Applications of the Nonlinear Schrödinger Equation to Optical Fiber Communications

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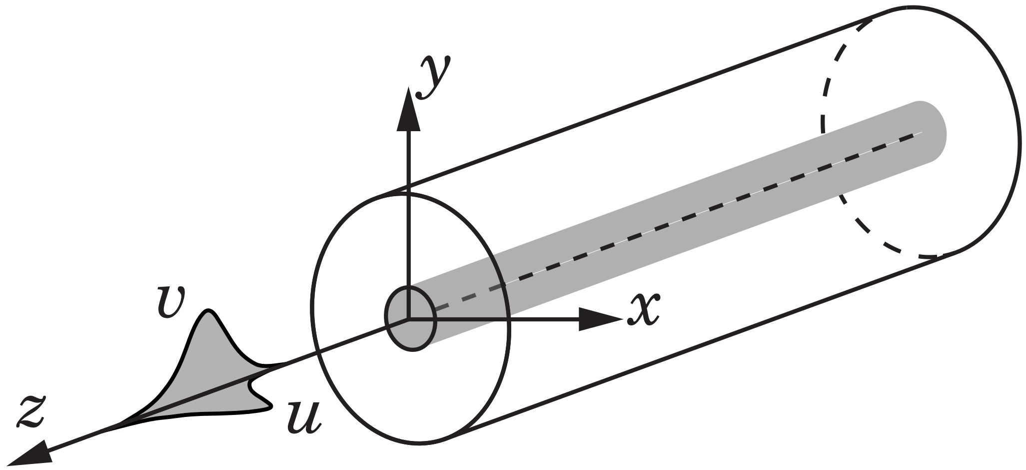
The key component of high-speed communications



Single mode optical fibers[1]:

- inner core diameter $\approx\; 2-5\,\mu$m
- outer cladding diameter $\approx\; 100\,\mu$m
- wavelength $\approx\; 1.3-1.5\,\mu$m
- *weakly guiding*:

$$\frac{n_{\mathrm{core}} - n_{\mathrm{clad}}}{n_{\mathrm{clad}}} \ll 1$$

Mathematical description of signals in an optical fiber

Nonlinear Schrödinger (NLS) equation[1, 2]:

$$\frac{\partial u}{\partial z} = -\frac{i}{2}k''(\omega)\frac{\partial^2 u}{\partial t^2} + i\gamma|u|^2 u\,,$$

- u is signal envelope, $|u|^2$ is power
- $\gamma = \omega n_2 / cA_{\text{eff}}$ is nonlinear coefficient (units $\text{m}^{-1}\text{W}^{-1}$)
- $n_2 =$ nonlinear index coeff.; $A_{\text{eff}} =$ effective mode area,

$$A_{\text{eff}} = \frac{\left[\iint |F(x,y)|^2\,dx\,dy\right]^2}{\iint |F(x,y)|^4\,dx\,dy}\,,$$

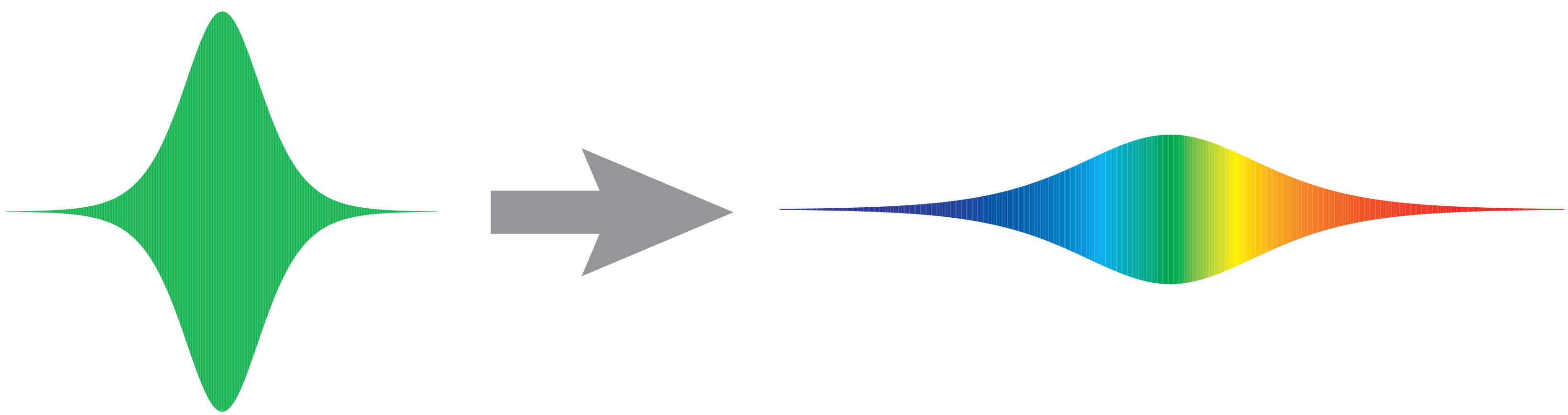
  where $F(x,y)$ is transverse mode shape
- $k''(\omega)$ is the dispersion coefficient

The first term in the NLS equation: dispersion

$k''(\omega)$ is the dispersion coefficient

$$\frac{\partial u}{\partial z} = -\frac{i}{2} k''(\omega) \frac{\partial^2 u}{\partial t^2}$$

2nd derivative makes pulse widen (disperse) with distance since different frequencies have different group velocities

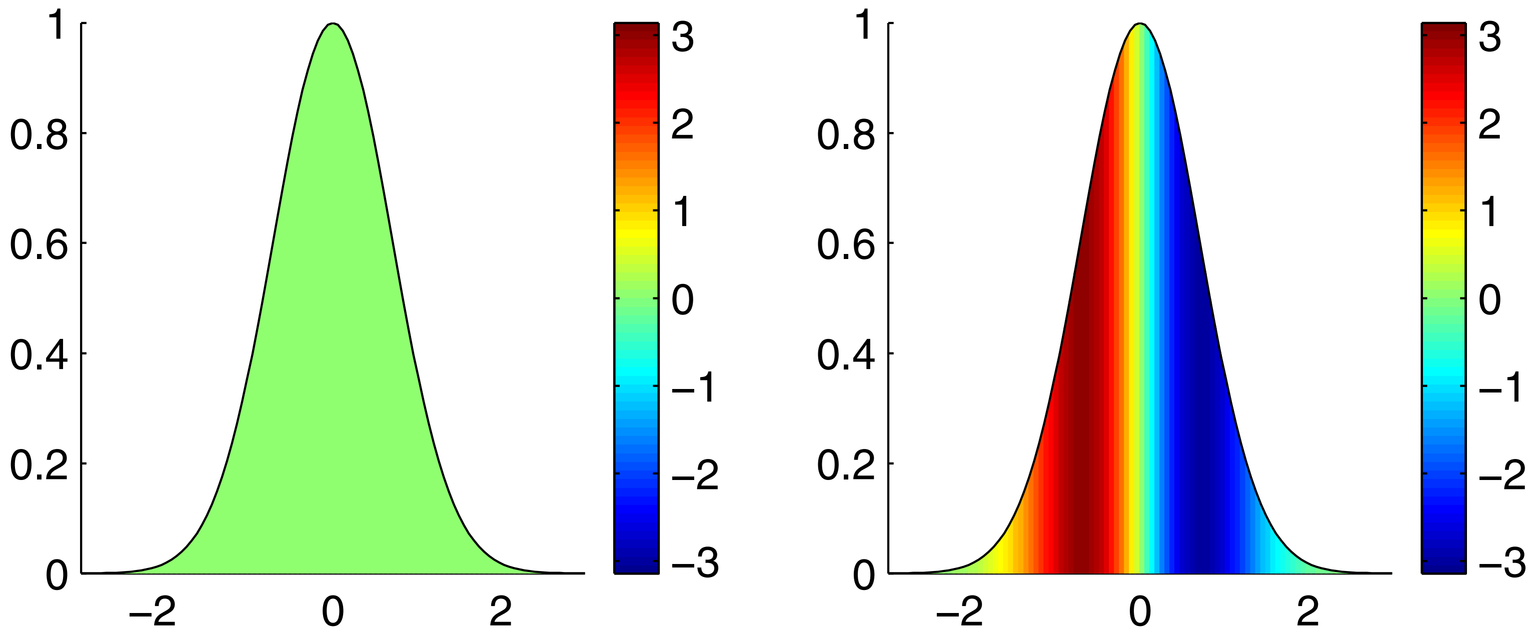


$k''(\omega) > 0$, normal/defocusing; $k''(\omega) < 0$, anomalous/focusing

The second term in the NLS equation: nonlinearity

$$\frac{\partial u}{\partial z} = i\gamma |u|^2 u$$

- Intensity-dependent phase rotation
- Creates new signal frequencies:



Balance of dispersion + nonlinearity produce solitons

Dimensionless NLS (anomalous dispersion):

$$i\frac{\partial u}{\partial z} + \frac{1}{2}\frac{\partial^2 u}{\partial t^2} + |u|^2 u = 0$$

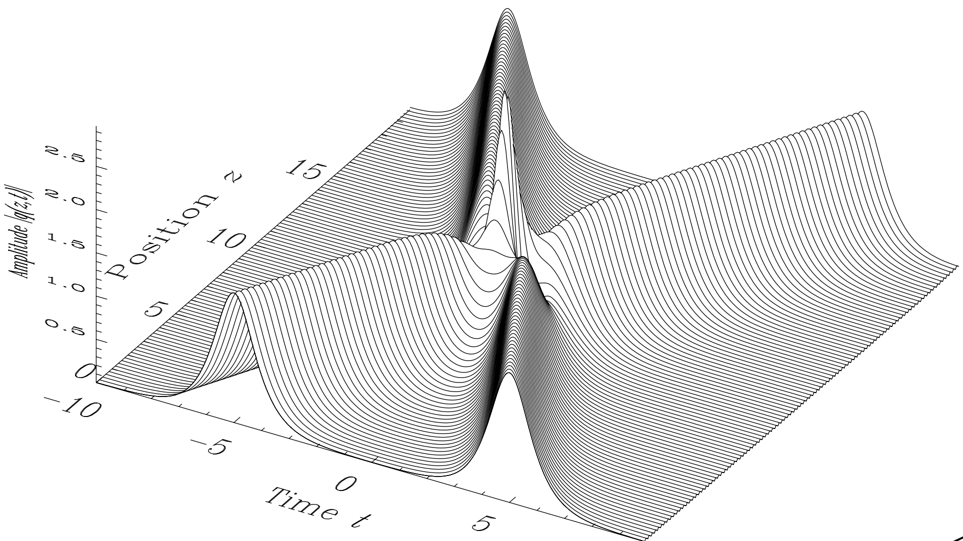
soliton — nonlinearity cancels dispersion; robust to disturbances:

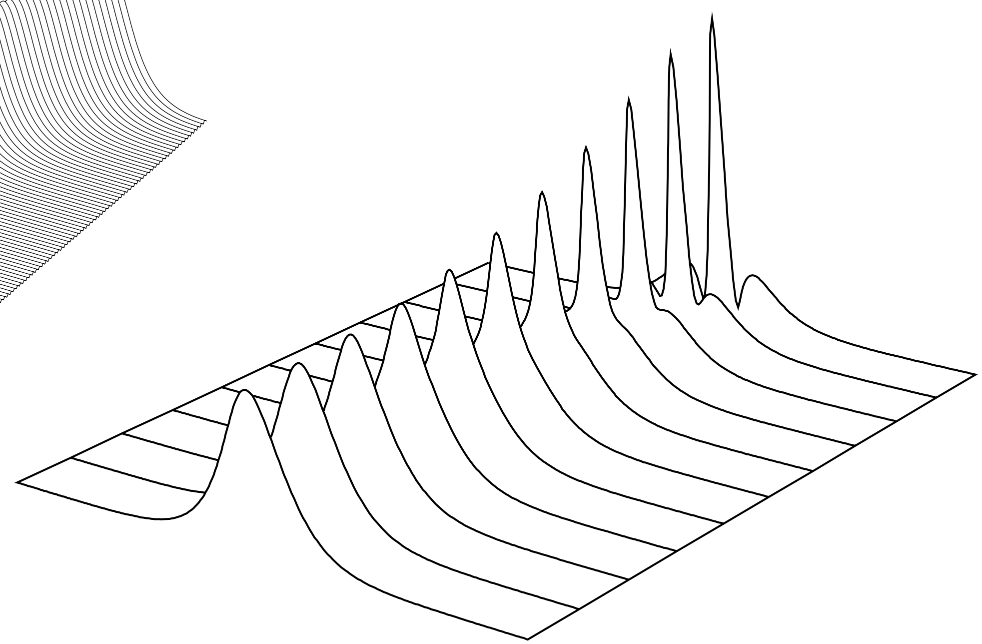
$$u = A\,\mathrm{sech}[A(t-T-\Omega z)]e^{i[\Omega t + \frac{1}{2}(A^2-\Omega^2)z+\varphi]}$$

- made practical by laser (1960), & making of low-loss fiber
- NLS exactly solvable by the inverse scattering transform: Zakharov and Shabat, 1971 [3]
- NLS proposed for fibers by Hasegawa and Tappert, 1973 [4]
- experimental verification by Mollenauer, 1983 [5]

Insight from more complicated NLS solutions

Soliton interaction: wavelength-division-multiplexing (WDM) and collision-induced timing shifts

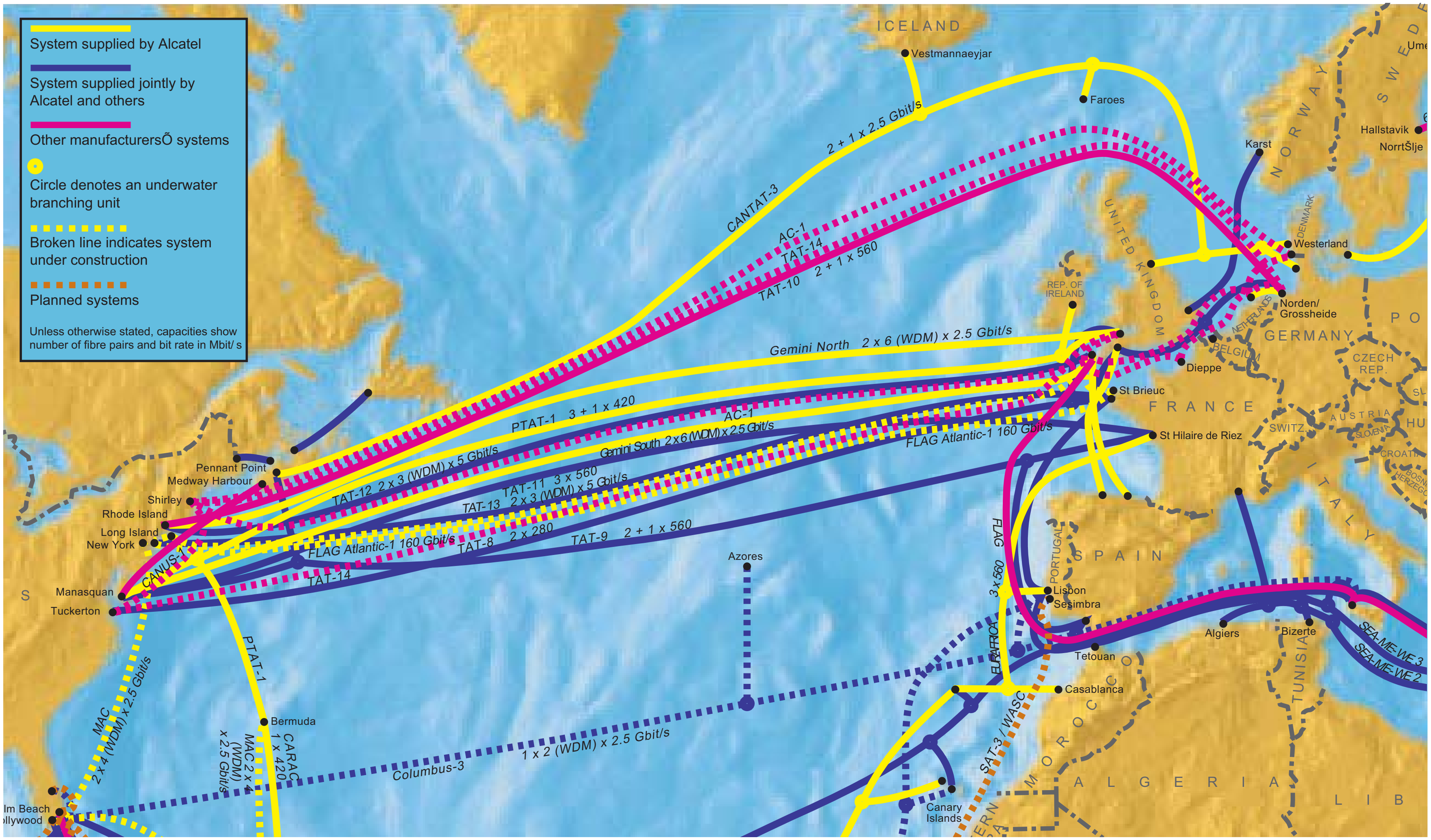




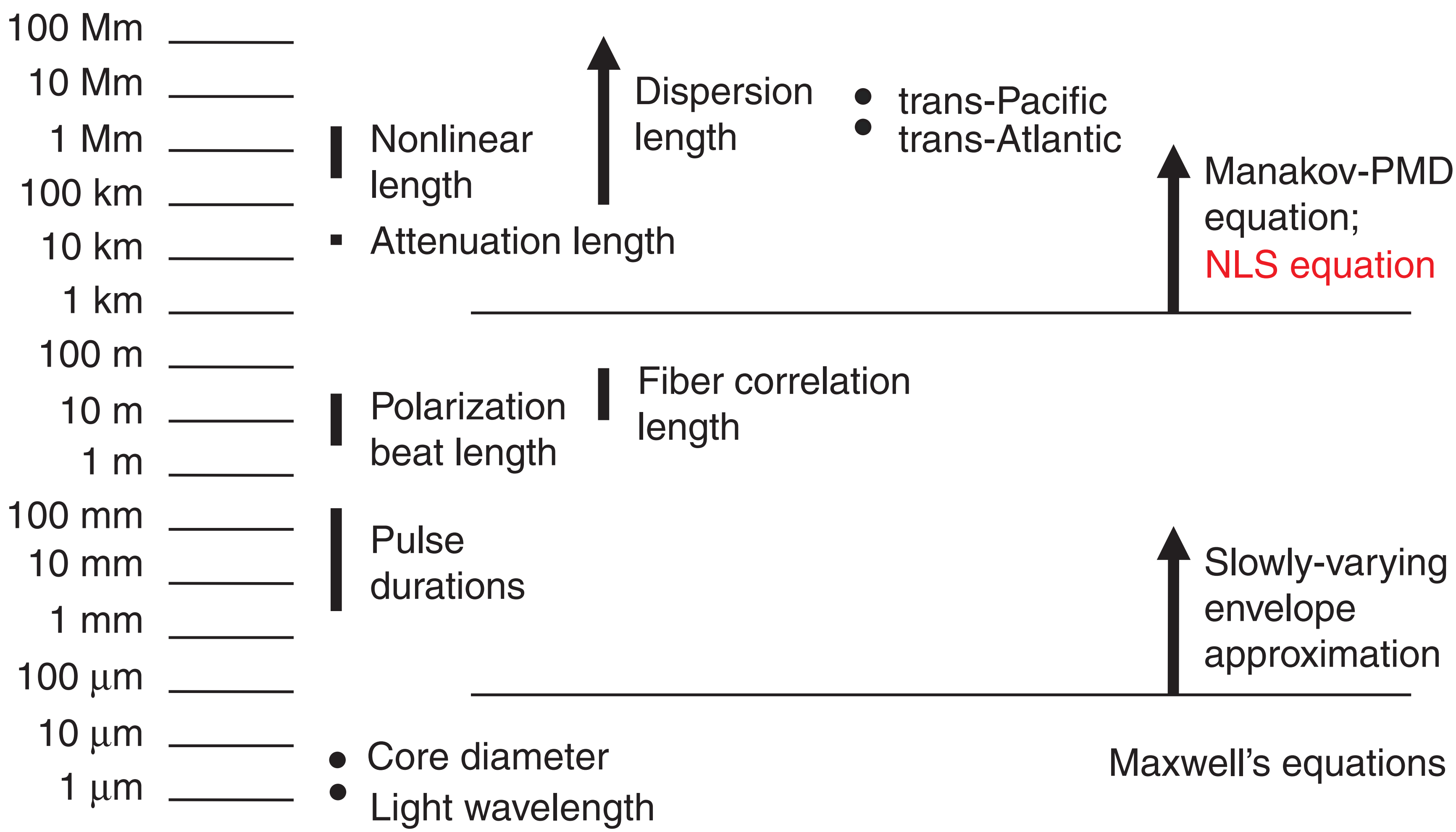
Bound *N*-soliton: model of pulse compression

The global significance of the NLS equation

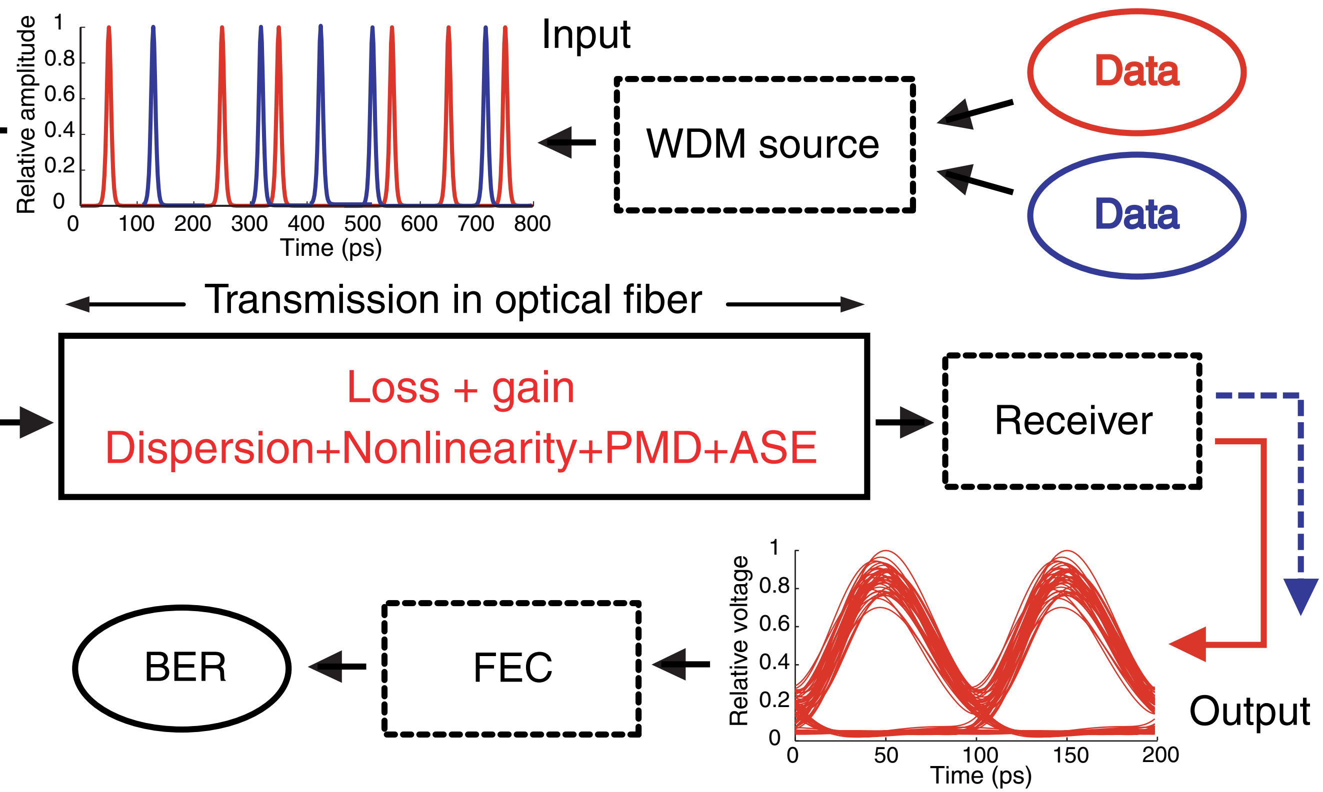
Optical Fibre Submarine Systems
North Atlantic



Transmission system length scales [6]



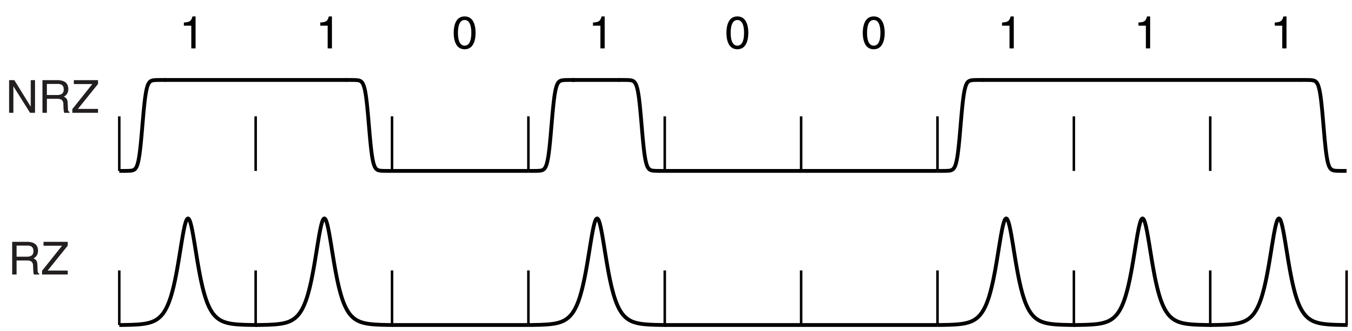
The elements of an optical transmission system



WDM, wavelength-division multiplexing
PMD, polarization mode dispersion
ASE, amplified spontaneous emission noise
FEC, forward error correction
BER, bit-error ratio

Note here *amplitude shift keying* is shown

Some examples of transmission formats



Non-return-to-zero (NRZ):

- work with low powers and small dispersion
- lots of engineering experience

Return-to-zero (RZ) and/or solitons:

- soliton only when nonlinearity and dispersion balance
- lower-powered pulses: chirped return-to-zero (CRZ)

Other formats: differential phase-shift keying,. . .

Impairments in optical communication systems

Solitons:

- with loss/amplification, soliton collisions become inelastic $\Rightarrow$ permanent frequency shifts, resonant four-wave mixing

NRZ and CRZ:

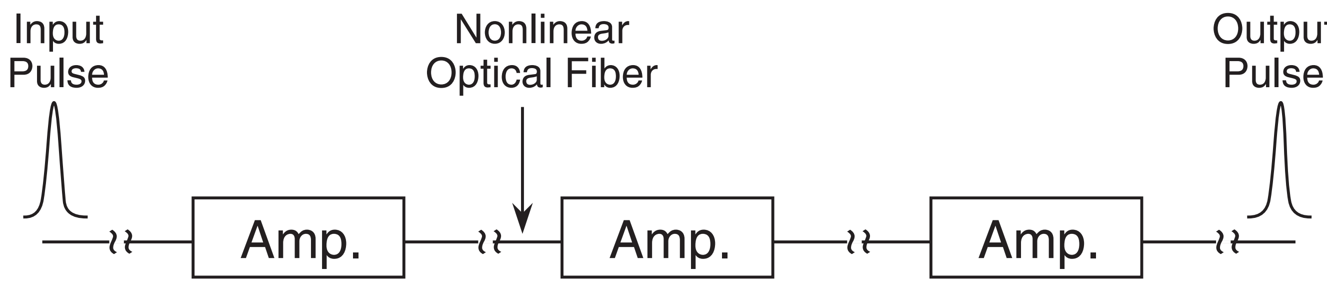
- need to compensate for accumulated dispersion
- overcoming noise with larger signal powers $\Rightarrow$ nonlinearity

All formats:

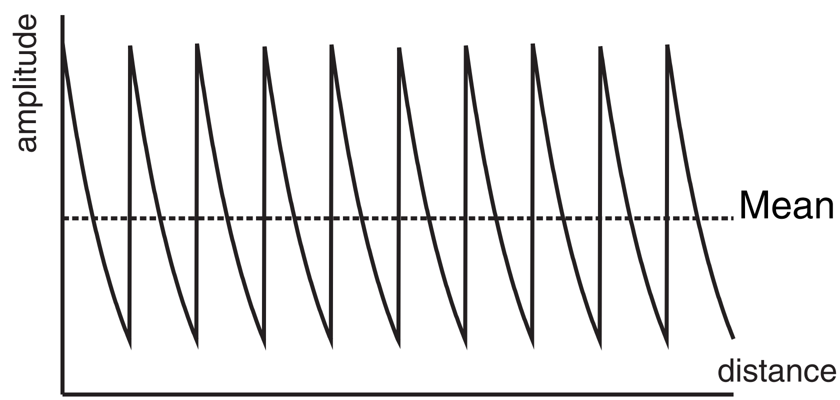
- amplifier noise $\Rightarrow$ finite signal-to-noise ratio
- net dispersion + noise $\Rightarrow$ Gordon-Haus timing jitter
- random birefringence $\Rightarrow$ polarization-mode dispersion

Nonlinearity, dispersion, noise and polarization effects

Loss is compensated by amplifiers



- Erbium-doped fiber amplifiers (EDFAs), periodically spaced (every 50 km or so), are ideal at 1.55 μm
- power loss $\alpha = 0.24$ dB/km; $\Gamma = (\alpha/20) \ln 10$



Raman (distributed) amplification now also used

Side effects due to the amplifiers

Transformed NLS (due to Hasegawa and Kodama):[2, 7-9]

$$\frac{\partial \bar{u}}{\partial z} = \frac{i}{2}\frac{\partial^2 \bar{u}}{\partial t^2} + i a^2(z/\epsilon)|\bar{u}|^2 \bar{u}$$

- $a^2(z/\epsilon)$ due to power variations
- Can be replaced by its average (H&K), giving NLS propagation for the mean of $\bar{u}$ (to leading order)

But,

- Amplifiers amplified spontaneous emission (ASE) noise produces jitter in soliton parameters A, φ, Ω and T
- Jitter leads to transmission errors

The NLS equation with additive noise [10, 11]

$$i\frac{\partial u}{\partial z} + \frac{1}{2}\frac{\partial^2 u}{\partial t^2} + |u|^2 u = \sum f_n(t)\delta(z - nz_a)\,.$$

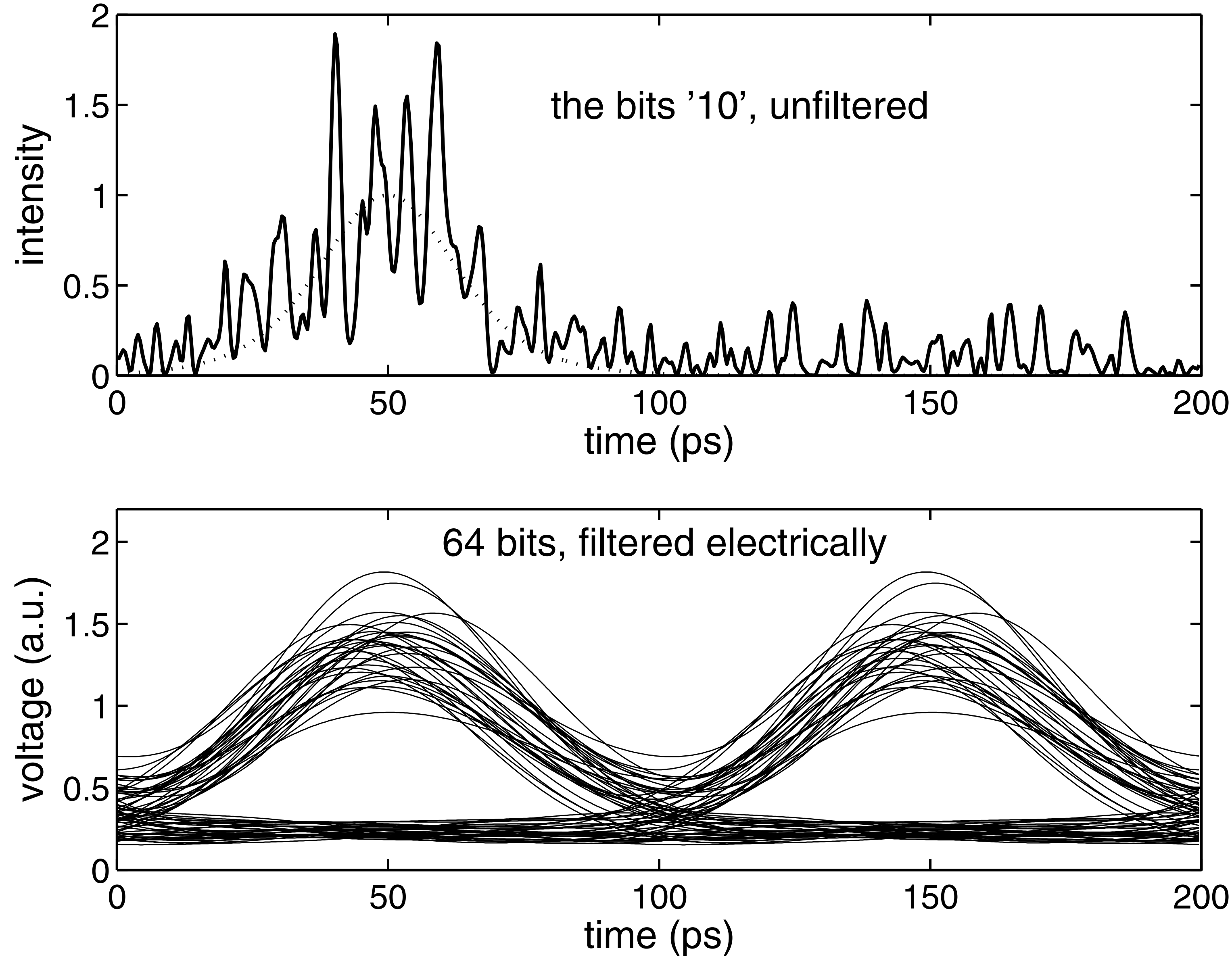
Here, $f(t)$ is Gaussian white noise added at each amplifier

$$\langle f_i(t) f_j^*(t')\rangle = \frac{(G-1)^2}{G\ln G}\frac{\eta_{\text{sp}} T_w \gamma}{|\beta''|}\delta(t-t')\delta_{ij}\,.$$

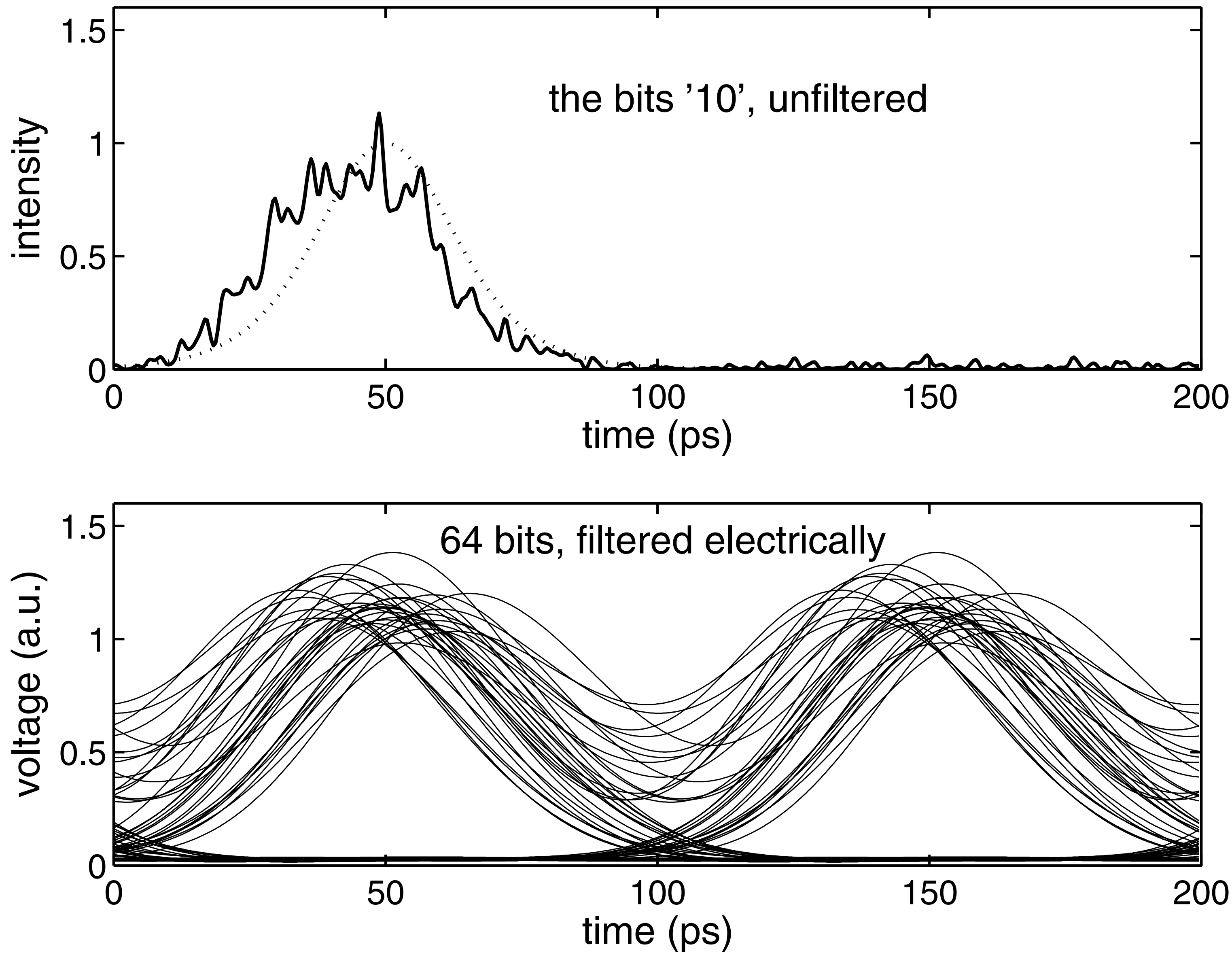
G = amplifier gain; η_{sp} = spontaneous emission factor;
T_w = pulse width; γ, β'' = nonlinear, dispersion coefficients

(Actually, noise must be a version with finite spectral extent)

Amplitude jitter example



Timing jitter example



Filtering to reduce timing jitter [12-17]

Add frequency reference to stop growth of jitter

$$F(\omega) \approx F(0) + \frac{1}{2}F''(0)\omega^2 + \ldots \Leftrightarrow F(0) - \frac{1}{2}F''(0)\frac{\partial^2}{\partial t^2} + \ldots$$

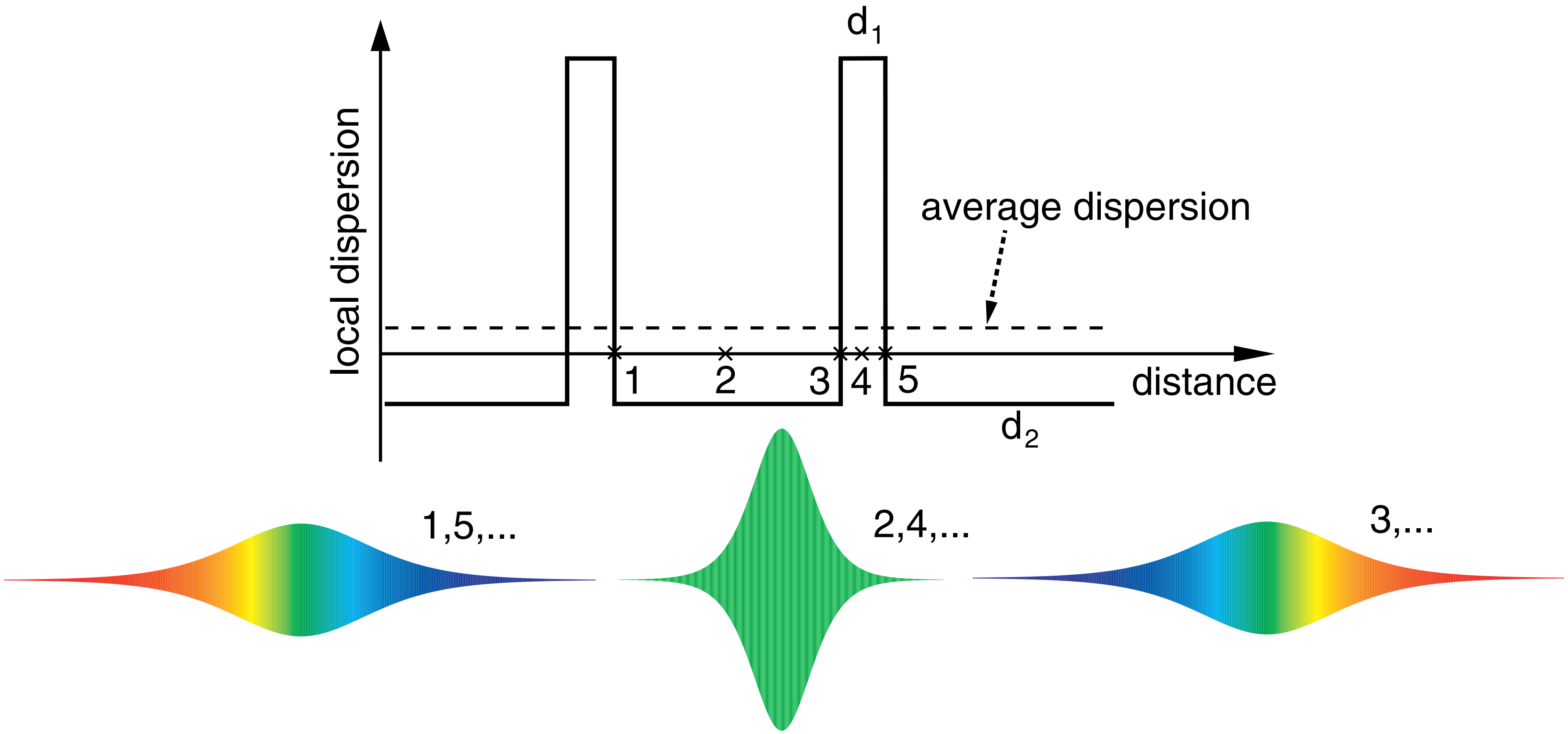
$$\Rightarrow \quad \frac{\partial u}{\partial Z} = i\frac{1}{2}\frac{\partial^2 u}{\partial T^2} + i|u|^2 u + \alpha u + \beta \frac{\partial^2 u}{\partial T^2}$$

- filtering acts like diffusion $\Rightarrow$ Ginzburg-Landau equation
- extra gain needed to compensate diffusive loss
- solitons stabilized to frequency and amplitude fluctuations, but direct timing noise still present

Need for stable pulses (in fiber loop): fiber lasers

Dispersion management

Periodic concatenation of fibers with alternating dispersion



Dispersion map $=$ specific choice of parameters ($d_{1,2}$, z_a, $z_{1,2}$)

Dispersion management (continued)

NLS equation with rapidly varying dispersion

$$\frac{\partial u}{\partial z} = \frac{i}{2} d(\frac{z}{z_a}) \frac{\partial^2 u}{\partial t^2} + g(\frac{z}{z_a}) |u|^2 u$$

For all formats:

- low average dispersion reduces Gordon-Haus jitter
- high local dispersion reduces four-wave mixing

For non-soliton pulses:

- dispersion compensation
- compression/expansion cycle reduces peak nonlinearity

For solitons:

- power enhancement further reduces Gordon-Haus timing jitter

Analytical/numerical methods for dispersion management

The same factors which make dispersion management so useful also make it so much harder to analyze

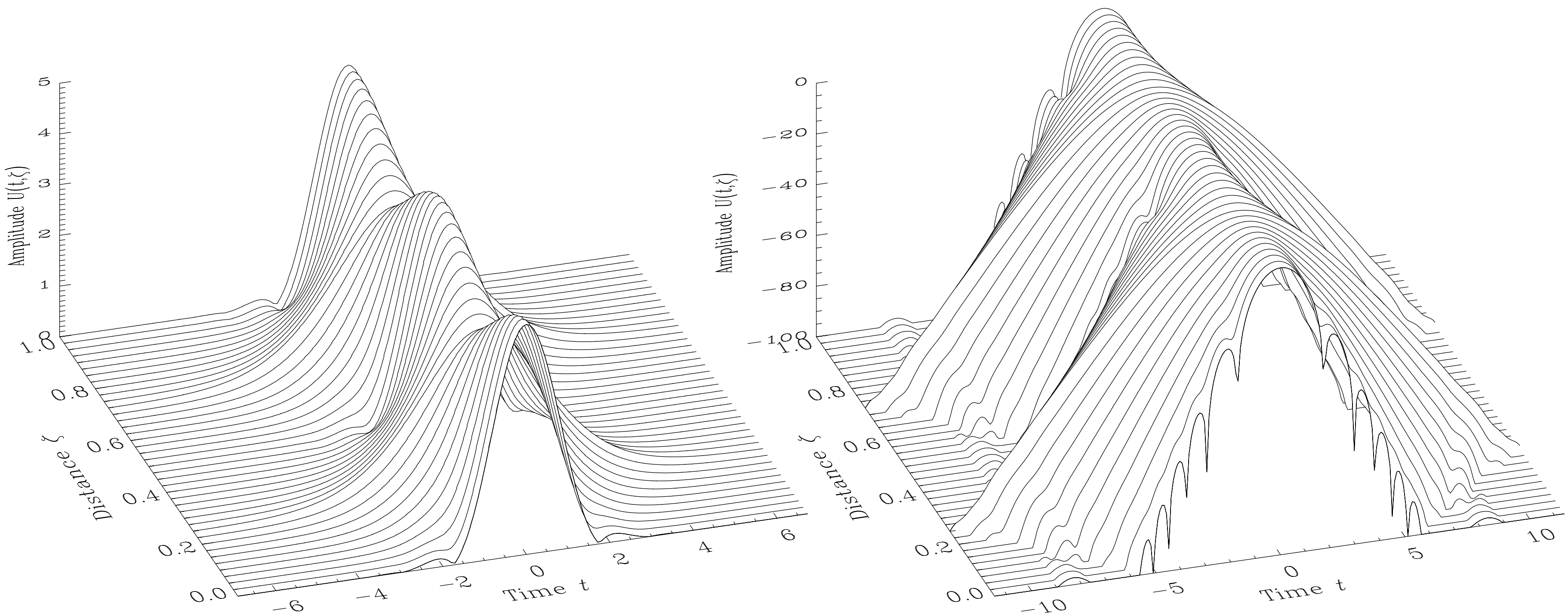
One has a PDE with large and rapid perturbations

Analytical/numerical methods:

- Lie transform, multiple scales or averaging methods [18-26]
- Variational or moment methods [27-30]
- Numerical simulations [31-34]

Reviews: [35-39]

Breathing of the full dispersion-managed (DM) pulse



DM solitons recover their profile stroboscopically (up to a phase)

Non-soliton pulses have non-periodic evolution

Radiation loss of DM solitons [40]

Dispersion-managed NLS without loss and gain:

$$\frac{\partial u}{\partial Z} = \frac{i}{2}\sigma\left(\frac{Z}{\epsilon}\right)\frac{\partial^2 u}{\partial T^2} + i|u|^2 u$$

Here σ is $O(1) \Rightarrow$ weak dispersion management

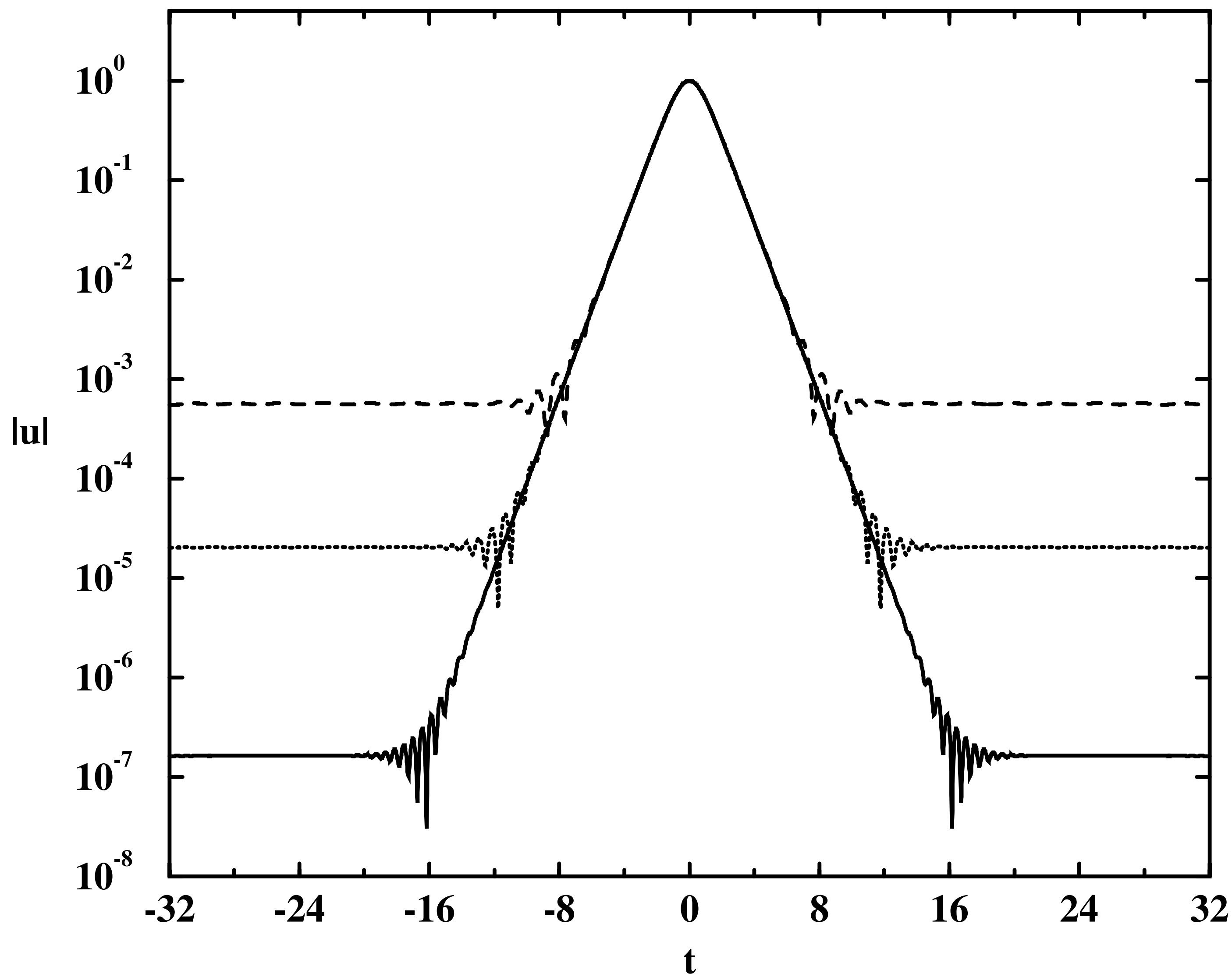
Formal asymptotic expansion; can show for small ϵ that

$$I_r = |u|^2_{t\to\pm\infty} \sim \frac{\pi}{4\epsilon}|C|^2 \exp\left(-2\pi^{3/2}/\epsilon^{1/2}\right)$$

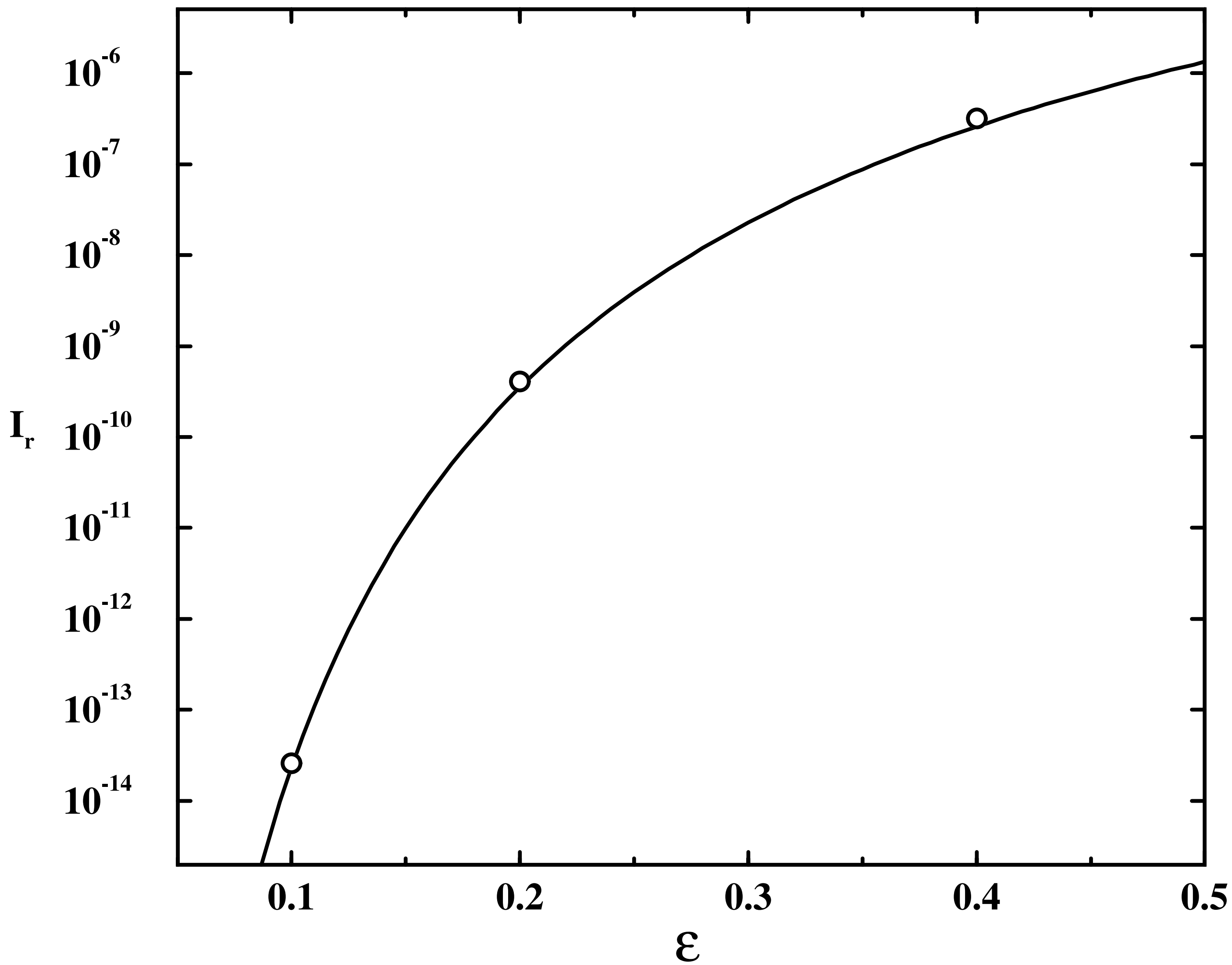
where C is an $O(1)$ constant determined numerically

Thus, radiation loss is beyond all orders

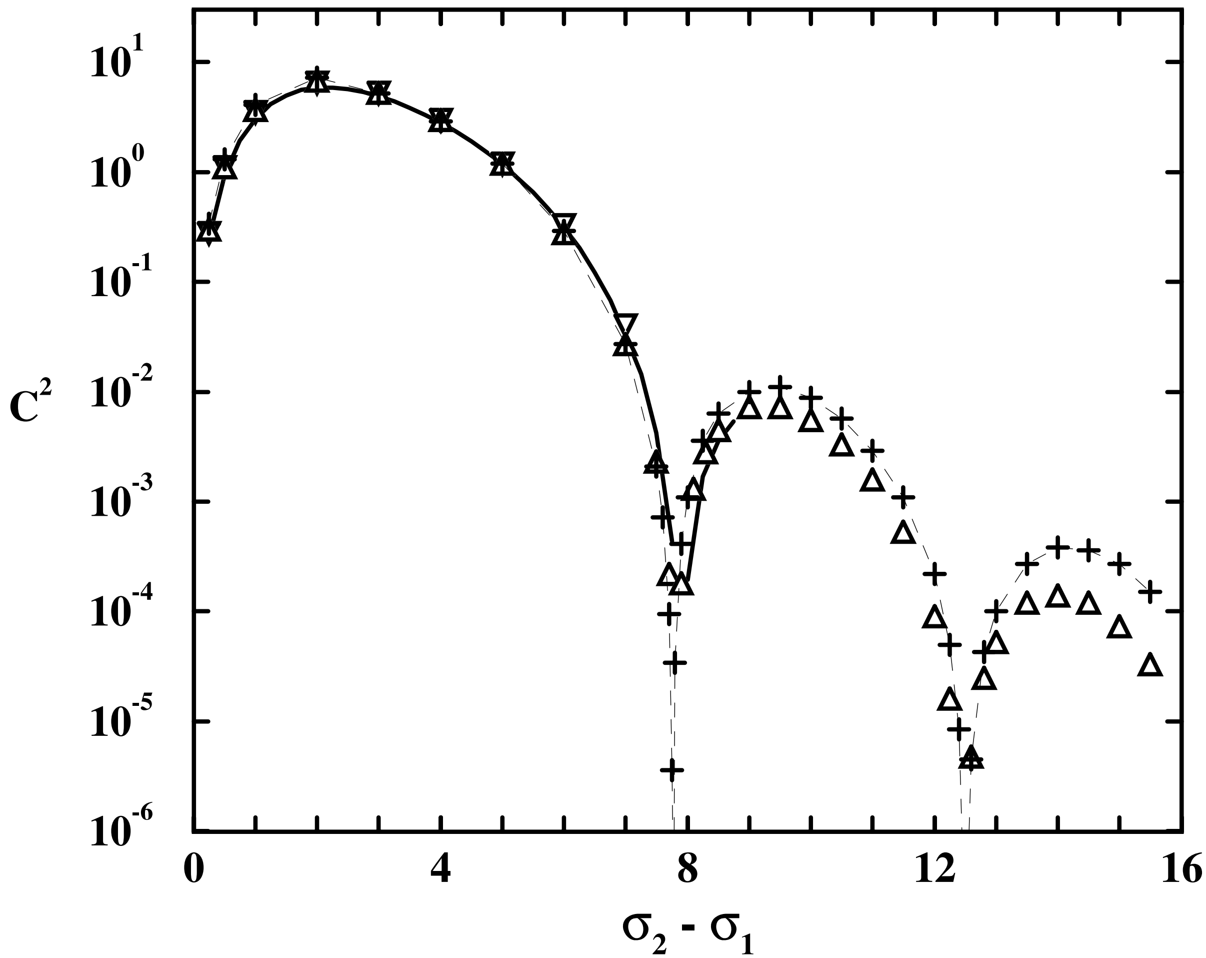
Results of refined numerical simulations for 2-step maps



Comparison between theory and simulations

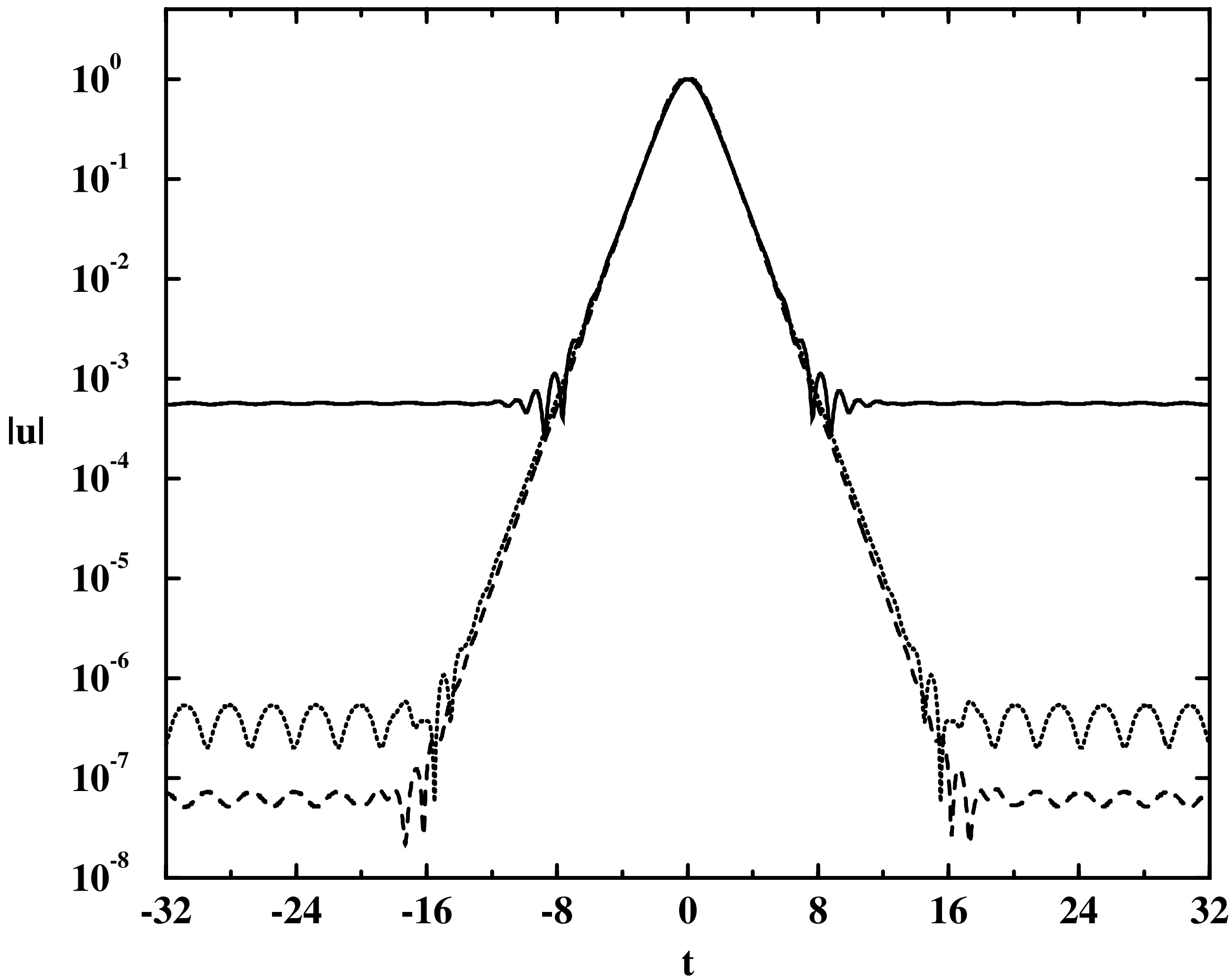


Coefficient C shows radiation nulls for some parameters



$\epsilon = 0.1$: triangles down; $\epsilon = 0.2$: triangles up; $\epsilon = 0.4$: + signs
theory: solid line

Simulations showing radiation nulls



$\sigma_1 - \sigma_2 =$ 2 (solid), 7.75 (dotted), 12.5 (dashed) for $\epsilon = 0.4$

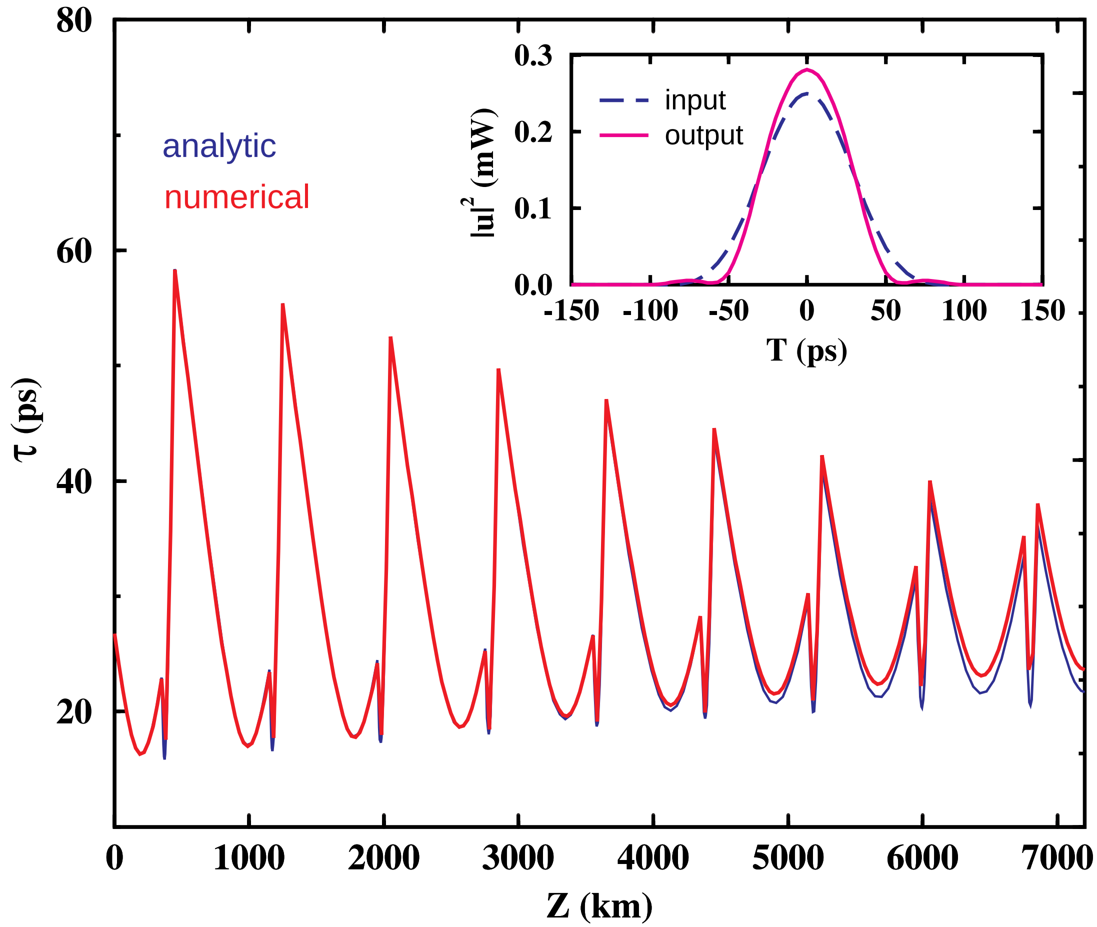
Non-periodic chirped return-to-zero (CRZ) pulses

Characterize using RMS pulse parameters (moments)

- pulse energy: $E = \int_{-\infty}^{\infty} |u|^2 dt$ (constant)
- temporal width: $\tau^2 = \int_{-\infty}^{\infty} t^2 |u|^2 dt \,/\, E$
- spectral width: $(\Delta\omega)^2 = \int_{-\infty}^{\infty} |u_t|^2 dt \,/\, E$
- chirp: $b = \int_{-\infty}^{\infty} t \; \mathrm{Im}\{u^* u_t\} \, dt \,/\, (E\tau^2)$
- average power: $P = \int_{-\infty}^{\infty} |u|^4 dt \,/\, E$

Can obtain and study ODEs for these parameters [27-29, 41-43]

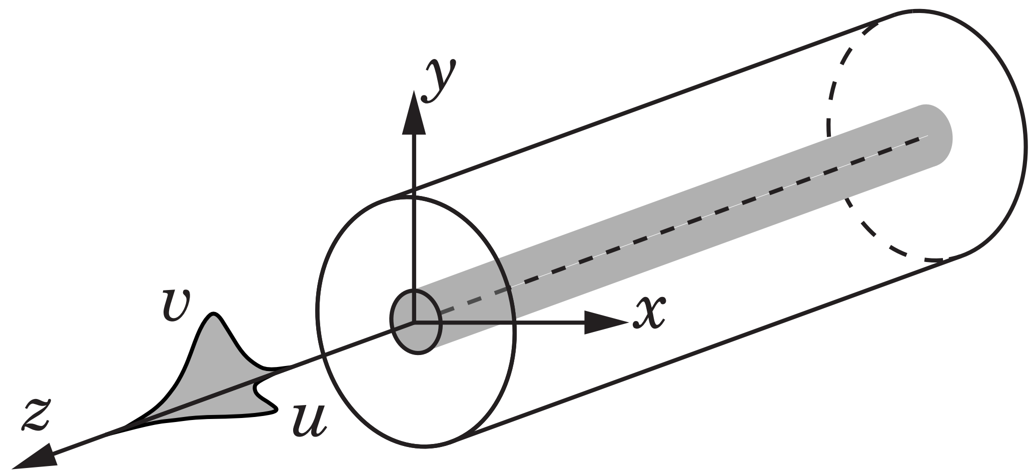
CRZ pulse dynamics [44]



Optimize chirp+transmission to minimize pulse width *at receiver*

Can show analytically that nonlinearity reduced by $O(\log s/s)$ [26]

Basic polarization effects



Coupled NLS equations for E-field envelope [45]:

$$i\frac{\partial \boldsymbol{u}}{\partial z} + \boldsymbol{\Delta\beta}\,\boldsymbol{u} + i\boldsymbol{\Delta\beta}'\frac{\partial \boldsymbol{u}}{\partial t} + \frac{1}{2}\frac{\partial^2 \boldsymbol{u}}{\partial t^2} + \boldsymbol{N}(\boldsymbol{u}) = 0$$

$$\boldsymbol{u} = \begin{bmatrix} u \\ v \end{bmatrix}, \qquad \boldsymbol{N}(\boldsymbol{u}) = \begin{bmatrix} (|u|^2 + \frac{2}{3}|v|^2)u + \frac{1}{3}v^2u^* \\ (|v|^2 + \frac{2}{3}|u|^2)v + \frac{1}{3}u^2v^* \end{bmatrix}$$

Coupled NLS (CNLS) equations

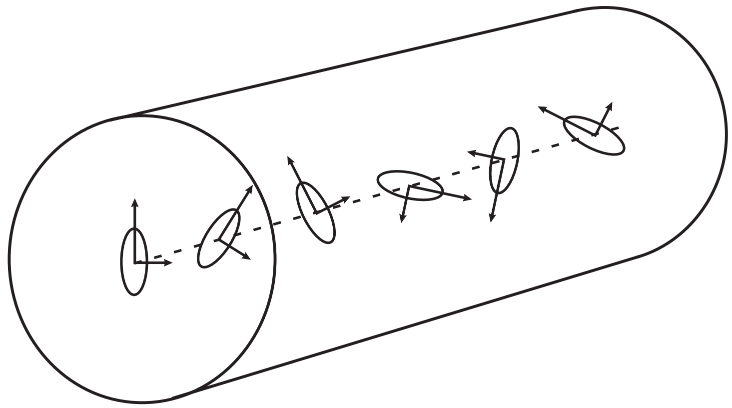
$$i\frac{\partial \boldsymbol{u}}{\partial z} + \boldsymbol{\Delta\beta}\,\boldsymbol{u} + i\boldsymbol{\Delta\beta}'\frac{\partial \boldsymbol{u}}{\partial t} + \frac{1}{2}\frac{\partial^2 \boldsymbol{u}}{\partial t^2} + \boldsymbol{N}(\boldsymbol{u}) = 0$$

$$\boldsymbol{\Delta\beta} = b\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}, \quad \boldsymbol{\Delta\beta}' = b'\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

- $\boldsymbol{\Delta\beta}$ is phase velocity difference. $b = ||\boldsymbol{\Delta\beta}||$
- $\boldsymbol{\Delta\beta}'$ is group velocity difference
- θ is angle between the coordinate & fiber principal axes
- these terms vary randomly with distance and over time $\Rightarrow$ polarization-mode dispersion (PMD).

Polarization-mode dispersion

birefringence: phase, group velocity polarization dependent



Two kinds of birefringence variations: small- and large-scale:

- *small-scale* from internal fiber perturbations
  (e.g., from core cross-section fluctuations from manufacturing imperfections, stress variations from differential cooling, etc.)

- *large-scale* from macroscopic effects
  (e.g., bending and twisting)

Phase and group birefringence

Birefringence $=$ velocity difference

- Phase birefringence produces random rotation of the polarization state
- Group birefringence produces random pulse splitting (differential group delay, DGD; fast vs. slow axes)

Typical length scales:

- Random birefringence: 10’s or 100’s of *meters*
- Dispersion and nonlinearity: 100’s of *kilometers*

Resolve linear evolution over short distances first; interaction with dispersion and nonlinearity later

Manakov-PMD equations [46]

Use fundamental solution $\boldsymbol{U}$ for part of linear evolution ,

$$i\frac{\partial \boldsymbol{U}}{\partial z} + \boldsymbol{\Delta\beta}\,\boldsymbol{U} = 0\,,$$

and transformation $\boldsymbol{u} = \boldsymbol{U}\boldsymbol{\Psi}$ to remove fast polarization rotation

$$\Rightarrow \quad i\frac{\partial \boldsymbol{\Psi}}{\partial z} + \frac{1}{2}\,\frac{\partial^2 \boldsymbol{\Psi}}{\partial t^2} + \frac{8}{9}|\boldsymbol{\Psi}|^2\boldsymbol{\Psi} \;=\; -i\widehat{\boldsymbol{\Delta\beta}'}\,\frac{\partial \boldsymbol{\Psi}}{\partial t} - \hat{\boldsymbol{N}},$$

where

$$\widehat{\boldsymbol{\Delta\beta}'} = \boldsymbol{U}^{\dagger}\boldsymbol{\Delta\beta}'\,\boldsymbol{U}\,,$$

and $\hat{\boldsymbol{N}}$ represents the fluctuating part of the nonlinearity

Manakov-PMD equations, continued

- Under reasonable assumptions, $\langle \widehat{\Delta\beta'} \rangle = 0$ and $\langle \hat{N} \rangle = 0$
- Thus, mean evolution obeys the Manakov equation,

$$i\frac{\partial \boldsymbol{\Psi}}{\partial z} + \frac{1}{2}\frac{\partial^2 \boldsymbol{\Psi}}{\partial t^2} + \frac{8}{9}|\boldsymbol{\Psi}|^2\boldsymbol{\Psi} = 0$$

- Manakov equation also completely integrable by the IST
- If only one polarization present, reduces to scalar NLS
- Full analysis requires dealing with coupled system and perturbing random birefringence fluctuations

Wavelength-division-multiplexing

Recall explicit N-soliton solution:

$$u(z,t) = \sum_{j,k=1}^{N} (Q^{-1})_{jk}\,,$$

$$Q_{jk} = \frac{\exp[-i\chi_j - S_j] + \exp[-i\chi_k + S_k]}{A_j + A_k + i(\Omega_j - \Omega_k)}\,,$$

$$S_j(z,t) = A_j(t - T_j - \Omega_j z)\,,$$

$$\chi_j(z,t) = \Omega_j t - \tfrac{1}{2}(\Omega_j^2 - A_j^2)z + \Phi_j\,.$$

$4N$ soliton parameters: A_j, Ω_j, T_j and Φ_j.

Wavelength-division multiplexing:

$$|A_j + A_k| \ll |\Omega_j - \Omega_k| \quad \text{for } k \neq j$$

Several frequency channels simultaneously travel across the fiber.

WDM interactions and four-wave mixing

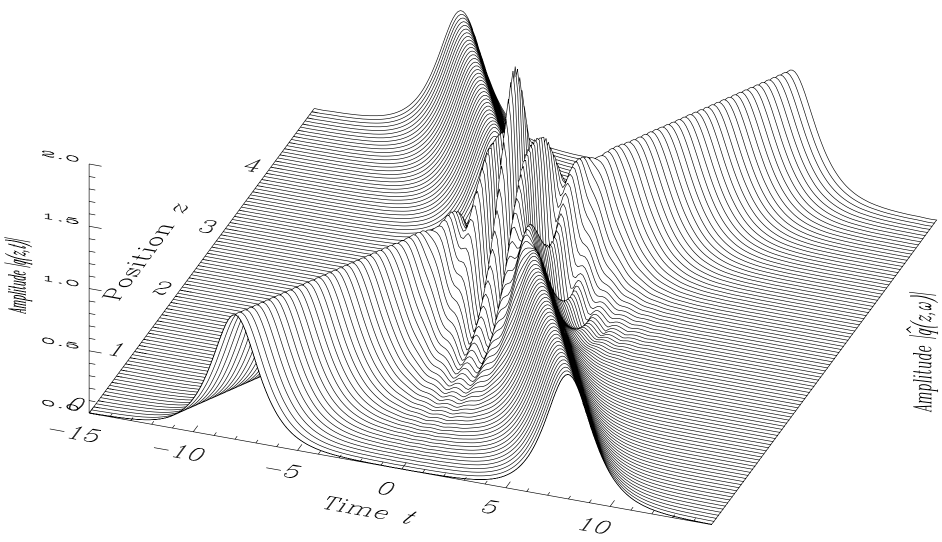
Soliton interactions in lossless fibers

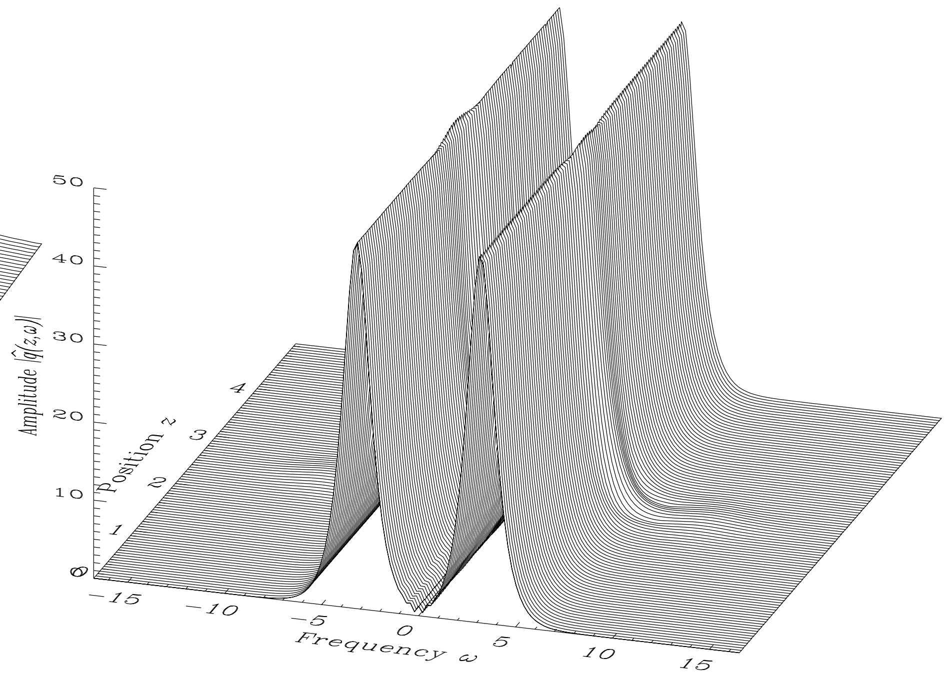
- WDM regime: well-separated frequency channels
- Expand the N-soliton solution in powers of

$$\max_{j,k=1,\dots,N} |A_j + A_k| / |\Omega_j - \Omega_k|$$

- $u^{(0)}(z,t) = \sum_{j=1}^{N} u_j(z,t)$, superposition of N one-soliton solutions.
- To leading order, WDM solitons traverse each other as linear pulses.
- $O(\epsilon)$: permanent timing shifts due to collisions, corresponding to a temporary shift of the soliton frequency.
- Soliton interactions are pairwise to leading order.

Soliton interactions and four-wave mixing





$A_1 = A_2 = 1$, $\Omega_2 = -\Omega_1 = 3.2$, $T_2 = -T_1 = 8$.

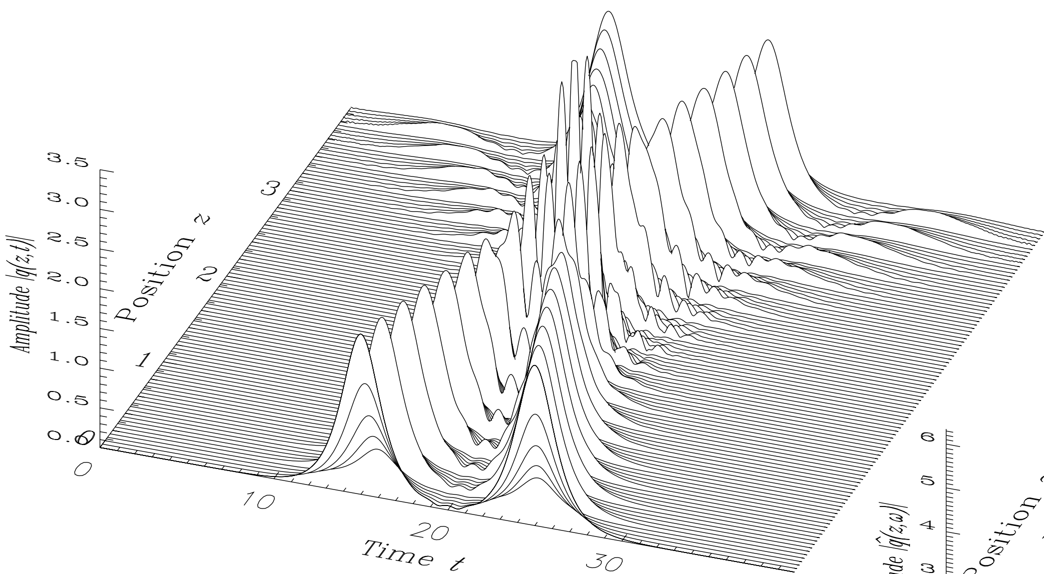
$O(\epsilon^2)$: four-wave mixing terms, at $N^2(N-1)/2$ frequencies (not necessarily distinct):

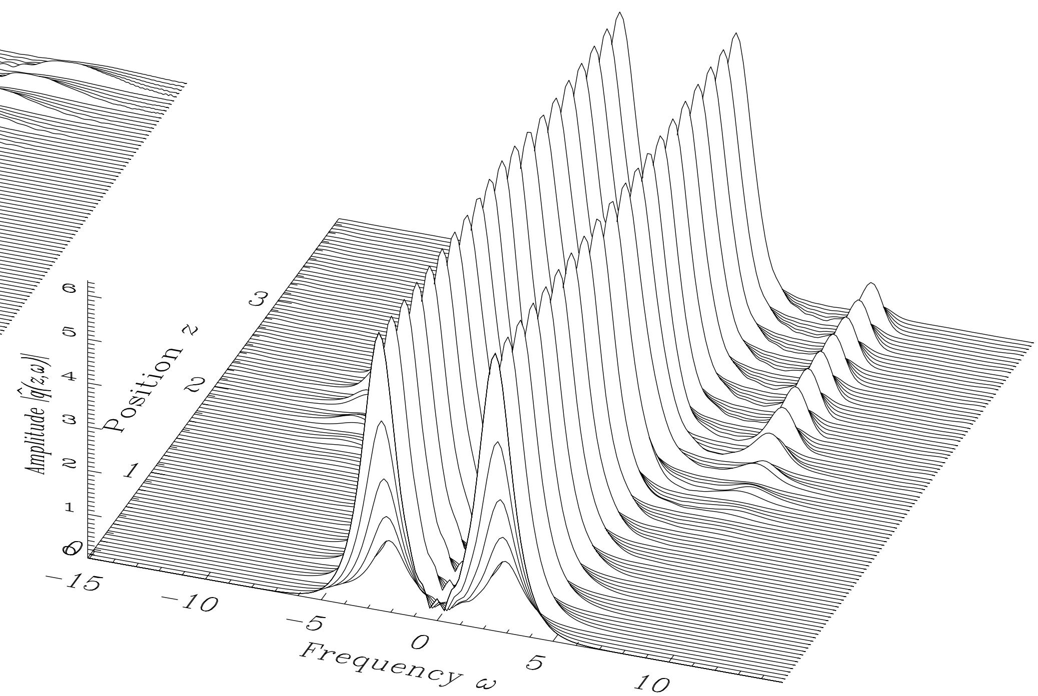
$$\Omega_{klj} = \Omega_k + \Omega_l - \Omega_j\,, \qquad k, l \neq j\,.$$

FWM terms are produced during soliton interactions, and their energy flows back into the solitons after the collision.

Four-wave mixing in real fibers [47]

2-soliton collision with loss and amplification:





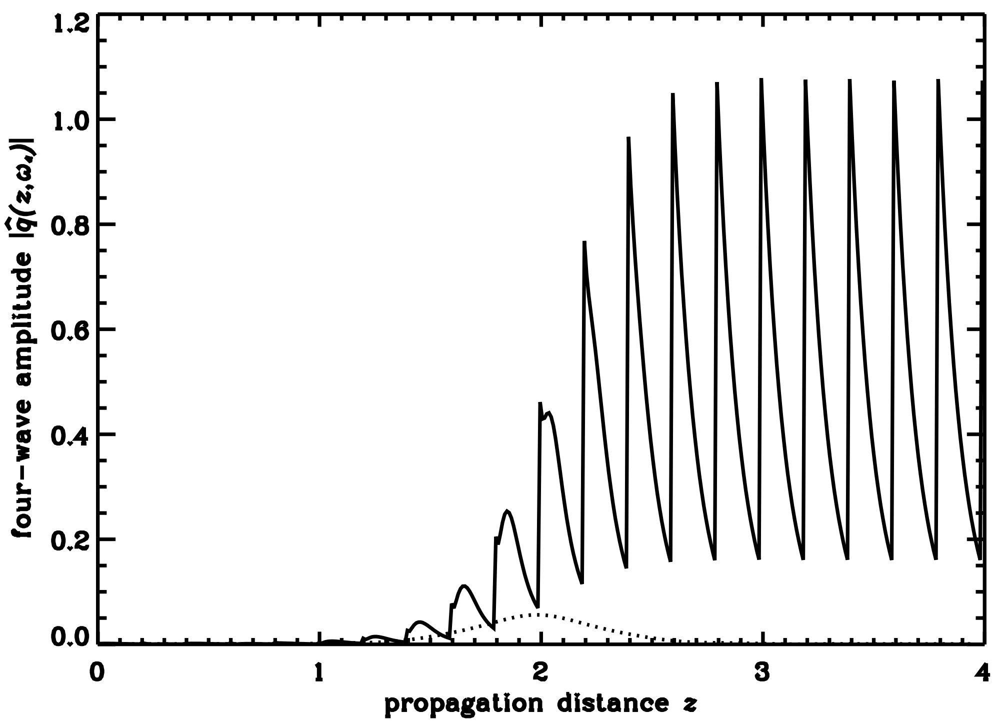
$\Gamma = 10$, $z_a = 0.2$, $A_1 = A_2 = 1$

$\Omega_1 = -\Omega_2 = -3$, $T_1 = -T_2 = 5$.

Similar problem independent of transmission format.

Four-wave mixing in real fibers (continued)

Solid line: the four-wave mixing amplitude as a function of z.
Dashed line: the corresponding amplitude in the lossless case.



Four-wave mixing in real fibers

- Two-soliton collision: $u = u_s + u_{\mathrm{fwm}} + \cdots$

  $u_s = u_1 + u_2 =$ solitons, with frequencies Ω_1, Ω_2
  $u_{\mathrm{fwm}} = u_{112} + u_{221} =$ FWM terms, located at Ω_{112}, Ω_{221}:

$$\Omega_{112} = 2\Omega_1 - \Omega_2 \qquad \Omega_{221} = 2\Omega_2 - \Omega_1$$

- Growth of anti-Stokes FWM:

$$i\frac{\partial u_{221}}{\partial z} + \frac{1}{2}\frac{\partial^2 u_{221}}{\partial t^2} = -g(z)\,u_2^2 u_1^*$$

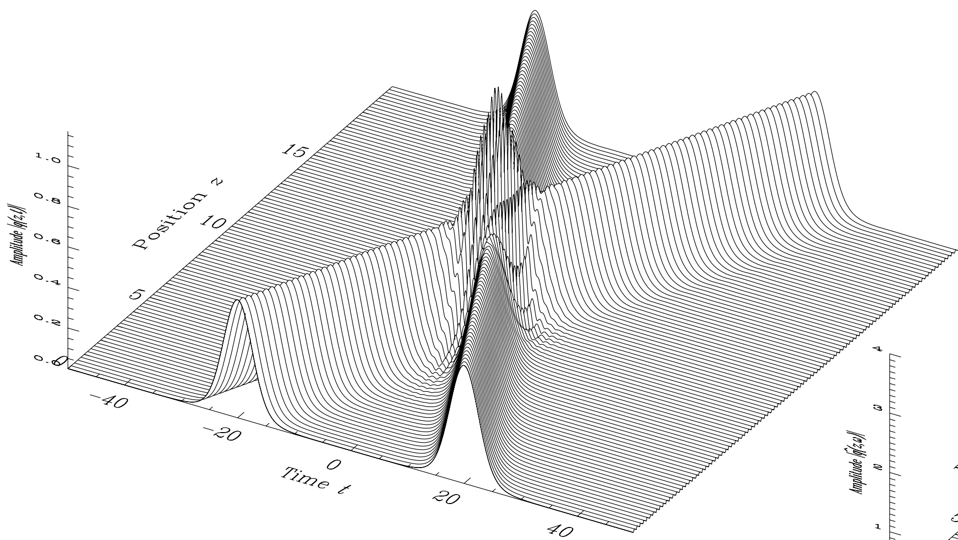
- Resonance condition:

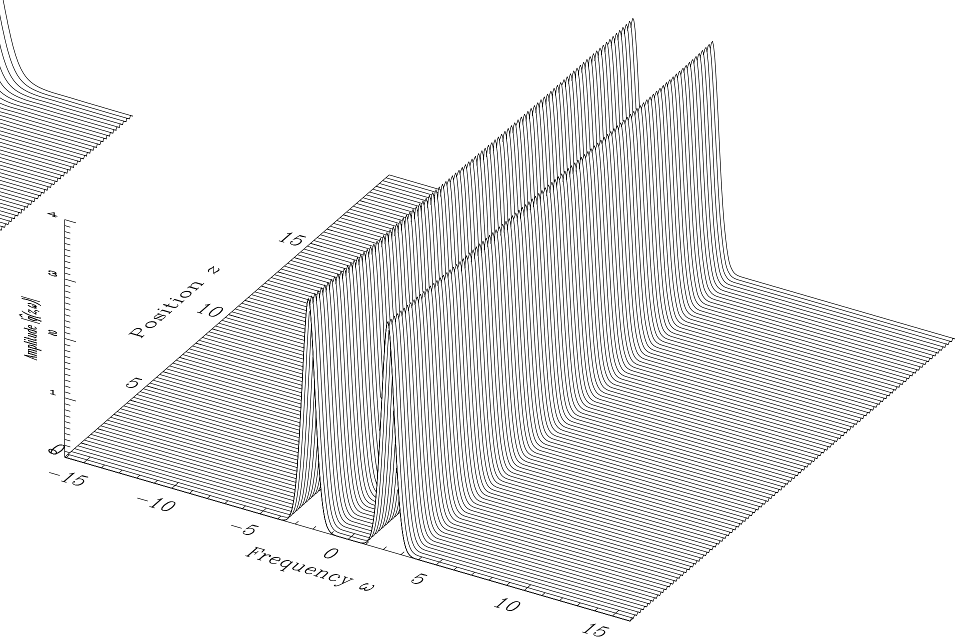
$$\frac{2n\pi}{\langle d\rangle z_a} = \Delta\Omega^2 + \frac{1}{2}A^2$$

- Similar phenomenon for all transmission formats [48]

Dispersion management reduces growth of sidebands

system parameters from WDM experiments [49]





The interactions are sometimes almost better than for pure NLS!

Large, rapid phase variations responsible for FWM reduction [50]

Collision-induced timing jitter theory is much more involved [51-53]

Summary

- Optical fibers are the backbone for today's communications
- NLS equation provides the fundamental framework for *many* models
- Much more work has been done with NLS than with Manakov
- Perturbations almost always involved:
  loss/gain, noise, dispersion management, polarization effects...
- The rich structure of NLS leads to lots of interesting behavior
- New technologies continue to produce new mathematical questions

$$\frac{\partial u}{\partial z} = \frac{i}{2}\frac{\partial^2 u}{\partial t^2} + i|u|^2 u$$

References

[1] Govind P. Agrawal. *Nonlinear Fiber Optics*. Academic Press, San Diego, 2nd edition, 1995.

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