

Wave Packet Dynamics and 'Granulations' in Wave Kinetics

- Review of Phase Space Density
Granulation in Vlasov Turbulence
(Dupree, et.al. 70's, 80's)
- Applications to Turbulence with
Disparate Scale Interaction - Nonlinear
Modulational Dynamics
(P.D., T.S. Hahm, K.Itoh, S.-I. Itoh)

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Outline

c.) Motivations (Ex tended)

- Disparate Scale Interaction + Nonlinear Modulation
- $k-S$ Parameter Space
⇒ 'Turbulent Trapping'?
- Relation to Vlasov Turbulence

c') Theory of Granulations in Vlasov Turbulence - 1D

- why? → 'coherent', 'incoherent' fluctuations
- Basic Ideas and Descriptions
- Impact on Relaxation, 'Instability'
- Structure of Theory
- Higher Dimensions?

III.) 'Granulation in Wave Kinetics'

- 'mapping' \rightarrow 1D Vlasov $\left\langle \begin{array}{l} 1D \text{ Langmuir *} \\ \text{Zonal Flow - DWT} \end{array} \right.$
- meaning / necessity of granulation in wave population
- description - statistical, Bok
- Momentum exchange and relaxation - 'new' route to Langmuir turbulence
- Differences from Vlasov
 - 'ray accumulation points'
 - caustics

IV.) Future Plans

I.) Motivation

- 'Disparate Scale Interaction' Ubiquitous



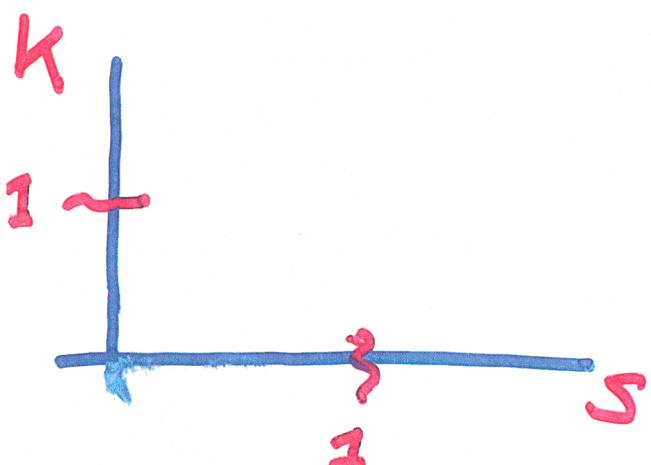
- Langmuir
 - SWE - Zonal Flow
 - Dynamo
 - 'ONLS'
- ⋮

→ large scale self organization?

→ Leverage:
↓
facilit 'small parameters'

Scale Separation -
Envelope Formulation
Adiabatic Theory -
→ Wave Kinetics

- Useful 'Catalogue Framework'



$S \equiv$ Ray Ginzburg Parameter

$$S = \frac{\sigma V_g(k)}{\Delta(\Omega/2)}$$

→ stochastic?

$k \equiv$ tube Number

→ Self-trapping

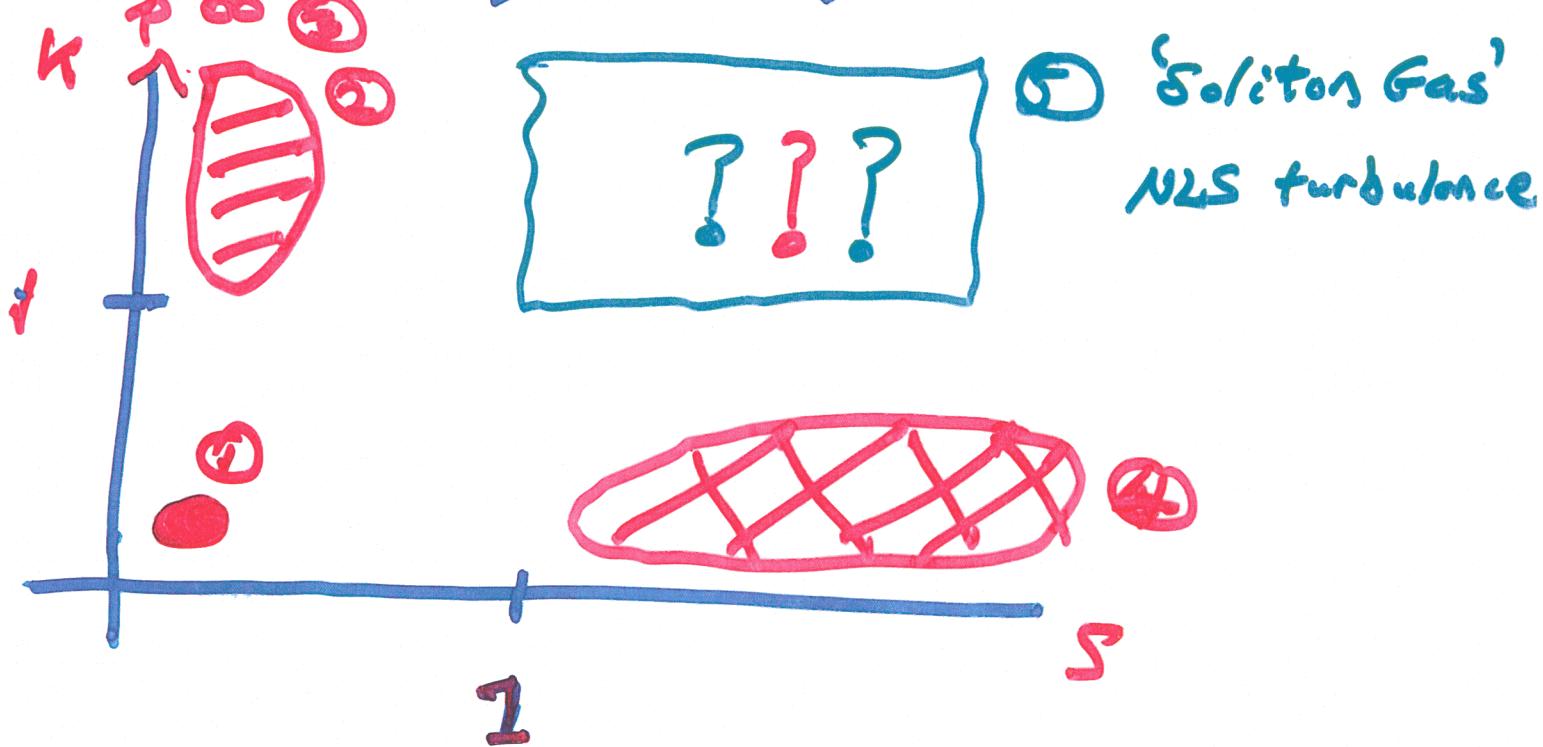
$$k = \bar{T}_{\text{ac}} / \bar{T}_{\text{bounce}}$$

- Eg: Langmuir Turbulence (Zakharov & Sagdeev 1972)

$$\frac{\partial^2 \phi}{\partial t^2} - c_s^2 \phi = \frac{\sigma^2 \Sigma}{nm} \quad \Sigma = \omega N$$

$$\frac{\partial N}{\partial t} + \nabla \cdot \mathbf{v} N - \frac{\partial}{\partial x} [\omega_R (1 + \delta \rho)] \cdot \nabla_x N = 0$$

or envelope equation.



① coherent parametric interaction

② BGK approach to wave kinetics

Langmuir Solitons

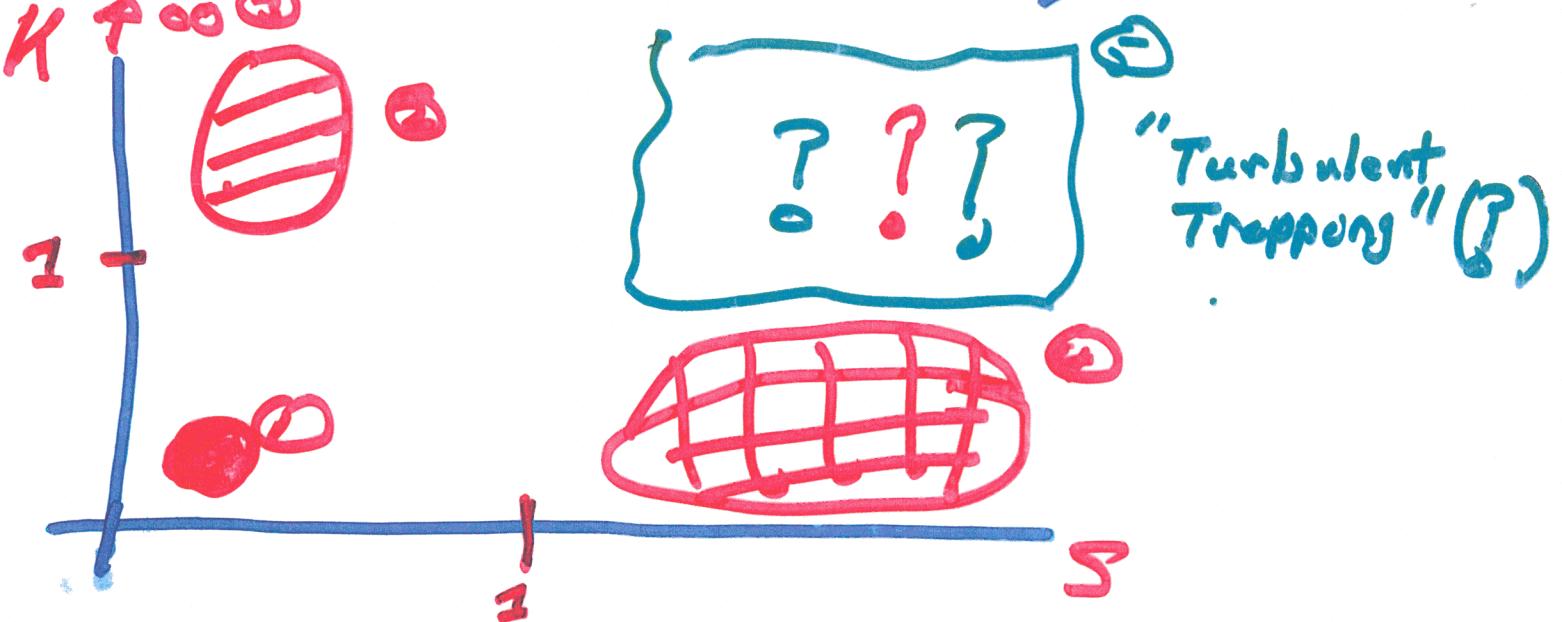
③ Langmuir Collapse

④ Induced Diffusion - QL wave kinetics

e.g. Drift Wave - Zonal Flow

$$\partial_t \langle v_y(x) \rangle = -\partial_x \langle \hat{v}_x \hat{v}_y \rangle - u \langle v_y \rangle \quad \xrightarrow{\text{NDSW}}$$

and WKE or Envelope Equations



- ① Coherent parametrics - Chen, et. al.
- ② BGK wave kinetics - Smolyakov et. al.
Kondratenko, et. al.
- ③ Singular Shear Layer - O. Gurcan, P.D.
- ④ "Random Shearing" - P.D., et. al.
- ⑤ "Turbulent Trapping" - indicated by Balcerowicz

↳ {Very Relevant to Z.F.
{Saturation}.

- Prototype of 'Turbulent Trapping' -
 1 D Vlasov Turbulence

$$\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x} + \frac{e}{m} E \frac{\partial f}{\partial v} = 0 \quad f = \langle \rho \rangle + \delta f$$

$$\nabla^2 g = -4\pi n_0 Z \int f dv$$

- Observe Scalability:

1D Langmuir:

$$\frac{\partial N}{\partial t} + v_g(k) \frac{\partial N}{\partial x} - \frac{e}{m} \omega \frac{\partial N}{\partial k} = 0$$

$$\partial_t^2 \rho = + \partial_x^2 [c_e^2 \rho + \int dk \omega N]$$

(Dynamic Poisson Eqn.)

- Simple Zonal Flow - Drift Wave

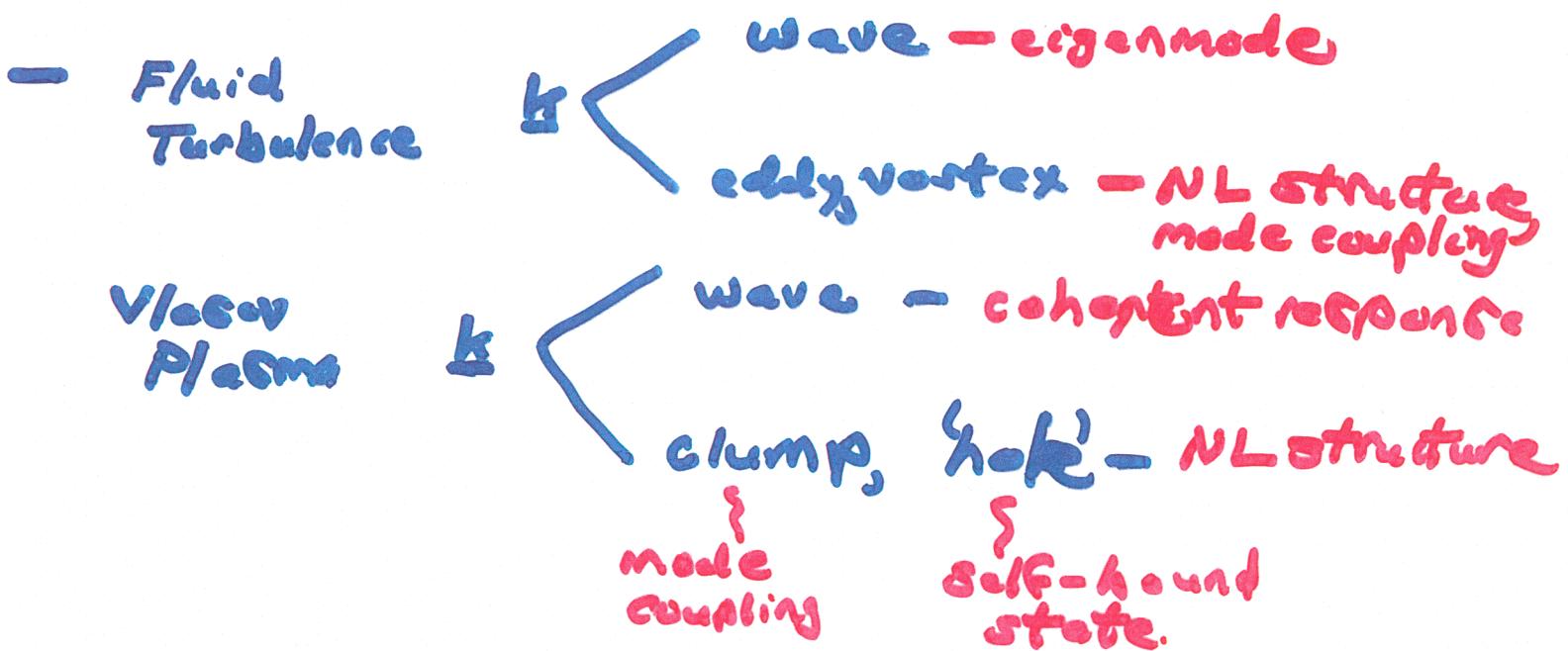
$$\frac{\partial N}{\partial t} + v_{gy} \frac{\partial N}{\partial x} - \frac{\partial}{\partial x} (k_y \langle v_y \rangle) \frac{\partial N}{\partial k_x} = \underline{\underline{\delta N + C(A)}}$$

$$\partial_t \langle v_y \rangle = -\mu \langle v_y \rangle - \partial_x \langle J_x J_y [N] \rangle \rightarrow$$

N.B. Symmetry: $\left\{ \begin{array}{l} \frac{dk_y/dt}{dk_x} = 0, \quad v_{gy} = \text{const.} \\ z_y = 0 \end{array} \right.$

II.) Theory of Phase Space Density

Granulation = 1D V/acev



N.B.: Scales: k^{-1} ; $\Delta V \sim 1/kT_e$
 $\sim (kT_e)^{1/2}/k$

Wave-particle resonance

- Granulation (Pure V/acev)

- portion of fluctuation not represented by response to \vec{E}

coherent incoherent

$$-\delta f_{k\omega} = f_{k\omega}^c + \tilde{f}_{k\omega}$$

$\sim R_E E_S$

→ everything else (granulation)

Calculate? Impact?

→ Is \tilde{F} 'real'?

- $\langle \delta F(1) \delta F(2) \rangle$ - 2 Pt, 1 Time Correlation

$$v_{\pm} = (v_1 \pm v_2)/2 \quad + \rightarrow \text{"slow"}$$

$$x_{\pm} = (x_1 \pm x_2)/2 \quad - \rightarrow k^{-1}, \Delta V$$

$$\left[\dot{x}_i + v_{\pm} \frac{\partial}{\partial x_{\pm}} + \frac{2}{m} (E(1) - E(2)) \frac{\partial}{\partial v_{\pm}} \right] \langle \delta F(1) \delta F(2) \rangle \\ = - \frac{2}{m} \langle E(1) \delta F(2) \rangle \frac{\partial v}{\partial v} + 1 \leftrightarrow 2$$

$$\Rightarrow (\dot{x}_i + T_{1,2}) \langle \delta F \delta F \rangle = S$$

$\stackrel{S}{\text{at evolution}}$

$\stackrel{\text{2 source}}{\text{(gradient relaxation)}}$

Key Point: $\lim \tau \rightarrow 2$

$$T_{1,2} \rightarrow 0, \quad \delta \rightarrow \text{finite}$$

, ∴ $\langle \delta F(1) \delta F(2) \rangle$ MUST diverge

But

$$\langle f'(1) f'(2) \rangle = \sum_{k, \omega} \left| \left(\frac{i}{\omega - \hbar \nu + i\tau_0} \right) E_k \frac{\partial \langle e \rangle}{\partial V} \right|^2$$

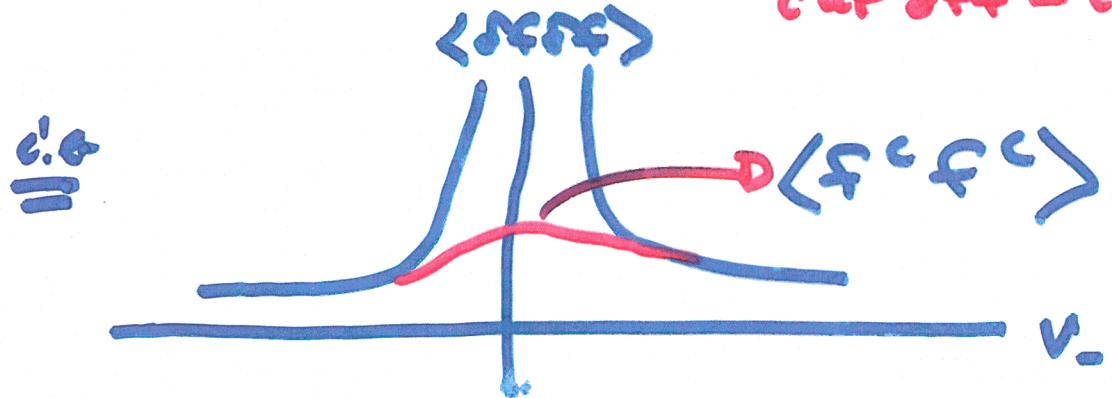
$$= \tilde{c}_v D_{\text{SL}} \left(\frac{\partial \langle e \rangle}{\partial V} \right)^2$$

$\therefore \langle d^* f d f \rangle - \langle f^* f \rangle \rightarrow \text{finite}$

"Something Else is at
work"

\rightarrow incoherent
correlation func
and cross term.

cut off - coll/abs.



Issues:

- How does granulation impact relaxation / inst. ability?
- Theoretical Approaches — How calculate something?

→ Impact on Relaxation - { Look Beyond Bump-on-Tail }

$$\frac{df}{dt} = 0 \Rightarrow \int dv \partial_t \langle \delta f^2 \rangle = \int dv \langle f \rangle \frac{\partial \langle f \rangle}{\partial t}$$

Fluctuation
Correlation
growth

$$= -\frac{2}{m} \int dv \langle E \delta f \rangle \frac{\partial \langle f \rangle}{\partial v}$$

relaxation
→ $S_{1,2}$

$$\partial_t \langle f \rangle = -\partial_v \langle E \delta f \rangle$$

$$\delta f = f^c + \tilde{f}$$

coherent

granulation

$$\therefore \partial_t \langle f \rangle = \partial_v D \partial_v \langle f \rangle - \partial_v \langle E \tilde{f} \rangle$$

$\alpha \alpha' \delta$ QLT

'Friction' or
'Drag'

with:

$$\sigma^2 \phi = -4\pi n_0 Z \int dv \delta f$$

$$\epsilon(k, \omega) \phi_{k, \omega} = \frac{4\pi n_0 Z}{k^2} \int dv \tilde{f} \rightarrow \text{relate } \tilde{f}, f^c \text{ and } \phi$$

Structural Similarity to Lenard-Balescu Theory!

• Relaxation, cont'd

$$\langle E \delta f \rangle \xleftarrow[F]{D} \delta(v - \omega/k)$$

particle velocity fluctuation phase velocity

- relaxations also "collisions" in 1D (test-particle, "field-particle")
- recall stable 1D plasma with discreteness (also $L = \Delta$)

1D collision \rightarrow conserved
 - momentum \rightarrow
 - energy

for like particles \rightarrow final state = initial state

$\therefore e-e$ diffusion \gg cancel.
 $e-e$ drag

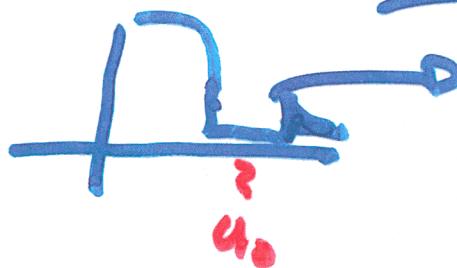
(stable \rightarrow over-saturated)

but, can drag on ions via Cerenkov

i.e. $\partial_t \langle f_e \rangle \sim \epsilon_{IM}^{\text{ion}} \langle \phi^2 \rangle$, etc.

• Alternatively

- Consider localized fluctuation
granule



i.e. 'hole'

$$\frac{d}{dt} (f_0 + \tilde{f}) = 0$$

$$\partial_t \langle \delta f^2 \rangle = -2 \frac{d}{dt} (f_0 \tilde{f})$$

$$\partial_t \int \langle \delta f^2 \rangle = -2 \frac{d}{dt} \int \underbrace{\nabla \tilde{f}}_{P_e} f_0'$$

P_e → granulation momentum

$$= -2 \frac{d P_e}{dt} f_0'$$

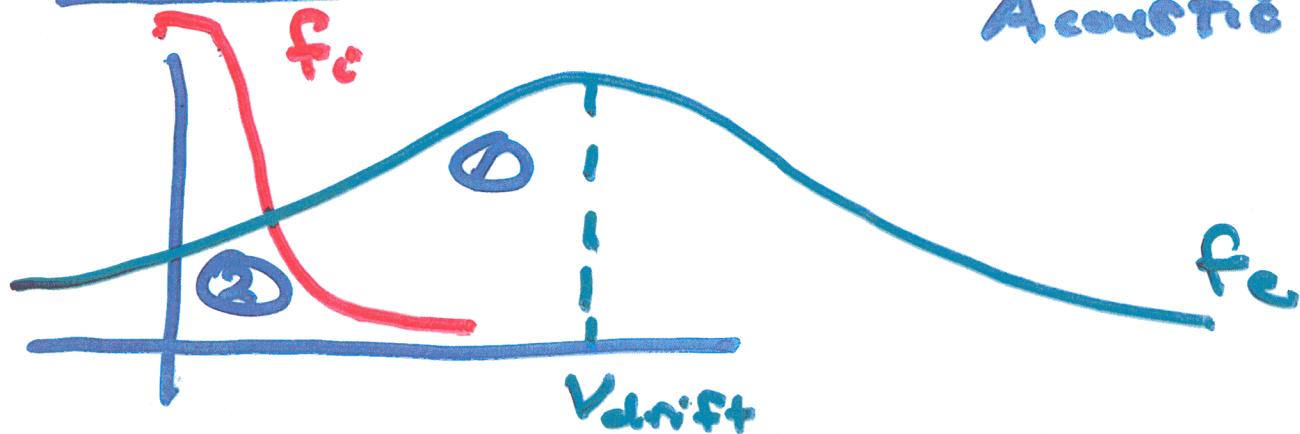
but:

$$\frac{d}{dt} (P_e + \underbrace{P_{\text{wave}} + P_i}_{\text{exchangers}}) = 0$$

① Single species \Rightarrow bump-on-tail
 $P_i \rightarrow 0$ slight enhancement of L.
(Liang, A.O.)

② Two species (Ion-Acoustic) \Rightarrow $\frac{d}{dt} \int \langle \delta f^2 \rangle = 2 \frac{d P_i}{dt} f_0'$
 $\sim \text{SIM.}$

- Implications : Current Driver Ion Acoustic



- ① - CDIA "Waves" - feel electron pressure L.Q.
- avoid ion L.Q.
- ② - CDIA "Granules" (electrons) - wave damped by ion L.Q.
- can exchange momentum with ions.
- Different Thresholds i.e. $V_d/V_{th,e}$, $\frac{m_e}{m_i}$, T_e/T_i
- R.H. Berman, et.al. '87
- fluctuations grow, f_e relaxes when CDIA instabilities stable
- growth non-linear

Granulations \Rightarrow Novel Relaxation Mechanisms !!

N.B. Energies in Vlasov Turbulence

QLT - (no mode coupling)

$$\partial_t (RPKED) + \partial_x (TWED) = 0$$

RPKED - resonant particle kinetic energy density

TWED - total wave energy density

$$\partial_t (TWED) = \gamma_h \sum_k \omega_k \frac{\partial E}{\partial \omega} / \frac{|E_k|^2}{\omega_k^2}$$

but: What happens at stationary state?

Granulations - (mode coupling)

$$\partial_t (RPKED) + \partial_x (TWED) + Q_{cc}^{\text{collisions}} = 0$$

→ allows stationary extraction of RP energy from 1 species to another

→ "anomalous resistivity" - CDA

- Vlasov Turbulence Theory / Granulations
 → Approach

a) Statistical

$$(\partial_t + T_{1,2}) \langle \delta f^2 \rangle = S_{1,2}$$

↓

$$\left\langle E(\mathbf{r}) \frac{\partial f(\mathbf{r}, \mathbf{v})}{\partial \mathbf{v}_i} \delta f(\mathbf{r}) \right\rangle$$

↳ also L-B theory,
 discussed.

↳ closure problem

- 'DIA' for Vlasov eqn. extractable \leftrightarrow
 dbl integrals - diff. Eqn.

↳ stochastic orbits \Rightarrow bivariate Fokker-
 Planck Eqn.

$$(\partial_t + \mathbf{v}_- \cdot \partial_{\mathbf{x}_-} - \partial_{\mathbf{v}_-} \cdot \mathbf{D}_- \cdot \partial_{\mathbf{v}_-}) \langle \delta f \delta f \rangle = S_{1,2}$$

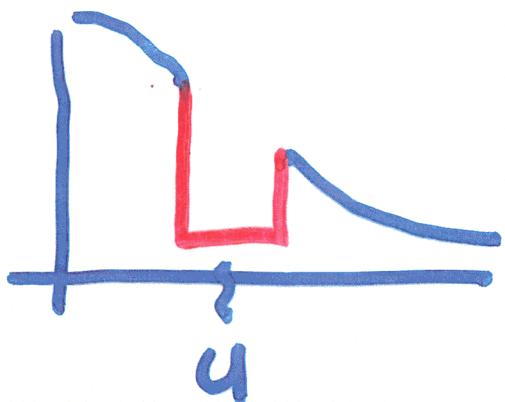
$$\mathbf{D}_- = \sum_{k, \omega} |E_k|^2 (1 - \cos(k \mathbf{x}_-)) R_{k\omega}$$

- $T_{1,2}$ set by orbit divergence: k -S entropy

- $\langle \delta f^2 \rangle$: divergence vs. Relaxation

- $x_- < (\bar{k}^2)^{-1/2}$, $v_- < \Delta v$ 'Clumps' -
 Many orbit divergence film

b) coherent - 'holes'



$$\langle f \rangle = f_m + \tilde{f}$$

$$u_0 \pm \Delta v$$

Then : $\epsilon(k, ku_0) = -\omega_p^2 \int_{u-\Delta v}^{u+\Delta v} \frac{\tilde{f} dv}{(\omega - kv)^2}$

$\left. \begin{array}{l} \\ \end{array} \right\}$
a/c' kinetic

Jeans instab.
Calculations

$$\gamma^2 = \frac{-\omega_p^2 \tilde{f} \Delta v}{\epsilon(k, ku_0)} - k^2 \Delta v^2$$

$\left. \begin{array}{l} \\ \end{array} \right\}$

Self - binding,
attracting
(a/c' Self Gravity)

$\left. \begin{array}{l} \\ \end{array} \right\}$
dispersion
via streaming

$\epsilon > 0 \rightarrow$
hole binds

$\epsilon < 0 \rightarrow$
'blob' binds

u vs v_{th} .

\rightarrow self-bound ; particle modes

\rightarrow a/c' Jeans Equilibrium;
B & K modes

\rightarrow skew PdF(\tilde{f})

\rightarrow observed in simulations

→ What of Higher Dimensions?

- with strong \underline{B} :

- becomes dynamics structure
ala' 1D

$$\omega - \omega_0 - k_{\parallel} V_{\parallel} \rightarrow \omega < \Omega$$

drift,
gyro
kinetics

$$\omega - k_{\parallel} V_{\parallel} - n\Omega \rightarrow \omega \gtrsim \Omega$$

Much of story persists...

- \perp direction \rightarrow spatial vortex

$$\underline{V_E} = \frac{\underline{E} \times \underline{B}}{B^2}, \quad \underline{V_E} \cdot \nabla f(V_{\parallel}^2 + 2\phi) = 0$$

- so $\begin{cases} \parallel \rightarrow \text{phase space} \\ \text{vortex } z, V_{\parallel} \\ \perp \rightarrow \text{spatial vortex, moden' etc.} \end{cases}$

- several applications examined.

III.) Granulation in Wave Kinetics - 'Modulational' Turbulence Dynamics

- Consider 1D Langmuir Problem (simple)

$$\frac{\partial}{\partial t} \tilde{N} + v_g \frac{\partial \tilde{N}}{\partial x} - \frac{\partial}{\partial x} \omega_p \delta \rho \frac{\partial \tilde{N}}{\partial k} = \frac{\partial \omega_p \delta \rho}{\partial x} \frac{\partial \langle N \rangle}{\partial k}$$

$\frac{\partial}{\partial t}$
dispersive
propagation
 $\frac{\partial}{\partial x}$
refraction
 $\frac{\partial}{\partial k}$
relaxation

$$\frac{\partial^2 \rho}{\partial t^2} + \frac{\partial \rho}{\partial t} = - \frac{\partial^2}{\partial x^2} [c_s^2 \rho + f_{dk} \omega \langle N \rangle]$$

$\frac{\partial}{\partial t}$
dissipation
i.e. ion-Landau
damping
 $\frac{\partial}{\partial x}$
acoustic
coupling
 $\frac{\partial}{\partial k}$
plasma
radiation
pressure.

- sub or supersonic : $(v_z/v_x)^2 \leq c_s^2$

subsonic \rightarrow NLS

supersonic \rightarrow energy coupled to waves
and cons.

- far less intensively studied

- chaotic rays \rightarrow traditional approach
via soliton theory, etc.
dubious.

- saturated state?

But Also

- granulations form in N :

$$\left\langle \frac{\partial \rho}{\partial x} \frac{\partial}{\partial k} \tilde{N} \tilde{N} \right\rangle \rightarrow \left\langle \left(\frac{\partial \rho(c)}{\partial x} - \frac{\partial \rho(s)}{\partial x} \right) \frac{\partial}{\partial k} \tilde{N}(c) \tilde{N}(s) \right\rangle$$

$\rightarrow 0 \quad 1 \rightarrow 2$

c.e. \tilde{N} $\begin{cases} \text{accelerated by} \\ \text{conserved along rays} \end{cases}$

\Rightarrow mode coupling enters wave kinetics evolution \rightarrow strongly localized packets (proto-cavitation)

- N granulations: $\tilde{N} = N^c + \hat{N}$

but plasmon relaxation/depletion \Rightarrow I.A scale structures (cavitation) (NL noise in reflection)

$$\frac{\partial_t}{\partial t} \langle N \rangle = \frac{\partial}{\partial k} \left\langle \frac{\partial \tilde{\rho}}{\partial x} \tilde{N} \right\rangle \quad \tilde{N} = N^c + \hat{N}$$

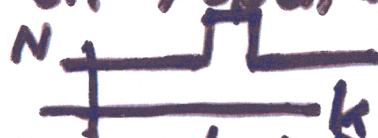
$$= \frac{\partial}{\partial k} \left[D_k \frac{\partial \langle N \rangle}{\partial k} + \left\langle \frac{\partial \tilde{\rho}}{\partial x} \hat{N} \right\rangle \right]$$

$$D_k = \sum_{\mu} \omega_{\mu}^2 I^2 |\tilde{P}_2|^2 R (\Omega - \omega_{\mu}) \quad \hookrightarrow \text{friction}$$

\hookrightarrow random refraction

→ What does \tilde{N} Mean?

- Bk/Ballistic Mode in Population
of Wave Packets



B

- 'Protocavitation' — self-bound structure

$$\text{i.e. } \left[\Omega^2 / z^2 - c_s^2 + i\frac{\omega}{\tau} \right] = - \int dk \frac{z \omega_p^2}{(\Omega - z \gamma_g(k)) \Delta k} \tilde{N}$$

**Screening by IA
response**

$$\rightarrow \epsilon \in (\Omega, z)$$

force induced by
wave packet on \hat{e}

$$\epsilon(\Omega, z) = \omega_p^2 z^2 \int_{k_0 - \Delta k}^{k_0 + \Delta k} dk \frac{\partial \gamma_g(k)/\partial k}{(\Omega - z \gamma_g)^2} \tilde{N}$$

⇒

$$\gamma^2 = - \frac{z^2 \omega_p^2 \tilde{N} \Delta k \gamma'}{\epsilon(\Omega, z)} - z^2 \gamma'^2 \Delta k^2$$

'self-binding'

**Packet
dispersion**

$$\epsilon > 0 \rightarrow \Omega/z > c_s \rightarrow \tilde{N} < 0$$

$$\epsilon < 0 \rightarrow \Omega/z < c_s \rightarrow \tilde{N} > 0$$

**Packet
can bind
by own
radiation**

- Relaxation : { New Route to Growth
in Langmuir Turbulence. }

$$\frac{d}{dt} N = \frac{d}{dt} (N_0 + \tilde{N}) = 0$$

$$\partial_t \int dk \tilde{N}^2 = -2 \frac{d}{dt} \int dk N_0 \tilde{N}$$

\tilde{N} localized : within $k_0 \pm \Delta k \Rightarrow$

$$\partial_t \int dk \tilde{N}^2 = -2 \frac{d}{dt} \int dk (k - k_0) \tilde{N} \left. \frac{\partial N_0}{\partial k} \right|_{k_0}$$

but: $P_{PL} = \tilde{N}(k - k_0) \equiv$ plasma momentum

$$\text{i.e. } P = k N = k \frac{\partial N}{\partial k} \propto P^2$$

$$\therefore \partial_t \int dk \tilde{N}^2 = -2 \frac{d}{dt} P_{PL} \left. \frac{\partial N_0}{\partial k} \right|_{k_0}$$

but momentum budget \Rightarrow

$$\frac{d}{dt} (P_{PL} + P_{IA} + P_{cusp}) = 0$$

IA wave cusp f₀

$$\text{Now: } \partial_t (P_{PL} + P_{IA} + P_{Ions}) = 0$$

if $\dot{P}_{Ions} = 0$ c.e. no dissipative momentum input to particles

$$\therefore \partial_t P_{PL} = -\partial_t P_{IA} \rightarrow \text{'usual'}$$

subsonic: $\partial_x [\rho C_s^2 + \tilde{\epsilon}] = 0$ ^{Lagrangian} Turbulence
IA stationary $\Rightarrow \tilde{\epsilon}$ adjusts \leftrightarrow NLS

supersonic: Plasmon momentum coupled to Ion-Acoustic Wave.

if $\dot{P}_{IA} \approx 0 \Rightarrow$ ion acoustic wave damped (c.e. L.D.)

$$\partial_t (P_{PL} + P_{Ions}) = 0$$

\Rightarrow (Plasmon momentum) coupled to ions.

New route to \tilde{N} growth, $\frac{\partial N_e}{\partial k}$ relaxn.

$$\text{for } \partial_t (P_{PL} + P_{con}) = 0$$

$$\partial_t \int dk \tilde{N}^2 = 2 \frac{\partial P_i}{\partial t} \frac{\partial N}{\partial k}$$

but: $\frac{\partial A_i}{\partial t} = \frac{1}{m} \langle \hat{E} \hat{n}_i \rangle$

$$\Rightarrow = - \int D_i^{QL} \frac{\partial \langle f \rangle}{\partial v} dv$$

$$\therefore \partial_t \int dk \tilde{N}^2 = - \left. \frac{\partial N}{\partial k} \right|_{k_0} \int dv D_i^{QL} \frac{\partial \langle f \rangle}{\partial v}$$

$$\approx - \frac{\partial N}{\partial k} \left\langle D_i^{QL} \frac{\partial \langle f \rangle}{\partial v} \right\rangle$$

$$> 0 \qquad < 0$$

\Rightarrow no 'free lunch' $\rightarrow \frac{\partial N}{\partial k} > 0$ required

(population inversion)

\rightarrow packet growth \sim Ion Landau Damping.

\rightarrow distinct from usual Langmuir instability.

- Theoretical Approaches

①

→ bi-variate Fokker - Planck
Equation for $\langle \hat{N} \hat{N} \rangle$

$$\rightarrow (\partial_t + T_{1,2}) \langle \hat{N} \hat{N} \rangle = S_{1,2}$$

$\left\{ \begin{array}{l} \text{ray divergence} \\ \text{due to } \underline{\text{ray chaos}} \end{array} \right.$

$\left\{ \begin{array}{l} \langle \partial_x \hat{N} [N + \hat{N}] \rangle \\ \downarrow \\ k\text{-diffusion and friction} \end{array} \right.$

easily generate:

- correlation func
- relaxation rate

[N.B. Cancellation]

— Langmuir 'stable': $\partial_t \hat{N}^2 \sim N^{2.01}$

new route to $N^{\text{pl.}}$ relaxation
(const. wave momentum)

②

→ single packet - as earlier.

→ The Differences.

- Vlasov plasma - $\propto \Delta V$

Packets - $\propto V_g' \Delta k$

$\therefore V_g' = 0 \rightarrow$ ray accumulation points
(caustic' pts.)

\rightarrow dispersion weak

\rightarrow no counterpart
in Vlasov plm.

$\rightarrow V_g' \Delta k + \frac{V_g''}{2} (\Delta k)^2 \Leftrightarrow$

$\Delta k \sim (\tilde{N})^{1/4}$ instead $(\hat{N})^{1/2}$

\rightarrow stronger interaction \Leftrightarrow wh. NL
theory dubious

\rightarrow seeds: soliton formation, soliton
gas

- higher D: caustics \rightarrow fronts/shocks.

→ Future Plans

- Ray accumulation
caustic \rightarrow problem
 ⇒ quantify WTT breakdown
■ statistics? ↓
relation to collapse?
- accommodate diffraction \rightarrow
 - GTD, α/α' Keller !
 - impact on $\langle \hat{N} \hat{N} \rangle$ structure
- higher dimensions ?!
- quantitative implication for Zonal flow problem.