T^{(0)}_{\perp r}} = -\frac{T^{(0)}_{\perp r}}{T^{(0)}_{\parallel r}}c_r\mathcal{W}_r|b^{(1)}|.$$

The plasma response function writes

$$\mathcal{W}_r \equiv \mathcal{W}(c_r) = \frac{1}{\sqrt{2\pi}}\mathrm{P}\int\frac{\zeta e^{-\zeta^2/2}}{\zeta - c_r}d\zeta + \sqrt{\frac{\pi}{2}}c_r e^{-c_r^2/2}\mathcal{H},$$

where $c_r = \lambda/v_{th,r}$, $v_{th,r} = \sqrt{T^{(0)}_{\parallel r}/m_r}$ and $\mathcal{H}$ is the Hilbert transform.

Extension to more general situations:

- c_r replaced by $-\frac{1}{v_{th,r}}\partial_t\partial_z^{-1}$.
- equations for the heat fluxes replaced by

$$
\begin{aligned}
\frac{q^{(1)}_{\|r}}{v_{th,r}p^{(0)}_{\|r}} &= \mathcal{F}_{\|}\left(-\frac{1}{v_{th,r}}\partial_t\partial_z^{-1}\right)\frac{T^{(1)}_{\|r}}{T^{(0)}_{\|r}} \\
\frac{q^{(1)}_{\perp r}}{v_{th,r}p^{(0)}_{\perp r}} &= \mathcal{F}^1_{\perp}\left(-\frac{1}{v_{th,r}}\partial_t\partial_z^{-1}\right)\frac{T^{(1)}_{\perp r}}{T^{(0)}_{\perp r}} + \mathcal{F}^2_{\perp}\left(-\frac{1}{v_{th,r}}\partial_t\partial_z^{-1}\right)|b^{(1)}|
\end{aligned}
$$

with HOMOGRAPHIC APPROXIMANTS (to finally get a 1st order initial value pb)

$$
\begin{aligned}
\mathcal{F}_{\|}(X) &= (q^3_{\|} + q^4_{\|}X\mathcal{H})^{-1}(q^1_{\|}X + q^2_{\|}\mathcal{H}) \\
\mathcal{F}^1_{\perp}(X) &= (q^3_{\perp} + q^4_{\perp}X\mathcal{H})^{-1}(q^1_{\perp}X + q^2_{\perp}\mathcal{H}) \\
\mathcal{F}^2_{\perp}(X) &= (q^3_{\perp} + q^4_{\perp}X\mathcal{H})^{-1}(q^5_{\perp}X + q^6_{\perp}\mathcal{H}).
\end{aligned}
$$

The coefficients $q^i_{\|}$ and $q^i_{\perp}$ are chosen in a way that ensures the correct asymptotic behavior of the heat fluxes in both isothermal ($c_r \ll 1$, $\mathcal{W}_r \approx 1 - c_r^2 + \sqrt{\frac{\pi}{2}}c_r\mathcal{H}_\xi$, $q^{(1)}_{\|r} = -\sqrt{\frac{8}{\pi}}v_{th,r}n^{(0)}\mathcal{H}_\xi T^{(1)}_{\|r}$, $q^{(1)}_{\perp r} \ll 1$) and adiabatic ($c_r \gg 1$, $\mathcal{W}_r \approx -1/c_r^2 - 3/c_r^4$, heat fluxes are negligible) limits.
This reduces to use appropriate Pade approximants for $\mathcal{W}_r$.

Heat flux closure

$$
\left(\frac{d}{dt} + \frac{v_{th,r}}{\sqrt{\frac{8}{\pi}}(1 - \frac{3\pi}{8})}\mathcal{H}\nabla_{\|}\right)\frac{q_{\|r}}{v_{th,r}p_{\|}^{(0)}} = \frac{1}{1 - \frac{3\pi}{8}} v_{th,r}\nabla_{\|}\frac{T_{\|r}}{T_{\|r}^{(0)}}
$$

$$
\left(\frac{d}{dt} - \sqrt{\frac{\pi}{2}} v_{th,r}\mathcal{H}\nabla_{\|}\right)\frac{q_{\perp r}}{v_{th,r}p_{\perp r}^{(0)}} = -v_{th,r}\nabla_{\|}\left(\frac{T_{\perp r}}{T_{\perp r}^{(0)}} + \left(\frac{T_{\perp r}^{(0)}}{T_{\|r}^{(0)}} - 1\right)\frac{|b|}{B_0}\right),
$$

with $p_{\|r} = nT_{\|r}$ and $p_{\perp r} = nT_{\perp r}$.

These equations can also be viewed as a linearized version of the exact equation for the heat fluxes with source terms arising from an appropriate closure on the next moment (r). See Snyder, Hammett and Dorland (1997).

For Oblique AW, these equations are modified

One has in general:

$$(\frac{d}{dt} + \frac{v_{th,r}}{\sqrt{\frac{8}{\pi}}(1-\frac{3\pi}{8})}\mathcal{H}\nabla_{\parallel})\frac{q'_{\parallel r}}{v_{th,r}p^{(0)}_{\parallel r}} = \frac{1}{1-\frac{3\pi}{8}}v_{th,r}\nabla_{\parallel}\frac{T^{(1)}_{\parallel r}}{T^{(0)}_{\parallel r}}$$

with

$$\frac{q'_{\parallel r}}{v_{th,r}p^{(0)}_{\parallel r}} = \frac{q_{\parallel r}}{v_{th,r}p^{(0)}_{\parallel r}} - 3[(1+\frac{v^2_{\Delta e}+v^2_{\Delta p}}{v^2_A})(\frac{\Omega_p}{\Omega_r}-1) + \frac{v^2_{\Delta p}-v^2_{\Delta r}}{v^2_A}]\frac{j_{\parallel}}{nqv_{th,r}}$$

and

$$\left(\frac{d}{dt} - \sqrt{\frac{\pi}{2}}v_{th,r}\mathcal{H}\nabla_{\parallel}\right)\frac{q'_{\perp r}}{v_{th,r}p^{(0)}_{\perp r}} = v_{th,r}\nabla_{\parallel}\left((1-\frac{T^{(0)}_{\perp r}}{T^{(0)}_{\parallel r}})\frac{|b|}{B_0} - \frac{T_{\perp r}}{T^{(0)}_{\perp r}} + \sqrt{\frac{\pi}{2}}\frac{v^2_{th,r}}{v^2_A}\frac{\Omega_p}{\Omega_r}\mathcal{H}\frac{j_{\parallel}}{nqv_{th,r}}\right),$$

with

$$\frac{q'_{\perp r}}{v_{th,r}p^{(0)}_{\perp r}} = \frac{q_{\perp r}}{v_{th,r}p^{(0)}_{\perp r}} - [(1+\frac{v^2_{\Delta e}+v^2_{\Delta p}}{v^2_A})(\frac{\Omega_p}{\Omega_r}-1) + \frac{v^2_{\Delta p}}{v^2_A} + \frac{v^2_{th,r}-2v^2_{\Delta r}}{v^2_A}\frac{\Omega_p}{\Omega_r}]\frac{j_{\parallel}}{nqv_{th,r}}$$

and $v^2_{\Delta r} = \dfrac{p_{\perp r}-p_{\parallel r}}{\rho^{(0)}}$

• (b) Equations for the pressures

It reads

$$\mathbf{p}_r \times \widehat{b} - \widehat{b} \times \mathbf{p}_r = \mathbf{k}_r$$

where

$$\mathbf{k}_r = \frac{1}{\Omega_r}\frac{B_0}{|b|}[\frac{d\mathbf{p}_r}{dt} + (\nabla \cdot u)\mathbf{p}_r + \nabla \cdot \mathbf{q}_r + (\mathbf{p}_r \cdot \nabla u)^{\mathcal{S}}].$$

This equation involves a fast time scale of order Ω_r^{-1} except for the part of the pressure tensor that lies in the kernel of the operator on the l.h.s. It is spanned by $(\mathbf{I} - \widehat{b} \otimes \widehat{b})$ and $\widehat{b} \otimes \widehat{b}$.

We thus split $\mathbf{p}_r = \mathbf{p}_r^G + \boldsymbol{\pi}_r$ as the sum of an element of the kernel

$$\mathbf{p}_r^G = p_{\perp r}(\mathbf{I} - \widehat{b} \otimes \widehat{b}) + p_{\| r}\widehat{b} \otimes \widehat{b}$$

and of a non-gyrotropic component $\boldsymbol{\pi}_r = \overline{\mathbf{p}}_r$ where, for any (3×3) rank two tensor $\mathbf{a}$,

$$\overline{\mathbf{a}} = \mathbf{a} - \frac{1}{2}\mathbf{a} : (\mathbf{I} - \widehat{b} \otimes \widehat{b})(\mathbf{I} - \widehat{b} \otimes \widehat{b}) - (\mathbf{a} : \widehat{b} \otimes \widehat{b})\widehat{b} \otimes \widehat{b}.$$

Thus $\mathrm{tr}\, \boldsymbol{\pi}_r = 0$ and $\boldsymbol{\pi}_r : \widehat{b} \otimes \widehat{b} = 0$.

Applying the trace operator and the contraction with $\widehat{b} \otimes \widehat{b}$ on both sides of the pression equation gives equations for the gyrotropic pressures

$$\partial_t p_{\perp r} + \nabla \cdot (u\, p_{\perp r}) + p_{\perp r} \nabla \cdot u - p_{\perp r} \widehat{b} \cdot \nabla u \cdot \widehat{b} + \frac{1}{2}(\mathrm{tr}\, \nabla \cdot \mathbf{q}_r - \widehat{b} \cdot (\nabla \cdot \mathbf{q}_r) \cdot \widehat{b})$$

$$+\frac{1}{2}(s_{1r} - s_{2r} + s_{3r}) = 0$$

$$\partial_t p_{\parallel r} + \nabla \cdot (u\, p_{\parallel r}) + 2 p_{\parallel r} \widehat{b} \cdot \nabla u \cdot \widehat{b} + \widehat{b} \cdot (\nabla \cdot \mathbf{q}_r) \cdot \widehat{b} + s_{2r} - s_{3r} = 0.$$

CGL eqs. with heat fluxes and coupling to non-gyrotropic components
$s_{1r} = \mathrm{tr}\, (\boldsymbol{\pi}_r \cdot \nabla u)^S$, $s_{2r} = (\boldsymbol{\pi}_r \cdot \nabla u)^S : \widehat{b} \otimes \widehat{b}$, $s_{3r} = \boldsymbol{\pi}_r : \frac{d}{dt}(\widehat{b} \otimes \widehat{b})$.

For weak perturbations of an equilibrium state with uniform density, gyrotropic pressures and uniform magnetic field, s_{1r}, s_{2r} and s_{3r} are SUBDOMINANT at all the relevant orders of the present analysis.

Both gyrotropic and non-gyrotropic heat flux components a priori contribute to the gyrotropic components of $\nabla \cdot \mathbf{q}_r$.

Finite Larmor radius corrections:

The other part of the pressure satisfies

$$\boldsymbol{\pi}_r \times \widehat{b} - \widehat{b} \times \boldsymbol{\pi}_r = \overline{\mathbf{k}}_r$$

Because of the proximity of $\widehat{b}$ and $\widehat{z}$ (along $\vec{B}_0$), it is convenient to rewrite

$$\boldsymbol{\pi}_r \times \widehat{z} - \widehat{z} \times \boldsymbol{\pi}_r = \mathbf{k}'_r, \qquad \text{where} \quad \mathbf{k}'_r = \overline{\mathbf{k}}_r - (\boldsymbol{\pi}_r \times (\widehat{b} - \widehat{z}) - (\widehat{b} - \widehat{z}) \times \boldsymbol{\pi}_r).$$

We split

$$\mathbf{k}'_r = \overline{\boldsymbol{\kappa}_r} + L(\boldsymbol{\pi}_r) \qquad \text{with} \qquad \overline{\boldsymbol{\kappa}_r} = \frac{1}{\Omega_r}\frac{B_0}{|b|}[\overline{\frac{d\mathbf{p}_r^G}{dt}} + \overline{\nabla \cdot \mathbf{q}_r} + \overline{(\mathbf{p}_r^G \cdot \nabla u)^{\mathcal{S}}}]$$

where

$$\overline{\frac{d\mathbf{p}_r^G}{dt}} = (p_{\|r} - p_{\perp r})\frac{d}{dt}(\widehat{b} \otimes \widehat{b}) = (p_{\|r} - p_{\perp r})\frac{1}{|b|^2}(\frac{db}{dt} \otimes b + b \otimes \frac{db}{dt} - \frac{2}{|b|}\frac{d|b|}{dt} b \otimes b)$$

(computed using the induction equation)

$$L(\boldsymbol{\pi}_r) \quad = \quad \frac{1}{\Omega_r}\frac{B_0}{|b|}[\overline{\frac{d\boldsymbol{\pi}_r}{dt} + (\nabla \cdot u)\boldsymbol{\pi}_r + (\boldsymbol{\pi}_r \cdot \nabla u)^{\mathcal{S}}]} - (\boldsymbol{\pi}_r \times (\widehat{b} - \widehat{z}) - (\widehat{b} - \widehat{z}) \times \boldsymbol{\pi}_r)$$

$$\ll \quad \boldsymbol{\pi}_r \quad \text{in a weakly nonlinear regime.}$$

This enables perturbative calculations.

One expands

$$\begin{aligned}\overline{\boldsymbol{\kappa}_r} &= \boldsymbol{\chi}_r^{(1)} + \boldsymbol{\chi}_r^{(2)} + \cdots \\ \boldsymbol{\pi}_r &= \boldsymbol{\pi}_r^{(1)} + \boldsymbol{\pi}_r^{(2)} + \cdots\end{aligned}$$

that obey

$$\begin{aligned}\boldsymbol{\pi}_r^{(1)} \times \widehat{z} - \widehat{z} \times \boldsymbol{\pi}_r^{(1)} &= \boldsymbol{\chi}_r^{(1)} \\ \boldsymbol{\pi}_r^{(2)} \times \widehat{z} - \widehat{z} \times \boldsymbol{\pi}_r^{(2)} &= \boldsymbol{\chi}_r^{(2)} + L(\boldsymbol{\pi}_r^{(1)}),\end{aligned}$$

with

$$\begin{aligned}&\pi_{r\,zz}^{(1)} = 0 \\ &\pi_{r\,zz}^{(2)} = -\widehat{z} \cdot \boldsymbol{\pi}_r^{(1)} \cdot (\widehat{b} - \widehat{z}) - (\widehat{b} - \widehat{z}) \cdot \boldsymbol{\pi}_r^{(1)} \cdot \widehat{z} - (\widehat{b} - \widehat{z}) \cdot \boldsymbol{\pi}_r^{(1)} \cdot (\widehat{b} - \widehat{z}).\end{aligned}$$

• $m_e/m_i \ll 1$: only non-gyrotropic corrections due to ions are relevant.

• Leading order $\boldsymbol{\pi}_p^{(1)}$ reproduces Yajima's (1966) result

$$\pi^{(1)}_{p\,xx} = -\pi^{(1)}_{p\,yy} = -\frac{p_{\perp p}}{2\Omega_p}(\partial_y u_x + \partial_x u_y)$$

$$\pi^{(1)}_{p\,zz} = 0$$

$$\pi^{(1)}_{p\,xy} = \pi^{(1)}_{p\,yx} = -\frac{p_{\perp p}}{2\Omega_p}(\partial_y u_y - \partial_x u_x)$$

$$\pi^{(1)}_{p\,yz} = \pi^{(1)}_{p\,zy} = \frac{1}{\Omega_p}[2p_{\| p}\partial_z u_x + p_{\perp p}(\partial_x u_z - \partial_z u_x)]$$

$$\pi^{(1)}_{p\,xz} = \pi^{(1)}_{p\,zx} = -\frac{1}{\Omega_p}[2p_{\| p}\partial_z u_y + p_{\perp p}(\partial_y u_z - \partial_z u_y)].$$

• $\boldsymbol{\chi}_r^{(2)}$ (needed to decribe oblique and kinetic Alfvén waves) involves the non-gyrotropic part of $\nabla \cdot \mathbf{q}_r$ (to be modelized) and also nonlinear terms such as the contributions originating from the last term of $L(\boldsymbol{\pi}_r^{(1)})$ that it is important to retain in order to prevent appearence of spurious nonlinearities (making the problem ill-posed) in the reduced equation for weakly nonlinear long oblique Alfvén waves.

Next order contribution given by

$$\boldsymbol{\pi}_p^{(2)} \times \widehat{z} - \widehat{z} \times \boldsymbol{\pi}_p^{(2)} = L(\boldsymbol{\pi}_p^{(1)}) + \boldsymbol{\chi}_p^{(2)}$$

with the condition

$$\pi_{p\,zz}^{(2)} = -\widehat{z} \cdot \boldsymbol{\pi}_p^{(1)} \cdot (\widehat{b} - \widehat{z}) - (\widehat{b} - \widehat{z}) \cdot \boldsymbol{\pi}_p^{(1)} \cdot \widehat{z} - (\widehat{b} - \widehat{z}) \cdot \boldsymbol{\pi}_p^{(1)} \cdot (\widehat{b} - \widehat{z}).$$

Denoting by an overline the projection on the subspace orthogonal to $(\mathbf{I} - \widehat{b} \otimes \widehat{b})$ and $\widehat{b} \otimes \widehat{b}$, and by a double overline the projection on the subspace orthogonal to $(\mathbf{I} - \widehat{z} \otimes \widehat{z})$ and $\widehat{z} \otimes \widehat{z}$, one has

$$L(\boldsymbol{\pi}_p^{(1)}) = \frac{1}{\Omega_p} \overline{\overline{[\frac{d\boldsymbol{\pi}_p^{(1)}}{dt} + (\nabla \cdot u)\boldsymbol{\pi}_p^{(1)} + (\boldsymbol{\pi}_p^{(1)} \cdot \nabla u)^{\mathcal{S}}]}} + ((\widehat{b} - \widehat{z}) \times \boldsymbol{\pi}_p^{(1)})^{\mathcal{S}}$$

and

$$\boldsymbol{\chi}_p^{(2)} = \frac{1}{\Omega_p}[\overline{\frac{d\mathbf{p}_p^G}{dt}} + \overline{\overline{\nabla \cdot \mathbf{q}_p}} + \overline{(\mathbf{p}_p^G \cdot \nabla u)^{\mathcal{S}}}] - \boldsymbol{\chi}_p^{(1)}.$$

where

$$\boldsymbol{\chi}_p^{(1)} = \frac{1}{\Omega_p}(\overline{\overline{((p_{\perp p}(\mathbf{I} - \widehat{z} \otimes \widehat{z}) + p_{\| p}\widehat{z} \otimes \widehat{z}) \cdot \nabla u)^{\mathcal{S}}}} + (p_{\| p} - p_{\perp p})\partial_z[u \otimes \widehat{z} - (\widehat{z} \cdot u)\widehat{z} \otimes \widehat{z}]^{\mathcal{S}}).$$

Another modelization of the non-gyrotropic heat fluxes is here necessary. They contribute to $\boldsymbol{\chi}_p^{(2)}$ in the form

$$\overline{\overline{\nabla \cdot \mathbf{q}_p}} = \frac{p_{\perp p}}{2}(\nabla_\perp \otimes u_{dp} - (\widehat{z} \times \nabla_\perp) \otimes (\widehat{z} \times u_{dp}))$$

$$+[\widehat{z} \otimes (\nabla_\perp q_{\perp p} - p_{\perp p}\frac{v_{\Delta p}^2}{2\Omega_p}\widehat{z} \times \Delta_\perp \widehat{b} - 2p_{\parallel p}\widehat{z} \times (\nabla \times u_{dp}))]^{\mathcal{S}},$$

where we introduce the diamagnetic drifts $u_{d,r} = \dfrac{c}{nq|b|^2} b \times \nabla \cdot \mathbf{p}_r$.

They also enter the gyrotropic pressure equations which obey

$$\partial_t p_{\perp r} + \nabla \cdot (u\, p_{\perp r}) + p_{\perp r} \nabla \cdot u - p_{\perp r} \widehat{b} \cdot \nabla u \cdot \widehat{b} + \nabla \cdot (\widehat{b}\, q_{\perp r}) + q_{\perp r} \nabla \cdot \widehat{b}$$
$$+ \frac{1}{2} (\mathrm{tr}\, (\nabla \cdot \mathbf{q}_r^{NG})^G - \widehat{b} \cdot (\nabla \cdot \mathbf{q}_r^{NG})^G \cdot \widehat{b}) = 0$$
$$\partial_t p_{\parallel r} + \nabla \cdot (u\, p_{\parallel r}) + 2 p_{\parallel r} \widehat{b} \cdot \nabla u \cdot \widehat{b} + \nabla \cdot (\widehat{b}\, q_{\parallel r}) - 2 q_{\perp r} \nabla \cdot \widehat{b}$$
$$+ \widehat{b} \cdot (\nabla \cdot \mathbf{q}_r^{NG})^G \cdot \widehat{b} = 0,$$

with

$$\frac{1}{2} (\mathrm{tr}\, (\nabla \cdot \mathbf{q}_e^{NG})^G - \widehat{b} \cdot (\nabla \cdot \mathbf{q}_e^{NG})^G \cdot \widehat{b}) = 2 \nabla_\perp \cdot [p_{\perp e} (u_{d,e} - \frac{j}{qn})]$$
$$\frac{1}{2} (\mathrm{tr}\, (\nabla \cdot \mathbf{q}_p^{NG})^G - \widehat{b} \cdot (\nabla \cdot \mathbf{q}_p^{NG})^G \cdot \widehat{b}) = 0$$
$$\widehat{b} \cdot (\nabla \cdot \mathbf{q}_e^{NG})^G \cdot \widehat{b} = \nabla_\perp \cdot [p_{\parallel e} (u_{d,e} - \frac{j}{qn})]$$
$$\widehat{b} \cdot (\nabla \cdot \mathbf{q}_p^{NG})^G \cdot \widehat{b} = 2 \nabla_\perp \cdot [p_{\parallel p} (u_{d,p} - \frac{j}{qn})].$$

Energy conservation

Is usual energy $E = \int (\rho \frac{u^2}{2} + \frac{b^2}{8\pi} + p_\perp + \frac{1}{2} p_\parallel) d^3x$ conserved by the above mono-fluid model?

The delicate contributions originate

$\star$ from the electron pressure gradient in the induction equation

$\star\star$ from the second order non-gyrotropic pressure corrections.

$\star$ The electronic pressure that affects the magnetic field evolution only in the case of pressure anisotropy, contributes in a long wave theory at the level of the linear dispersion relation. In this limit, it can thus be replaced by $\frac{1}{\rho_0} \nabla \cdot p_{\perp e} - \frac{v_{\Delta e}^2}{B_0^2} \nabla \cdot (b \otimes b)$, that does not contribute to the energy budget.

$\star\star$ Concerning the non-gyrotropic pressure contributions, while the leading order $\boldsymbol{\pi}^{(1)}$ preserves energy, the effect of $\boldsymbol{\pi}^{(2)}$ is still unclear.

In fact, this contribution also only arises in the linear dispersion relation of oblique and kinetic Alfvén waves. In the case it affects the energy budget, this effect will be subdominant.

V. Validation

Landau-fluid description of long dispersive parallel Alfvén waves

The long wave reductive perturbative expansion performed on the Landau-fluid model reproduces the KDNLS equations derived from Vlasov-Maxwell up to the replacement in the transverse pressure fluctuations of the plasma response function $\mathcal{W}$ by the corresponding two- or four-pole approximants.

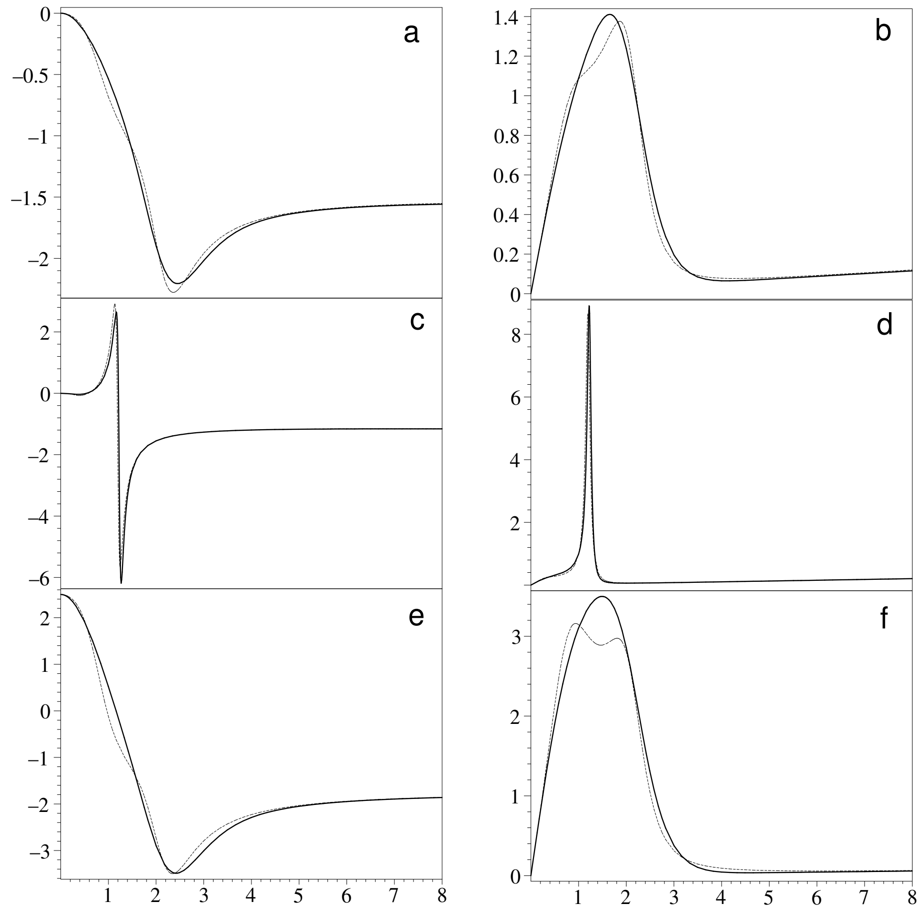

Figure 1: Contributions p^R (left column) and p^I (right column) to the perpendicular pressure $\frac{\widetilde{p}_\perp^{(1)}}{p_\perp^{(0)}} = -(p^R + p^I \mathcal{H})|b^{(1)}|$ versus $\beta^{-1/2}$, where $\beta = T_{\|e}^{(0)}/(m_i\lambda^2)$, for $T_{\|e}^{(0)} = T_{\|i}^{(0)} = T_{\perp e}^{(0)} = T_{\perp i}^{(0)}$ (a and b), $T_{\|e}^{(0)} = 8T_{\|i}^{(0)}$ and $T_{\perp e}^{(0)} = T_{\|e}^{(0)}$, $T_{\perp i}^{(0)} = T_{\|i}^{(0)}$ (c,d) and $T_{\perp e}^{(0)} = T_{\|e}^{(0)} = T_{\|i}^{(0)}$ and $T_{\perp i}^{(0)} = 3T_{\|i}^{(0)}$ (e, f). Kinetic calculation (solid line) Landau-fluid closure (dashed line).

• For magnetosonic waves with propagation angle α, the Landau damping rate is (assuming $\frac{m_e}{m_p} \ll \beta \ll \frac{T_e}{T_p}$)

$$\gamma = -\sqrt{\beta}\sqrt{\frac{\pi}{8}}\sqrt{\frac{m_e}{m_p}}\frac{\sin^2\alpha}{\cos\alpha}\frac{(\omega^2-\beta\cos^2\alpha)^2+\beta^2\cos^4\alpha}{(2\omega^2-\beta-1)(\omega^2-\beta\cos^2\alpha)},$$

an expression identical to that found by a direct derivation from the Vlasov-Maxwell equations. The long-wave equation is KdV+damping term.

• For Alfvén waves at finite angle of propagation , finite Larmor radius corrections of order $1/\Omega_p^2$ have to be retained. The governing equation is linear and reads, assuming $\frac{m_e}{m_p} \ll \beta \ll \frac{T_e}{T_p}$, (adiabatic protons and isothermal electrons) and $\beta \ll 1$ (ξ: stretched coordinate along the propagation)

$$\partial_\tau \frac{b_y}{B_0} + \frac{v_A^3}{2\Omega_p^2}\left[\frac{\cos^3\alpha}{\sin^2\alpha} + \sqrt{\beta}\sqrt{\frac{\pi}{2}}\sqrt{\frac{m_e}{m_p}}\cos^3\alpha\left(\tan^2\alpha + \frac{1}{\tan^2\alpha}\right)\mathcal{H}\right]\partial_{\xi\xi\xi}\frac{b_y}{B_0} = 0,$$

• For Kinetic Alfvén waves $(\cos^2\alpha \ll \beta)$,

$$\partial_\tau \frac{b_y}{B_0} + \frac{v_A^3}{2\Omega_p^2}\cos\alpha\left[-\beta\left(1+\frac{3}{4}\frac{T_p^{(0)}}{T_e^{(0)}}\right) + \sqrt{\beta}\sqrt{\frac{\pi}{2}}\sqrt{\frac{m_e}{m_p}}\mathcal{H}\right]\partial_{\xi\xi\xi}\frac{b_y}{B_0} = 0.$$

The dispersion and damping coefficients agree with classical results (Akhiezer et al. 1975, Hasegawa and Chen 1976).

VI. Conclusion

Collisionless dissipation (Landau damping) of dispersive MHD waves can be described using a monofluid model.

The model thus reproduces the correct dispersion relation for MHD waves for any value of the β parameter and for any angle of propagation, provided the wavelength is large compared to the ion inertial length.

Secondary instabilities as well as the resulting nonlinear dynamics are also captured.

For example small-amplitude oblique Alfvén waves obey a linear dynamics while parallel Alfvén waves are governed by the KDNLS equation.

Perspectives

• Benchmark the model by comparison with gyrokinetic simulations and possibly Vlasov-Maxwell simulations.

• In particular explore the nonlinear stage of parametric instabilities.

• Modelisation of coherent structures (magnetic holes and shocklets) observed in the solar wind and magnetosheath.

• Simulation of dispersive Alfvén wave turbulence:

  ⋆ Generation of KAW at small scales: importance of higher-order FLR corrections that are to be described in a computationally manageable way.

  ⋆ Self-consistent computation of turbulent dissipation

  ⋆ Self-consistent determination of the fast wave spectrum: important for cosmic ray scattering

  ⋆ Possible emergence of coherent structures

• Treat electrons as a Landau fluid in hybrid simulations.

• Explore the possible description of nonlinear Landau damping (see Prakash and Diamond, Nonlinear Proc. Geophys. **6**, 161 (1999)).

References:

T. Passot and P.L. Sulem:

- A long-wave model for Alfvén wave trains in a collisionless plasma:

I. Kinetic theory, Phys. Plasmas, **10**, 3887 (2003)

II. A Landau-fluid approach, Phys. Plasmas, **10**, 3906 (2003)

- Filamentation instability of long Alfvén waves in warm collisionless plasmas, Phys. Plasmas, **10**, 3914 (2003)
- A fluid description for Landau damping of dispersive MHD waves, Nonlinear Proc. Geophys., in press.
- A Landau fluid model for dispersive magnetohydrodynamics, *Phys. Plasmas*, submitted.

Can be downloaded from : http://www.obs-nice.fr/passot/filamentation/AWfil.html