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Program on the Geometry of String Theory

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1. Basic Introduction to String Theory

• In string theory, a particle is modelled not by a point in space, but by a string (a one-dimensional object):

•S^1 (closed string)

or

•$[0, 1]$ (open string)

Strings propagate in spacetime (sweep out a two-dimensional trajectory called a worldsheet)

•Different particles appear as different vibrational modes of the string.

• Strings live naturally in a space M_{10} with 10 dimensions. 4-dimensional spacetime is replaced by a product of 4-dimensional spacetime M_4 with a compact manifold M_6 of dimension 6 which is extremely small.

In some types of string theory, M_6 is required to be a Calabi-Yau manifold CY_3 (i.e. a Kähler manifold in complex dimension 3 whose canonical bundle is trivial).

Analogy: garden hose $\mathbf{R} \times S^1$

•$\mathbf{R}$: analogue of M_4

•S^1: analogue of M_6

•An ant crawling on the surface of the hose will think (if the radius of the circle is extremely small compared to the ant's size) that the surface is one-dimensional.

•Winding modes: A string may have nonzero winding number around the S^1.

2. Advantages of string theory:

• String theory allows unification of quantum mechanics and general relativity.

• String theory automatically incorporates gravitation. (If we had discovered string theory before we discovered gravitation, we would think this was very interesting; of course gravity was known long before string theory.)

•Newton's gravitational constant can be computed from the fundamental constants of string theory (physicists say Newton's constant is a "derived quantity").

• String theory eliminates singularities that occur in the standard quantum field theory.

3. Supersymmetry

Physicists distinguish between particles with integer spin (called *bosons*) and particles with spin $1/2, 3/2, \ldots$ (called *fermions*). An example of a boson is the photon.

Examples of fermions are electrons and quarks.

The functions representing bosons commute, while the functions representing fermions anti-commute

(cf. even and odd degree elements in an exterior algebra)

Superstring theory is equipped with *supersymmetry*, a symmetry group.

Analogue: representations of $SU(2)$

An operator Q takes the fundamental (2-dimensional) representation $V_2 \cong \mathbf{C}^2$ of $SU(2)$ to the 3-dimensional representation $V_3 \cong \mathbf{C}^3$ of $SU(2)$

(via the double cover $SU(2) \to SO(3)$).

If $v \in V_2$ then $Qv \in V_3$.

V_2 is the analogue of the space of bosons, while V_3 is the analogue of the space of fermions.

In supersymmetry, every boson (resp. fermion) v has a partner Qv which is a fermion (resp. boson).

Supersymmetry gives more calculational control and facilitates cancellations.

Partner particles have not yet been detected. But the Large Hadron Collider (LHC) is scheduled to begin operating in 2007-8 and one of its main objectives is to search for supersymmetric partner particles.

4. Types of string theory:

In terms of weak coupling perturbation theory there appear to be only **five** different consistent superstring theories known as **Type I SO(32)**, **Type IIA**, **Type IIB**, **SO(32) Heterotic** and **E8 x E8 Heterotic**.

	Type IIB	**Type IIA**	**E8 x E8 Heterotic**	**SO(32) Heterotic**	**Type I SO(32)**
String Type	Closed	Closed	Closed	Closed	Open (& closed)
10d Supersymmetry	N=2 (chiral)	N=2 (non-chiral)	N=1	N=1	N=1
10d Gauge groups	none	none	E8 x E8	SO(32)	SO(32)
D-branes	-1,1,3,5,7	0,2,4,6,8	none	none	1,5,9

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5. D-Branes

•Endpoints of an open string must live on a *D-brane* (extended object of some specified dimension, in 10-dimensional manifold)

•An open string may be thought of as a function on $[0, 1]$ (or a map from $[0, 1]$ to the 10-manifold M_{10}).

•The values (or positions) at the endpoints obey the wave equation.

•They may have Neumann or Dirichlet boundary conditions.

•Imposing Dirichlet boundary conditions corresponds to requiring that the endpoints of the string lie on D-branes ("D" stands for Dirichlet).

6. Duality

Duality: Different string theories are related by duality transformations.

• One duality (large/small radius duality, or T-duality) takes a theory where the radius of the target space is R (recall the garden hose) to a theory where the radius is $1/R$.

• Another (strong-weak duality) exchanges a theory with strong coupling constant with a different theory with a weak coupling constant.

• The two dual theories may be different, but are sometimes the same (as for type IIB theories: *self-dual*)

$$\begin{array}{ccccc} & & \mathrm{M} & & \\ \mathrm{I} \longleftrightarrow \mathrm{HO} < \cdots > \mathrm{HE} & \nearrow & & \nwarrow & \mathrm{IIA} < \cdots > \mathrm{IIB} \\ & & & & \qquad\qquad\qquad \bigcirc \end{array}$$

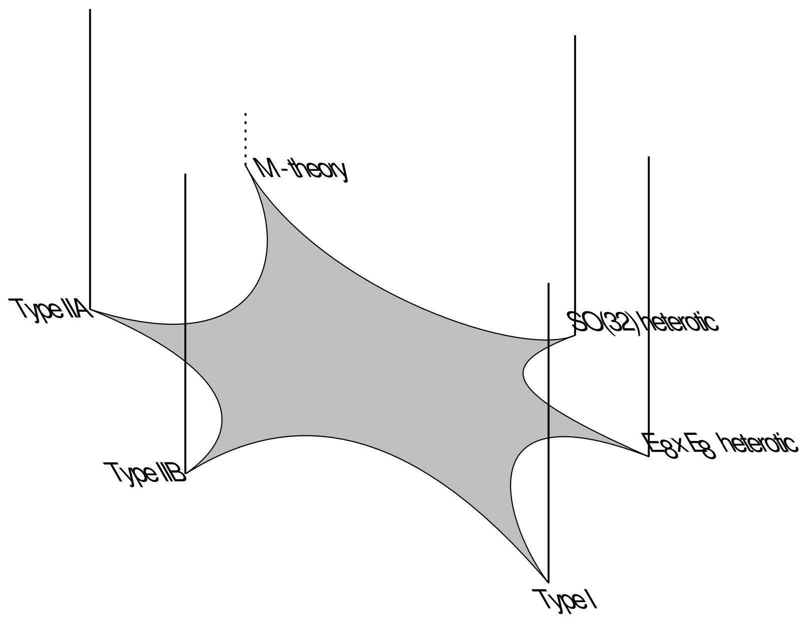

Figure 1: All string theories, and M-theory, as limits of one theory. Energy increases vertically.

Here:

$\longleftrightarrow$: strong/weak coupling duality

$< \cdots >$: large/small radius duality

Events

- Graduate courses
- Boris Dubrovin, Minicourse on Frobenius Manifolds and Integrable Systems
- Distinguished Lecture Series: Edward Witten, April 2005
- Coxeter Lecture Series:
  - Nigel Hitchin (November 2004)
  - Robbert Dijkgraaf (January 2005)
- CMS Summer Meeting, Special Session on String Theory and Integrable Systems (U. of Waterloo, June 2005)
- Summer School program (June-July 2005)
- Strings 2005: The premier international conference in the field of string theory is held an-

nually in a different location each year, and will be held in Toronto in July 2005.

Organizers:

- Kentaro Hori (Mathematics Department and Physics Department, University of Toronto)
- Lisa Jeffrey (Mathematics Department, University of Toronto)
- Mikhail Kapranov (Yale University and University of Toronto)
- Boris Khesin (Mathematics Department, University of Toronto)
- Rob Myers (Perimeter Institute)
- Amanda Peet (Physics Department, University of Toronto)

Workshop M1: Forms of Homotopy Theory, Elliptic Cohomology and Loop Spaces

Organizers: M. Ando, M. Hopkins, H. Miller, J. Morava

Deep results in homotopy theory in the 1980's indicate that the topology of compact manifolds has a rich structure (the "chromatic filtration") whose geometric meaning is not yet understood. The main problem is to find algebraic invariants which detect the layers in the chromatic filtration (these are indexed by integers $n \geq 0$). The invariants which detect the 0 and 1 layers are singular cohomology and K-theory; these are classical and well-understood.

Elliptic cohomology is the name of a family of invariants which detect the 2 layer. It has generated a lot of interest, because it relates the second layer of the chromatic filtration to 1) the arithmetic of elliptic curves and modular forms and 2) string theory.

Elliptic cohomology was first discovered in the early 70's by Morava, and then rediscovered in the mid-80's. After slow but steady progress in the period 1985–1995, the subject moved forward very rapidly in the late 1990's, thanks to work of Hopkins and his collaborators, whose results give a striking account of the relationship between stable homotopy and elliptic curves; his work has led to new results not only in homotopy theory but also arithmetic.

There has recently been substantial work by various groups to understand the relationship between elliptic cohomology and string theory:

1. Stolz and Teichner have very recently made considerable progress in understanding the geometry of elliptic cohomology, by refining and substantially enriching Segal's notion of "elliptic object". Not only do they appear to have constructed a cohomology theory based on string-theoretic ideas, but the geometry of their gadget has many features consistent with the results of Hopkins' program.

2. Sullivan, Cohen, and their collaborators have discovered a rich new structure in the algebraic topology of the space of smooth loops on a manifold, modeled on fusion in a closed string theory.

3. Freed, Hopkins, and Teleman have discovered that the Verlinde algebra of a compact Lie group G is just the Pontrjagin ring struc-

ture on the *twisted* equivariant K-homology of G. This is striking, because on the one hand equivariant elliptic cohomology is supposed to be related to the Verlinde algebra; and on the other hand twisted K-theory is supposed to be related to open string theory. This result suggests that the search for the geometry of elliptic cohomology should contemplate open as well as closed string theory.

Workshop M2: Mirror Symmetry

Organizers: D. Auroux, M. Gross, K. Hori, N. Yui

Mirror symmetry is a conjecture in string theory that certain “mirror pairs” of Calabi-Yau manifolds (compact Kähler manifolds with trivial canonical bundle) give rise to isomorphic physical theories. In some sense, mirror symmetry unifies symplectic geometry and algebraic geometry.

•Mirror symmetry has inspired many new developments in algebraic geometry, toric geometry, the theory of Riemann surfaces and infinite dimensional Lie algebras.

•For example, mirror symmetry has been used to tackle the problem of counting the number of rational curves on Calabi-Yau threefolds.

•The deformations of the complex structure of one manifold are identified with the deformations of the Kähler structure of its mirror manifold.

•Special Lagrangian varieties have attracted enormous attention after the work of Strominger, Yau and Zaslow who used them to provide the first explicit conjectural definition of mirror manifolds. Subsequent work (for example, by M. Gross and S. Katz) followed.

•The workshop will treat both the topological approach (M. Gross) and the systematic study of special geometry from the point of view of differential geometry (Hitchin, Joyce) and analysis (McNeal).

Workshop M3: Topological Strings

Organizers: E. Getzler, K. Hori, S. Katz

•Gromov-Witten invariants involve the space of (pseudo)holomorphic maps from an oriented 2-manifold Σ (usually a 2-sphere) to a Kähler (more generally symplectic) manifold.

•The original quantum cohomology had to be enlarged to incorporate variation of the Riemannian metric.

•Work of Eguchi, Hori and Xiong and its development by Katz, Getzler and others identified the correct framework for Gromov-Witten invariants of higher genus.

•The conceptual point of view on quantum cohomology using Frobenius manifolds (advocated by Manin and others) does not quite extend to the case where the metric varies.

•There is a duality between open and closed topological strings (which has played a major role in recent physics).

Workshop P1: $N = 1$ Compactifications

Organizers: M. Douglas, K. Hori, S. Sethi

•The simplest vacua of string theory have ten flat dimensions of spacetime, with larger amounts of supersymmetry, but a theory in four dimensions can be realized via *compactification* of 6 extra dimensions.

•$\mathcal{N} = 1$ *compactifications* are those that preserve the minimal supersymmetry in four dimensions.

•During 1985-1995, this unification program in string theory was pursued in the sole framework available at that time – heterotic string theory compactified on Calabi-Yau manifolds.

•In 1995 (the year of the "Second String Revolution") *duality* and *branes* came onto the center stage, introducing *M theory* – an eleven dimensional theory that (conjecturally) unifies all string theories together into an overarching theory.

•Since then, new frameworks for producing theories of our real world have emerged. One idea is that we live on the world volume of D-branes (special types of higher-dimensional membranes) which naturally support non-abelian gauge groups. Another idea is the compactification of M theory on certain seven-dimensional manifolds.

Different approaches are often related by duality, where a weakness in one description is compensated by another.

There are several ways to realize $\mathcal{N} = 1$ compactifications, including

• Heterotic string theory on Calabi-Yau manifolds,
• Type II orientifolds (essentially orbifolds) and
• M theory on G_2 holonomy manifolds.
•The most important problem is to identify the low-energy effective theory, namely identify the light degrees of freedom and the interactions between them.

Workshop P2: String Phenomenology

Organizers: J. Louis, R. Myers, G. Shiu

String phenomenology is an emerging research area in the rich interdisciplinary boundaries of string theory, particle physics, and cosmology. Although research in string phenomenology is motivated by physical questions, progress often comes from connections to more formal and mathematical aspects of string theory. These connections include:

- How does the Standard Model of particle physics arise from string theory?
- Can string theory provide a realistic cosmology?: brane-antibrane instability and decay of unstable branes, thermodynamics of strings and branes, models of inflation, time-dependent

backgrounds

- What are the experimental signatures of string theory?: impact on particle physics, cosmology

Workshop P3: Gravitational Aspects of String Theory

Organizers: C. Johnson, P. Kraus, D. Marolf, A. Peet

•Recent successes in string theory have included a microscopic computation of the entropy of black holes, and the discovery of dualities between gauge theories and gravity (where the spacetimes corresponding to the gauge theory and the gravitational theory may have different dimensions).

•The workshop aims to explore and interpret gravitational aspects of string theory.

•The workshop will treat:

1. the role of black holes
2. two problems in general relativity (curvature singularities and closed timelike curves) which

may be cured by string theory

3. "warped compactifications": metrics on $M_4 \times M_6$ of the form

$$ds_4^2(x) + \sum_{i,j=1}^{6} f_{ij}(x) dy_i dy_j$$

where $x \in M_4$ and $y = (y_1, \ldots, y_6) \in M_6$

4. the route to constructing realistic cosmological models

5. the cosmological dark energy problem

6. non-commutative and matrix theories