

# **SPATIO-TEMPORAL PATTERNS in HEAT EXCHANGER TUBE ARRAYS**

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This presentation is based on  
the Doctoral Dissertation of

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## OUTLINE

1. Heat exchanger arrays
2. Symmetries and their implications  
(model-independent results)
3. Differential equation models,  
with a bifurcation parameter
4. Equivariant Hopf bifurcation  
analysis
5. Numerical simulations
6. Conclusions

## MOTIVATION

- ▶ Heat exchangers are an essential component of nuclear-electric power generators and many other devices.
- ▶ AECL (Atomic Energy Canada Limited) nuclear power generators use heat exchangers with thousands of parallel tubes in regular arrays.
- ▶ Flow-induced vibrations of these tubes can cause metal fatigue, cracks and dangerous leaks.
- ▶ Any shut-down for repairs of heat exchanger tubes costs millions of dollars.
- ▶ Engineers who design such heat exchangers want to understand the causes of the vibrations and they seek strategies to avoid them.
- ▶ Experiments at AECL show that flow-induced oscillations of tube arrays occur in distinctive spatio-temporal patterns.

# EXPERIMENTS OF M. PETTIGREW

(AECL Video 1986)

- ▶ The apparatus contains a rectangular array of flexible cylinders in cross-flow.
- ▶ Experiments were performed with different fluids (air, water, ...) and different flow velocities  $U$ .
- ▶ For sufficiently large flow velocity  $U$ , the cylinders oscillate in coherent spatio-temporal patterns.
- ▶ Two typical patterns observed:
  - (a) The cylinders oscillate transversely (across the flow), synchronized in vertical columns but anti-phase between columns.
  - (b) The cylinders oscillate longitudinally (in the direction of flow), in a "travelling wave" along the columns and in-phase between columns.

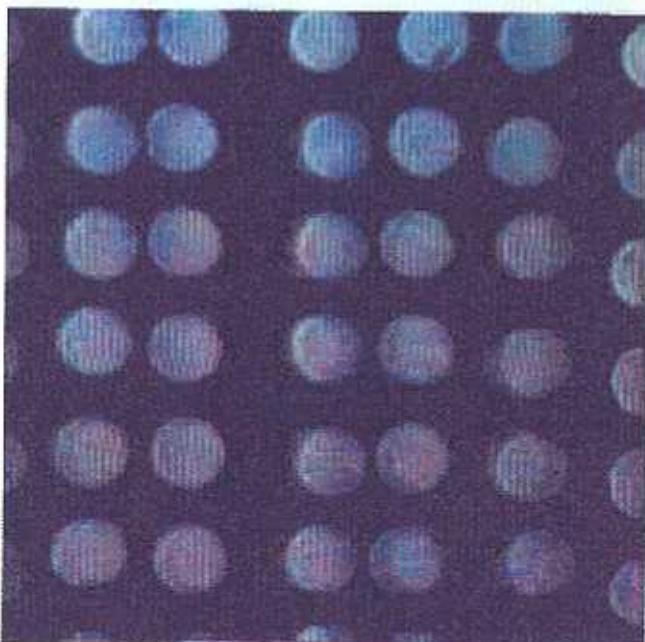
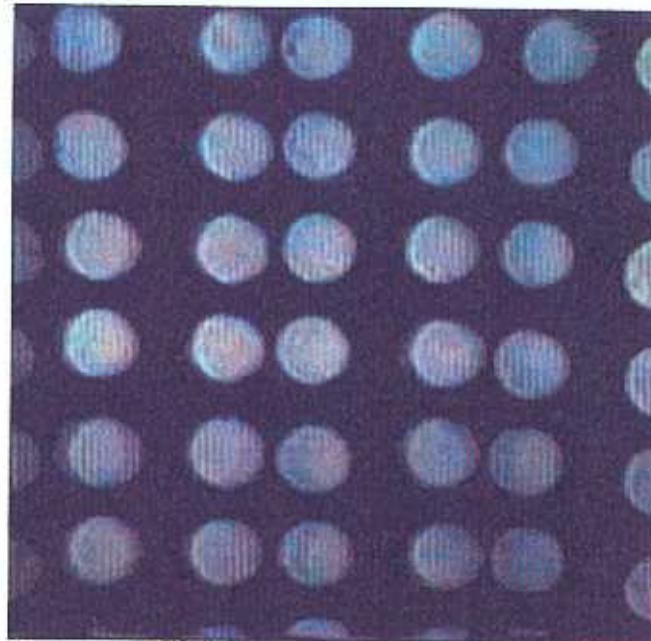
## EXPERIMENTAL APPARATUS



Flow-Induced Vibrations of Heat Exchanger Tube Arrays  
Atomic Energy Canada Limited, 1986

# **TRANSVERSE VIBRATION MODE**

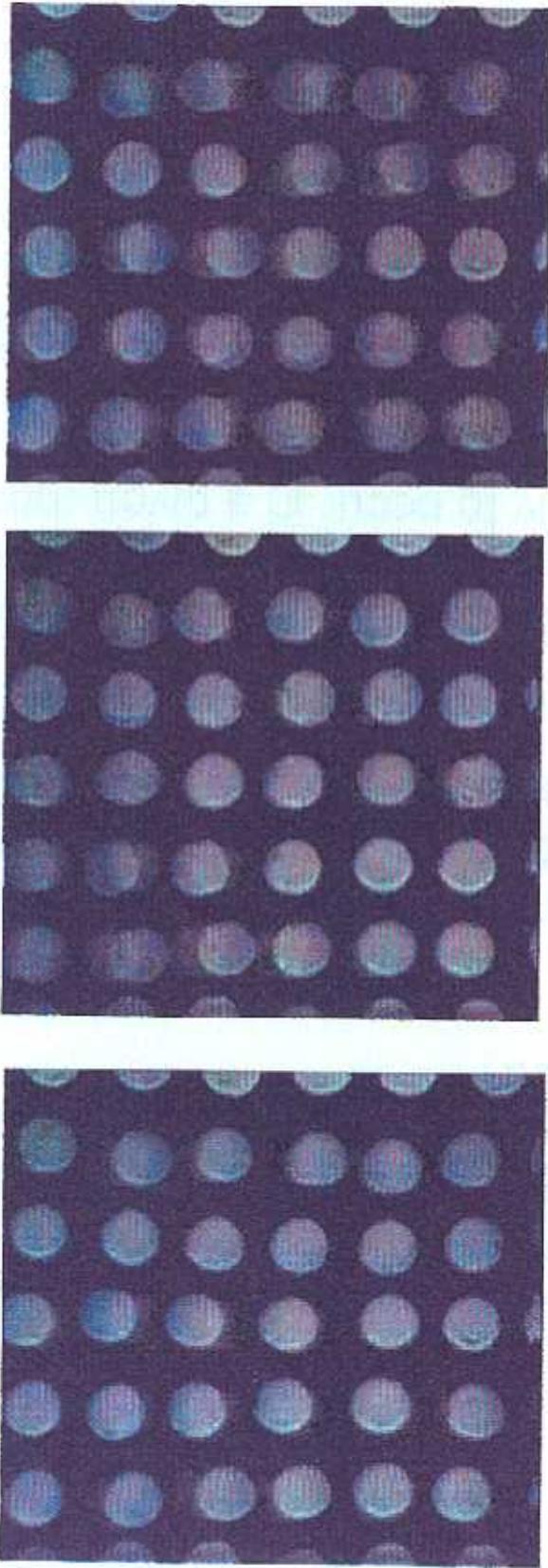
of Heat Exchanger Tube Array



Experiments of M. Pettigrew, AECL, 1986.

# **LONGITUDINAL VIBRATION MODE**

## of Heat Exchanger Tube Array



Experiments of M. Pettigrew, AECL, 1986.

## QUESTIONS

1. Can the possible types of spatio-temporal patterns be determined from the geometry and symmetries of the apparatus, using the “model independent” group-theoretic methods of Golubitsky, Stewart and others?
2. Can a simple ODE model be constructed, that incorporates the essential geometry, symmetries and physical design of a heat exchanger array?
3. Can the Equivariant Hopf Bifurcation Theorem be used to predict which spatio-temporal patterns are most likely to occur in a given model?
4. If the answer is “yes” to 1. 2. and 3., then:  
Do the predictions agree with the physical experiments?

# SYMMETRIES of the SYSTEM

0. All cylinders are identical and equally spaced, in each of  $x$  and  $y$  directions.  
 $\Rightarrow$  translational symmetry.
1. The coupling between cylinders has a left-right reflection symmetry.  
 $\Rightarrow \mathbb{Z}_2(\alpha) = \langle 1, \alpha \rangle, \alpha^2 = 1.$
2. Each individual cylinder has an "internal" left-right reflection symmetry.  
 $\Rightarrow \mathbb{Z}_2(\sigma) = \langle 1, \sigma \rangle, \sigma x = -x.$
3. Experiments suggest "travelling waves" of period  $N$  ( $N$  unknown) in the longitudinal  $y$ -direction,  
 $\Rightarrow \mathbb{Z}_N(\rho) = \langle 1, \rho \rangle, \rho^N = 1, (\rho^{k+1}, k < N)$

NOTES:

- (a) symmetries 1. and 2. are independent.
- (b)  $\mathbb{Z}_N(\rho)$  commutes with  $\mathbb{Z}_2(\alpha)$  and  $\mathbb{Z}_2(\sigma)$ .
- (c) There is no reflectional symmetry in the longitudinal (flow) direction.

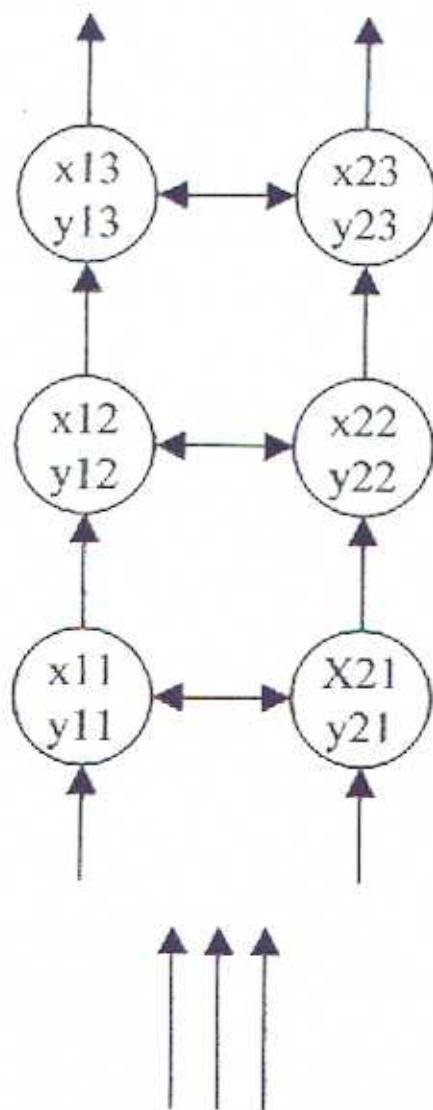
# GEOMETRICAL ASSUMPTIONS

- The array is infinite in both  $x$  and  $y$ .
  - this eliminates the physical B.C.'s.
  - in reality, arrays are very large.
- A  $2 \times N$  block "tiles" the infinite array.
  - assume periodic B.C.'s between blocks.
  - thus reduce  $\infty$ -system to  $2N$  cylinders.
- Each cylinder has 2 degrees-of-freedom ( $x$  and  $y$  directions).
- The total dimension of model state space is:  
$$(2N) \times 2 \times 2$$

(No. of cylinders)    (degrees of freedom)    (displacement + velocity)
- We consider the cases:  
 $N = 2, 3, 4;$        $N = \text{prime}$ .

## 3X2 ARRAY OF OSCILLATORS

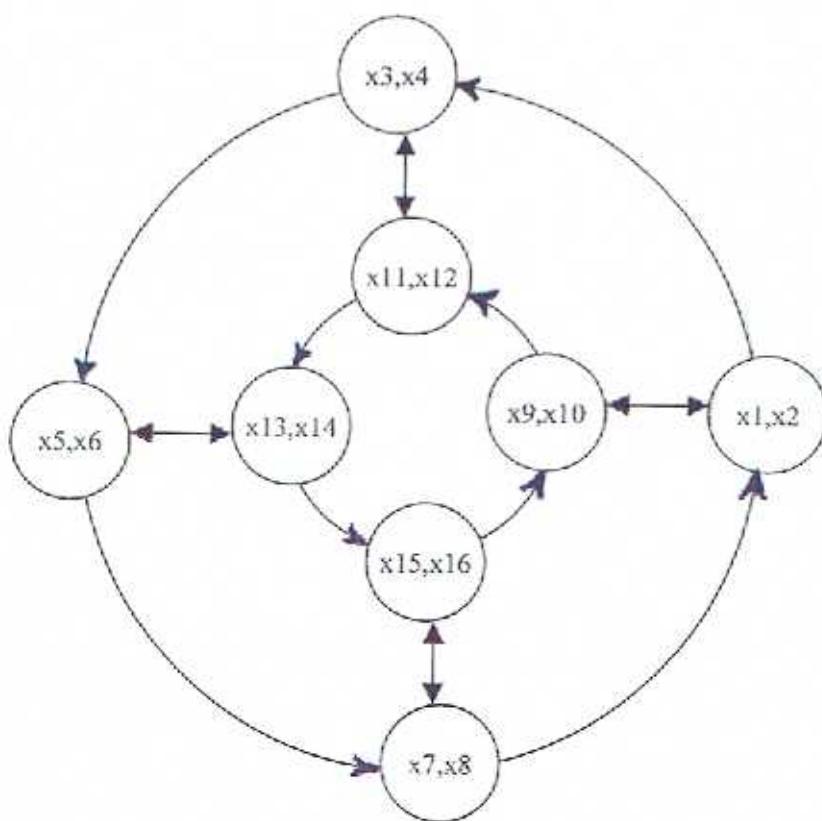
with symmetry:  $G_N \equiv Z_2 \times Z_N$  ( $N = 3$ )



The flow velocity

## TWO RINGS of $N$ OSCILLATORS SYMMETRICALLY-COUPLED

$$G_N = Z_N \times Z_2 \quad (N = 4)$$



Theorem 1 (H-K Theorem). Let  $\Gamma$  be a finite group acting on  $\mathbb{R}^n$ . There is a periodic solution to some  $\Gamma$ -equivariant system of ordinary differential equations on  $\mathbb{R}^n$  with spatial symmetries  $K$  and spatio-temporal symmetries  $H$  if and only if

- (a)  $H/K$  is cyclic,
- (b)  $K$  is an isotropy subgroup,
- (c)  $\dim(Fix(K)) \geq 2$ . If  $\dim(Fix(K)) = 2$ , then either  $H = K$  or  $H = N(K)$ ,
- (d)  $H$  fixes a connected component of  $Fix(K) \setminus L_K$ .

Here,  $H$  and  $K$  are subgroups of  $\Gamma$  defined by:

$$K = \left\{ \gamma \in \Gamma : \gamma x(t) = x(t), \forall t \right\}$$

$$H = \left\{ \gamma \in \Gamma : \gamma \{x(t)\} = \{x(t)\} \right\}$$

Possible isotropy subgroups and their corresponding fixed-point subspaces for the case when  $N = 2$ .

Twisted isotropy subgroup $\Sigma$	Fixed-Point Subspace $Fix(\Sigma)$	$\dim(Fix(\Sigma))$
$\mathbb{G}_2 \times \{1\}$	$\{(z, z), (z, z)\}$	2
$\mathbb{G}_2(\kappa_1, -\kappa_2)$	$\{(z, z), (-z, -z)\}$	2
$\mathbb{G}_2(-\kappa_1, \kappa_2)$	$\{(z, -z), (z, -z)\}$	2
$\mathbb{G}_2(-\kappa_1, -\kappa_2)$	$\{(z, -z), (-z, z)\}$	2
$\mathbb{Z}_2(\kappa_1)$	$\{(z, z), (w, w)\}$	4
$\mathbb{Z}_2(-\kappa_1)$	$\{(z, -z), (w, -w)\}$	4
$\mathbb{Z}_2(\kappa_2)$	$\{(z, w), (z, w)\}$	4
$\mathbb{Z}_2(-\kappa_2)$	$\{(z, -w), (-z, w)\}$	4
$\mathbb{Z}_2(\kappa_1\kappa_2)$	$\{(z, w), (w, z)\}$	4
$\mathbb{Z}_2(-\kappa_1\kappa_2)$	$\{(z, w), (-w, -z)\}$	4
$I$	$\{(z_1, z_2), (w_1, w_2)\}$	8

## Symmetry Breaking Hopf Bifurcation

We say that an ordinary differential equation

$$\frac{dx}{dt} = f(x, \mu), \quad f(x_0, \mu_0) = 0 \quad (12)$$

where  $f : \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}^n$  is smooth, undergoes a Hopf bifurcation at  $\mu = \mu_0$ , if in the linearized equation of (12),  $(df)_{(x_0, \mu_0)}$  has a conjugate pair of purely imaginary eigenvalues.

- When considering Hopf bifurcation in the presence of symmetry, one generically expects the eigenspace is “ $\Gamma$ -simple”.
- The vector space  $W$  is  $\Gamma$ -simple if either:
  - (a)  $W = V \oplus V$  where the representation of  $\Gamma$  on  $V$  is absolutely irreducible or
  - (b)  $\Gamma$  acts irreducibly but not absolutely irreducibly on  $W$ .

**Theorem 2 (Equivariant Hopf Theorem).** Let a compact Lie group  $\Gamma$  act  $\Gamma$ -simply on  $\mathbb{R}^{2n}$ . Assume that

(a)  $f : \mathbb{R}^{2n} \times \mathbb{R} \rightarrow \mathbb{R}^{2n}$  is smooth and  $\Gamma$ -equivariant,  $f(x_0, \mu) = 0$  and  $(df)_{x_0, \mu}$  has eigenvalues  $\alpha(\mu) \pm i\beta(\mu)$  each of multiplicity  $n$ .

(b)  $\alpha(\mu_0) = 0$  and  $\beta(\mu_0) = 1$ .

(c)  $\alpha'(\mu_0) \neq 0$  — the eigenvalue crossing condition.

(d)  $\Sigma \subseteq \Gamma \times S^1$  is a  $C$ -axial isotropy subgroup.

Then there exists a unique branch of small amplitude periodic solutions to system (12) with period near  $2\pi$ , emanating from  $x_0$  with spatio-temporal symmetries  $\Sigma$ .

## SPATIO-TEMPORAL MODES:

$$G_3 \equiv Z_2 \times Z_3 \text{ Symmetry}$$

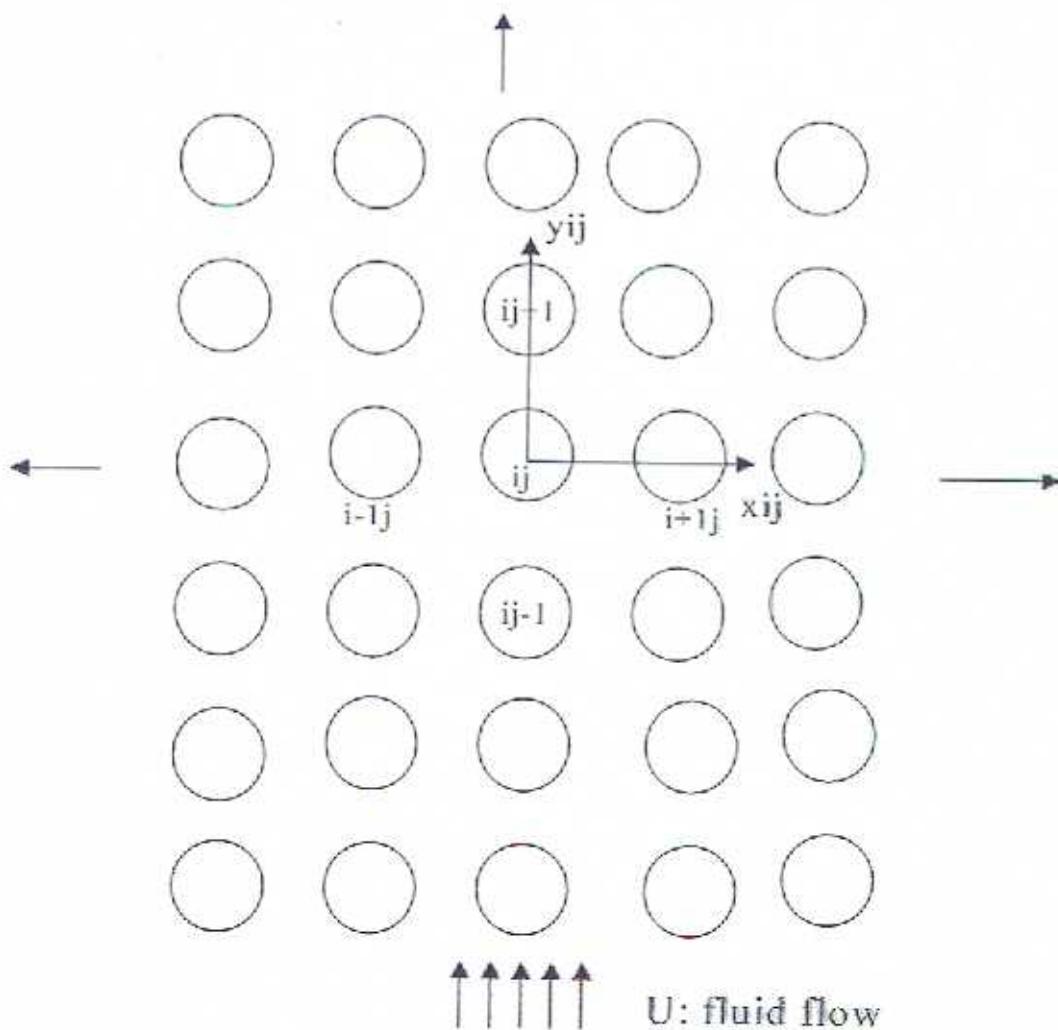
Twisted Isotropy Subgroup $\Sigma$	$\text{Fix } \Sigma$	Oscillators	Description
$G_3(\rho, \kappa)$	$H_1$	$((x, x, x), (x, x, x))$	All oscillators behave identically.
$G_3(\rho, \kappa\pi)$	$H_2$	$((x, x, x), (x + \pi, x + \pi, x + \pi))$	All oscillators in each ring are in-phase with each other, but $\pi$ out-of-phase with oscillators in the other ring.
$G_3(\zeta\rho, \kappa)$	$H_3$	$((x, x + \frac{2\pi}{3}, x + \frac{4\pi}{3}), (x, x + \frac{2\pi}{3}, x + \frac{4\pi}{3}))$	Oscillators are phase-shifted by $2\pi/3$ within each ring, but are in-phase with the corresponding oscillators in the other ring.
$G_3(\zeta^2\rho, \kappa)$	$H_4$	$((x, x + \frac{4\pi}{3}, x + \frac{2\pi}{3}), (x, x + \frac{4\pi}{3}, x + \frac{2\pi}{3}))$	Oscillators are phase-shifted by $4\pi/3$ within each ring, but are in-phase with the corresponding oscillators in the other ring.
$G_3(\zeta\rho, \kappa\pi)$	$H_5$	$((x, x + \frac{2\pi}{3}, x + \frac{4\pi}{3}), (x + \pi, x + \frac{5\pi}{3}, x + \frac{7\pi}{3}))$	Oscillators are phase-shifted by $2\pi/3$ within each ring, but $\pi$ out of phase with the corresponding oscillators in the other ring.
$G_3(\zeta^2\rho, \kappa\pi)$	$H_6$	$((x, x + \frac{4\pi}{3}, x + \frac{2\pi}{3}), (x + \pi, x + \frac{7\pi}{3}, x + \frac{5\pi}{3}))$	Oscillators are phase-shifted by $4\pi/3$ within each ring, but $\pi$ out of phase with the corresponding oscillators in the other ring.

## SPATIO-TEMPORAL MODES:

$$G_4 \equiv Z_2 \times Z_4 \text{ Symmetry}$$

Twisted Isotropy Subgroup $\Sigma$	$Fix \Sigma$	Oscillators	Description
$G_4(\rho, i\kappa)$	$E_1$	$((x, x, x, x), (x, x, x, x))$	All oscillators behave identically.
$G_4(\rho, \kappa\pi)$	$E_2$	$((x, x, x, x), (x + \pi, x + \pi, x + \pi, x + \pi))$	All oscillators in the same ring are in-phase; adjacent oscillators in the two rings are anti-phase.
$G_4(\rho\pi, \kappa)$	$E_3$	$((x, x + \pi, x, x + \pi), (x, x + \pi, x, x + \pi))$	Around each ring oscillators alternate in-phase and anti-phase; the two rings behave identically.
$G_4(i\rho, \kappa)$	$E_4$	$((x, x + \pi, x, x + \pi), (x + \pi, x, x + \pi, x))$	Around each ring oscillators alternate in-phase and anti-phase; adjacent oscillators in the two rings are anti-phase.
$G_4(i\rho, \kappa)$	$E_5$	$((x, x + \frac{\pi}{2}, x + \pi, x + \frac{3\pi}{2}), (x, x + \frac{\pi}{2}, x + \pi, x + \frac{3\pi}{2}))$	Around both rings successive oscillators are $\pi/2$ out-of-phase; the two rings behave identically.
$G_4(i\rho\pi, \kappa)$	$E_6$	$((x, x + \frac{3\pi}{2}, x + \pi, x + \frac{\pi}{2}), (x, x + \frac{3\pi}{2}, x + \pi, x + \frac{\pi}{2}))$	Around both rings successive oscillators are $3\pi/2$ out-of-phase; the two rings behave identically.
$G_4(i\rho\pi, \kappa\pi)$	$E_7$	$((x, x + \frac{\pi}{2}, x + \pi, x + \frac{3\pi}{2}), (x + \pi, x + \frac{3\pi}{2}, x, x + \frac{\pi}{2}))$	Around each ring successive oscillators are $\pi/2$ out-of-phase; adjacent oscillators in the two rings are anti-phase.
$G_4(i\rho\pi, \kappa\pi)$	$E_8$	$((x, x + \frac{3\pi}{2}, x + \pi, x + \frac{\pi}{2}), (x + \pi, x + \frac{\pi}{2}, x, x + \frac{3\pi}{2}))$	Around each ring successive oscillators are $3\pi/2$ out-of-phase; adjacent oscillators in the two rings are anti-phase.

A Rectangular  
**HEAT EXCHANGER ARRAY**  
in Cross-Section



# PHYSICAL ASSUMPTIONS

- Von Karman vortices are NOT important, because the engineers have avoided these!
- The Bernoulli effect is unavoidable (for both  $x$ - and  $y$ -modes).
- Fluid-elastic inertial forces are important.
- Each cylinder oscillates as a weakly-nonlinear simple harmonic oscillator, (odd in  $x$ , but not in  $y$ ).
- Coupling is nearest-neighbor only (symmetric in  $x$  but not in  $y$ ).
- Transverse and longitudinal vibrations are independent. Thus, the model equations separate, into two systems of dimension  $4N$ .

# TRANSVERSE MODEL EQUATIONS

$$\ddot{x}_{ij} + (d_0 - d_1 \mu + \kappa_{ij}^2 + \dot{x}_{ij}^2) \dot{x}_{ij} + (e_0 + \kappa_{ij}^2 + \dot{\kappa}_{ij}^2) \kappa_{ij} \\ = -\mu a [\kappa_{i-1,j} - \kappa_{ij}] + (c_0 - \mu c_1) [\dot{x}_{i-1,j} - \dot{x}_{ij}] \\ + \mu c [\kappa_{i,j+1} - \kappa_{ij}] + \mu b [\dot{x}_{i,j+1} - \dot{x}_{ij}]$$

where:  $i = 1, 2 \pmod{2}$ ,  
 $j = 1, \dots, N \pmod{N}$ ,  
 $\mu \propto$  fluid velocity,

$a, b, c, c_0, c_1, d_0, d_1, e_0$  are physical constants.

- This is equivalent to a 1<sup>st</sup> order system of dimension  $4N$ , or a complex system in  $z = \dot{x} + i\dot{\kappa}$  of dimension  $2N$ .
- We study Hopf bifurcations in 2-dimensional invariant subspaces, (irreducible but not absolutely irreducible)
- For  $N=4$ , the bifurcation points corresponding to  $E_1, \dots, E_8$  are:  
 $\mu_1 = \frac{d_0}{d_1}, \quad \mu_2 = \frac{d_0 + 2c_0}{d_1 + 2c_1}, \quad \mu_3 = \frac{d_0}{d_1 - 2b}, \quad \mu_4 = \frac{d_0 + 2c_0}{d_1 - 2b + 2c},$   
 $\mu_5 = \frac{d_0}{d_1 + (c-b)}, \quad \mu_6 = \frac{d_0}{d_1 - (b+c)}, \quad \mu_7 = \frac{d_0 + 2c_0}{d_1 + c - b + 2c_1}, \quad \mu_8 = \frac{d_0 + 2c_0}{d_1 + c - b + 2c_1}$

# LONGITUDINAL MODEL EQUATIONS

$$\begin{aligned}\ddot{y}_{ij} + \alpha \dot{y}_{ij} + (\delta_0 + \delta_1 \mu^2) y_{ij} + \delta_2 \mu y_{ij}^2 + \delta_3 y_{ij}^3 \\ = \mu a [\dot{y}_{i,j+1} - \dot{y}_{ij}] + f_{ub} [\dot{y}_{i,j-1} - \dot{y}_{ij}] \\ + \mu c [y_{i-1,j} - y_{ij}] + \mu d [\dot{y}_{i+1,j} - \dot{y}_{ij}] \\ + \mu e [y_{ij} - y_{i,j-1}],\end{aligned}$$

where:  $i = 1, 2 \pmod{2}$   
 $j = 1, \dots, N \pmod{N}$   
 $\mu \propto \text{flow velocity}$ .

Equivalent to a real 1<sup>st</sup> order system of dimension  $4N$  or complex of dimension  $2N$ .

We study Hopf bifurcation in the 2D invariant subspaces  $H_1, H_2, \dots, H_6$ .

For  $N=3$ , the corresponding bifurcation points are:

$$M_2 = \frac{\alpha + \frac{3}{2}k_0}{\frac{3}{2}(b-a) - \frac{\sqrt{3}}{2}e}$$

$$M_5 = \frac{\alpha + \frac{3}{2}k_0}{\frac{3}{2}(b-a) - \frac{\sqrt{3}}{2}e - 2d}$$

$$M_3 = \frac{\alpha + \frac{3}{2}k_0}{\frac{3}{2}(b-a) + \frac{\sqrt{3}}{2}e}$$

$$M_6 = \frac{\alpha + \frac{3}{2}k_0}{\frac{3}{2}(b-a) + \frac{\sqrt{3}}{2}e - 2d}$$

( $M_1$  and  $M_4$  do not exist.)

# HOPF BIFURCATION RESULTS

For realistic parameter values in the model equations:

- All Hopf bifurcations are supercritical.
- The FIRST (and therefore stable) bifurcation to occur is as follows:

## Transverse Case with $N=4$ :

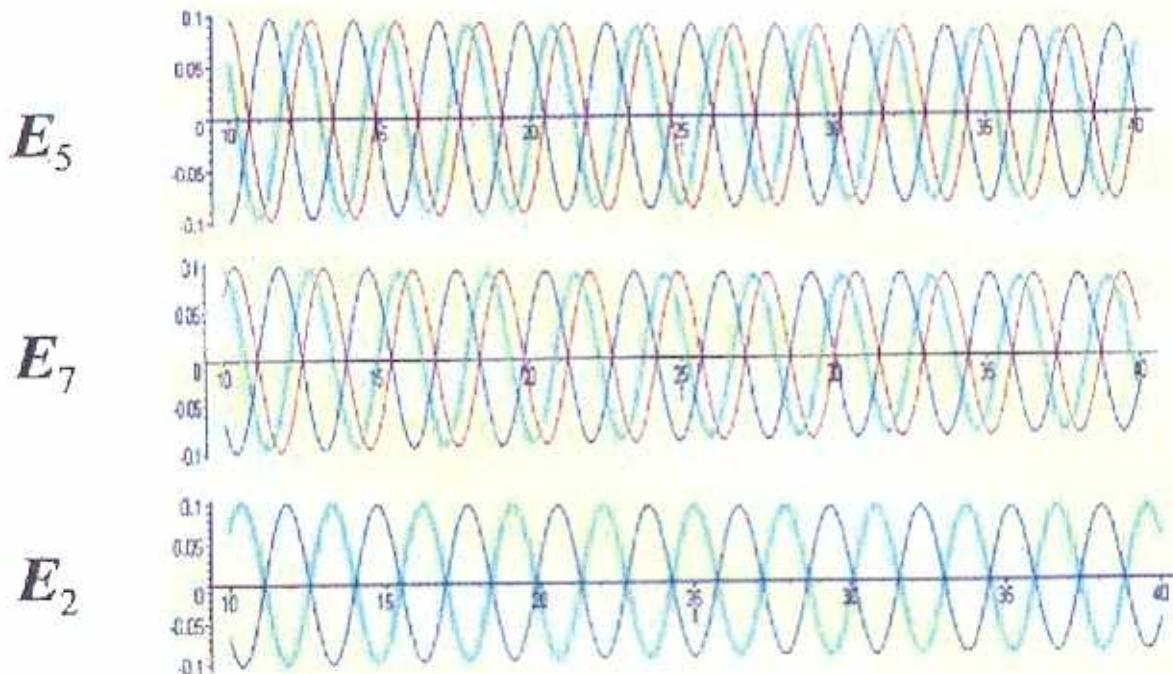
<u>Bif. Pt.</u>	<u>Conditions</u>	<u>Symmetry</u>
$\mu_2$	$c_0$ large	$G_4(p, 2\pi)$
$\mu_5$	$c_0$ small and $b < c$	$G_4(ip, \pi)$
$\mu_7$	$c_0$ small and $b > c$	$G_4(ip, \pi)$

## Longitudinal Case with $N=3$ :

$\mu_3$	(always)	$G_3(5p, \pi)$
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## NUMERICAL SIMULATIONS

Transverse Vibration with  $G_4$  Symmetry  
Modes most likely to occur:



$E_5$ :      Blue:  $x_1 = x_9$ ,    Green:  $x_3 = x_{11}$ ,

Red:  $x_5 = x_{13}$ ,    Yellow:  $x_7 = x_{15}$ .

$E_7$ :      Blue:  $x_1 = x_{13}$ ,    Green:  $x_3 = x_{15}$ ,

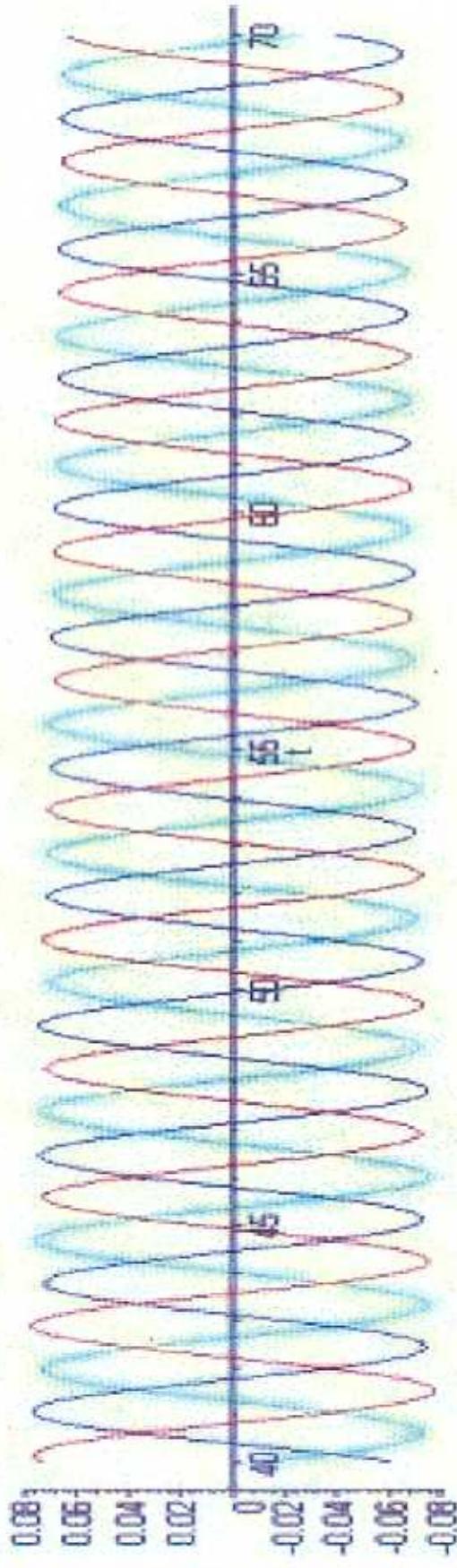
Red:  $x_5 = x_9$ ,    Yellow:  $x_7 = x_{15}$ .

$E_2$ :      Blue:  $x_1 = x_3 = x_5 = x_7$ ,

Green:  $x_9 = x_{11} = x_{13} = x_{15}$ .

## NUMERICAL SIMULATION:

Longitudinal Vibrations with  $G_3$  Symmetry



Red:  $y_1 = y_7$ , Blue:  $y_3 = y_9$ , Green:  $y_5 = y_{11}$ .

Bifurcation analysis shows that this is the mode **most likely to occur**. It corresponds to  $H_3 = Fix[G_3(\varsigma\rho, \kappa)]$ .

## CONCLUSIONS

- The spatio-temporal patterns of oscillations that *potentially may occur* in heat exchanger arrays are determined primarily by the symmetries of the system. These may be found by group theory methods, without using detailed knowledge of the physical system.
- Given a mathematical model and a choice of bifurcation parameter (here the flow velocity), equivariant Hopf bifurcation analysis of the model predicts which spatio-temporal patterns *actually will occur in the system*.
- For the present model (comparing columns):
  - ▶ Transverse vibrations are *anti-phase*
  - ▶ Longitudinal vibrations are *in-phase*