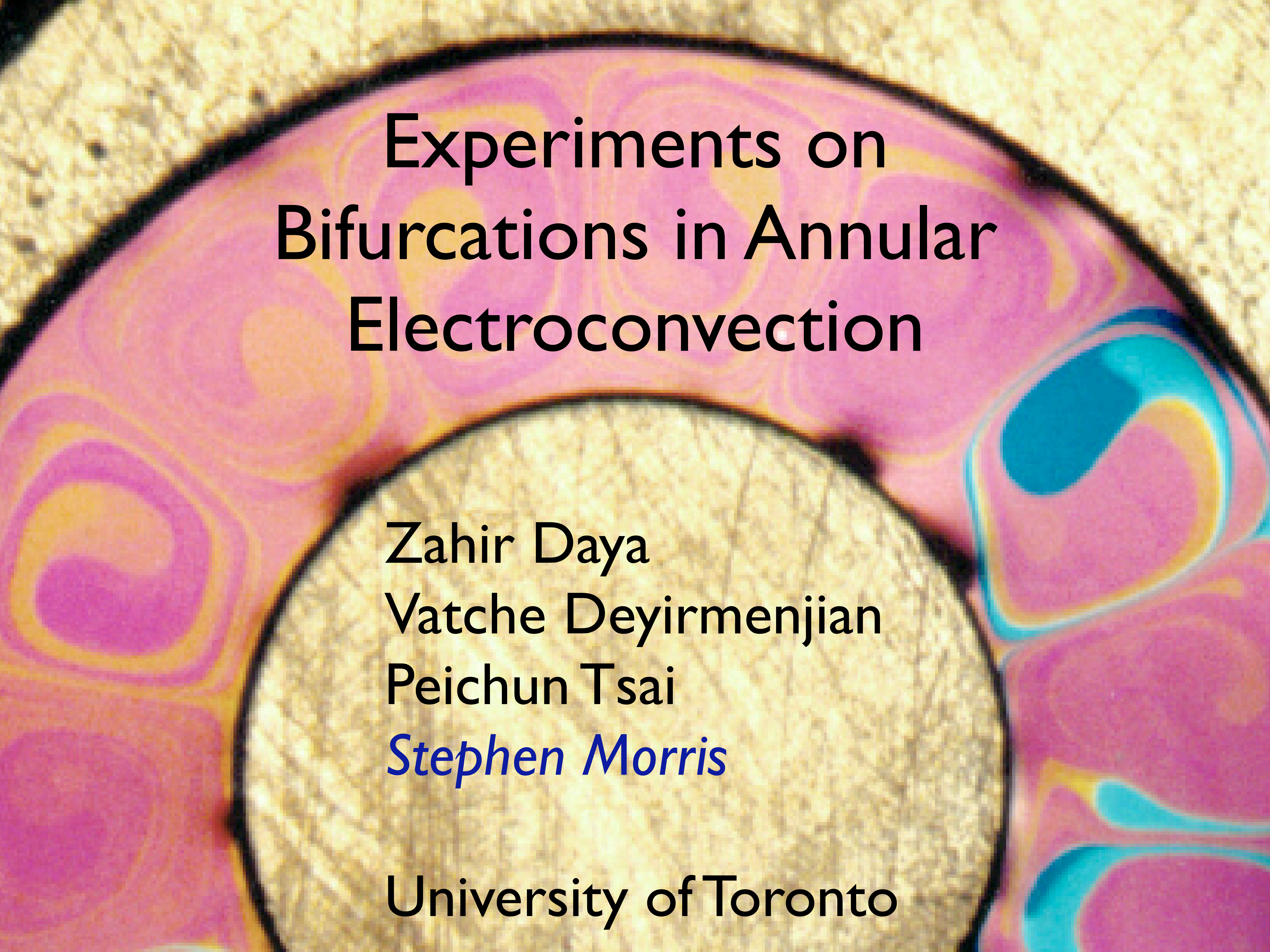
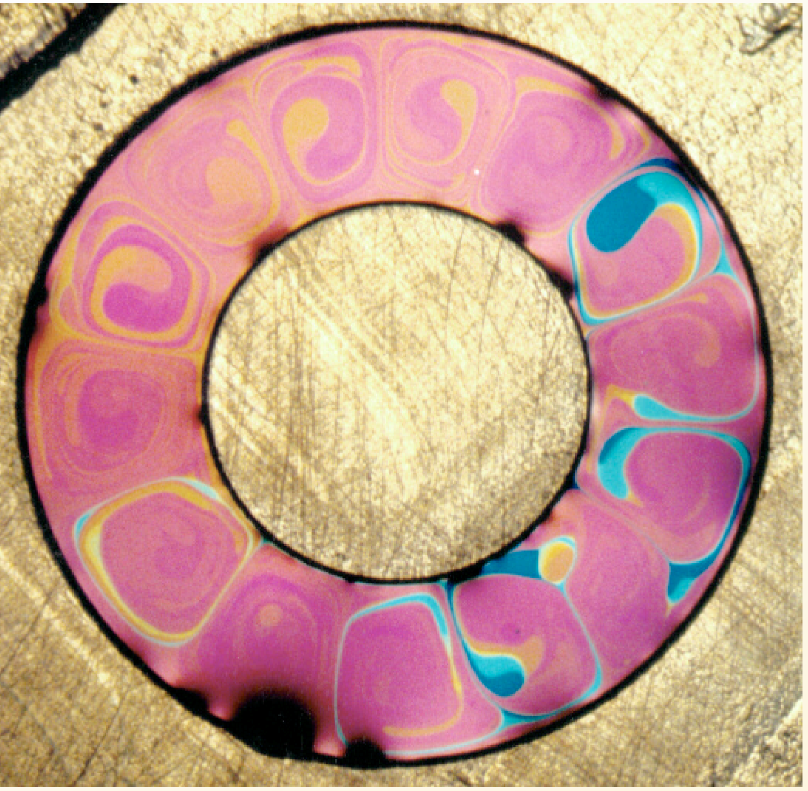


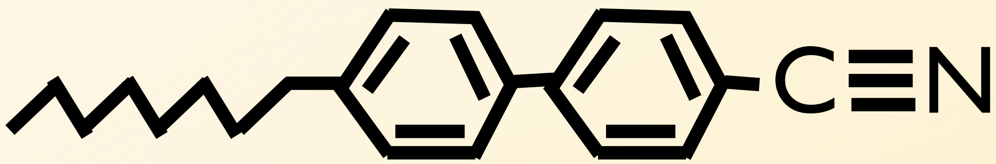
Experiments on Bifurcations in Annular Electroconvection

Zahir Daya
Vatche Deyirmenjian
Peichun Tsai
Stephen Morris

University of Toronto



Smectic liquid crystal films

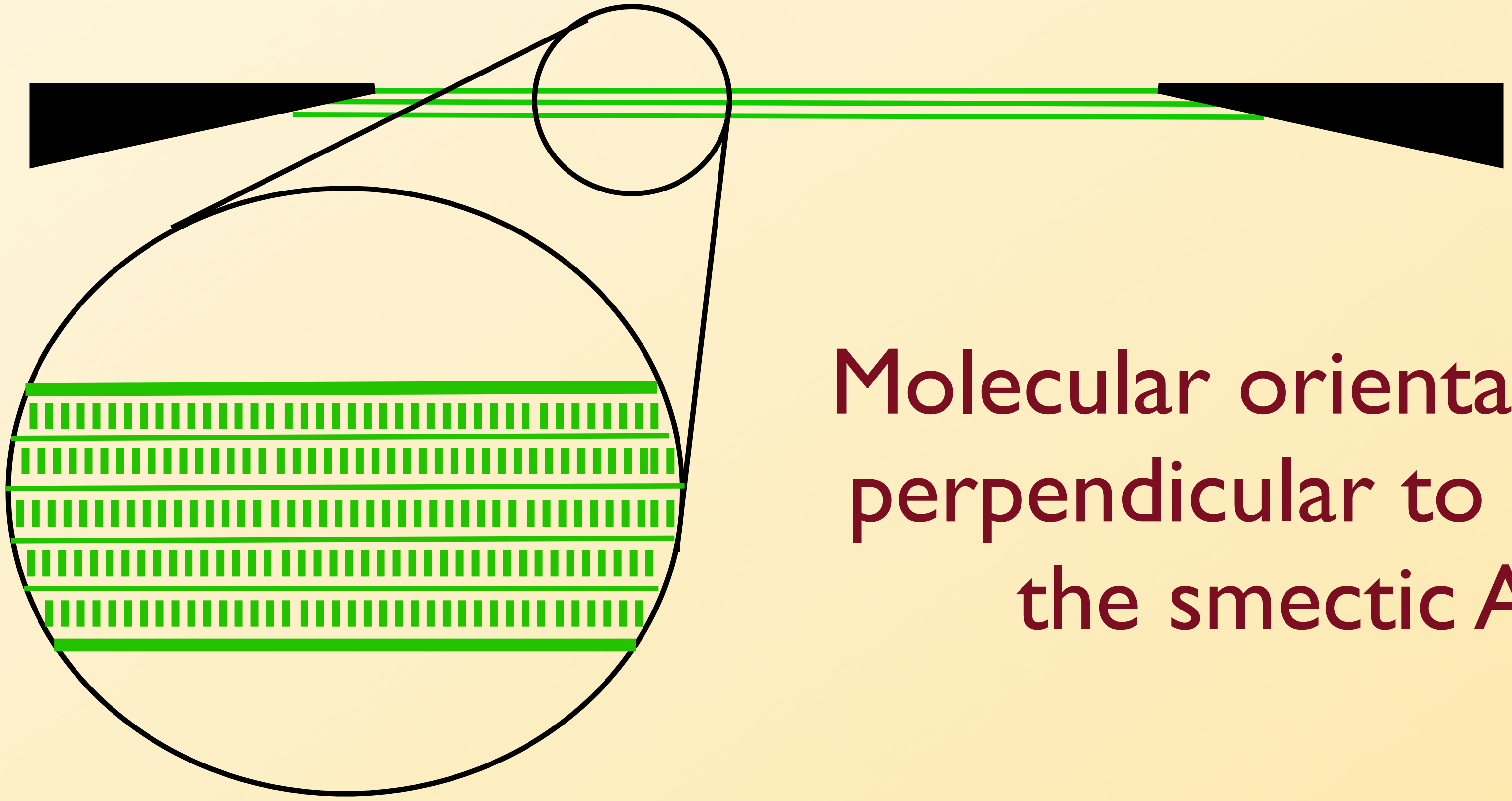


8CB

smectic = soapy

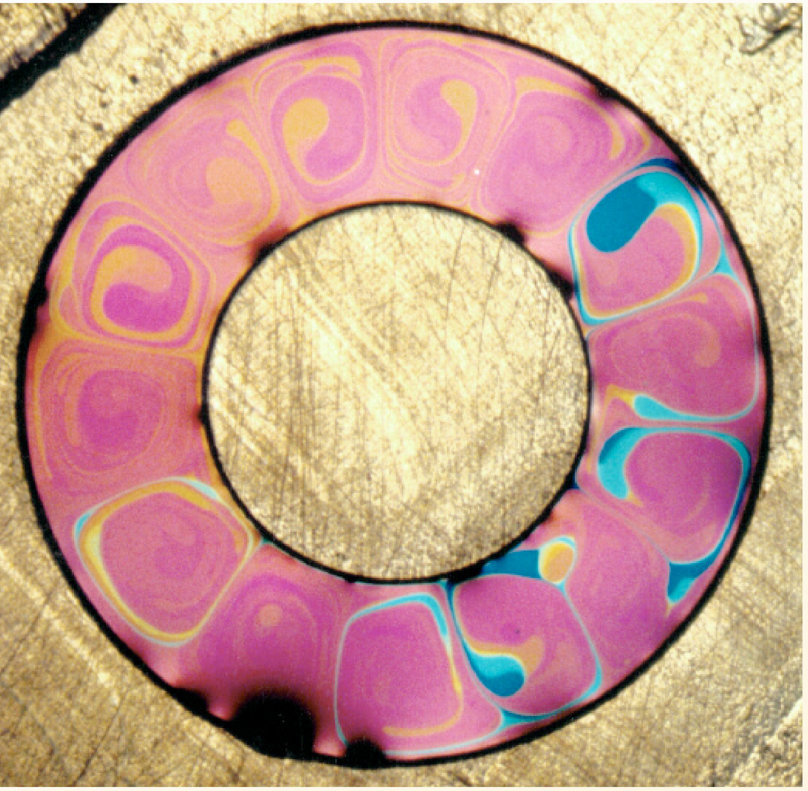
Smectics form robust submicron thick suspended films which are an integer number of smectic layers thick. They are Newtonian for flows in the plane of the film and resist thickness change.

side view



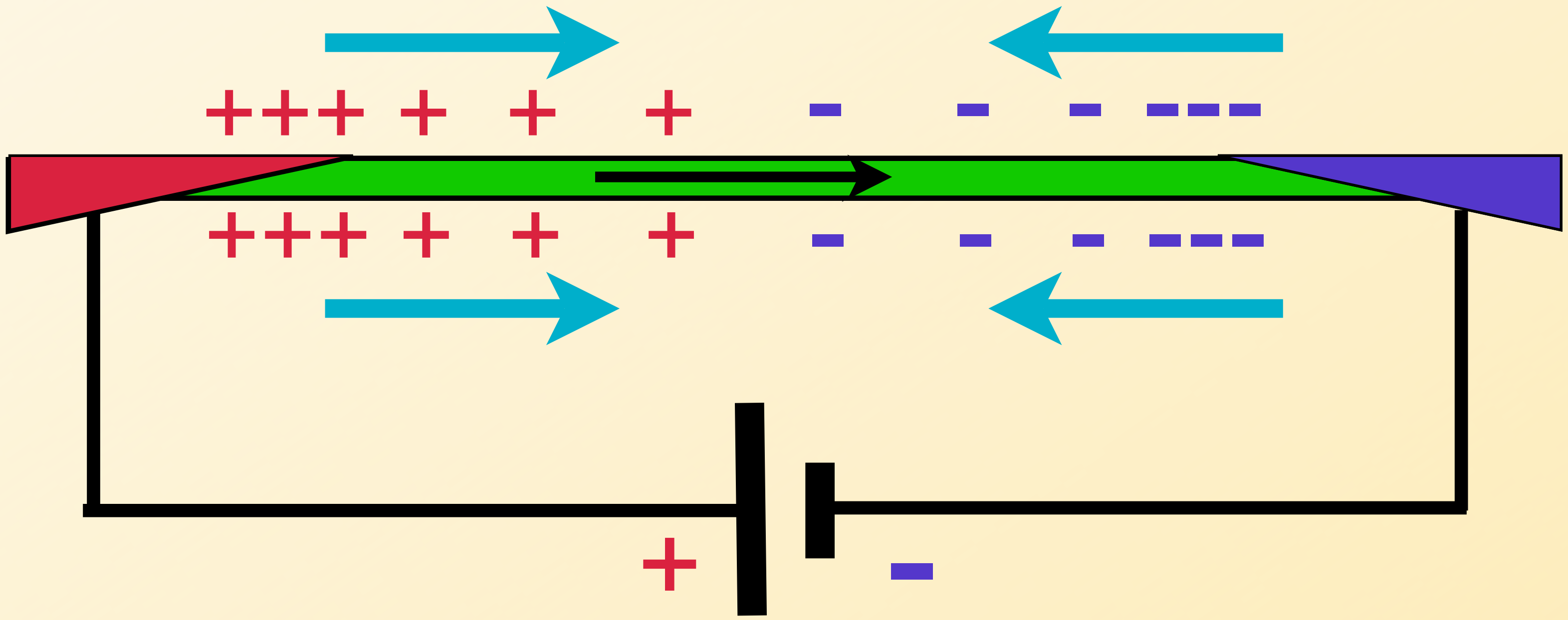
Each layer is 3.16nm thick.

Molecular orientation remains perpendicular to the layers in the smectic A phase.



Mechanism of electroconvection

Convection is driven by an unstable surface charge distribution on the two free surfaces

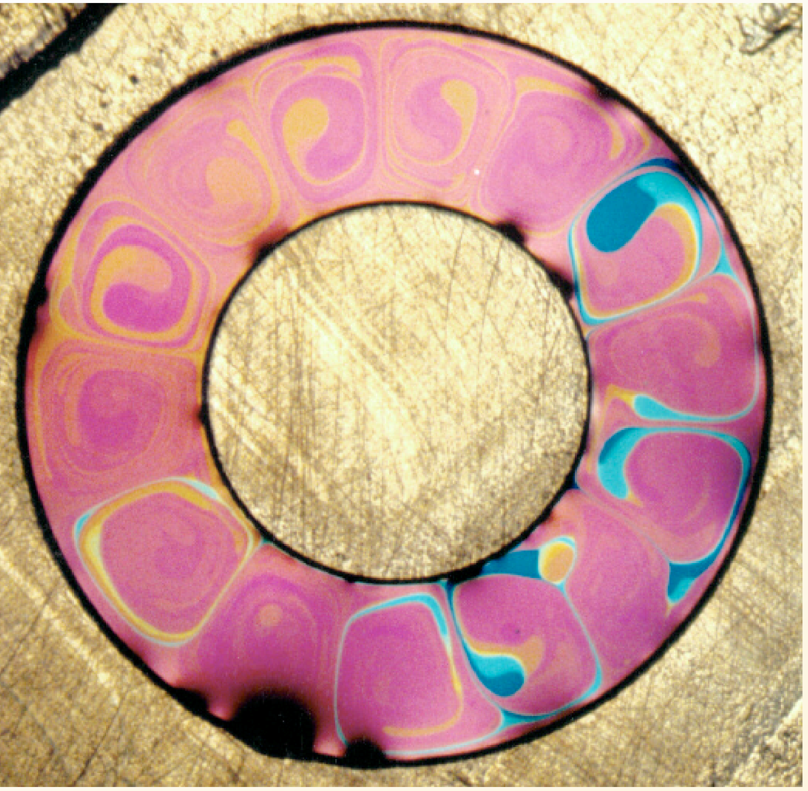


Apply a DC voltage.

Drive a current.

Produce surface charge

Surface forces appear that can drive convection



Mechanism of electroconvection

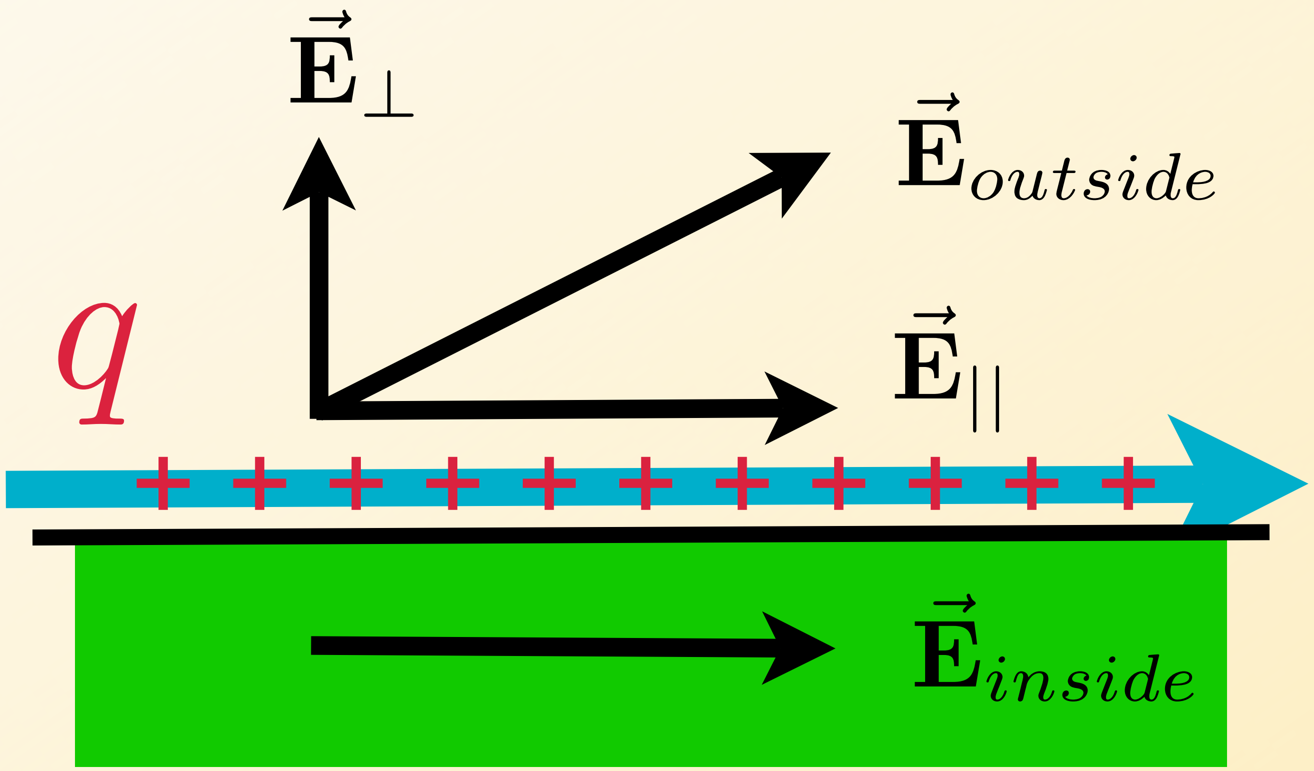
continued

Current density $\vec{\mathbf{J}} = \sigma \vec{\mathbf{E}}_{inside}$

$$\vec{\mathbf{E}}_{outside} = -\nabla\psi$$

with $\nabla^2\psi = 0$

outside the film.



Boundary conditions require

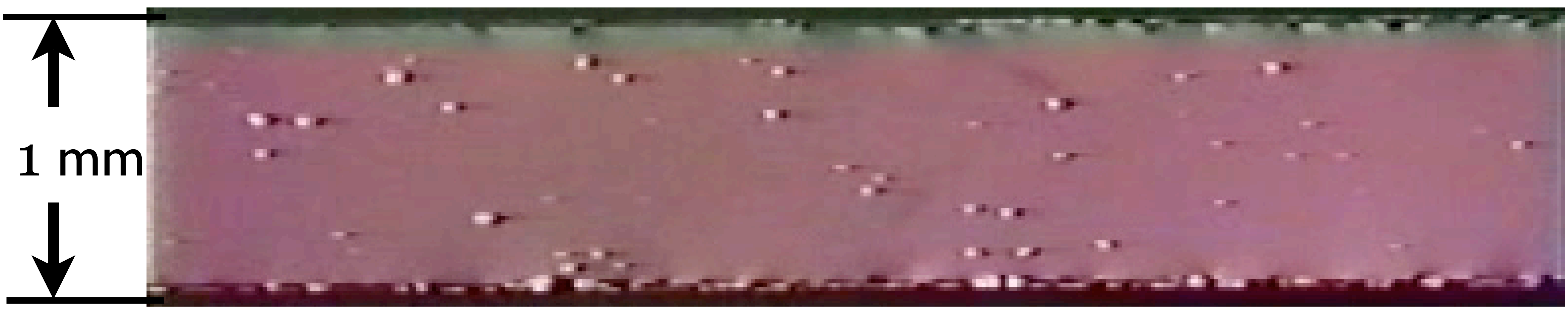
$$\vec{\mathbf{E}}_{||} = \vec{\mathbf{E}}_{inside}$$

Surface charge $q = \epsilon_o E_{\perp} = -\epsilon_o \partial_{\perp}\psi$

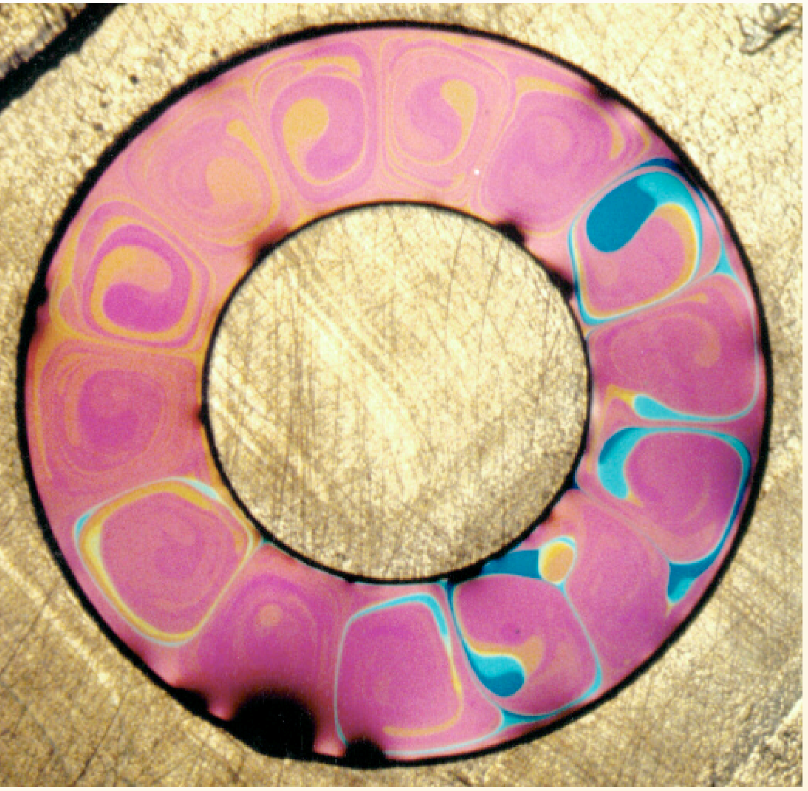
Surface force $q\vec{\mathbf{E}}_{||} = -\epsilon_o q \nabla_{||}\psi$

Convection in a rectangular film suspended between two wires

Uniform colour indicates uniform thickness



Flow visualized by backlit particles

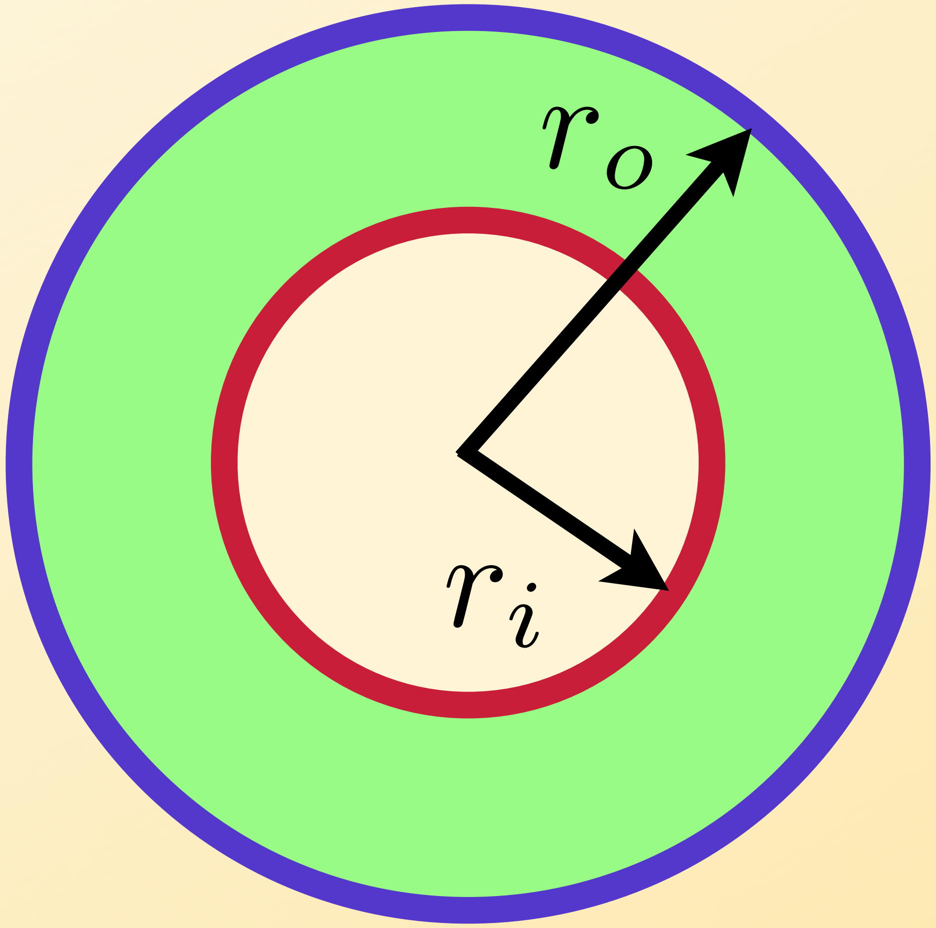


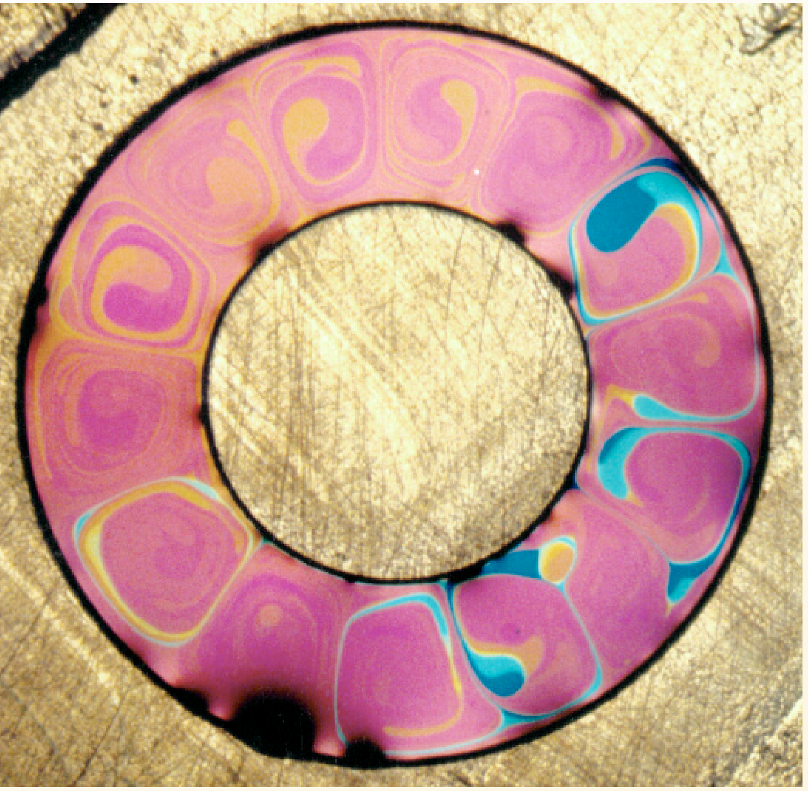
Annular electrodes

radius ratio $\alpha = r_i / r_o$

Naturally periodic boundary conditions

There is a unique current path through the film





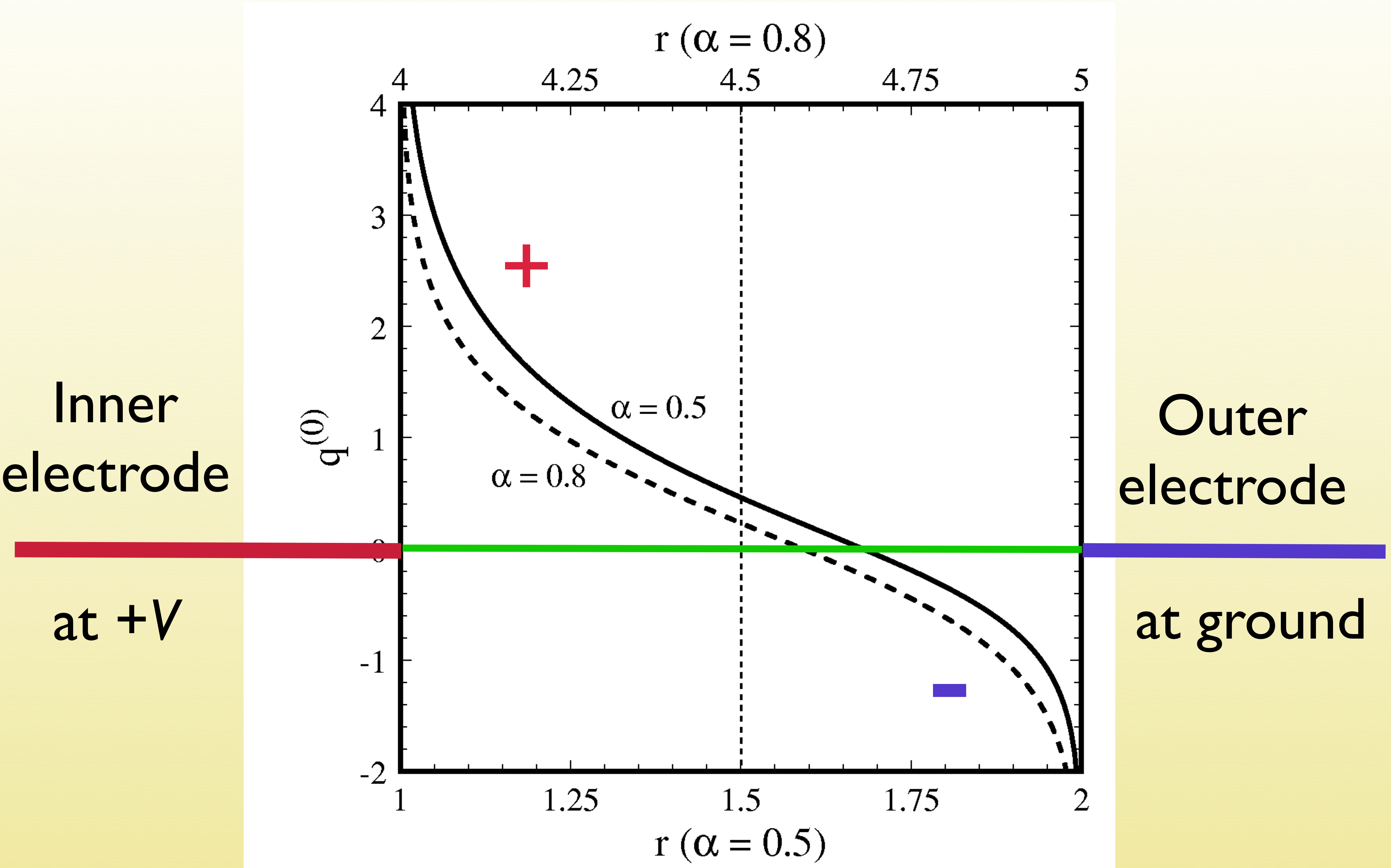
Annular electrodes

radius ratio $\alpha = r_i / r_o$

Base state charge density can be calculated analytically using a Green function.

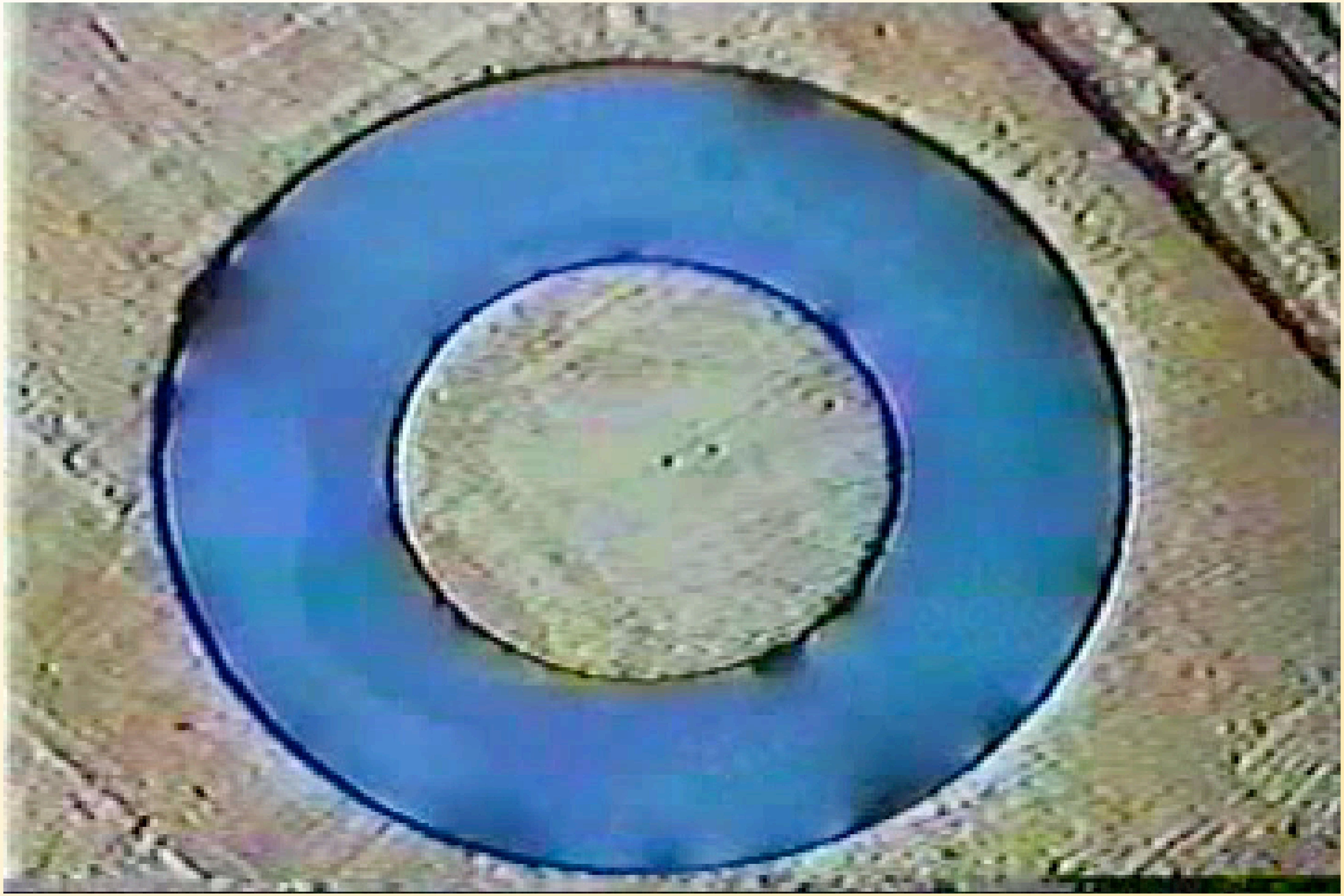
$$q(r) = \frac{2}{\ln \alpha} \left[\frac{1}{r} \; {}_2F_1\left(\frac{1}{2}, \frac{1}{2}; 1; \frac{r_o^2}{r^2}\right) - \frac{1}{r_i} \; {}_2F_1\left(\frac{1}{2}, \frac{1}{2}; 1; \frac{r^2}{r_i^2}\right) \right]$$

Base state (no convection) charge density



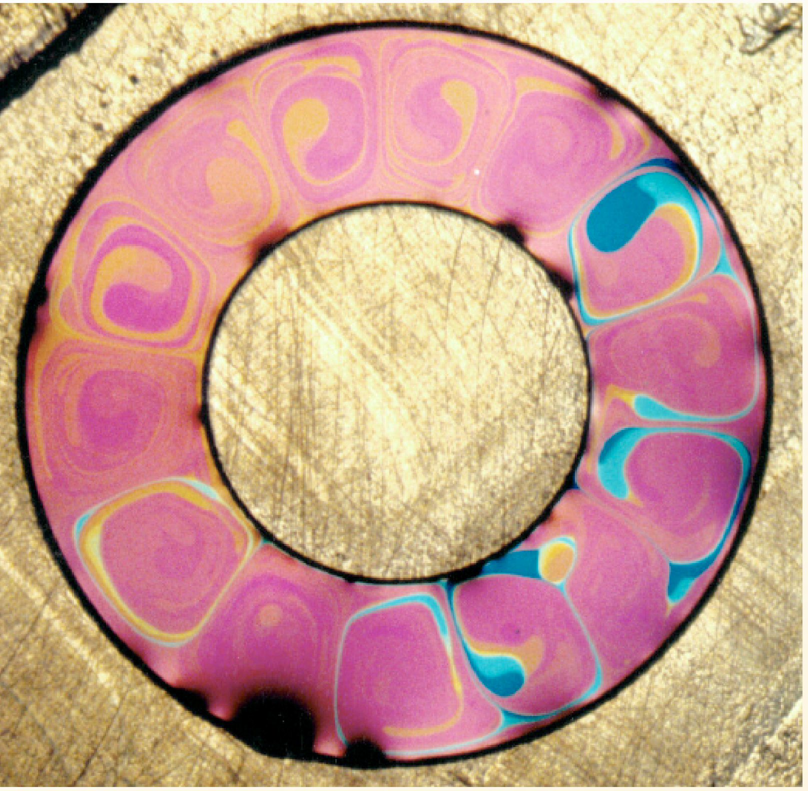
q becomes antisymmetric about midline as $\alpha \to 1$

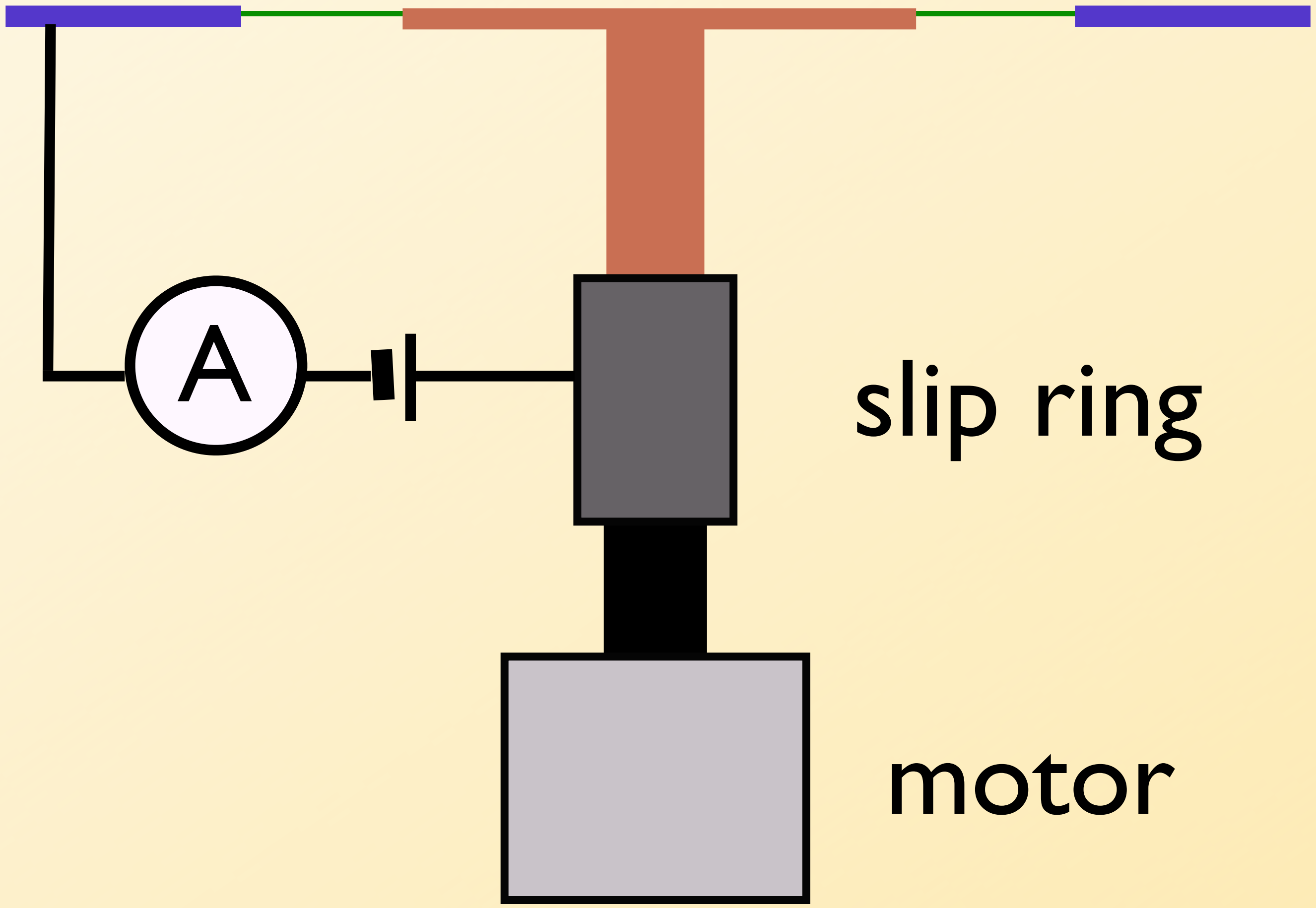
Convection in a nearly uniform annular film

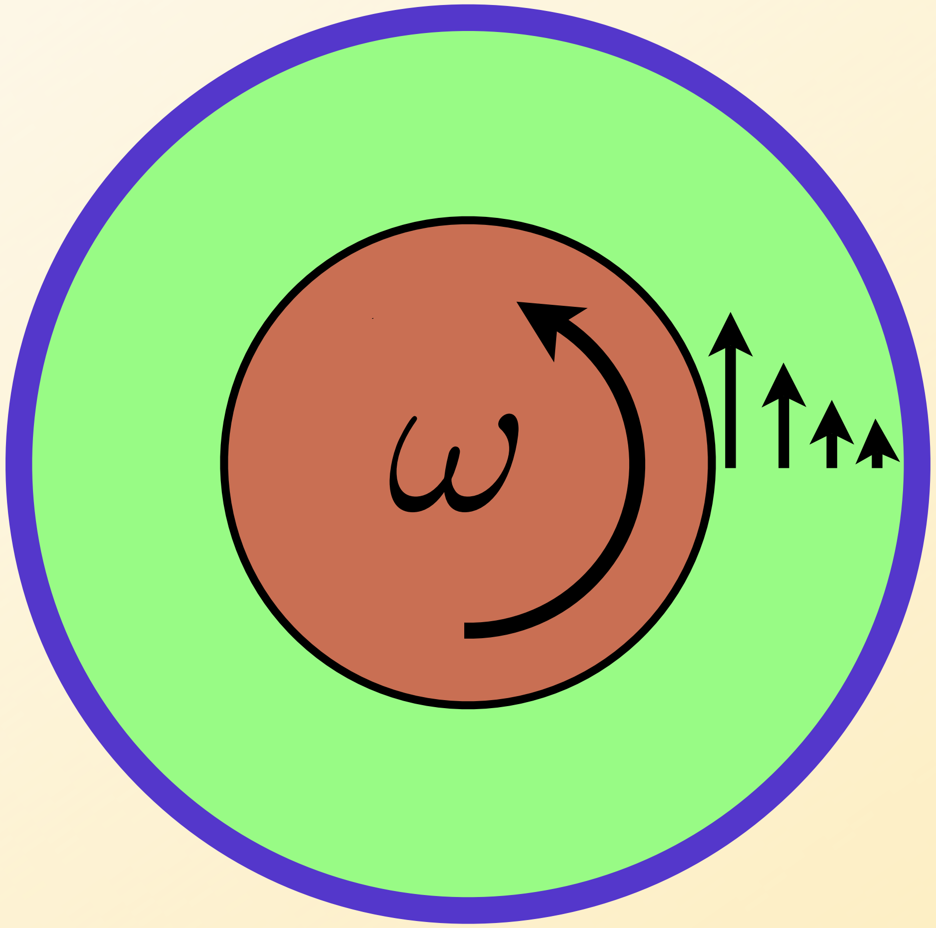


Shape distortion is a video format artifact!

Annular electrodes with superposed Couette shear

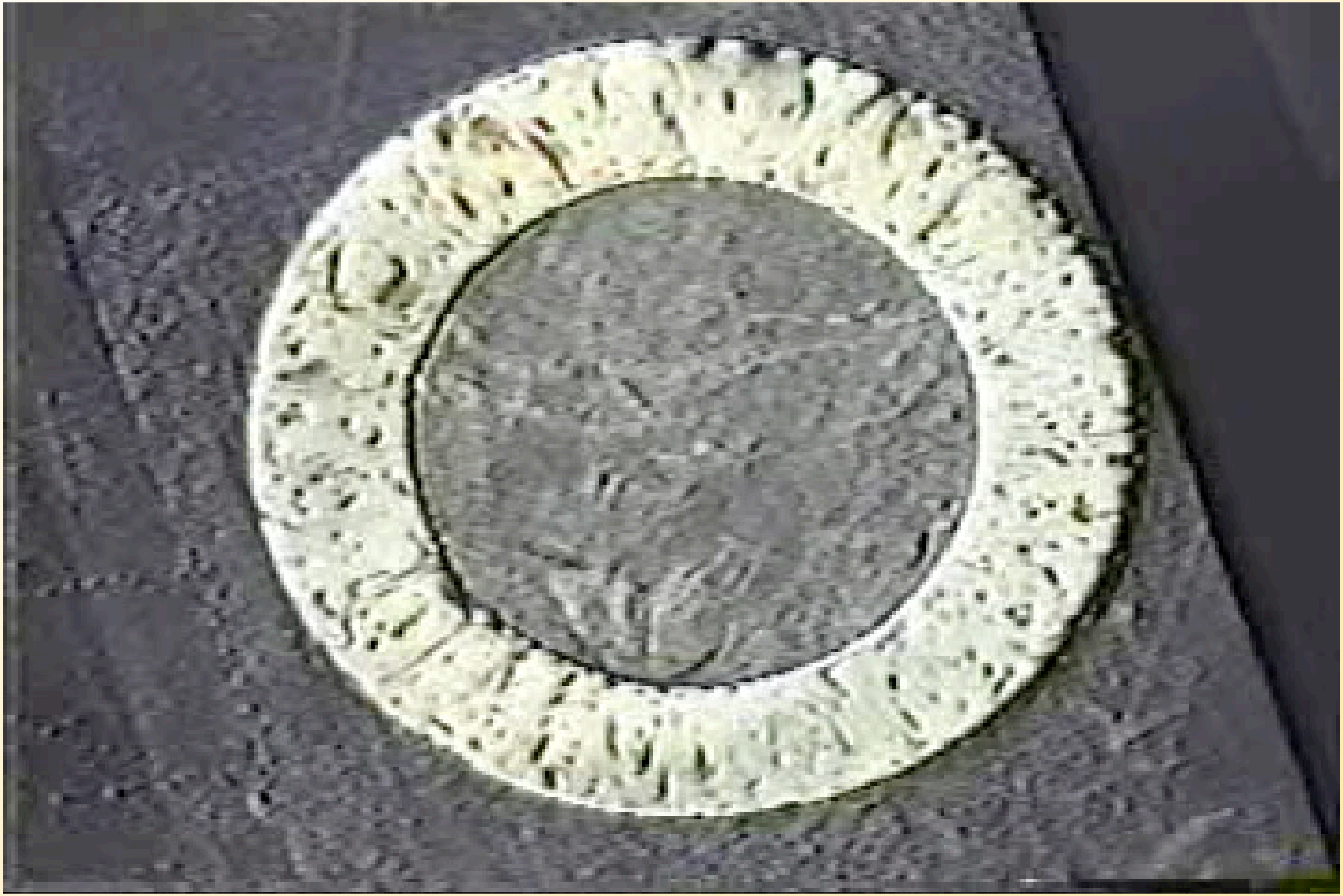




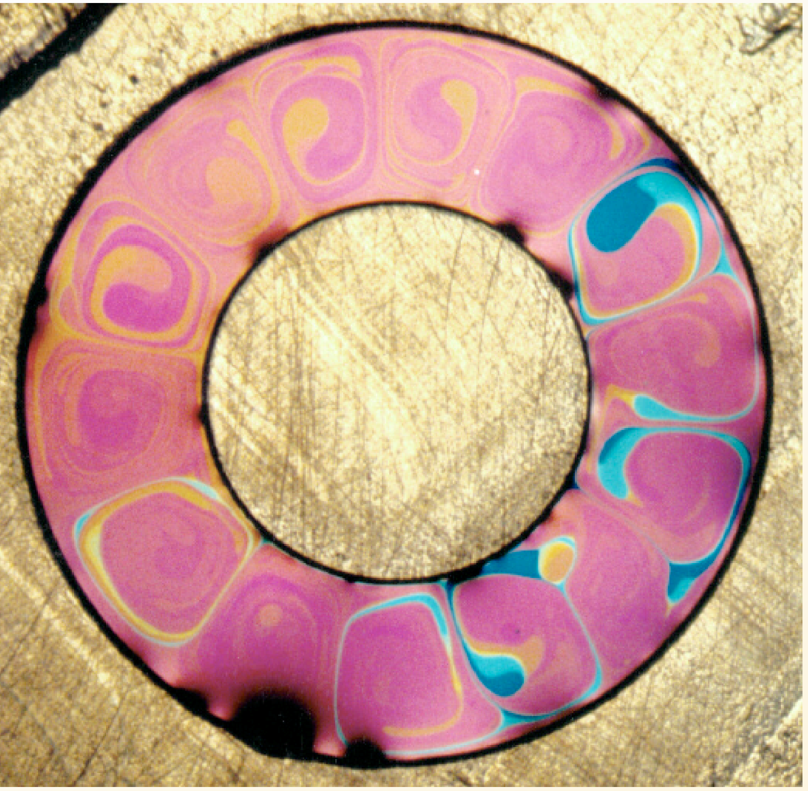


Purely azimuthal flow does not change electrical base state

Convection in a nonuniform annular film with shear



Shape distortion is a video format artifact!



Linear stability analysis

Closely analogous to radial Rayleigh-Bénard.
Treat fluid as 2D, potential as 3D.

$$\tilde{\rho}\left[\frac{\partial \vec{\mathbf{u}}}{\partial t} + (\vec{\mathbf{u}}\cdot\nabla_{||})\vec{\mathbf{u}}\right] = -\nabla_{||}P + \tilde{\eta}\nabla^2_{||}\vec{\mathbf{u}} + q\vec{\mathbf{E}}$$

$$\frac{\partial q}{\partial t} = -\nabla_{||}\cdot(q\vec{\mathbf{u}} + \tilde{\sigma}\vec{\mathbf{E}})$$

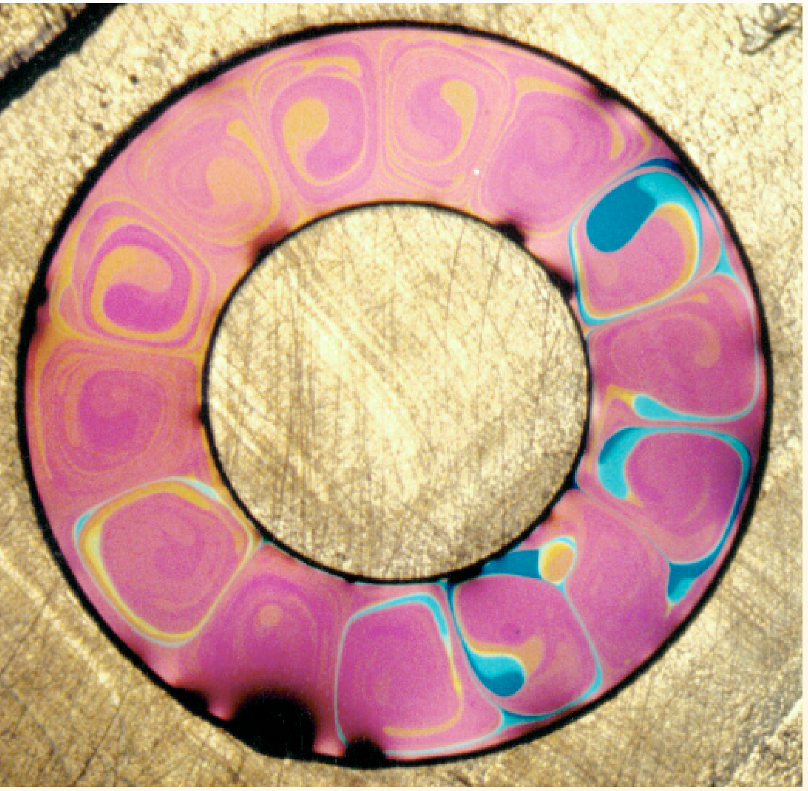
$$\nabla^2\psi = 0$$

$$q = -2\epsilon_0\partial_\perp\psi\Big|_{z=0^+}$$

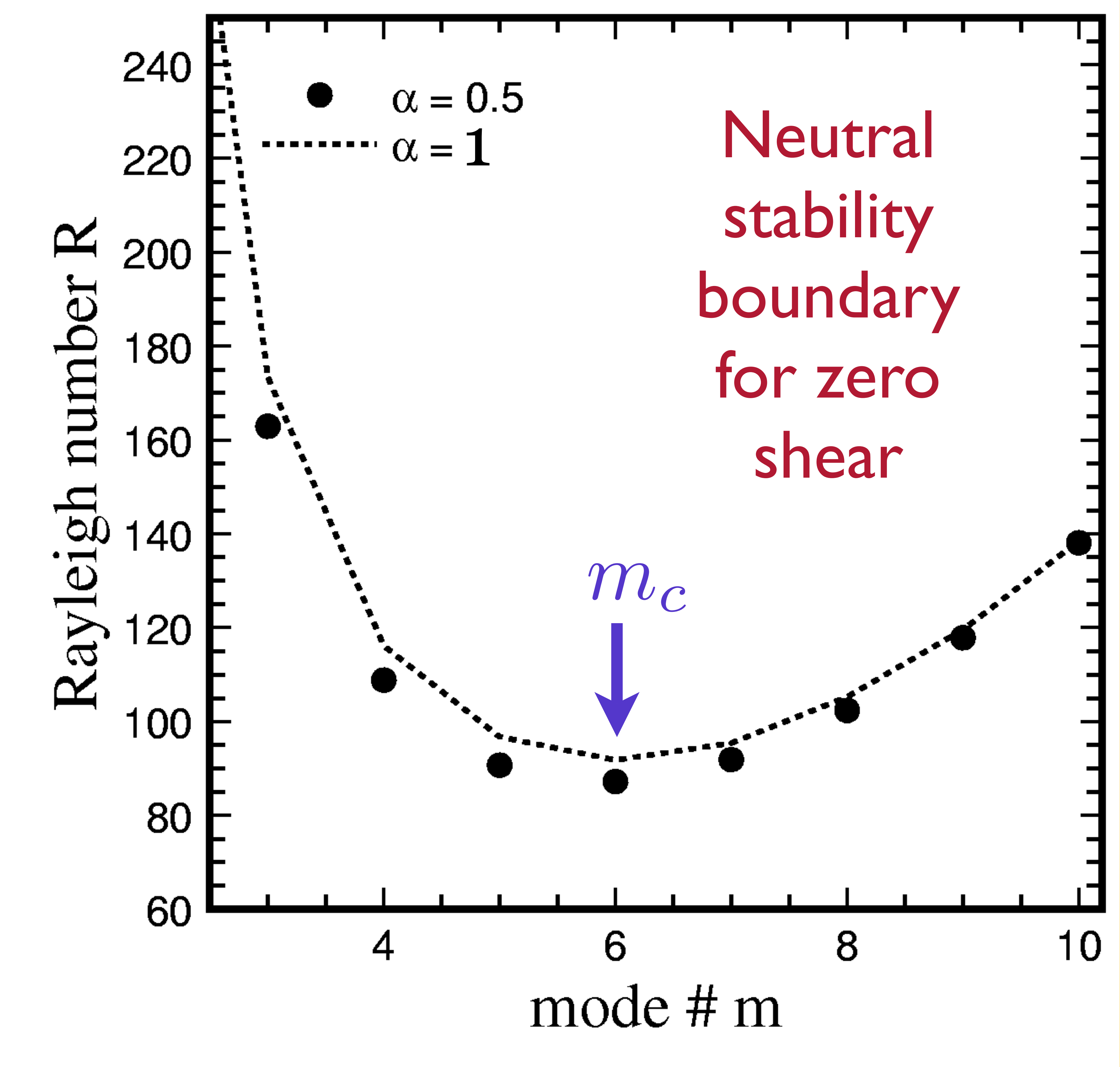
Dimensionless numbers

$$\mathcal{R} = \frac{\epsilon_0^2 V^2}{\tilde{\sigma}\tilde{\eta}} = \frac{\epsilon_0^2 V^2}{\sigma\eta s^2}$$

$$\mathcal{P} = \frac{\epsilon_0\tilde{\eta}}{\tilde{\rho}\tilde{\sigma}d} = \frac{\epsilon_0\eta}{\rho\sigma sd}$$



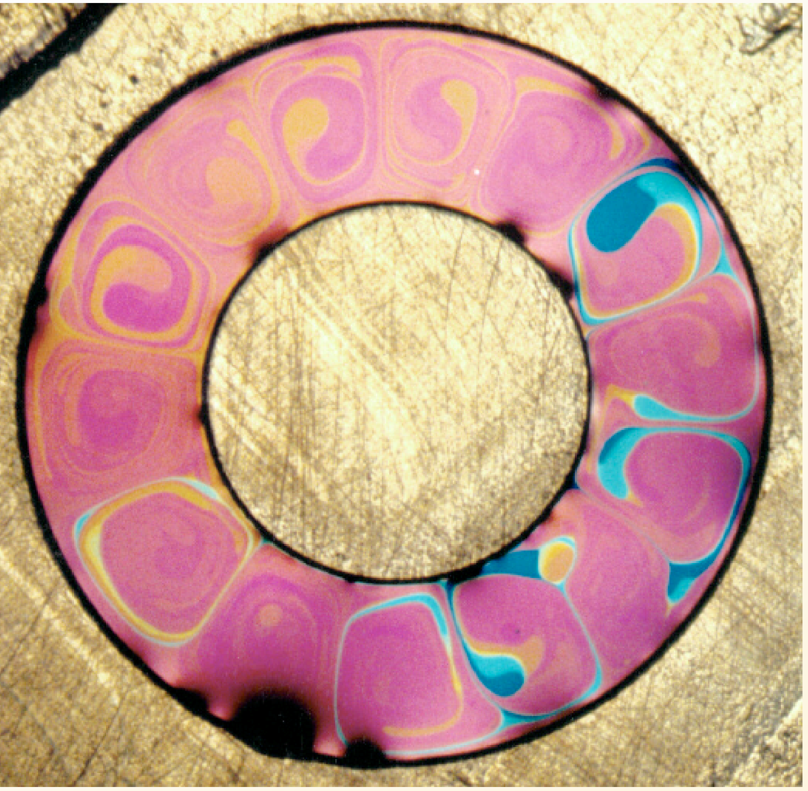
Linear stability analysis



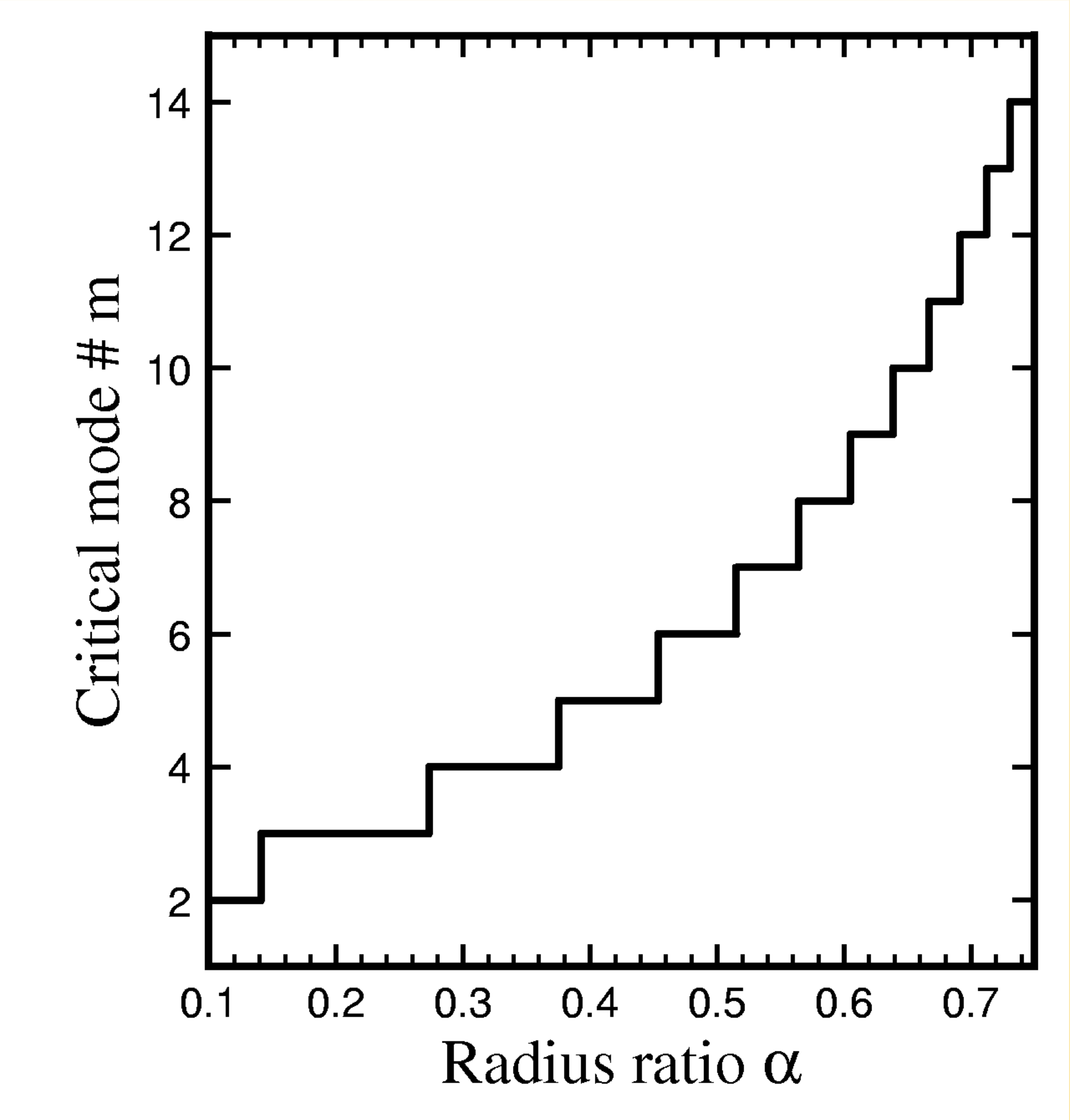
fields $\sim e^{im\theta}$

m = azimuthal mode number

$\mathcal{R}_c \sim 100$

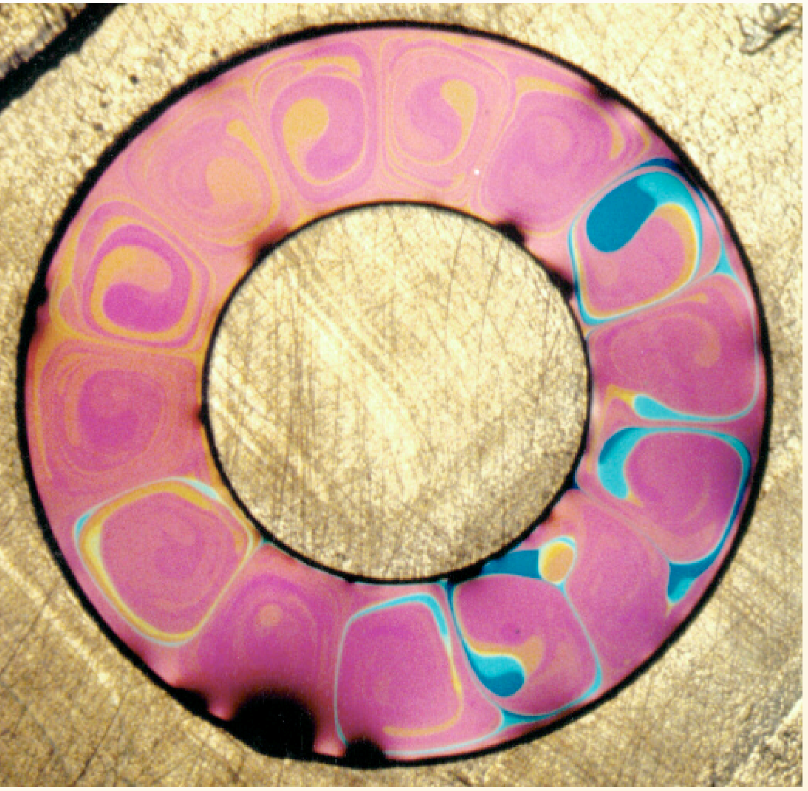


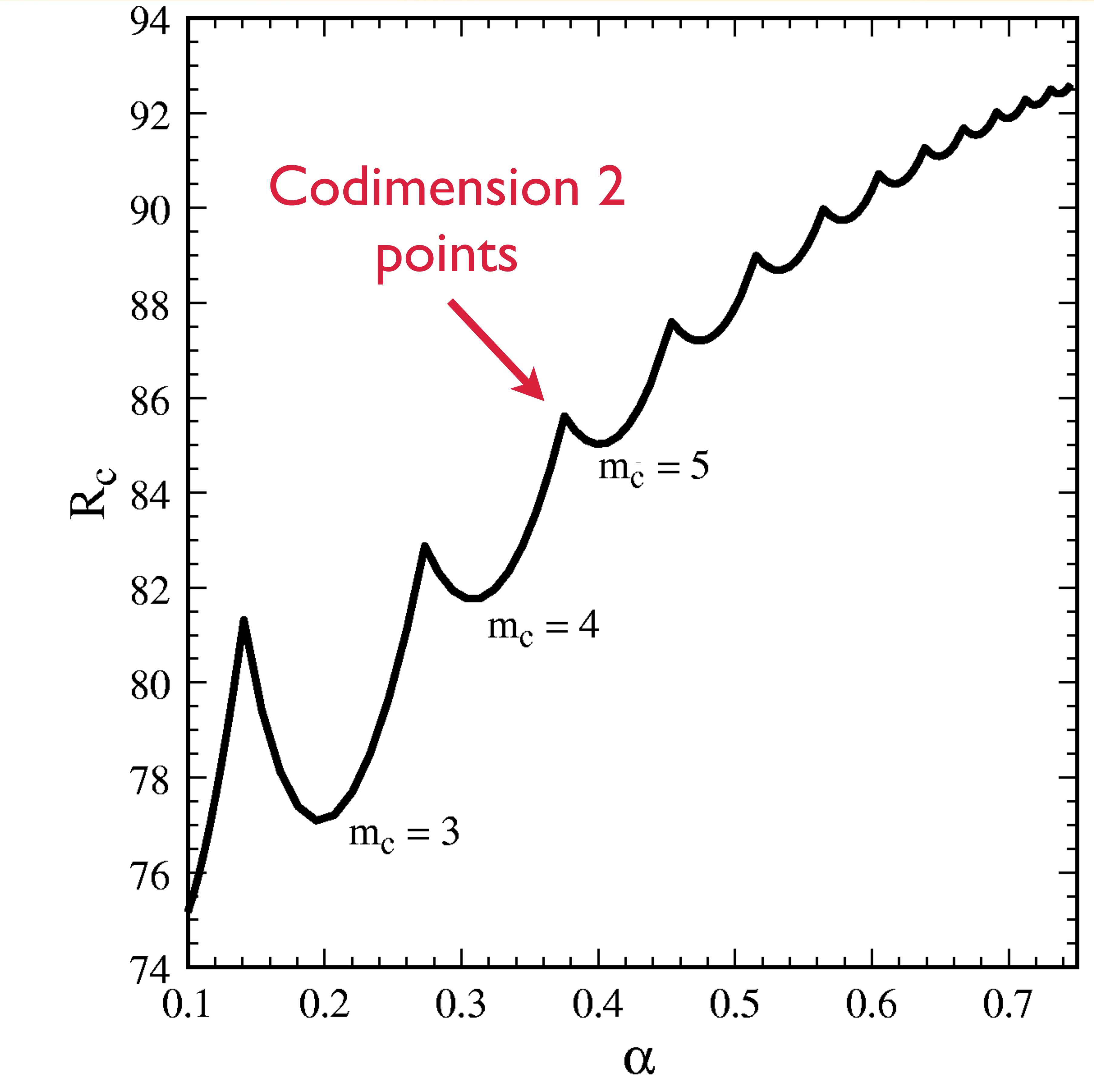
Linear stability analysis



Critical mode number
for zero shear
vs. radius ratio

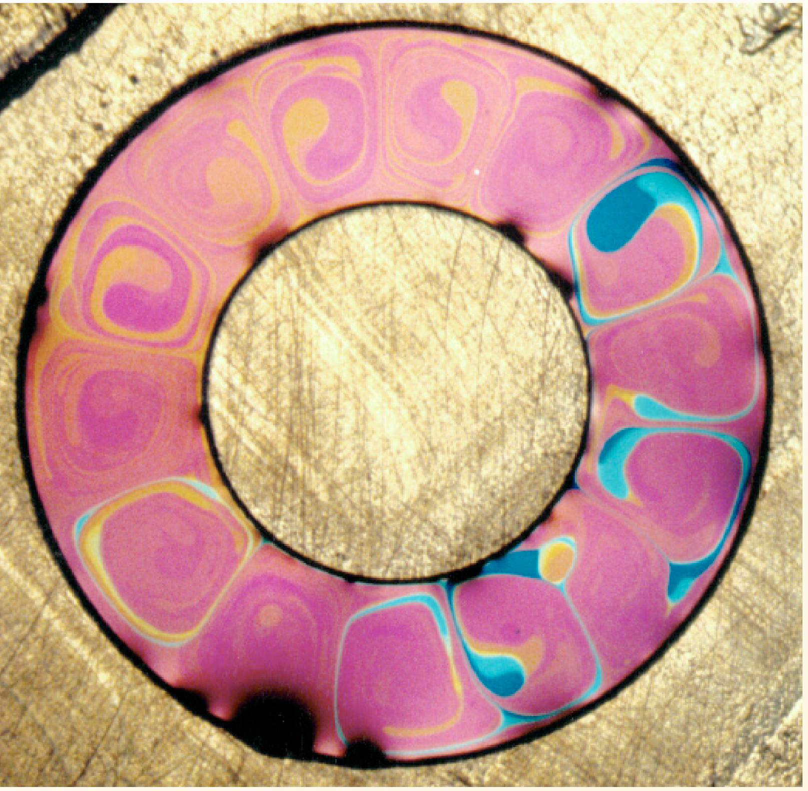
Linear stability analysis

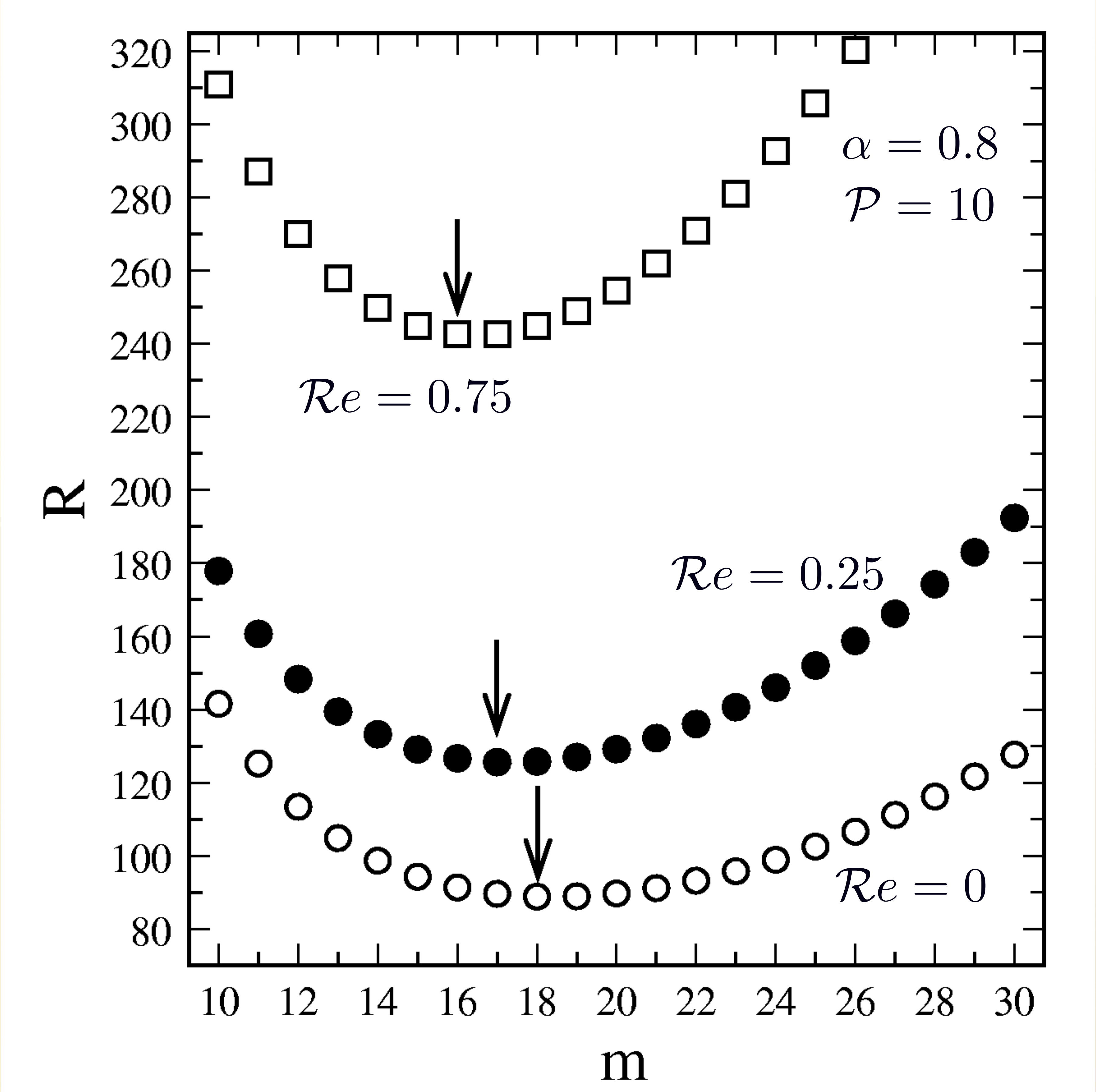




Critical Rayleigh number
for zero shear
vs. radius ratio

Linear stability analysis



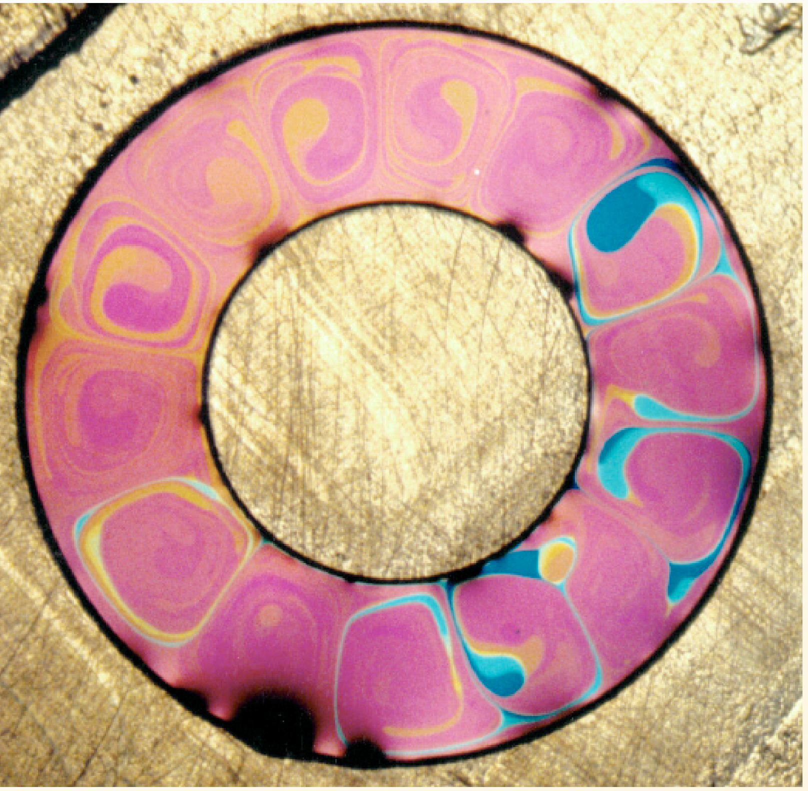


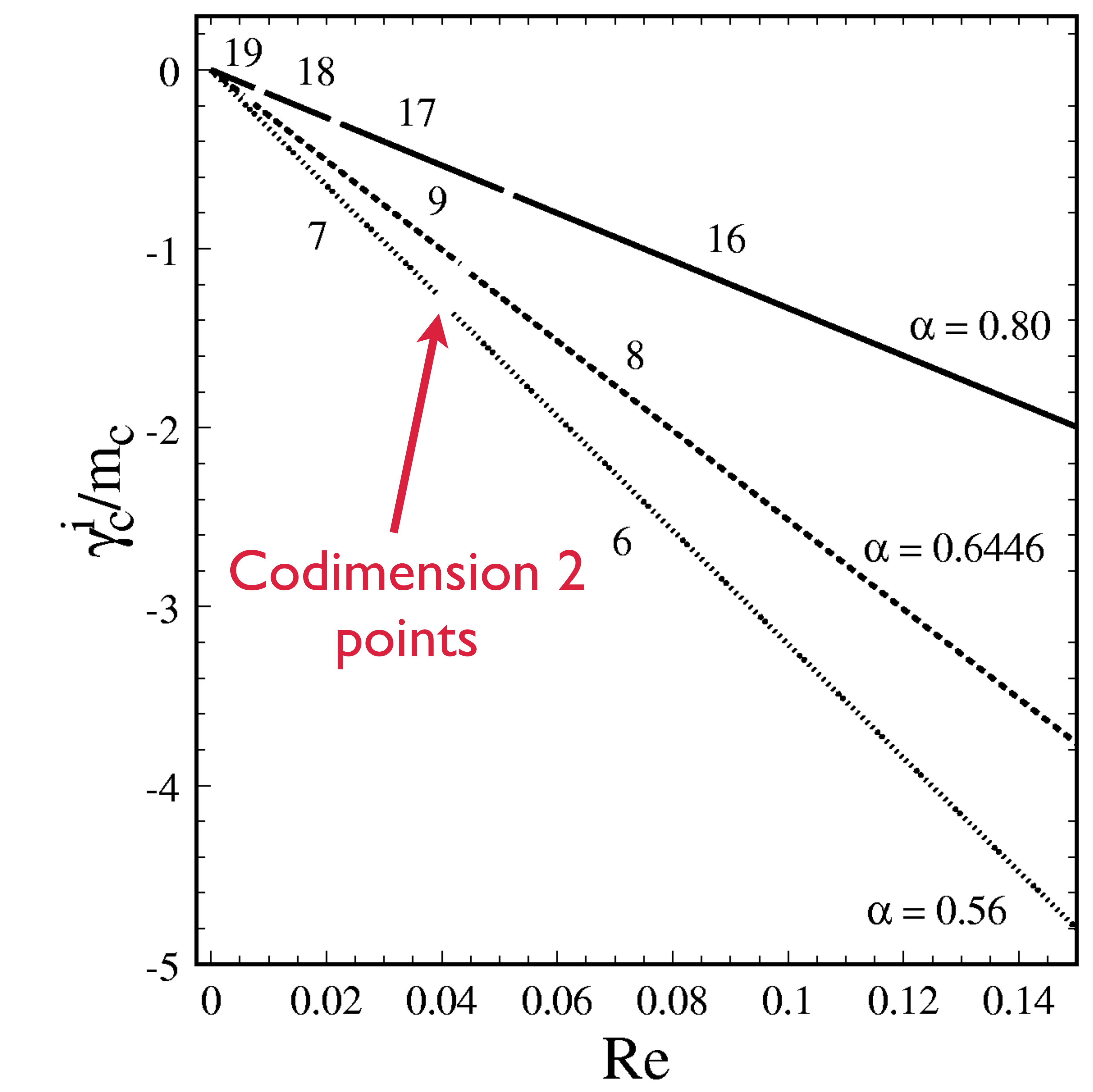
Under shear, onset is *suppressed* to higher *R*

Reynolds number

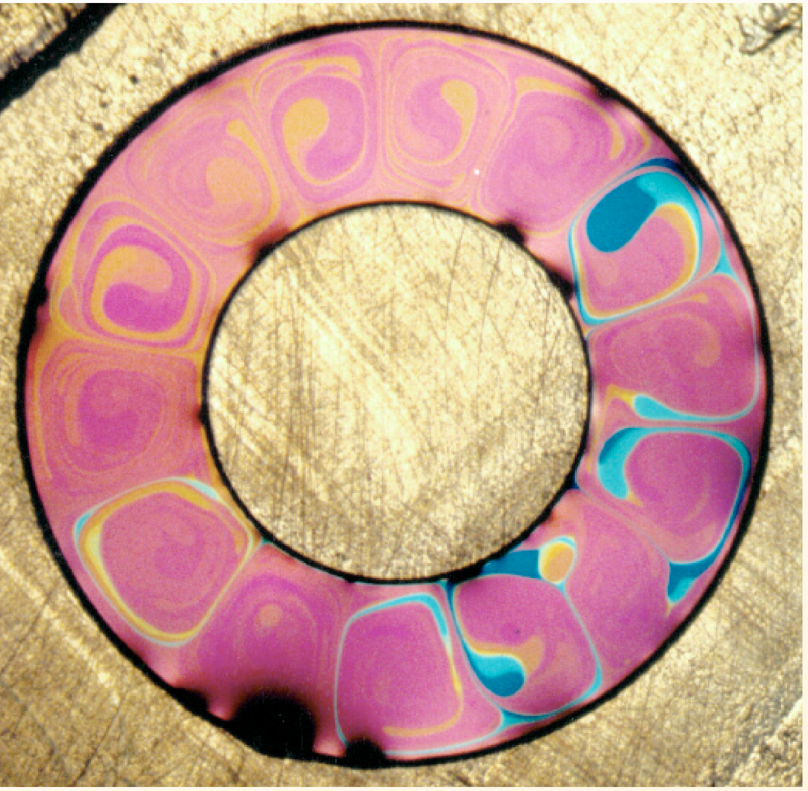
$\mathcal{R}e = \rho \omega r_i (r_o - r_i)/\eta$

Linear stability analysis

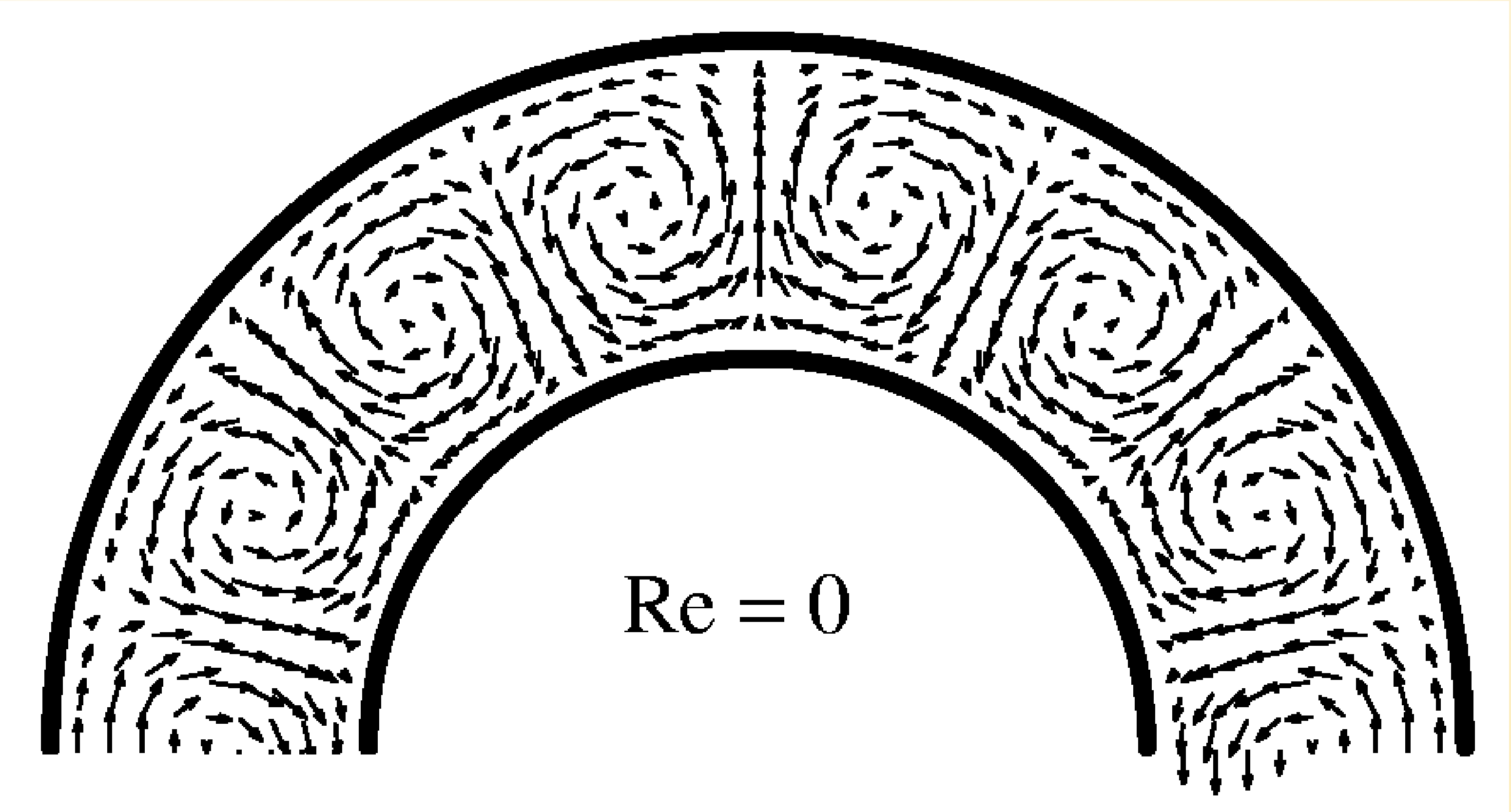


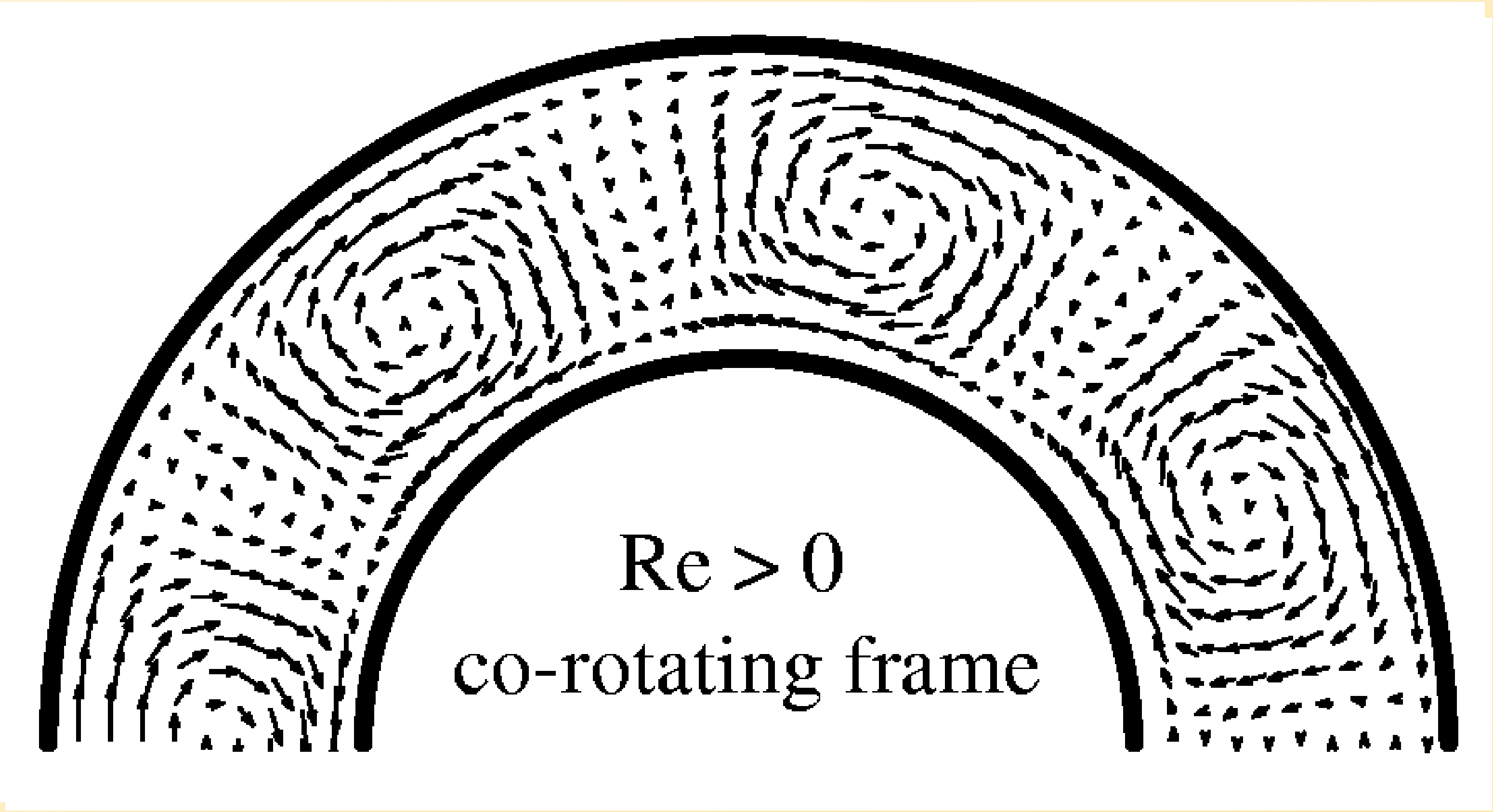


Most unstable linear modes travel azimuthally under shear.



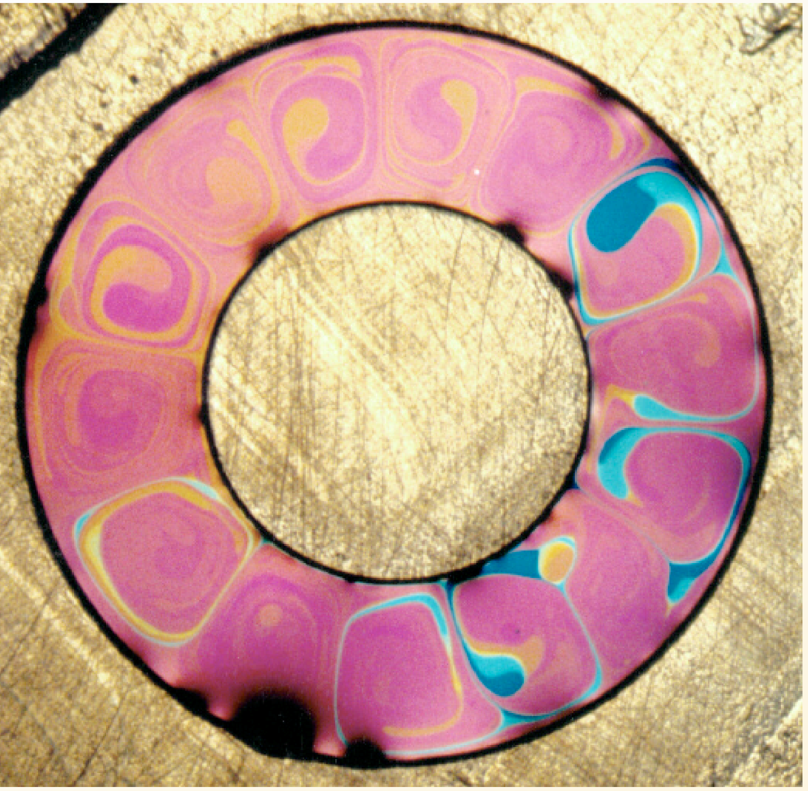
Linear stability analysis





Most unstable linear modes, with vortices visualized

Experiment



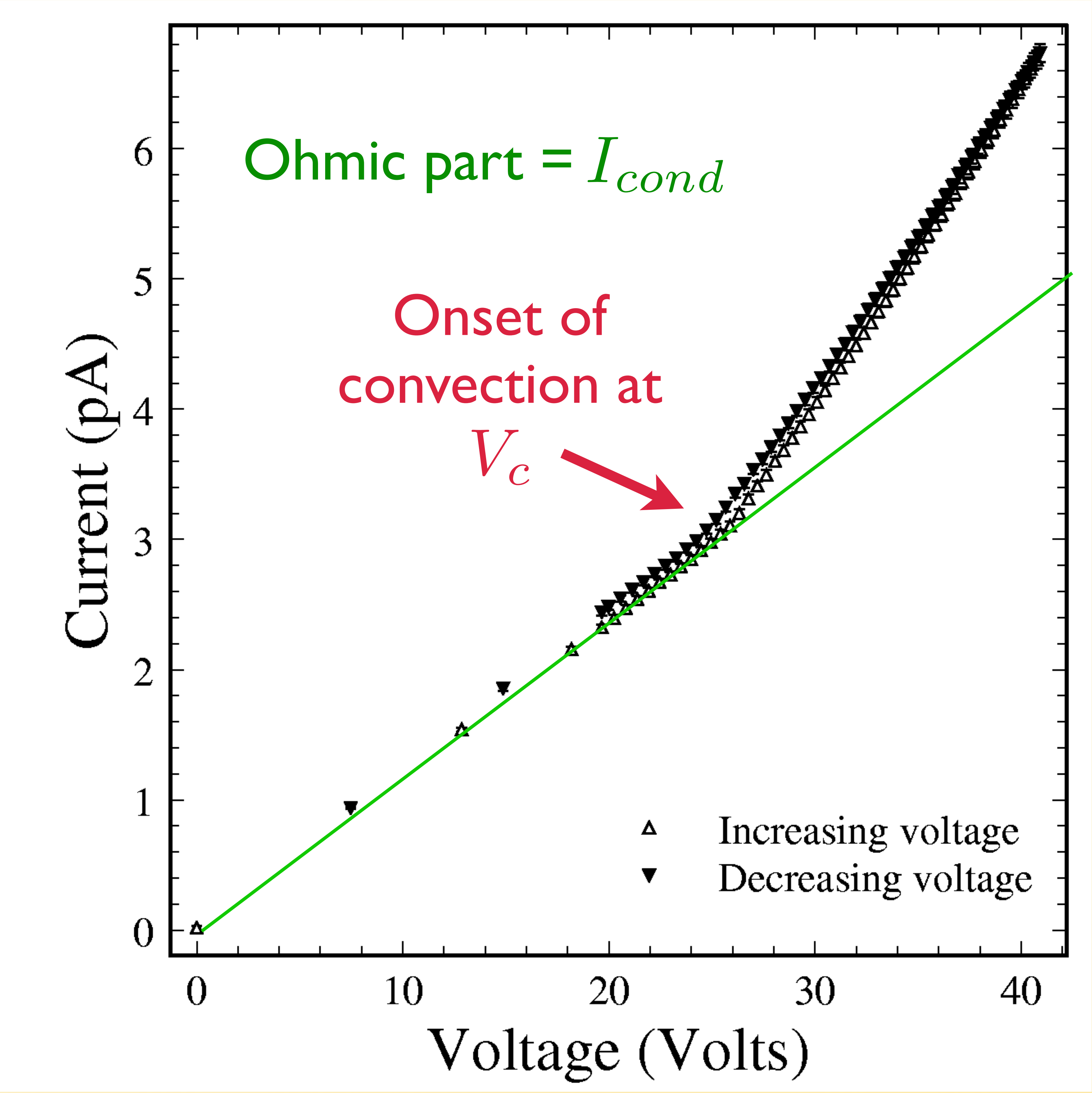
Measure the current through the film.

Nusselt number

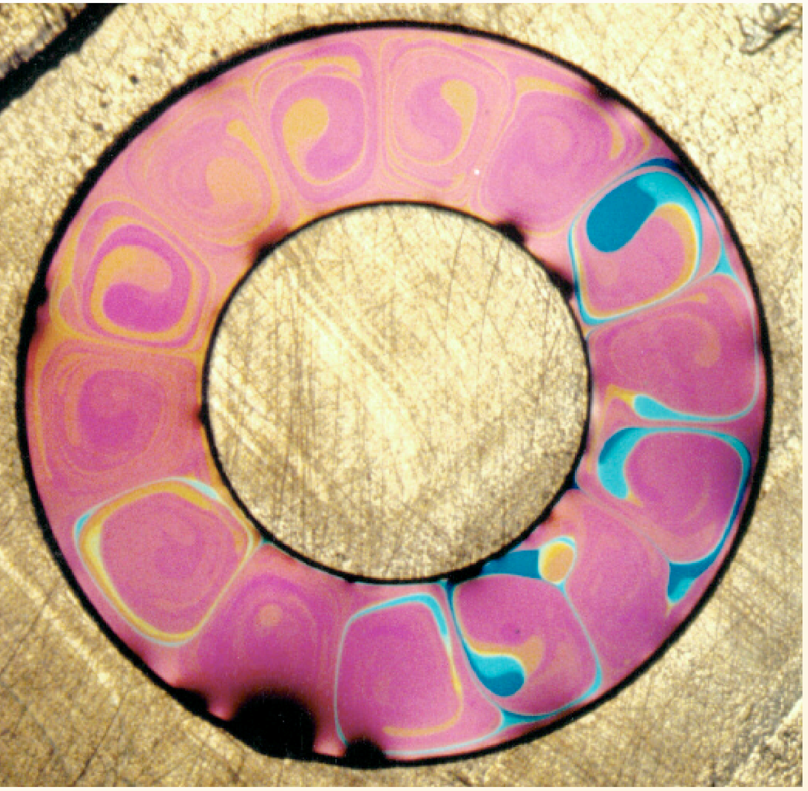
$$\mathcal{N}u = \frac{I_{total}}{I_{cond}}$$

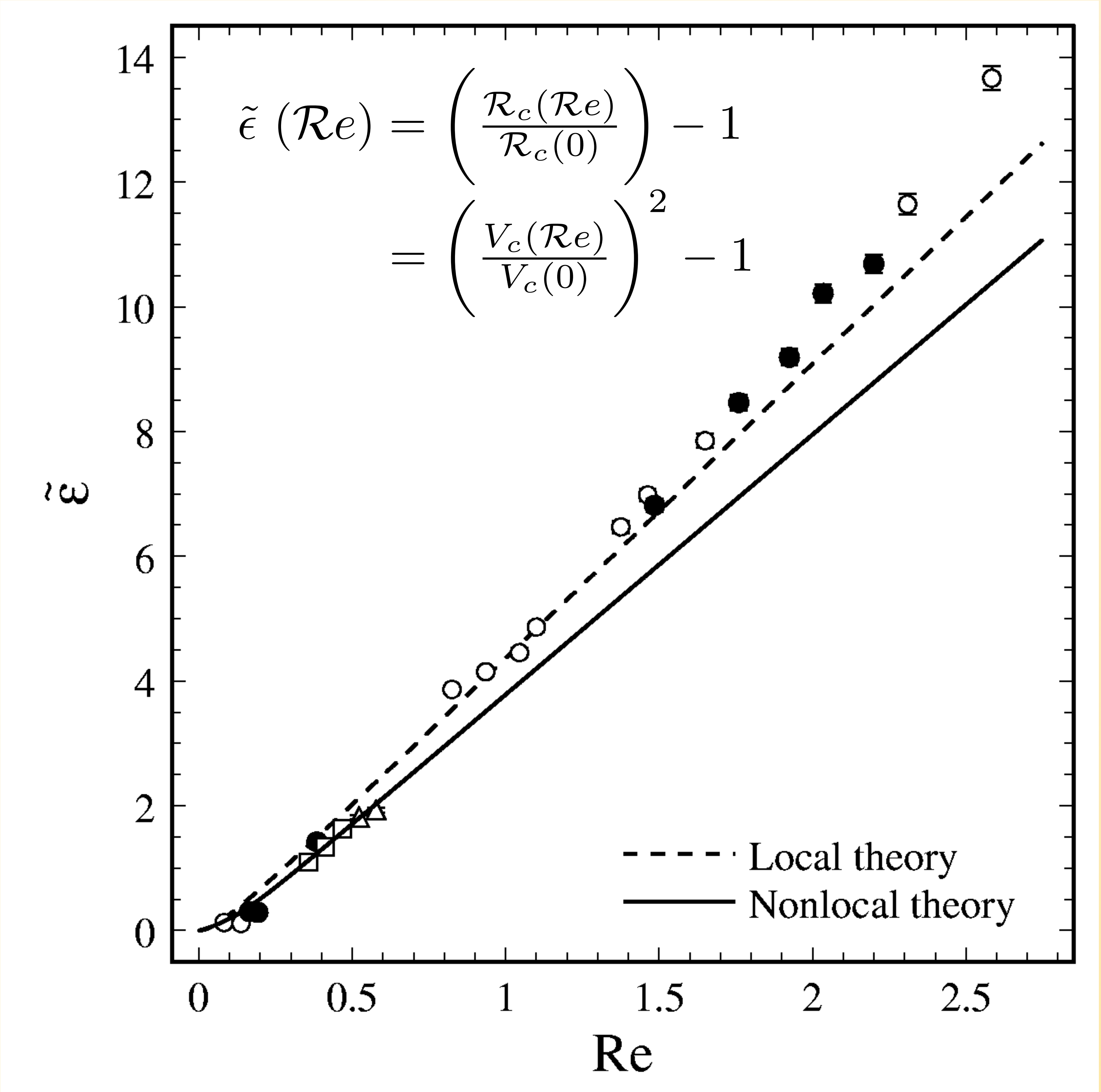
reduced Nusselt number

$$n = \mathcal{N}u - 1$$

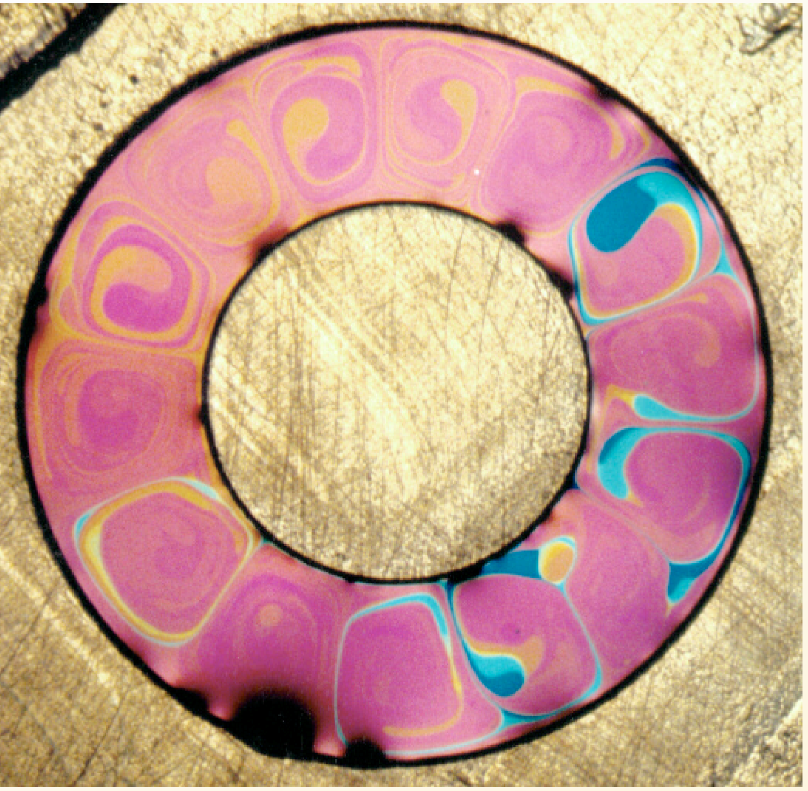


Experiment





Suppression of onset by shear agrees with linear theory



Nonlinear regime

Amplitude equation formulation gives a complex Landau equation

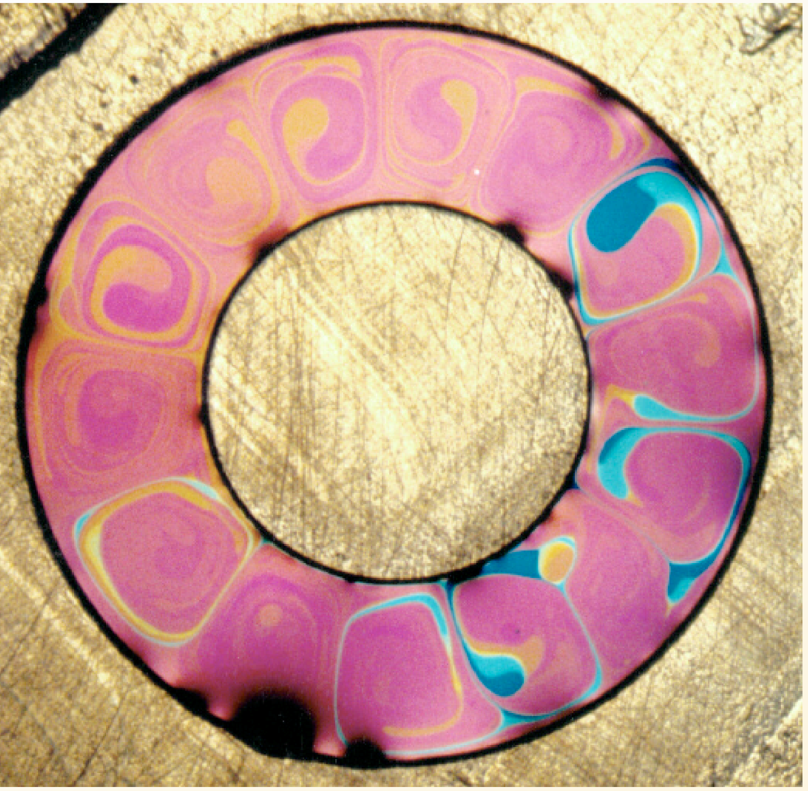
$$\epsilon = \left[\frac{\mathcal{R}}{\mathcal{R}_c}\right] - 1 \quad \text{Fields} \sim A_m e^{im\theta}$$

$$\tau(\partial_t - i a_{Im}) A_m$$
$$= \epsilon(1 + ic_0)A_m - g(1 + ic_2)|A_m|^2 A_m$$
$$+ h(1 + ic_3)|A_m|^4 A_m - \ldots$$

$$A_m(t) = A(t)e^{i\Phi(t)}$$

$$\tau\partial_t A = \epsilon A - gA^3 - hA^5 - \ldots$$ Real part

$$\tau(\partial_t \Phi - a_{Im}) = \epsilon c_0 - g c_2 A^2 + \ldots$$ Imaginary part



Nonlinear regime

We can use Nusselt number measurements to probe the magnitude of the complex amplitude.

time independent real amplitude equation

$$\tau \partial_t A = \epsilon A - g A^3 - h A^5 = 0$$

A can be scaled so that $\mathcal{N} - 1 = n = A^2$

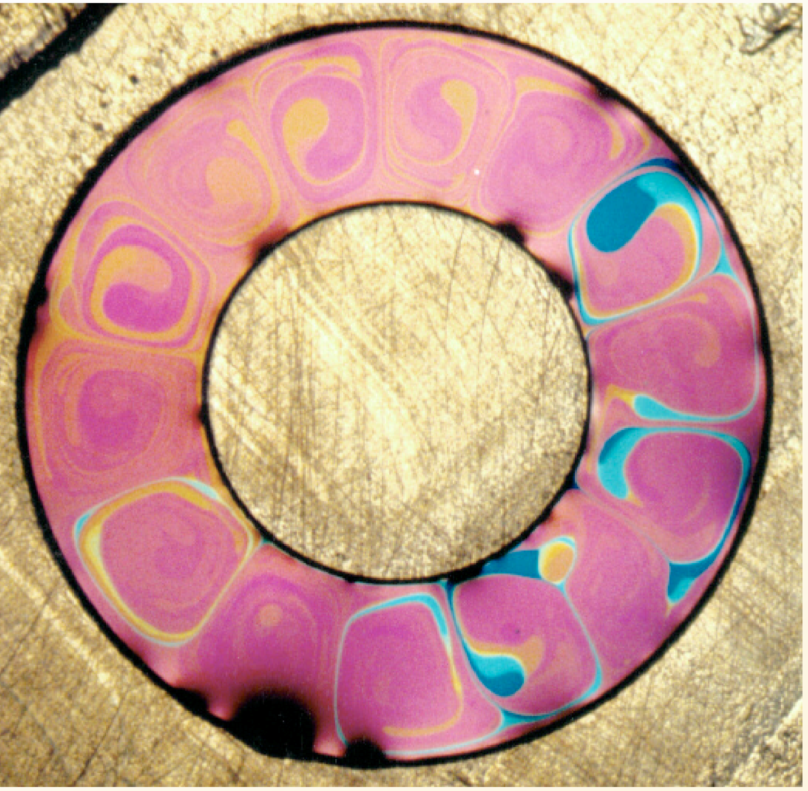
Fit n vs. ϵ to extract g and $h > 0$

$g > 0$ supercritical

$g = 0$ tricritical

$g < 0$ subcritical

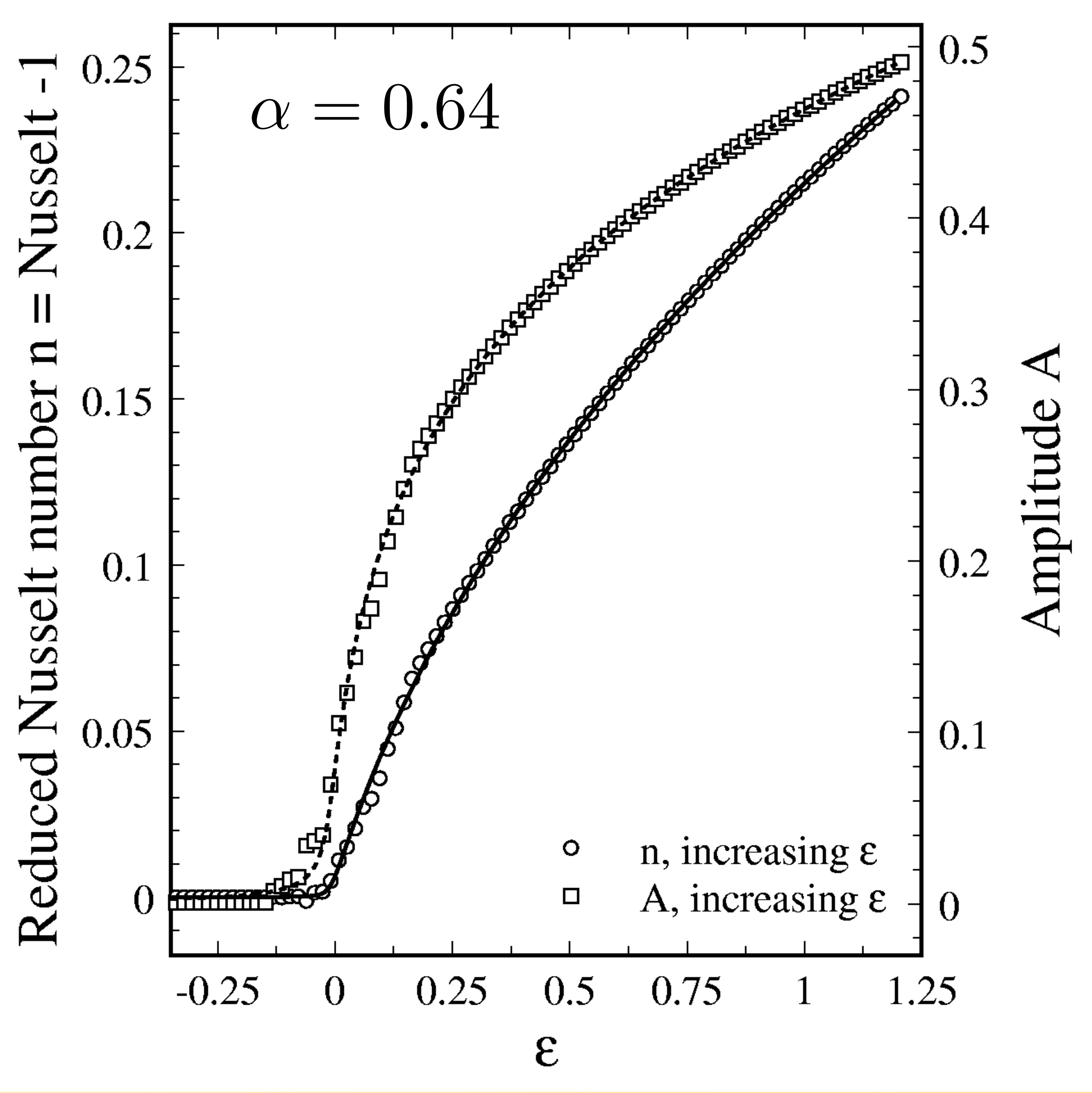
Nonlinear regime



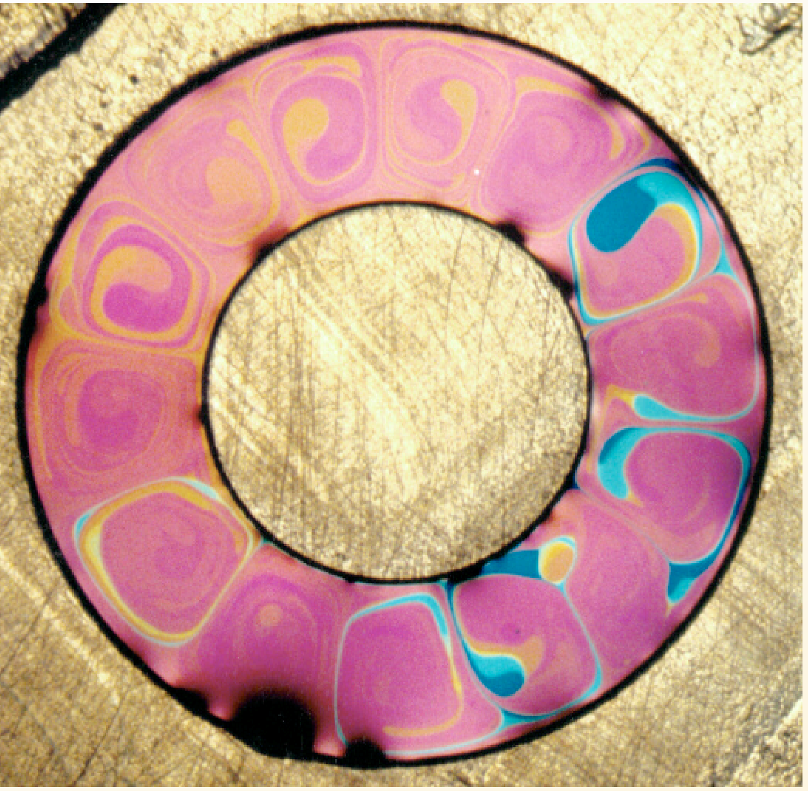
For small Reynolds number, we find

$$g > 0$$

supercritical bifurcations



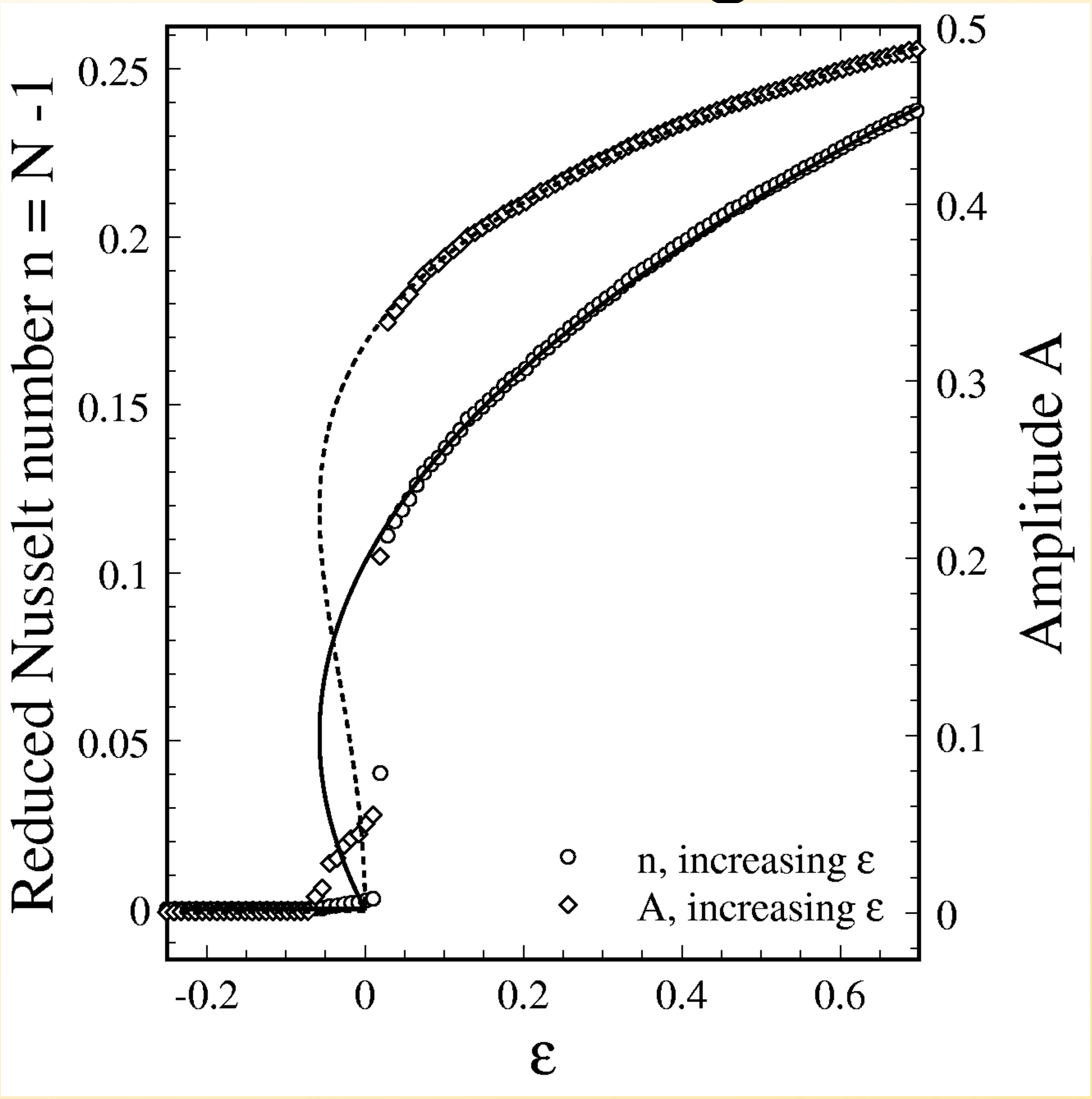
Nonlinear regime



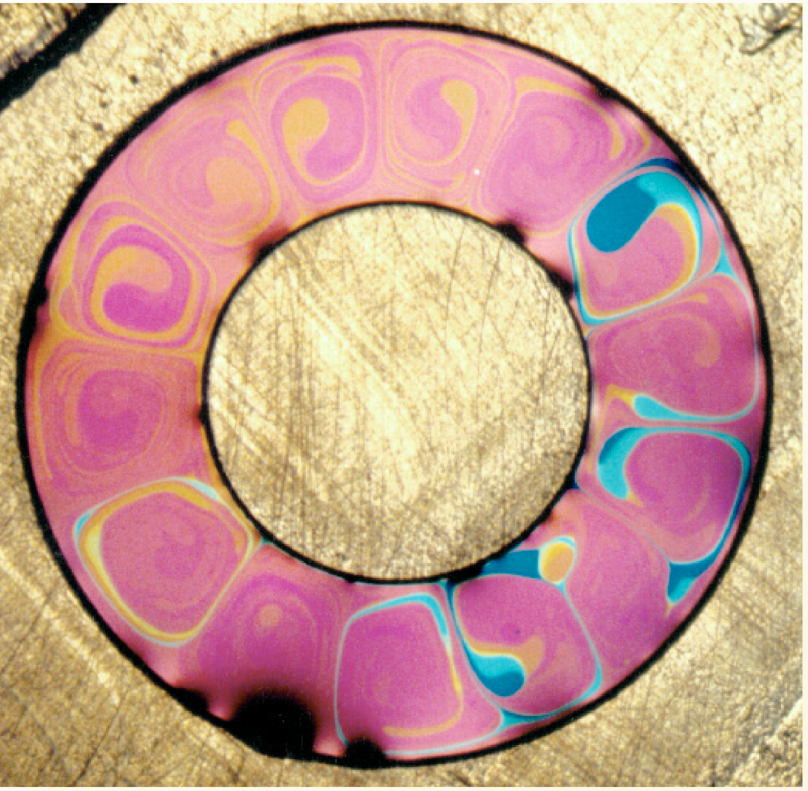
For larger Reynolds number, we find

$$g < 0$$

subcritical bifurcations

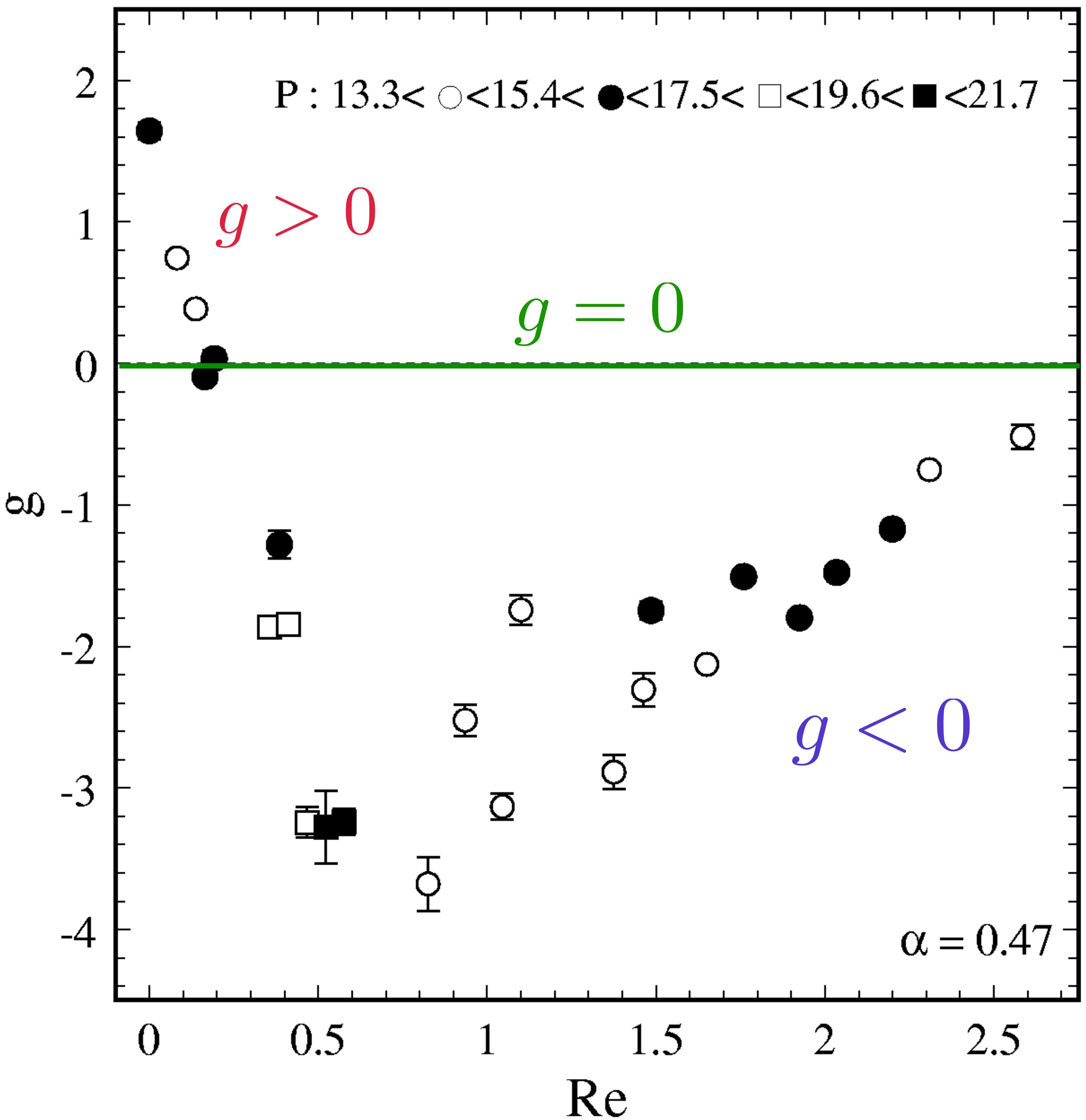


Nonlinear regime

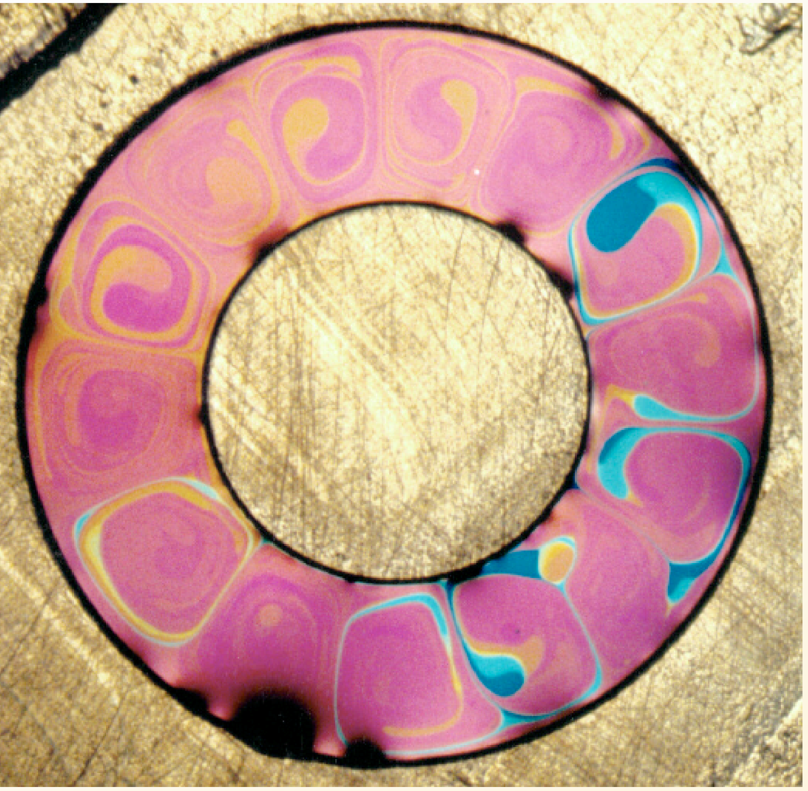


The cubic coefficient also depends on

$$\alpha, \mathcal{P}$$



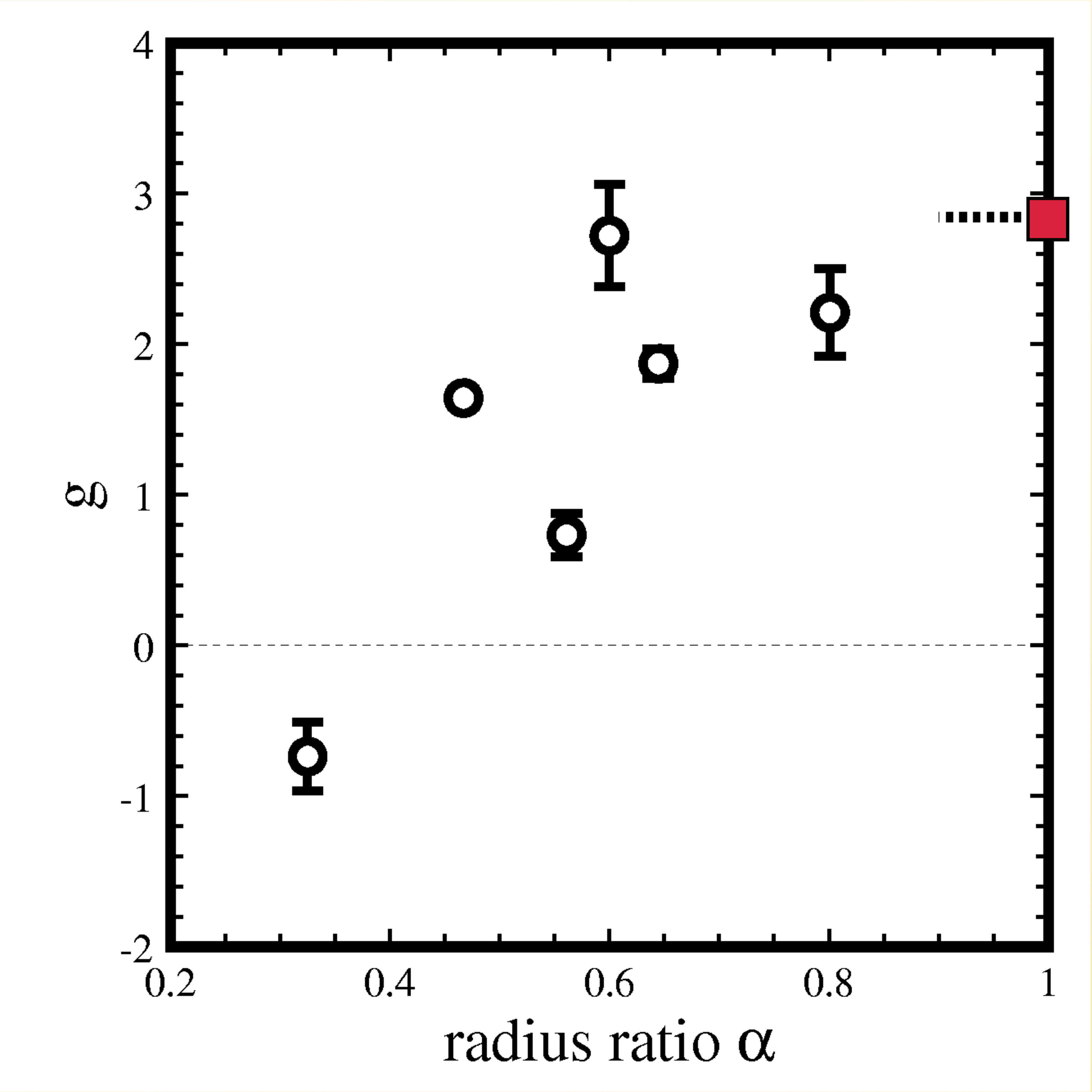
Nonlinear regime

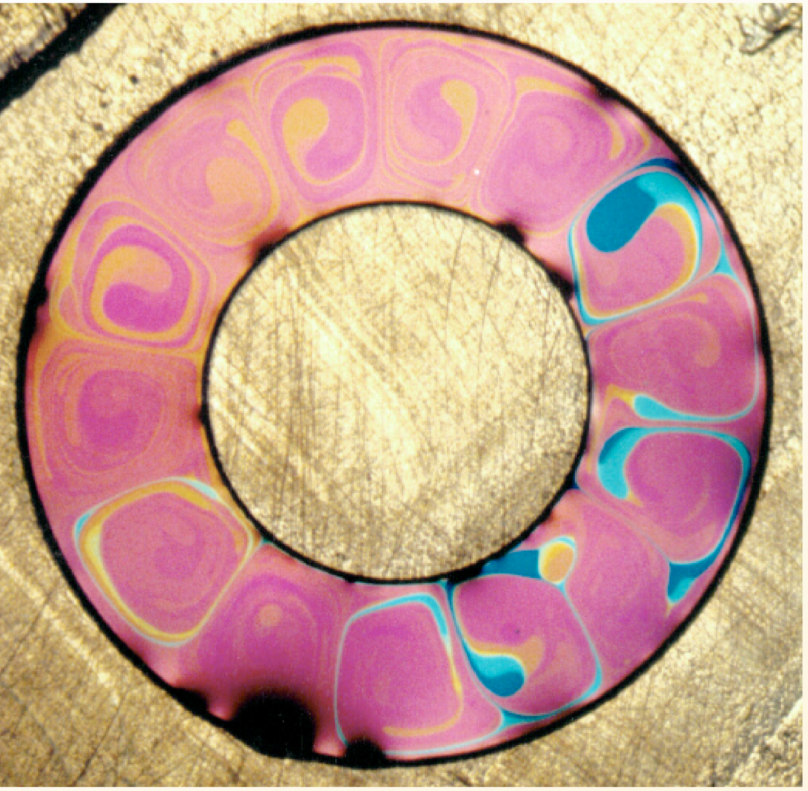


Bifurcation also becomes subcritical for *Re*=0 and small α

$$g = 2.84$$

for $\alpha = 1, \mathcal{P} \to \infty$

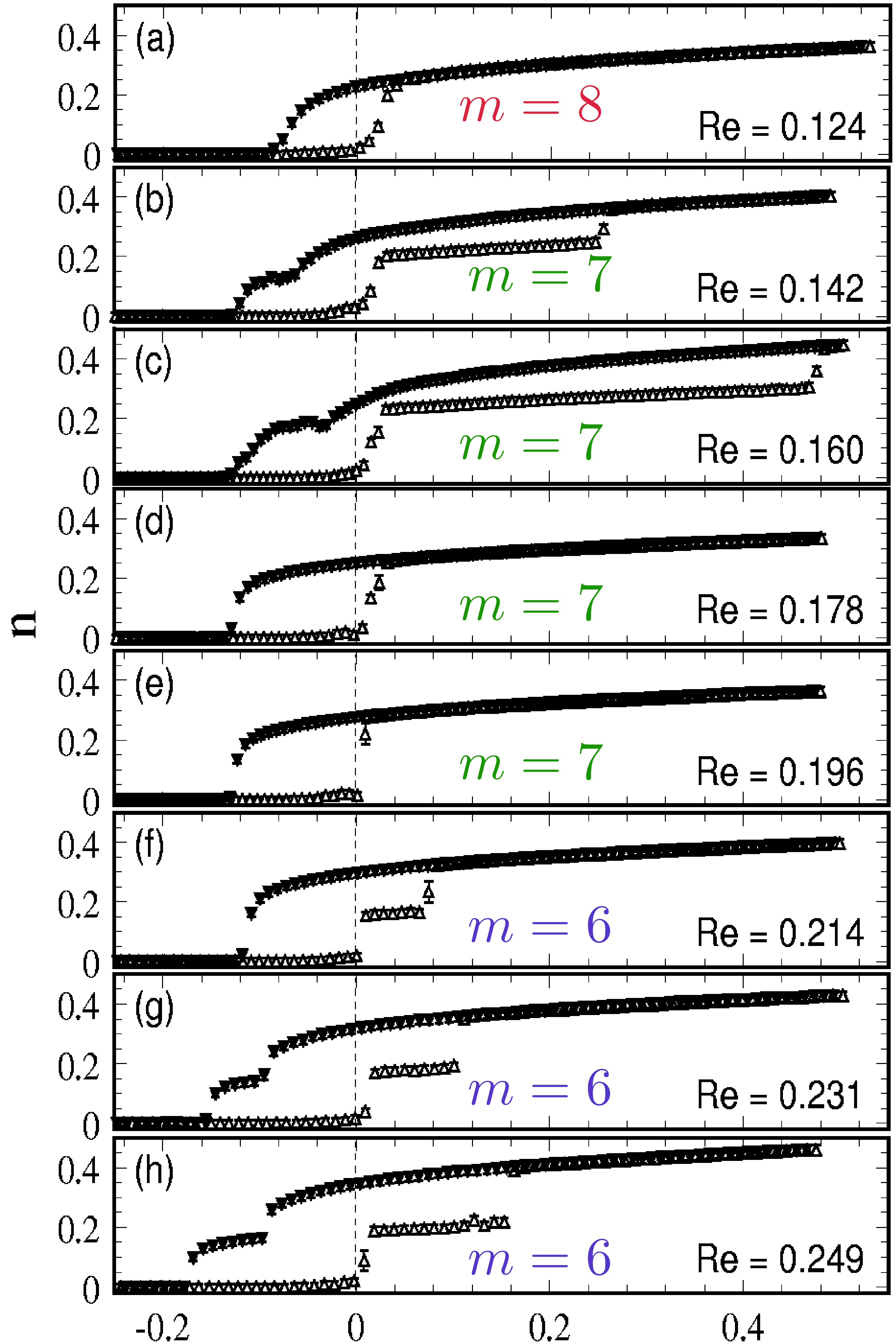




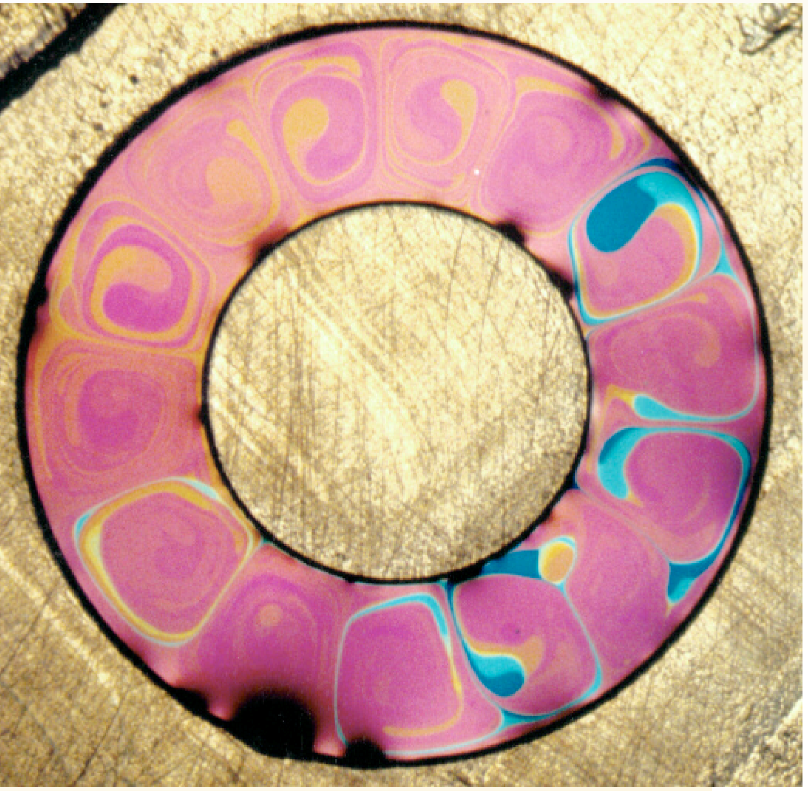
Secondary bifurcations

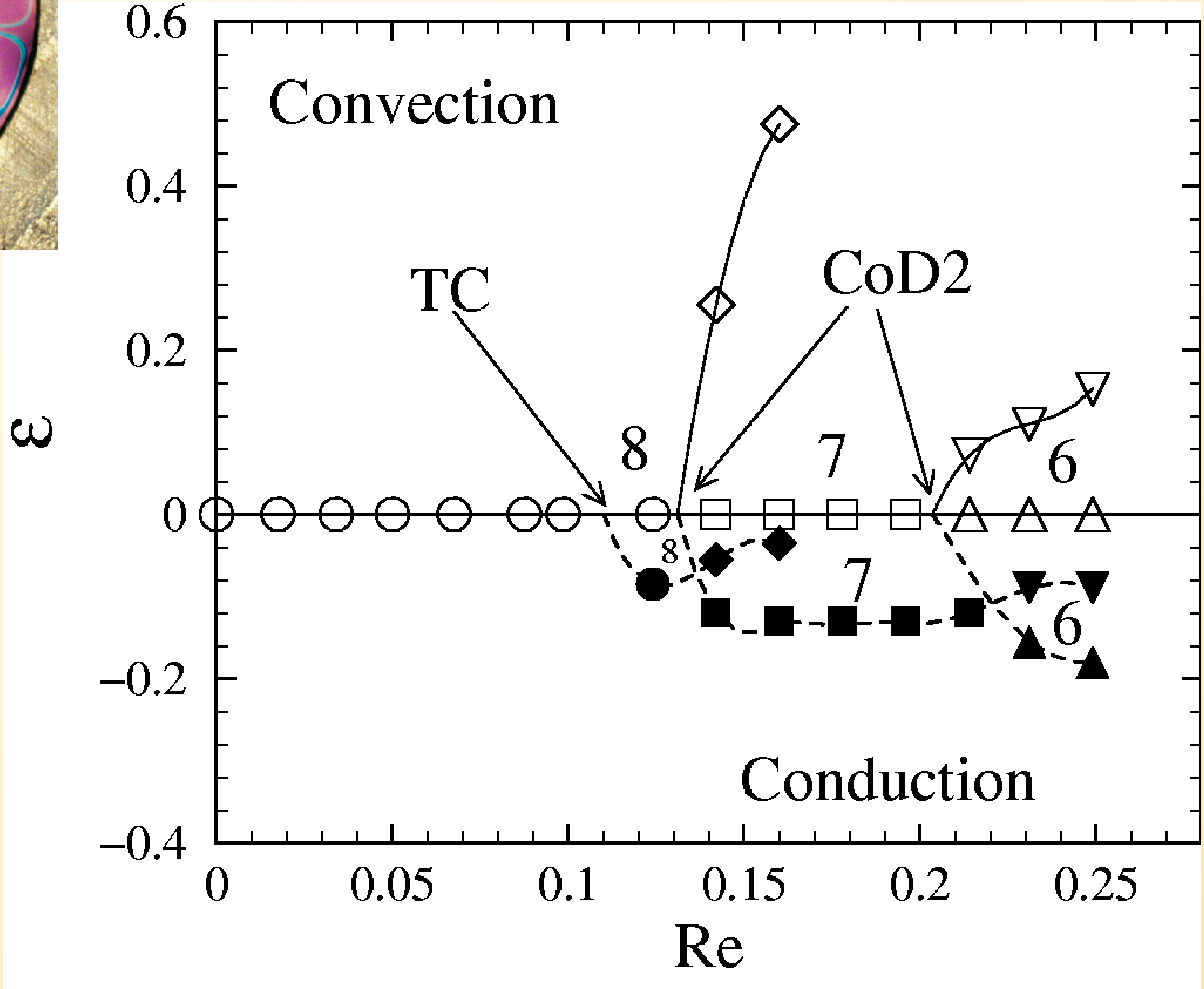
Consist of mode hopping transitions

$$m \rightarrow m \pm 1$$

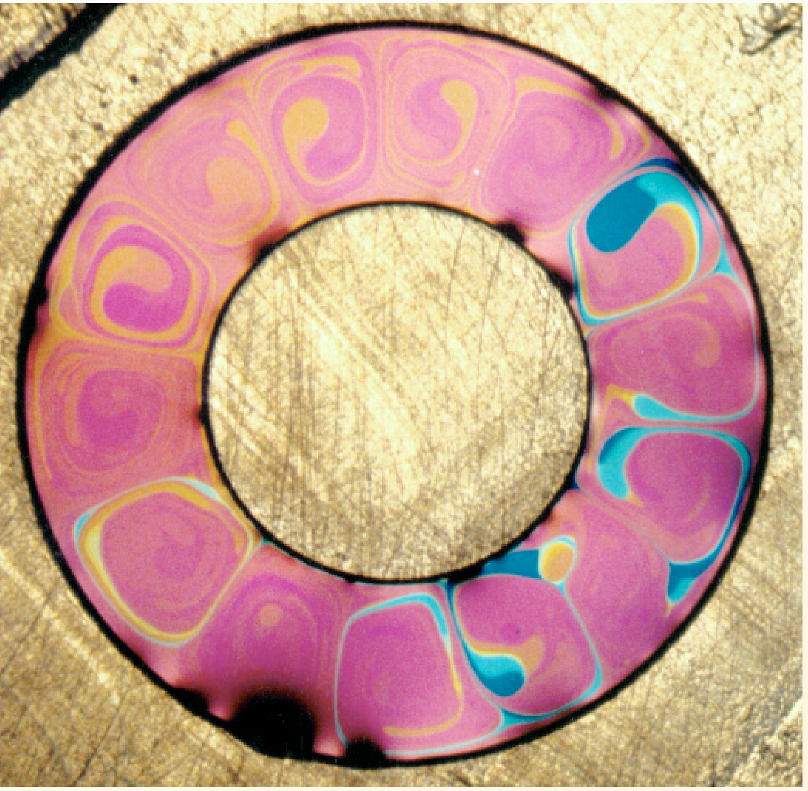


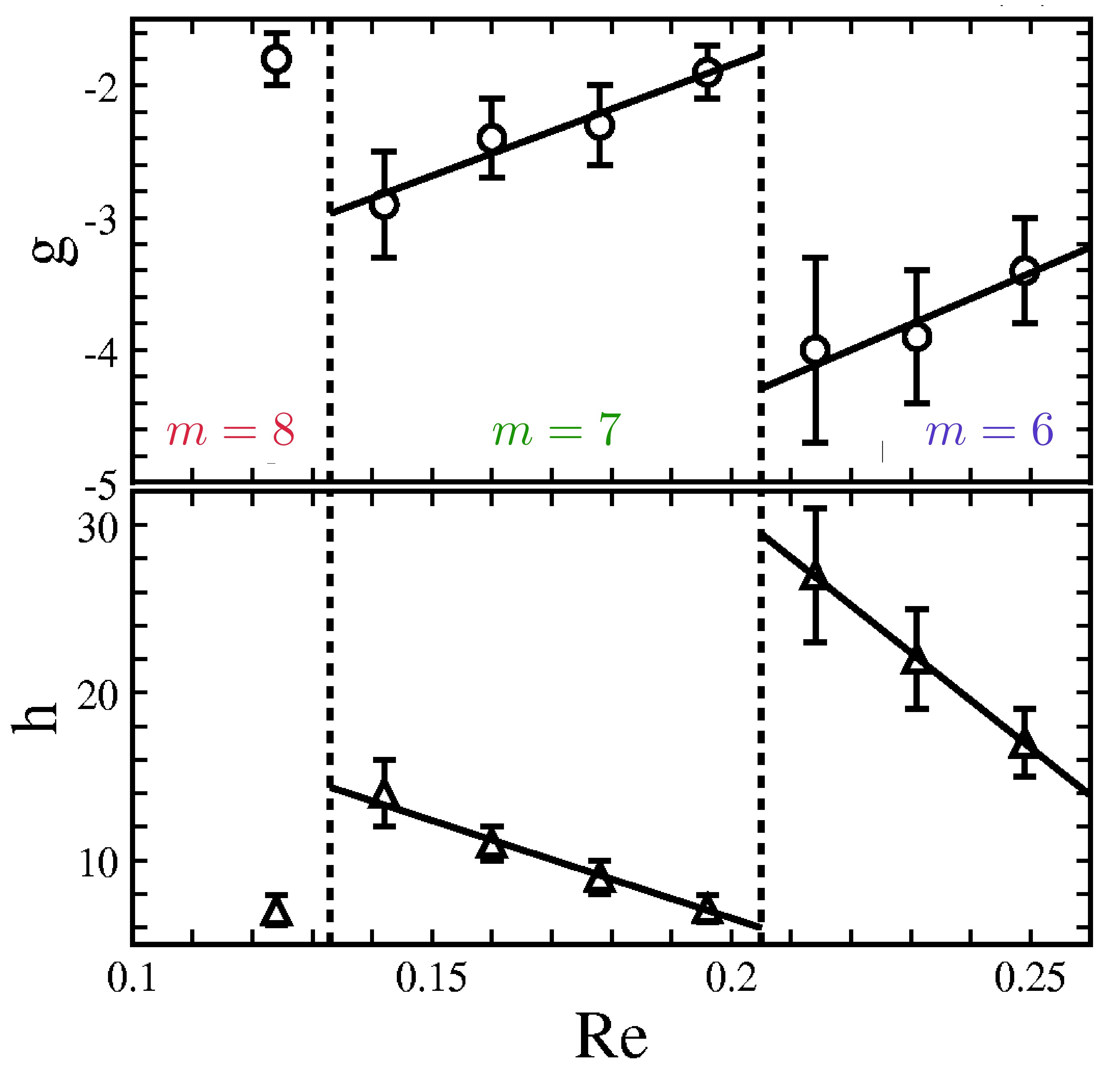
Secondary bifurcations





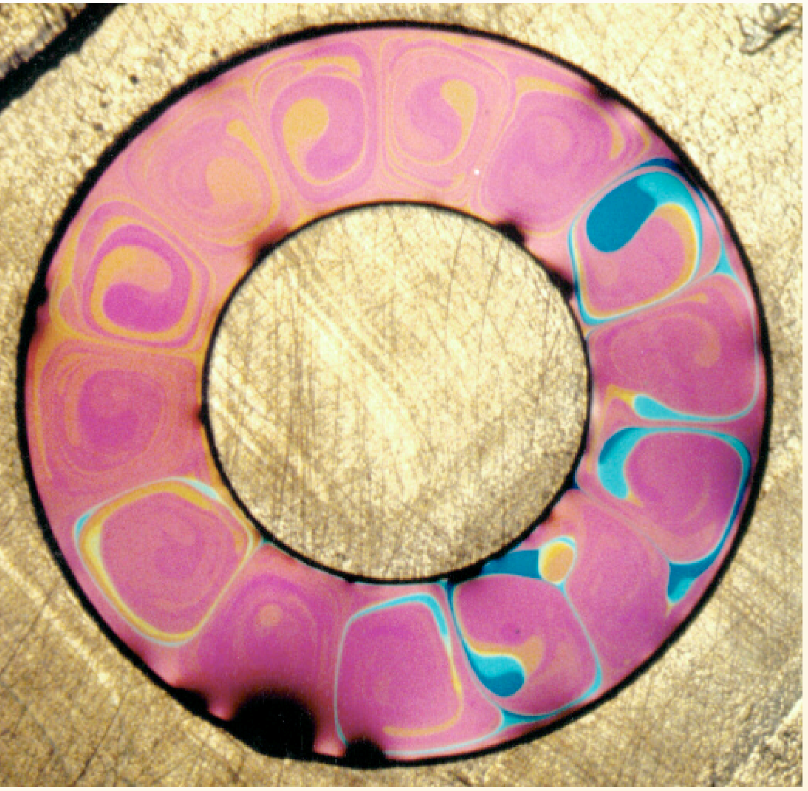
Tree of *m* states and subcritical bifurcations





Secondary bifurcations

We observe discontinuities in g and h as codimension 2 points are passed



Conclusion

- Smectic electroconvection with shear is a rich playground for weakly nonlinear analysis.
- Large parameter space has not been fully mapped experimentally.
- Full Navier Stokes simulations should be feasible.

You are invited to visit the nonlinear physics group lab!

Thursday at 1:00pm
Friday at 1:00pm

or anytime by appointment,
smorris@physics.utoronto.ca

60 St. George St., Room 090 (north basement). Go out the back door of Fields and past the big smokestack...

www.physics.utoronto.ca/nonlinear