IMPLICIT PLASMA SIMULATION CODES

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Workshop on Kinetic Theory
The Fields Institute, 2004

WHY USING MAXWELL-VLASOV THEORY FOR THE STUDY OF SPACE PLASMAS ?

Magnetospheres, solar/stellar coronae and winds, hot accretion discs...

- Most of the space plasmas are non collisional.
- Many interesting phenomena on boundaries (magnetopause, thin current sheets...) whose thicknesses (when measured onboard space probes) are found to be typically a few ion Larmor radii.
- Small scale turbulence, heating, and acceleration processes invoke phase space $(\vec{\mathbf{r}}, \vec{\mathbf{v}})$ structures. Usual MHD or multifluid variables not appropriate.

How to do ?

- Solve analytically the Maxwell-Vlasov system (Yves Elskens'talk on Landau absorbtion).
- Drop the Mawxell-Vlasov system, and solve Maxwell-Hamilton equations (Yves Elskens'talk on Landau absorbtion, QL theory)
- Derive fluid equations from Maxwell-Vlasov equations, through asymptotic expansions (Thierry Passot's talk on the kinetic theory of MHD waves, ...)
- Do Maxwell-Vlasov numerical simulations.

OTHER DOMAINS OF ASTROPHYSICAL INTEREST

Stars in a galaxy

The stars in a galaxy are the elementary particles of a self-gravitating collisionless medium that can be described by the Vlasov-Poisson equations.

Dark matter in cosmology

Non baryonic matter, only gravitational interactions : Vlasov-Poisson equations. Might represent 90% of the energy in the present universe. Case of a gase made of WIMPs (Weakly Interacting Massive Particles).

Different specific algorithms for plasmas and star gases

In star gases, all particles have the same charge. The thermal noise does not couple with waves. The implicit methods presented in this talk are based on the mode frequency analysis of the plasma. Therefore, they are not relevant for such systems. The explicit methods are similar for plasma and star gases. The cosmological codes include (for large scale phenomenology) a non euclidian metrics (Robertson-Walker for instance).

[René Pellat, impulsed researches and formed scientists both in the domains of plasmas and of cosmology].

MAXWEL-VLASOV SIMULATIONS AND PARTICLE IN CELL (PIC) CODES ?

General purpose Maxwell-Vlasov simulations methods :

- Vlasov codes. Compute the time evolution of the particle distribution function $f(t, \vec{\mathbf{r}}, \vec{\mathbf{v}})$ through Vlasov equation, coupled to Poisson or Maxwell equations. No numerical noise on the distribution function. But huge memory requirement (6D space) $\rightarrow$ low dimensionality, and poor velocity resolution.

- N body codes. The plasma is an assembly of particles. The individual interactions are considered between every particles. Require to take care of the divergence of the Poisson interaction. Computation time proportional to N^2 (hardware specific solutions, Beowulf).

- Particle In Cell (PIC) codes. Liouville : the caracteristics of the Vlasov function are "macroparticle" trajectories. The plasma is considered as an assembly of finite size macroparticles moving under the influence of an electromagnetic field that is computed on a grid. There is no explicit mention of $f(t, \vec{\mathbf{r}}, \vec{\mathbf{v}})$, but it can be computed for diagnostics. The interpolation between the particles and the grid is done in a way that damps close interactions between particles, i.e. retain only collective plasma interactions.

THE DATA IN A PARTICLE IN CELL CODE

[Birdsall and Langdon, 1985, Hockney and Eastwood, 1988].

An electromagnetic field $\vec{\mathbf{E}}_n(X_j)$, $\vec{\mathbf{E}}(X_j)$ computed on a grid $(X_j)_{j \in G}$, $G = Z/[1, NX] \times Z/[1, NY]$

Particle data

N_p particles of location $\vec{\mathbf{r}}_n$ and velocity $\vec{\mathbf{v}}_{n-1/2}$ at time step n.

Notations

The particles are indexed by i, the grid elements by j, or by their location X_j, time steps by n. To avoid overloaded equations, some of these indices may be omitted.

Plasma condition

In a real plasma, the condition $g = 1/n\lambda_D^3 \ll 1$ is necessesary to ensure a collective behavior.
In a PIC code, $\Delta X = L_x/NX \geq \lambda_D$ and the macroparticles must have a collective behavior. The condition $g \ll 1$ requires

$$\frac{N_p}{NX * NY} \gg 1$$

THE COMPUTATIONS IN A PARTICLE IN CELL CODE

- Initial values: compute $E_0(X_j)$, $B_0(X_j)$, $\vec{\mathbf{r}}_0$, $\vec{\mathbf{v}}_{-1/2}$ etc. (not trivial)
- Time evolution loop:
  - Interpolate the electromagnetic field (and related fields) at particle locations : $E_n(X_j) \rightarrow E_n(x_{n,i})$.
  - Push the particles : compute $\vec{\mathbf{v}}_{n+1/2}$, then $\vec{\mathbf{r}}_{n+1}$.
  - Interpolate the charge $\rho(X_j) = \Sigma_i q_i S(X_j - x_i)$ and current densities (and other source fields if necessary) on the grid.
  - Define the electric field $E(X_j)_{j\in G}$ equation (deduced from Maxwell Eq. and plasma densities).
  - Solve the electric field equation .
  - Compute the magnetic $B(X_j)_{j\in G}$, and other related fields.
  - Case of a predictor-corrector scheme : correct the particle's motions and locations.
  - Do diagnostics, sometimes.
- End: After a few 1000 time steps, stop the computation.

SECOND ORDER DIFFERENTIAL EQUATIONS AND THE LEAP FROG SCHEME (1/2)

leap frog (saute mouton) basics

For time $t_n = n\Delta t$, given x_0; find x_n such as

$$\ddot{x}_n = f(t_n, x_n, \dot{x}_n)$$

(Notation : f is not supposed to be a distribution function. It can be an Eq. of motion). Let $v_{n+1/2} = \dot{x}_{n+1/2}$,

$$v_{n+1/2} = v_n + \frac{\Delta t}{2}\dot{v}_n + \frac{\Delta t^2}{8}\ddot{v}_n + O(\Delta t^3)$$
$$v_{n-1/2} = v_n - \frac{\Delta t}{2}\dot{v}_n + \frac{\Delta t^2}{8}\ddot{v}_n + O(\Delta t^3)$$

by difference

$$v_{n+1/2} = v_{n-1/2} + \Delta t \dot{v}_n + O(\Delta t^3)$$

Can be used in the time domain for the particle motion, in the space domain for Maxwell's equations.

SECOND ORDER DIFFERENTIAL EQUATIONS AND THE LEAP FROG SCHEME (2/2)

the explicit leap-frog scheme

Then $\ddot{x}_n = f(t_n, x_n, \dot{x}_n)$ becomes

$$\begin{aligned} v_{n+1/2} &= v_{n-1/2} + \Delta t f(t_n, x_n, (v_{n+1/2} + v_{n-1/2})/2) + O(\Delta t^3) \\ x_{n+1} &= x_n + \Delta t v_{n+1/2} \end{aligned}$$

It is a $(\Delta t)^2$ acurate implicit scheme. It is easy to use if we can solve directly the above equation for $v_{n+1/2}$. The solution is correct as long as $\Delta t \omega_{HF} < 1$, where ω_{HF} is the highest characteristic frequency appearing in the (physical) solution of the differential equation.
Let f be the magnetic Lorents force in a unform magnetic field (depending only on v), then, for $\Delta\omega_c > 1$, the particle rotates in the plane perpendicular to the magnetic field, but with a wrong frequency ($\omega \neq \omega_c$) and a wrong Larmor radius.
Let f be an harmonic function of x, with a caracteristic frequency ω_h, it can be shown that for $\Delta\omega_h > 1$, the solution is an exponentially growing oscillation. As the plasma oscillations correspond, in the linear approximation, to an electron harmonic oscillator, we cannot use a time step such as $\Delta\omega_{pe} > 1$. Therefore, although implicit, this scheme is not unconditionally stable, it must be used under the conditions $\Delta\omega_c < 1$ and $\Delta\omega_{pe} < 1$.
The Maxwell's equations can be described with a leap frog scheme that involves both spatial and temporal derivatives.
If f represent the set of the Maxwell Eq. and of the Eq. of motion, we can solve explicitely the above equation (unless it is implicit). They are the basis of the sometimes (abusively called) Particle In Cell explicit codes.

THE EXPLICIT PIC CODE EQUATIONS (I/II)

Interpolate electromagnetic field at particles' location

$$\mathbf{E}(x)_n = \sum_j S(\mathbf{X}_j - \mathbf{x}_n)\mathbf{E}(X_j)_n$$
$$\mathbf{B}(x)_n = \sum_j S(\mathbf{X}_j - \mathbf{x}_n)\mathbf{B}(X_j)_n$$

Particle Motion

$$\begin{aligned}
\mathbf{v}_{n+1/2} &= \mathbf{v}_{n-1/2} + \Delta t[\frac{e}{m}\mathbf{E}(x)_n + \frac{e}{2mc}(\mathbf{v}_{n+1/2} + \mathbf{v}_{n-1/2}) \times \mathbf{B}_n(\mathbf{x}_n)] \quad (1) \\
\mathbf{x}_{n+1} &= \mathbf{x}_n + \Delta t \mathbf{v}_{n+1/2}
\end{aligned}$$

Charge and current densities

$$\begin{aligned}
\rho(X_j)_{n+1/2} &= \sum_i q_i S(\mathbf{X}_j - \mathbf{x}_n) \quad (2) \\
\mathbf{J}(X_j)_{n+1/2} &= \sum_i q_i \mathbf{v}_{n+1/2} \frac{1}{2}[S(\mathbf{X}_j - \mathbf{x}_n) + S(\mathbf{X}_j - \mathbf{x}_{n+1})]
\end{aligned}$$

THE EXPLICIT PIC CODE EQUATIONS (I/II)

Maxwell's Equations

$$
\begin{aligned}
\mathbf{E}_{n+1} - \mathbf{E}_n &= c\Delta t \nabla \times \mathbf{B}_{n+1/2} - 4\pi \Delta t \mathbf{J}_{n+1/2} \\
\mathbf{B}_{n+1/2} - \mathbf{B}_{n-1/2} &= -c\Delta t \nabla \times \mathbf{E}_n && \text{iterated value, for good} \\
\mathbf{B}_{n+1} - \mathbf{B}_{n+1/2} &= -\frac{c}{2}\Delta t \nabla \times \mathbf{E}_{n+1} && \text{extrapolation for particle pusher} \\
\Delta \Psi_n &= -4\pi \rho_n
\end{aligned}
$$

from which we deduce the electric field : $\mathbf{E}_{n+1} = \mathbf{E_M} - \nabla\Psi$ where

$$\mathbf{E}_{M,n+1} = \mathbf{E}_n + C\Delta t \times [\mathbf{B}_{n-1/2} - c\Delta t \nabla \times \mathbf{E}_n] - \Delta t \mathbf{J}_{n+1/2} = \mathbf{Q}$$

and (Poisson correction)

$$\Delta \Psi_{n+1} = -4\pi \rho_{n+1}.$$

The condition $\nabla \cdot B = 0$ is conserved through time evolution by the numerical scheme. Just settle it in the initial conditions.

NEED FOR BETTER IMPLICIT CODES

Let ω_{max} be the fastest caracteristic frequency in the plasma. The time step Δt used in an explicit code must satisfy $\omega_{max}\Delta t \ll 1$. Unless, we get wrong (sometimes unstable) solutions. This constraint is very tedious when were are interested only in low frequency phenomena. It is therefore interesting to use other implicit algorithms that allow to get rid of the above constraint. Implicit algorithms maybe stable, but, as Δt increases, they can converge toward "wrong" solutions. We are interested in wrong implicit solutions that damp the high frequency phenomena, and retain a good description of the long time scales physics.

IMPLICIT DECENTERED LEAP FROG SCHEME

Developping v around a non integer time step $n+\alpha-1/2$:

$$\begin{aligned}
v_{n+1/2} &= v_{n+\alpha-1/2} + (1-\alpha)\frac{\Delta t}{2}\dot{v}_{n+\alpha-1/2} + (1-\alpha)^2\frac{\Delta t^2}{8}\ddot{v}_{n+\alpha-1/2} + O(\Delta t^3) \\
v_{n-1/2} &= v_{n+\alpha-1/2} - \alpha\frac{\Delta t}{2}\dot{v}_{n+\alpha-1/2} + \alpha^2\frac{\Delta t^2}{8}\ddot{v}_{n+\alpha-1/2} + O(\Delta t^3)
\end{aligned}$$

If $\alpha = 1/2$ we recover the explicit second order scheme. We consider $1/2 < \alpha < 1$, by difference

$$v_{n+1/2} = v_{n-1/2} + \Delta t f(t_{n+\alpha-1/2}, x_{n+\alpha-1/2}, v_{n+\alpha-1/2}) + O(\Delta t^2).$$

The data at time $n+\alpha-1/2$ in f is obtained through numerical interpolation. The equation is implicit, because x_{n+1}, v_{n+1} lies in implicit form in the RHS of the equation, through the interpolation.
It is considered as unconditionally stable. But its solution are only accurate to the first order. That is a problem when we want to reach large time steps.

The leap-frog scheme is implicit up to Δt^2. As we use it, we would like to keep a second order accurate implicit algorithm.

DIRECT IMPLICIT LEAP FROG SCHEME

Let $f_n = f(t_n, x_n, v_n)$ and $(\frac{df}{dt})_n = (\frac{\partial f}{\partial t})_n + \dot{x}_n(\frac{\partial f}{\partial x})_n + \ddot{x}_n(\frac{\partial f}{\partial \vec{\mathbf{v}}})_n$, and develop:

$$f_{n+1} = f_n + \Delta t \frac{df}{dt}_n + \frac{\Delta t^2}{2}(\frac{d^2 f}{dt^2})_n + O(\Delta t^3), \qquad f_{n-1} = f_n - \Delta t \frac{df}{dt}_n + \frac{\Delta t^2}{2}(\frac{d^2 f}{dt^2})_n + O(\Delta t^3).$$

By addition $f_n = (f_{n+1} + f_{n-1})/2 + O(\Delta t^3)$, hence

$$v_{n+1/2} = v_{n-1/2} + \frac{\Delta t}{2}[f(t_{n+1}, x_{n+1}, v_{n+1}) + f(t_{n-1}, x_{n-1}, v_{n-1})] + O(\Delta t^3)$$

Second order accurate. Implicit, because x_{n+1}, v_{n+1} lies in implicit form in the RHS of the equation. It is considered as unconditionally stable.

Direct implicit scheme with a predictor-corrector

[Friedman et al. 1981, Barnes et al. 1983, Langdon et al. 1983]
How to compute x_{n+1}, v_{n+1} in the above Eq. ? Sometimes, it is very easy, when f is simple (for instance a linear function of v).
Otherwise, a solution consists of breaking the variables x_{n+1}, v_{n+1} into a prediction plus a correction

$$\begin{aligned} v_{n+1/2} = \tilde{v}_{n+1/2} + \delta v_{n+1/2} &= \text{prediction} + \text{correction} \\ \tilde{v}_{n+1/2} = v_{n-1/2} &+ \frac{\Delta t}{2} f(t_{n-1}, x_{n-1}, v_{n-1}) \\ \delta v_{n+1/2} &= \frac{\Delta t}{2} f(t_{n+1}, \tilde{x}_{n+1}, \tilde{v}_{n-1}) \end{aligned}$$

It is possible to iterate.
The following developpments are based on the leap frog direct time implicit schemes.

DIRECT IMPLICIT ALGORITHM

Direct implicit variables

Based on implicit variables $\bar{\mathbf{X}}$. Replaces one physical variable in one equation and damps one physical effect.

$$\bar{\mathbf{X}}_n = \frac{1}{2}(\mathbf{X}_{n+1} + \bar{\mathbf{X}}_{n-1})$$

Making a variable "direct implicit" do not consists only of introducing impliciteness in the equations, it also introduces a diffusion, via a low-pass filtering of $\mathbf{X}_{n+1}$. Therefore, using the direct implicit algorithm, we gain stability, but we loose reversibility.

A constraint for stability remains on the time step, because we use a predictor-corrector scheme to resolve the implicit equations. The prediction is based on explicit computations. With a too large time step, the prediction would not be good enough to solve the implicit equation, and the computation would fail. Nevertheless, the time step constraint is much weaker than for the classical PIC scheme.

Making all the variables in the code implicit would (maybe) allow for a "universal implicit" PIC code. But, the algebraic complexity, and the ill conditionement of the final electric field equation, increase with the introduction of new implicit variables.

It is wiser to introduce a low number of well choosen implicit variables. This choice is based on the analysis of the high frequency phenomena that we want to damp.

DIFFERENT ORDERINGS FOR IMPLICIT PIC CODES

The highest frequencies in a plasma

- The light waves (propagate at c, freq. ω_l, do not interact with the particles, the plasma is transparent).
- The electron plasma waves (electrostatic oscillations at $\omega_p = (ne^2/\epsilon_0 m_e)^{1/2}$).
- The gyromotion of the electrons (at $\omega_c = eB/m_e$).

Ordering

- Low magnetized plasmas (most parts of the magnetospheres and solar corona): $\omega_l \gg \omega_p \gg \omega_c$
- Highly magnetized plasmas (auroral zones, some astroϕ jets): $\omega_l \gg \omega_c \gg \omega_p$
- Intermediate case: $\omega_l \gg \omega_c \sim \omega_p$

HOW TO BE IMPLICIT ON THE LIGHT WAVES

Maxwell's Equations

Ampère:

$$\mathbf{E}_{n+1} - \mathbf{E}_n = c\Delta t \nabla \times \mathbf{B}_{n+1/2} - 4\pi \Delta t \mathbf{J}_{n+1/2}$$

Faraday:

$$\mathbf{B}_{n+1/2} - \mathbf{B}_{n-1/2} = -c\Delta t \nabla \times \bar{\mathbf{E}}_n$$

$$\mathbf{B}_{n+1} - \mathbf{B}_{n+1/2} = -\frac{c}{2}\Delta t \nabla \times \mathbf{E}_{n+1}$$

Implicit electric field:

$$\bar{\mathbf{E}}_n = \frac{1}{2}[\mathbf{E}_{n+1} + \bar{\mathbf{E}}_{n-1}]$$

no Darwin separation of transverse and longitudinal fields.

The impliciteness is naturally removed from the resolution of the equations, and the equations $\mathbf{E}_{n+1} = \mathbf{E_M} - \nabla \Psi$,

$$\mathbf{E}_{M,n+1} + \frac{c^2 \Delta t^2}{2} \nabla \times \nabla \times \mathbf{E}_{M,n+1} = \mathbf{Q}$$

and (Poisson correction)

$$\nabla^2 \Psi = \nabla \cdot \mathbf{Q}' - 4\pi \tilde{\rho}_{n+1}.$$

We get rid of light waves at no cost. (The above equations refer to the wave-plasma interaction, and there is no interactions between light waves and plasma).

HOW TO BE IMPLICIT ON THE ELECTRON PLASMA OSCILLATIONS

[Hewet & Langdon, J. Comput. Phys., 1987]

$$\mathbf{v}_{n+1/2} = \mathbf{v}_{n-1/2} + \Delta t[\bar{\mathbf{a}}_n - \frac{\mu}{m}\nabla B_n(\mathbf{x}_n) - \frac{e}{2mc}(\mathbf{v}_{n+1/2} + \bar{\mathbf{v}}_{n-1/2}) \times \mathbf{B}_n(\mathbf{x}_n),]$$

where an implicit electric acceleration $\bar{\mathbf{a}}_n$ is computed for each particle,

$$\bar{\mathbf{a}}_n = \frac{1}{2}(-\frac{e}{m}\mathbf{E}_{n+1}(\mathbf{x}_n) + \bar{\mathbf{a}}_{n-1}).$$

The impliciteness of this equation ($\mathbf{E}_{n+1}$ appears in the RHS of the equation) is solved through a predictor-corrector scheme. For each particle, we define predictions $\tilde{\mathbf{x}}_n$, $\tilde{\mathbf{v}}_{n-1/2}$, and corrections $\delta\mathbf{x}_n$, $\delta\mathbf{x}_{n-1/2}$ of the particle location and velocity.
The electric field Eq. $E = E_M - \nabla\Psi$ is given by

$$(\mathbf{I} + 4\pi\boldsymbol{\chi}) \cdot \mathbf{E}_{M,n+1} + \frac{c^2\Delta t^2}{2}[\nabla \times ((\mathbf{I}+\boldsymbol{\Gamma})] \cdot (\nabla \times \mathbf{E}_{M,n+1}) = \mathbf{Q}$$

and

$$\nabla \cdot (\mathbf{I} + 4\pi\boldsymbol{\chi}) \cdot \nabla\Psi = \nabla \cdot \mathbf{Q} - 4\pi\tilde{\rho}_{n+1}.$$

where the $(3,3)$ matrices $\boldsymbol{\chi}(X_j)$ and $\Gamma(X_j)$ and the vector $Q(X_j)$, are defined on each grid cell, and are the sum over the particles of functions that depend on the predicted quantities $\tilde{\mathbf{x}}_n$ and $\tilde{\mathbf{v}}_{n-1/2}$.

HOW TO BE IMPLICIT ON THE ELECTRON GYROMOTION: THE GUIDING CENTRE EQUATION OF MOTION (I)

The Classical Set of Guiding Centre Equations

(The guiding centre theory is more restrictive than the gyrokinetic approximation presented by Hong Qin and Alain Brizard. It belongs to the context of last week's P.L Sulem talk, references to it are in blue)

The most widely used equations (because the first is explicit) among plasma physicists

$$\mathbf{v}_\perp = \frac{\mathbf{b}}{B} \times [-c\mathbf{E} + \frac{\mu c}{e}\nabla B + \frac{mc}{e}(v_\parallel \frac{d\mathbf{b}}{dt} + \frac{d\mathbf{u}_E}{dt})] \quad = U_0 + U_1$$

$$\frac{dv_\parallel}{dt} = \frac{e}{m}\mathbf{E} - \frac{\mu}{m}\frac{d\mathbf{B}}{ds} + \mathbf{u}_E\frac{d\mathbf{b}}{dt} = \quad \frac{dv_\parallel}{dt}$$

where $\mathbf{b} = \mathbf{B}/B$ defines the local frame, $\mathbf{u}_E = U_0$ is the $\mathbf{E} \times \mathbf{B}/B^2$ drift velocity, and μ is the magnetic moment.

The magnetic moment μ, which is the first adiabatic invariant, is supposed to be constant.

But, when the equations are discretized, $\mathbf{v}_\perp$ and $v_\parallel$ are given respectivelely at times n and $n-1/2$ $\rightarrow$ how to deal with the current density $\vec{\mathbf{J}}_{n-1/2}$ computation at the second order in Δt ?

HOW TO BE IMPLICIT ON THE ELECTRON GYROMOTION: THE GUIDING CENTRE EQUATION OF MOTION (II)

[Mottez et al., Computer Phys. Communications, 1998]

A more convenient set of guiding centre equations

$$\frac{d\mathbf{v}}{dt} = -\frac{e}{m}[\mathbf{E}(\mathbf{r}) + \frac{\mathbf{v}}{c} \times \mathbf{B}(\mathbf{r})] - \frac{\mu}{m}\nabla B(\mathbf{r})$$ [Northrop's book, 1963]

This equation has been 'forgotten' by most scientists. It belongs to the theory presented last week by P.L. Sulem. The low frequency part of this solution is the guiding centre motion. It can be solved numerically with an implicit algorithm.
The above equation can be combined with the above parallel velocity equation (not necessary, but better in the case of fast magnetic variations).

THE IMPLICIT GUIDING CENTRE NUMERICAL EQUATIONS (1/2)

The guiding centre equation

$$\hat{\mathbf{v}}_{n+1/2} = \mathbf{v}_{n-1/2} + \Delta t[\frac{e}{m}\mathbf{E}(x)_n - \frac{\mu}{m}\nabla B_n(\mathbf{x}_n) - \frac{e}{2mc}(\hat{\mathbf{v}}_{n+1/2} + \bar{\mathbf{v}}_{n-1/2}) \times \mathbf{B}_n(\mathbf{x}_n)]$$

involves an implicit 'gyromotion velocity'

$$\bar{\mathbf{v}}_{n-1/2} = \frac{1}{2}(\hat{\mathbf{v}}_{n+1/2} + \bar{\mathbf{v}}_{n-3/2}).$$

The parallel velocity

$$v_{\parallel n+1/2} = v_{\parallel n-1/2} + \Delta t[\frac{e}{m}\mathbf{E}(x)_n - \frac{\mu}{m}\nabla B_n(\mathbf{x}_n)] \cdot \mathbf{b}_n$$

gives a correction to the first estimate $\hat{\mathbf{v}}_{n+1/2}$ of the guiding centre velocity

$$\mathbf{v}_{n+1/2} = v_{\parallel n+1/2}\mathbf{b}_{n+1/2} + (\mathbf{I} - \mathbf{b}_{n+1/2}\mathbf{b}_{n+1/2}) \cdot \hat{\mathbf{v}}_{n+1/2}.$$

The positions, the velocities, and the magnetic field direction $\mathbf{b}_{n+1/2}$ are cut into a prediction, and a correction.
The electron current is the sum of the guiding centre velocities current, and of the magnetization current. Each of these currents is cut into a prediction and a correction. The current prediction contribute to $\mathbf{Q}$. The current correction, when expressed in terms of $\mathbf{E}_{n+1}$, of predicted variables and injected into Maxwell's Eq. leads to $E = E_M - \nabla\Psi$ where

$$\mathbf{E}_{M,n+1} + \frac{c^2\Delta t^2}{2}[N + \nabla \times ((\mathbf{I} + \mathbf{\Gamma})] \cdot (\nabla \times \mathbf{E}_{M,n+1}) = \mathbf{Q}$$

and

$$\nabla^2\Psi = \nabla \cdot \mathbf{Q} - 4\pi\tilde{\rho}_{n+1}.$$

THE IMPLICIT GUIDING CENTRE NUMERICAL EQUATIONS (2/2)

The guiding centre current correction writes

$$\delta \mathbf{J}_{n+1/2,GC} = \frac{1}{4\pi}\frac{c^2\Delta t}{2}\mathbf{N}\cdot(\nabla\times\mathbf{E}_{n+1})$$

where

$$\mathbf{N} = -4\pi\sum_i \frac{e}{cB^*}S(\mathbf{X}_j - \tilde{\mathbf{x}}_{n+1})[\hat{\mathbf{v}}_{n+1/2}\mathbf{b}^* - (\hat{\mathbf{v}}_{n+1/2}\cdot\mathbf{b}^*)\mathbf{b}^*\mathbf{b}^*],$$

The magnetization current correction is

$$\delta \mathbf{J}_{n+1/2,M} = \frac{c^2\Delta t}{2}\nabla\times(\mathbf{\Gamma}\cdot(\nabla\times\mathbf{E}_{n+1}))$$

where $\mathbf{\Gamma}$ is the tensor defined by:

$$\mathbf{\Gamma} = \sum_i \frac{\mu_i}{cB^*}(\mathbf{I} - \mathbf{b}^*\mathbf{b}^*)S(\mathbf{X}_j - \tilde{\mathbf{x}}_{n+1}).$$

PLASMAS IN THE REGIME $\omega_{CE}/\omega_{PE} \sim 1$.

The guiding centre equation includes and implicit electric acceleration

$$\hat{\mathbf{v}}_{n+1/2} = \mathbf{v}_{n-1/2} + \Delta t[\bar{\mathbf{a}}_n - \frac{\mu}{m}\nabla B_n(\mathbf{x}_n) - \frac{e}{2mc}(\hat{\mathbf{v}}_{n+1/2} + \bar{\mathbf{v}}_{n-1/2}) \times \mathbf{B}_n(\mathbf{x}_n)]$$

involves an implicit 'gyromotion velocity'

$$\bar{\mathbf{v}}_{n-1/2} = \frac{1}{2}(\hat{\mathbf{v}}_{n+1/2} + \bar{\mathbf{v}}_{n-3/2}).$$

The parallel velocity

$$v_{\parallel n+1/2} = v_{\parallel n-1/2} + \Delta t[\bar{\mathbf{a}}_n - \frac{\mu}{m}\nabla B_n(\mathbf{x}_n)] \cdot \mathbf{b}_n$$

gives a correction to the first estimate $\hat{\mathbf{v}}_{n+1/2}$ of the guiding centre velocity

$$\mathbf{v}_{n+1/2} = v_{\parallel n+1/2}\mathbf{b}_{n+1/2} + (\mathbf{I} - \mathbf{b}_{n+1/2}\mathbf{b}_{n+1/2}) \cdot \hat{\mathbf{v}}_{n+1/2}$$

Again, the implicit particle pusher is solved through a predictor-corrector sheme.

$$\nabla \cdot (\mathbf{I} + 4\pi\boldsymbol{\chi}) \cdot \mathbf{E}_{M,n+1} + \frac{c^2\Delta t^2}{2}[N + \nabla \times ((\mathbf{I} + \boldsymbol{\Gamma})] \cdot (\nabla \times \mathbf{E}_{M,n+1}) = \mathbf{Q}$$

and

$$\nabla \cdot (\mathbf{I} + 4\pi\boldsymbol{\chi}) \cdot \nabla\Psi = \nabla \cdot \mathbf{Q} - 4\pi\tilde{\rho}_{n+1}.$$

APPLICATIONS

Time decentered plasma implicit code

[M. Tanaka, J. Comput. Phys., 1988] Implicit on light waves, plasma waves, electron guiding centre or full dynamics, full ion dynamics, accurate to the first order in Δt.
Used to study the collisionless magnetic reconection , [Tanaka, 1995]

Direct implicit code, full electron dynamics

For low magnetized plasmas. Retains full ion and electron dynamics. Accurate to the second order in Δt. Time step limit $\Delta t\ \omega_{ce} < 1, \Delta t\ \omega_{pe} < or \sim 10$
Study of the Earth's bow shock in the solar wind. Role of the whistler precursor in the cyclic reformation of quasi-parallel shocks, [F. Pantellini, A. Heron, J.C. Adam; A. Mangeney, J. Comput. Phys., 1992], [D. Krauss-Varban, F. Pantellini, D. Burgess, Geophys. Res. Let., 1995].
Adding Monte-Carlo ionisation collisions : Modelisation of the stationary regime of a plasma truster (used for satellite propulsion). [L. Garrigues, A. Heron,J.C. Adam, J.P. Boeuf, Plasma Sources Sci. and Tech.., 2000] +1 publication in press

Direct implicit code, electron guiding centre

For higly magnetized plasmas. Retains full ion dynamics. Accurate to the second order in Δt. Time step limit $\Delta t\ \omega_{pe} < 1$, reverse constraint on $\Delta t\ \omega_{ce} >\sim 1$
Study of acceleration processes and turbulence in the Earth auroral zone. [V. Génot, F. Mottez, P. Louarn, J. Geophys. Res., 2000, 2001, Ann. Geophys. submitted]

UNDER DEVELOPEMENT

Direct implicit code, electron guiding centre, implicit on light and on plasma waves

Especially usefull in the regime $\omega_{pe} \sim \omega_{ce}$.

Written. Not yet validated.

Some positive results on the Vlasov-Poisson code. Time step constraint seems to be $\Delta t\ \omega_{ce} >\sim 1$ and $\Delta t\ \omega_{pe} < or \sim 10$.

The electric field equation requires a special attention : with high magnetic field value and long time step, seem to be ill conditionned. Classical (and less classical) iterative methods do not work. Need for preconditioning or more robust algorithms. Further work is needed.