Rating System Dynamics and Bank-Reported Default Probabilities under the New Basel Capital Accord

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- Goal – To better understand the effects of differences in internal rating methodologies on Basel II
  - Minimum capital requirements
  - PD validation
- Approach
  - Analyze a stylized credit rating model
  - Illustrate results using historical simulations

Today's Talk

- The Basel II capital accord
- What is a PD?
- Rating philosophies
  - Point-in-time
  - Through-the-cycle
- Quantifying PDs under Basel II
- Dynamics of risk-based capital requirements
- Validating bank-reported PDs
  - Benchmarking
  - Backtesting
- Conclusions

Basel II

- Basel II is intended to more closely align regulatory capital requirements with underlying economic risks
- Timeline
  - Work begun in 1999
  - Third quantitative impact study completed in December 2002
  - Third consultative package (CP3) released for comment in June 2003
  - "Framework" document planned for June 2004

The Internal Ratings Based (IRB) Approach

- Supervisory risk-weight functions map bank-reported risk parameters to minimum capital requirements
- Capital charges are assigned on an exposure-by-exposure basis and are aggregated across exposures
- Bank-reported risk parameters include
  - Probability of default (PD)
  - Loss given default (LGD)
  - Maturity (M)
  - Exposure at default (EAD)

What is a PD?

- A PD is a forecast of an obligor's likelihood of default over a one-year time horizon
- Like all forecasts, PDs rely on currently observable information
  - Obligor-specific variables (e.g. balance sheet ratios)
  - Aggregate variables (e.g. GDP growth)

Unstressed vs. Stress PDs

- Unstressed PD (UPD) -- an unbiased estimate of an obligor's likelihood of default over the next year
  - Efficiently uses all available information
  - A "best guess" forecast
- Stress PDs (SPD) – a conditional estimate of an obligor's likelihood of default over the next year assuming an adverse macroeconomic "stress scenario"
  - Places less weight on observed aggregate data
  - A pessimistic forecast

A Stylized Default Model

- Obligor i defaults at date t if $Z_{it} < 0$

$$Z_{i,t+1} = \alpha + \beta_W W_i + \beta_X X_{it} + \beta_Y Y_t + U_{i,t+1}$$

$W_i \equiv$ Static obligor information

$X_{it} \equiv$ Dynamic obligor information

$Y_t \equiv$ Aggregate information

$U_{i,t+1} \equiv$ Unobservable information

- Unobservable information includes both idiosyncratic and systematic components

$$U_{i,t+1} = \omega V_{t+1} + \sqrt{1-\omega^2}\, E_{i,t+1}$$

$V_{t+1} \equiv$ Systematic risk factor

$E_{i,t+1} \equiv$ Idiosyncratic risk factor

Unstressed PD

- Date t forecast that obligor i will default at date t+1

$$
\begin{aligned}
UPD_{it} &= \Pr\left[Z_{i,t+1} < 0 \mid W_i = w_i, X_{it} = x_{it}, Y_t = y_t\right] \\
&= \Pr\left[\alpha + \beta_W w_i + \beta_X x_{it} + \beta_y y_t + U_{i,t+1} < 0\right] \\
&= \Phi\left(-\left(\alpha + \beta_W w_i + \beta_X x_{it} + \beta_Y y_t\right)\right)
\end{aligned}
$$

- UPD_{it} is negatively correlated with the business cycle

Stress PD

- Date t forecast that i will default at date t+1 given the adverse stress scenario

$$\beta_Y Y_t + \omega V_{t+1} = -\psi$$

$$\begin{aligned} SPD_{it} &= \Pr\left[Z_{i,t+1} < 0 \mid W_i = w_i, X_{it} = x_{it}, \beta_Y Y_t + \omega V_{t+1} = -\psi\right] \\ &= \Pr\left[\alpha + \beta_W w_i + \beta_X x_{it} - \psi + \sqrt{1-\omega^2}\,E_{i,t+1} < 0\right] \\ &= \Phi\left(-\frac{\alpha + \beta_W w_i + \beta_X x_{it} - \psi}{\sqrt{1-\omega^2}}\right) \end{aligned}$$

- SPD_{it} is uncorrelated with the business cycle

Rating Systems

- The rating grade assigned to an obligor is an assessment of that obligor's credit quality
- Rating systems can differ along many dimensions
  - Granularity
  - Time horizon
  - Dynamic rating philosophy
    - Point-in-time
    - Through-the-cycle
    - Hybrid

Dynamic Rating Philosophies

In a point-in-time process, an internal rating reflects an assessment of the borrower's current condition and/or most likely future condition over the course of the chosen time horizon. As such, the internal rating changes as the borrower's condition changes over the course of the credit/business cycle. In contrast, a "through-the-cycle" process requires assessment of the borrower's riskiness based on a worst-case, "bottom of the cycle scenario", i.e. its condition under stress. In this case, a borrower's rating would tend to stay the same over the course of the credit/business cycle.

-- Basel Committee Models Task Force
Range of Practices in Bank's Internal Rating Systems, 2000

Dynamic Rating Philosophies

- For analytical purposes, ratings are defined to reflect underlying PDs
- Point-in-time (PIT) rating
  - Tied to an obligor's unstressed PD
  - Changes rapidly as current macroeconomic conditions change
- Through-the-cycle (TTC) rating
  - Tied to an obligor's stress PD
  - Tends to be relatively insensitive to changing economic conditions

Point-in-Time Rating

- A PIT system maps observable obligor characteristics and aggregate information to a rating

$$\gamma = \Gamma^{PIT}\left(w_i, x_{it}, y_t\right) = \alpha + \beta_W w_i + \beta_X x_{it} + \beta_Y y_t$$

- All obligors with the same PIT rating share the same unstressed PD

$$UPD_{it}^{PIT}\left(\gamma\right) = \Phi\left(-\gamma\right)$$

Through-the-Cycle Rating

- A TTC system maps obligor-specific information to a rating grade that is insensitive to macroeconomic information

$$\gamma = \Gamma^{TTC}\left(w_i, x_{it}\right) = \alpha + \beta_W w_i + \beta_X x_{it}$$

- All obligors with the same TTC rating share the same stress PD

$$SPD_{it}^{TTC}\left(\gamma\right) = \Phi\left(-\frac{\gamma - \psi}{\sqrt{1-\omega^2}}\right)$$

Rating Philosophy and UPDs

- By construction, the unstressed PD associated with a PIT grade is stable over the business cycle

$$\mathrm{UPD}_t^{\mathrm{PIT}}(\gamma) = \Phi(-\gamma)$$

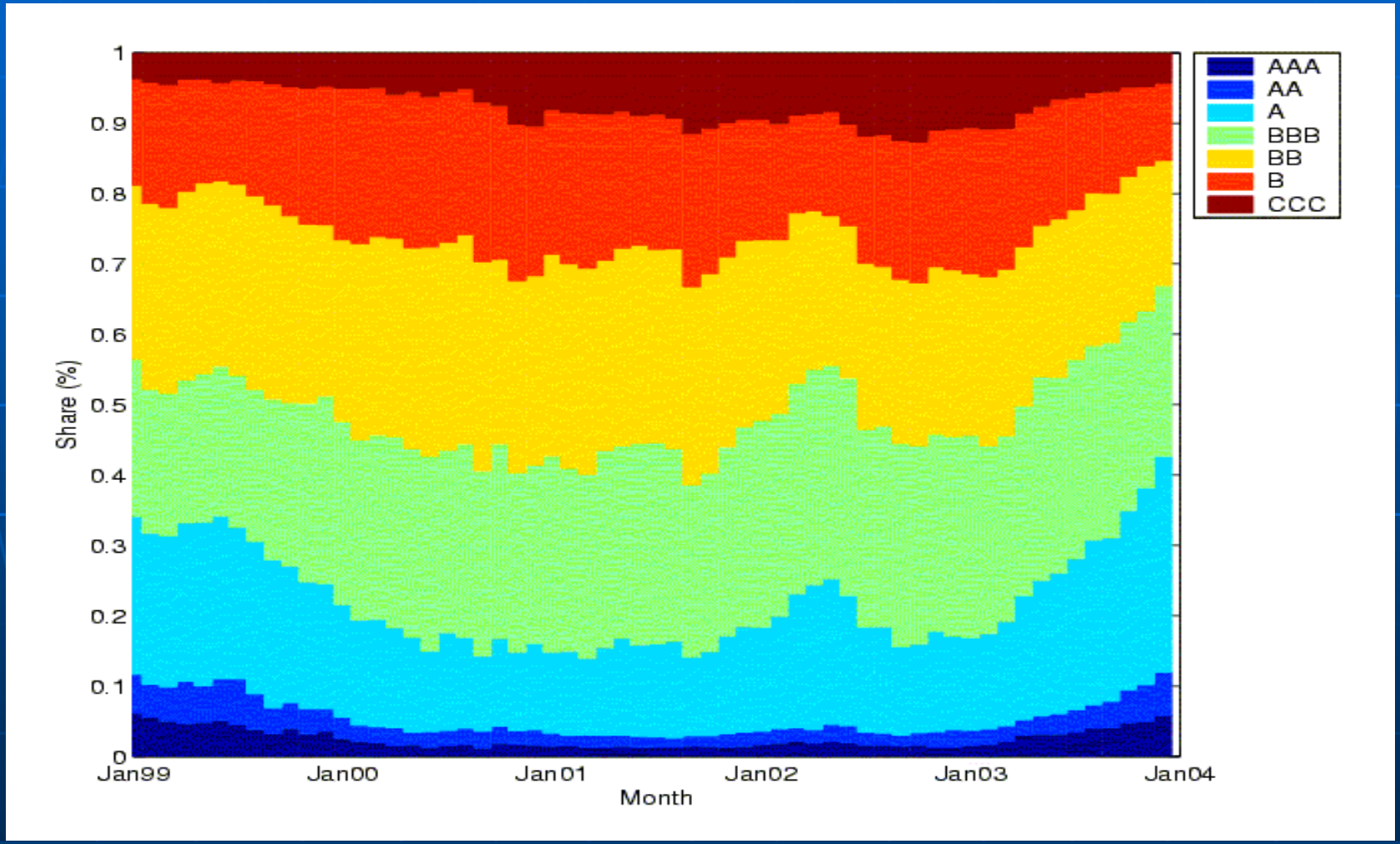
- The unstressed PD associated with a TTC grade is negatively correlated with the business cycle

$$\mathrm{UPD}_t^{\mathrm{TTC}}(\gamma) = \Phi(-\gamma - \beta_Y y_t)$$

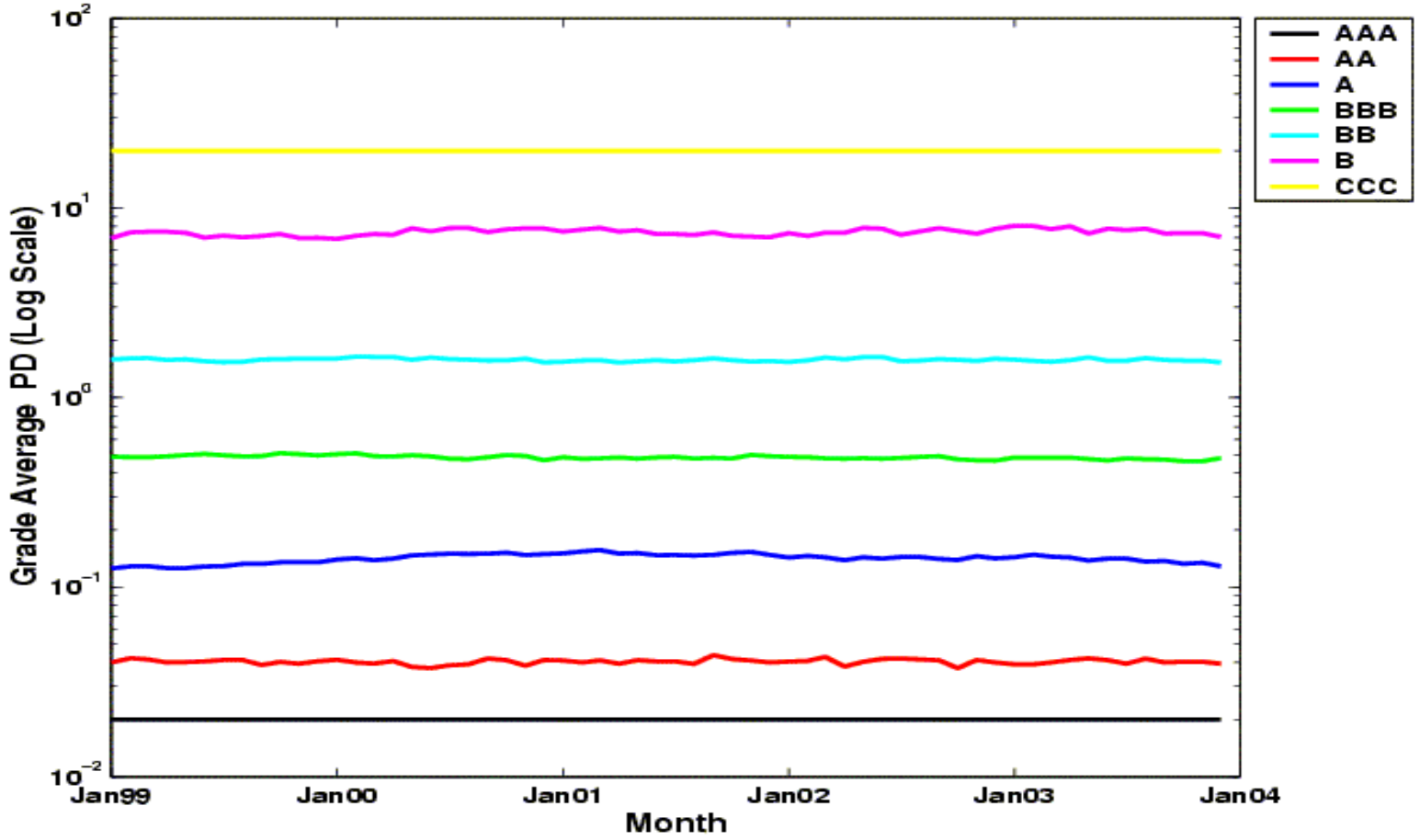
An Illustrative Simulation

- Data
  - Population of US corporate obligors with both KMV and S&P ratings
  - Monthly observations from January 1999 to December 2003
- Assume an obligor's KMV EDF is equal to its unstressed PD
- PIT grades are constructed by bucketing obligors according to their EDFs
- TTC grades are given by S&P ratings

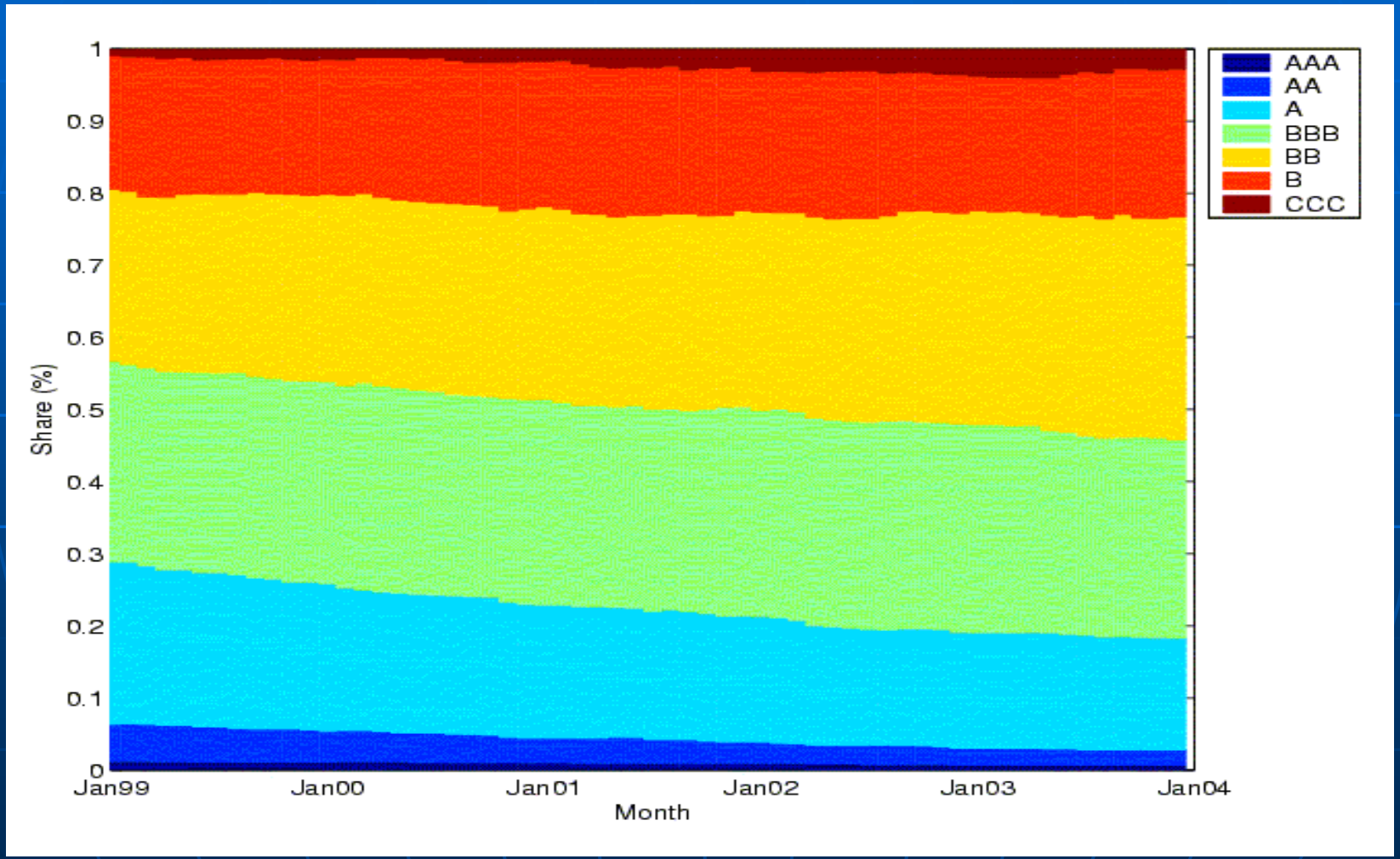
PIT Grade Distribution over Time



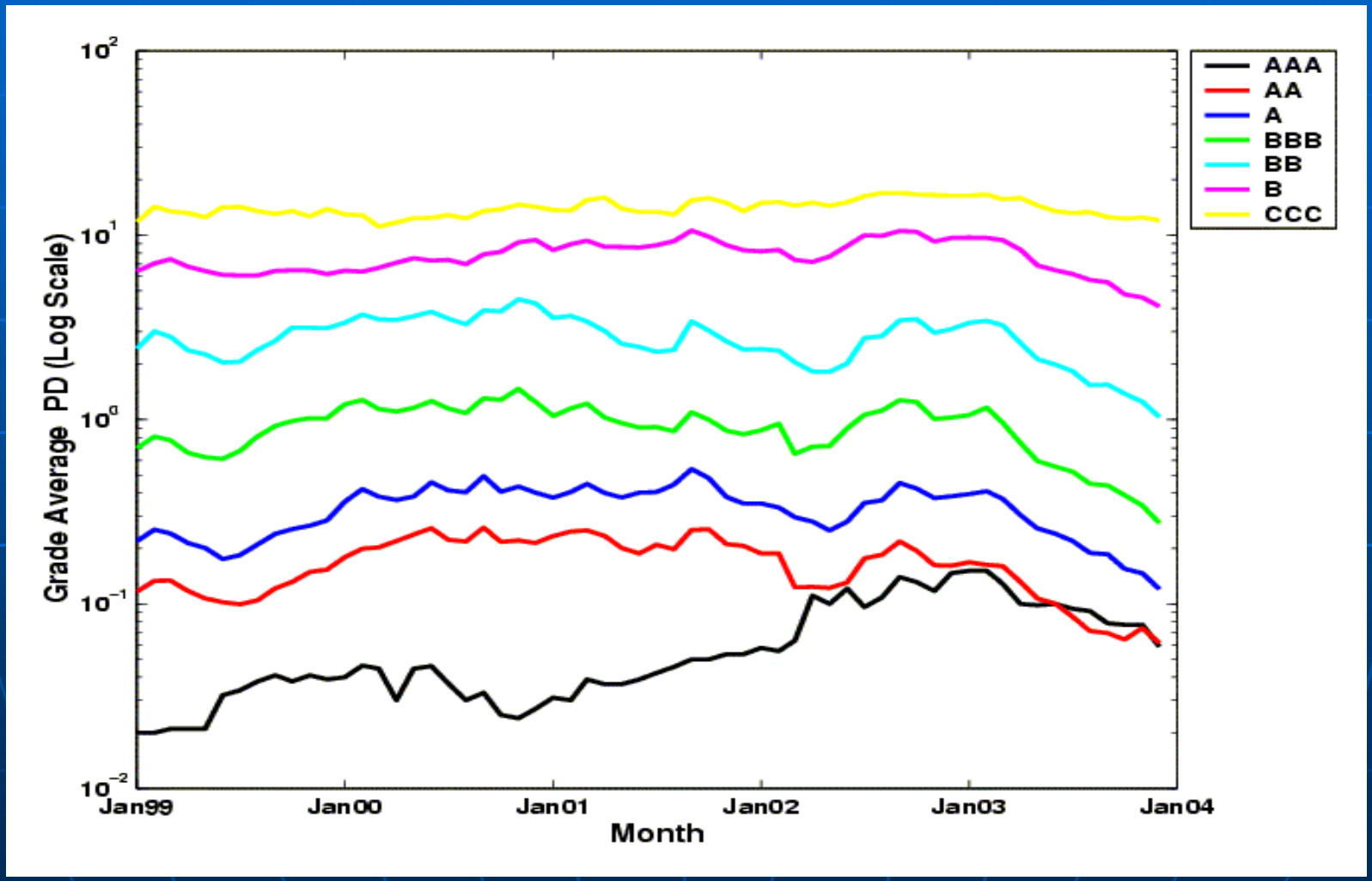
Average UPD by PIT Grade



TTC Grade Distribution over Time

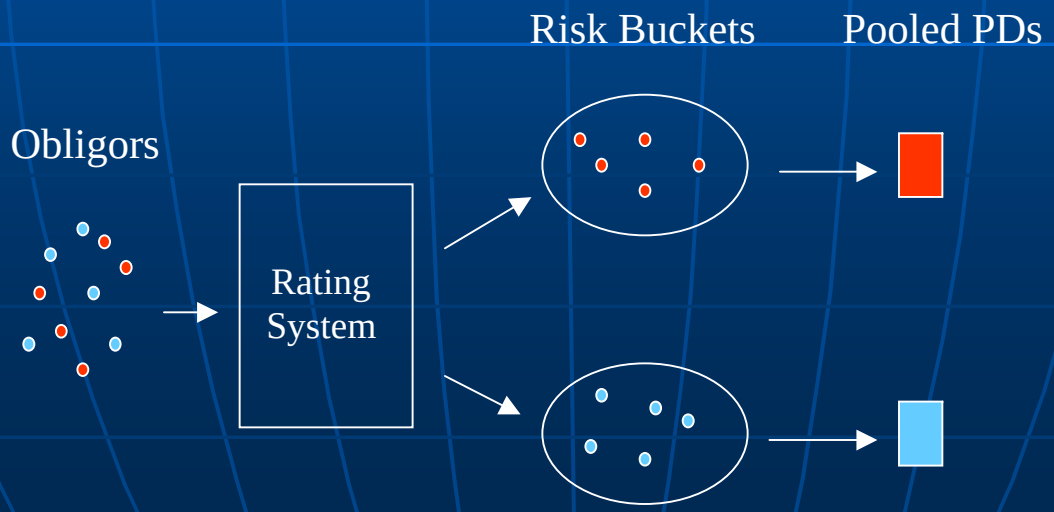


Average UPD by TTC Grade



Basel II's "Pooled" PDs

- Basel II rules stipulate a two-stage PD quantification process
  - Obligors are assigned discrete rating grades
  - A "pooled PD" is calculated for each grade
  - Each obligor within a grade receives the pooled PD associated with that grade



Basel II's Pooled PD

- According to CP3 ¶409 a grade's pooled PD must be "a long-run average of one-year realized default rates for borrowers in the grade"
- The pooled PD assigned to an obligor depends on its grade, not directly on its unstressed or stress PD
- A bank's rating philosophy affects the pooled PDs that obligors are assigned

Modeling Pooled PDs

- A grade's "true" pooled PD is the expected default rate for obligors assigned that grade over all years
- In practice, bank-reported pooled PDs will only be approximations of these "true" PDs
- To abstract from estimation issues, we examine the properties of these "true" PDs

Pooled PDs under a PIT System

- Since a PIT risk bucket is designed to maintain a fixed unconditional PD over time, the bucket's PPD must match its UPD

$$\begin{aligned} PPD^{PIT}(\gamma) &= E\left[D_{i,t+1} \mid \Gamma^{PIT}(W_i, X_{it}, Y_t) = \gamma\right] \\ &= Pr\left[Z_{i,t+1} < 0 \mid \alpha + \beta_W W_i + \beta_X X_{it} + \beta_Y Y_t = \gamma\right] \\ &= \Phi(-\gamma) \end{aligned}$$

- Under a continuous PIT rating system the PPD assigned to an obligor is equal its UPD

$$PPD_{it}^{PIT} = \Phi\left(-\left(\alpha + \beta_W w_i + \beta_X x_{it} + \beta_Y y_t\right)\right)$$

Pooled PDs under a TTC System

- The pooled PD for a TTC grade bares no direct relation to the unstressed PDs of the obligors assigned to that bucket

$$PPD^{TTC}(\gamma) = E\left[D_{i,t+1} \mid \Gamma^{TTC}(W_i, X_{it}) = \gamma\right]$$

$$= Pr\left[Z_{i,t+1} < 0 \mid \alpha + \beta_W W_i + \beta_X X_{it} = \gamma\right]$$

$$= \Phi\left(-\frac{\gamma}{\sqrt{1+\beta_Y^2}}\right)$$

- Likewise, the pooled PD assigned to an obligor matches neither its unstressed nor its stress PD

$$PPD_{it}^{TTC} = \Phi\left(-\frac{\alpha + \beta_W w_i + \beta_X x_{it}}{\sqrt{1+\beta_Y^2}}\right)$$

Rating Philosophy and Pooled PDs

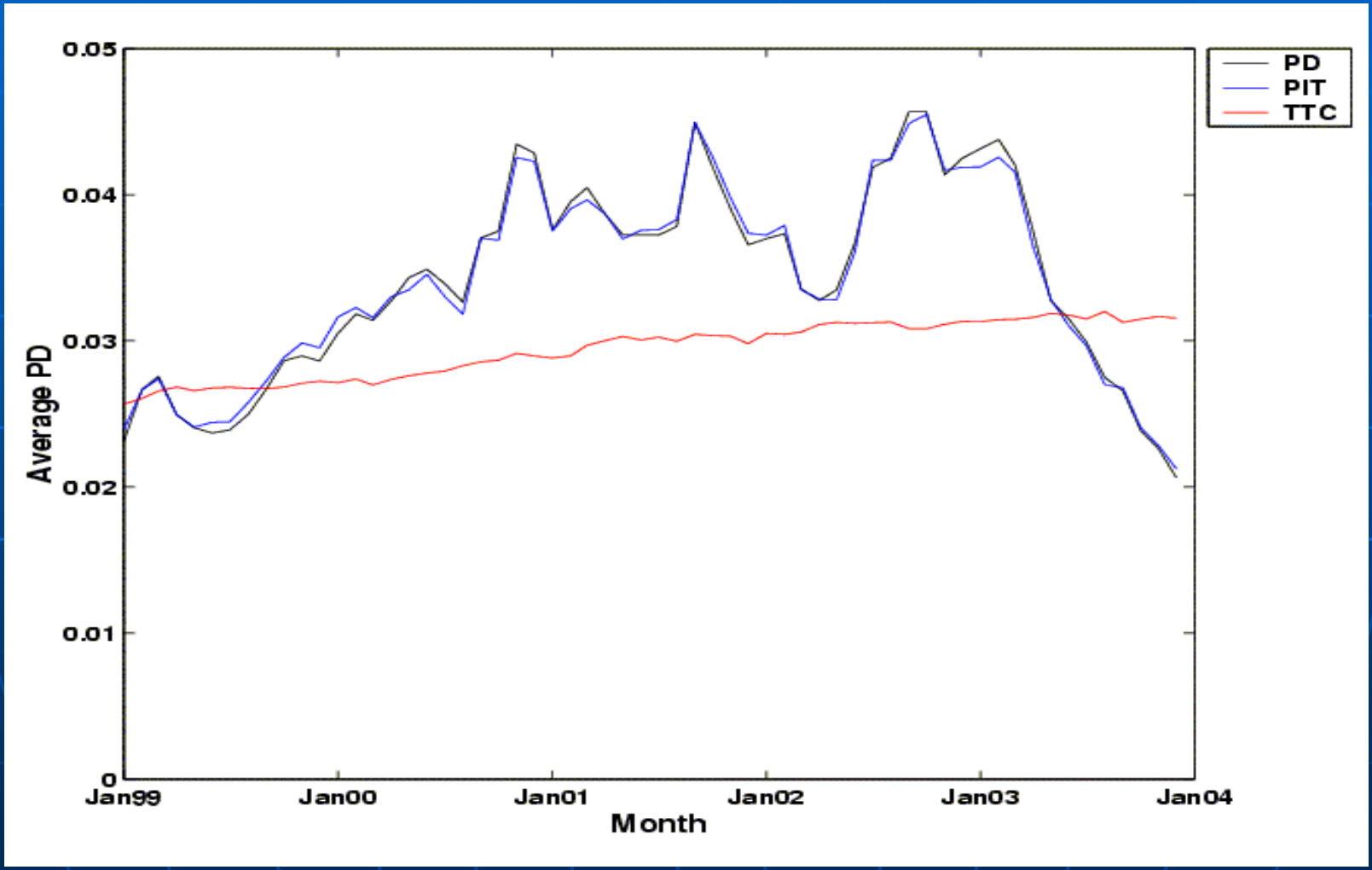
- The PIT pooled PD assigned to an obligor is negatively correlated with the business cycle

$$PPD_{it}^{PIT} = \Phi\left(-\left(\alpha + \beta_W w_i + \beta_X x_{it} + \beta_Y y_t\right)\right)$$

- The TTC pooled PD assigned to the same obligor is uncorrelated with the business cycle

$$PPD_{it}^{TTC} = \Phi\left(-\frac{\alpha + \beta_W w_i + \beta_X x_{it}}{\sqrt{1+\beta_Y^2}}\right)$$

Average PPDs over Time



Asymptotic-Single-Risk-Factor Capital Rule

- Gordy (2003) shows that a decentralized VaR capital rule can be derived if one assumes
  - A loan portfolio is well diversified
  - Cross-obligor dependence in loss rates is driven by a single systematic risk factor
- The capital charge for an exposure is equal to its conditional expected loss given an adverse draw of the systematic risk factor

ASRF Capital Rule

- To meet the VaR solvency target π at date t, we plug the 1-π percentile of the systematic risk factor V_{t+1} into the conditional expected loss function for each exposure

$$k_{it}^{\pi} = \Pr\left[Z_{i,t+1} < 0 \mid W_i = w_i, X_{it} = x_{it}, Y_t = y_t, V_{t+1} = \Phi^{-1}(1-\pi)\right] \cdot \lambda_i$$

$$= \Phi\left(\frac{-\left(\alpha + \beta_W w_i + \beta_X x_{it} + \beta_Y y_t\right) - \omega\Phi^{-1}(1-\pi)}{\sqrt{1-\omega^2}}\right) \cdot \lambda_i$$

- The Basel II capital function (CP3 ¶241) is derived from the same model, but is expressed in terms of an obligor's PD

$$k(PD) = \Phi\left(\frac{\Phi^{-1}(PD) + \omega\Phi^{-1}(1-\pi)}{\sqrt{1-\omega^2}}\right) \cdot \lambda_i$$

Capital Rules for PIT Pooled PDs

- The ASRF capital rule given a PIT pooled PD is

$$k_{it}^{\pi} = \Phi\left(\frac{\Phi^{-1}\left(PPD_{it}^{PIT}\right) + \omega\Phi^{-1}\left(1-\pi\right)}{\sqrt{1-\omega^2}}\right) \cdot \lambda_i$$

- This rule is fixed over the business cycle
- Using pooled PDs from a PIT rating system in the Basel II capital function ensures a 99.9% solvency target in every period

Capital Rules for TTC Pooled PDs

- The capital rule given a TTC pooled PD is

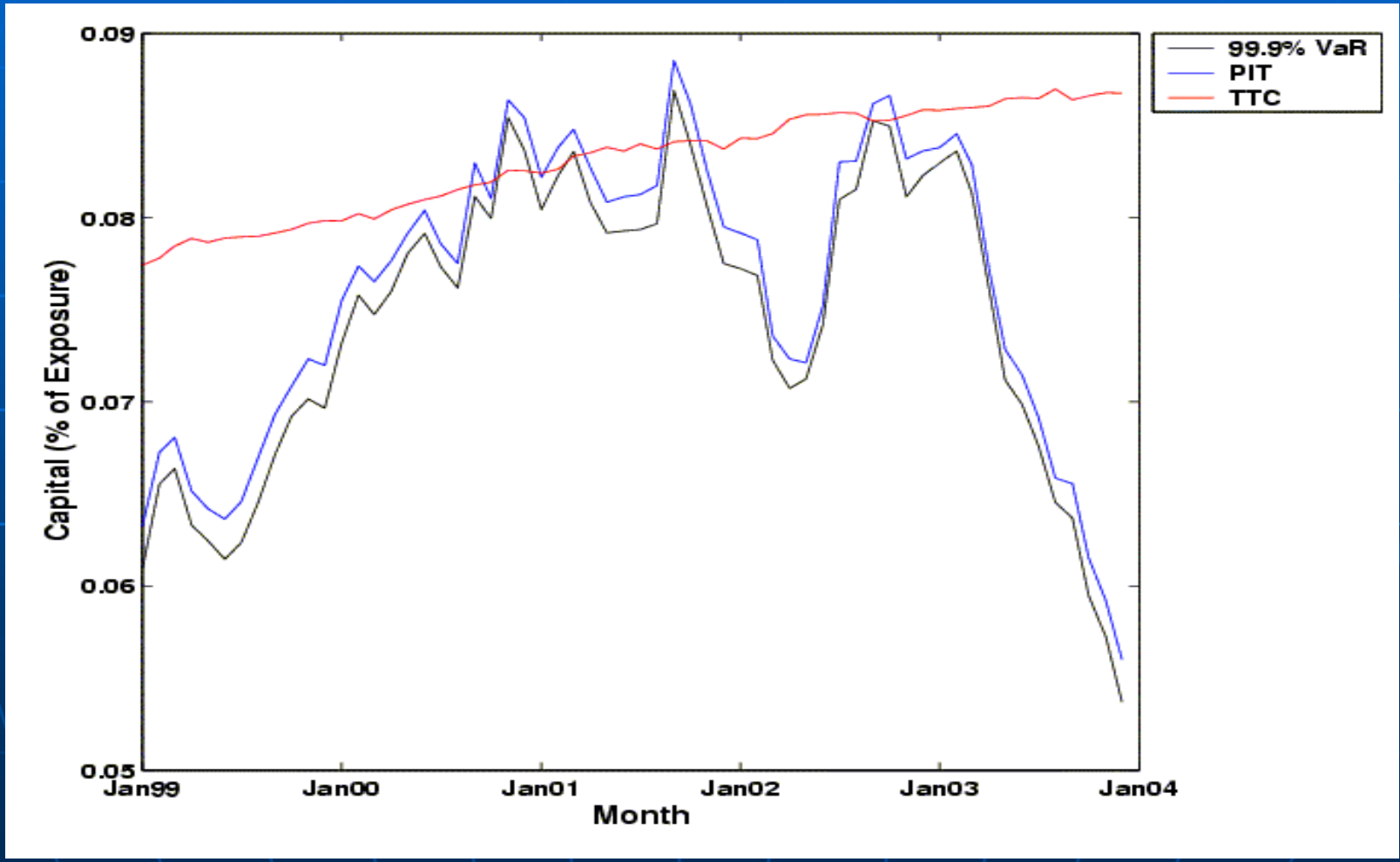
$$k_{it}^{\pi} = \Phi\left(\frac{\Phi^{-1}\left(PPD_{it}^{TTC}\right) - \beta_Y y_{it} + \omega\Phi^{-1}(1-\pi)}{\sqrt{1-\omega^2}}\right) \cdot \lambda_i$$

- The rule depends on both the pooled PD and the observable macroeconomic variable
- Using pooled PDs from a TTC rating system in the Basel II capital function will not ensure a fixed solvency target

Rating Philosophy and Capital

- PIT rating philosophy
  - Volatile Basel II capital requirement that rises during economic downturns
  - Capital is sufficient to satisfy a 99.9% solvency target in each period
- TTC rating philosophy
  - Stable Basel II capital requirement that is not correlated with the business cycle
  - Capital may not be sufficient to satisfy a 99.9% solvency target during economic downturns

Required Capital over Time



Rating Philosophy and Capital

- Problem
  - Basel II will not provide a level regulatory playing field for PIT and TTC banks
  - TTC banks may not hold sufficient capital during economic downturns
- Possible solutions
  - Apply different capital rules for PIT and TTC banks
  - Restrict the rating philosophy that banks can apply
  - Require that banks report unstressed PDs rather than pooled PDs

Validating PDs

- Supervisors must validate bank-reported pooled PDs
  - Ensure consistency across banks
  - Prevent gaming
- Two empirical approaches
  - BENCHMARKING – compare pooled PDs from different banks for similar exposures
  - BACKTESTING – compare a grade's pooled PD with the observed default frequency for that grade

Benchmarking Pooled PDs

- Benchmarking principle – all banks should report similar PDs for the same (or similar) obligors
- Pooled PD for obligor i under a PIT rating system

$$PPD_{it}^{PIT} = \Phi\left(-\left(\alpha + \beta_W w_i + \beta_X x_{it} + \beta_Y y_t\right)\right)$$

- Pooled PD for obligor i under a TTC rating system

$$PPD_{it}^{TTC} = \Phi\left(-\frac{\alpha + \beta_W w_i + \beta_X x_{it}}{\sqrt{1 + \beta_Y^2}}\right)$$

- Pooled PDs for the same obligors vary across rating systems

Benchmarking Pooled PDs

- Problem
  - Benchmarking may attribute differences in rating philosophy to errors in PD quantification
- Solutions
  - Restrict peer groups to banks with similar rating philosophies
  - Adjust reported PDs to reflect differences in rating philosophies
  - Require that banks report unstressed PDs rather than pooled PDs

Backtesting Pooled PDs

- Backtesting principle – a grade's pooled PD should match its long-run average default frequency
- Over the short-run systematic risk drives a wedge between the unstressed PD for a grade and its realized default frequency
- Over time average default frequencies should converge a grade's pooled PD

Backtesting under a PIT System

- Realized default frequency

$$DF_{t+1}^{PIT}(\gamma) = \Phi\left(-\frac{\gamma + \omega V_{t+1}}{\sqrt{1-\omega^2}}\right)$$

- Unstressed PD (best forecast)

$$UPD^{PIT}(\gamma) = \Phi(-\gamma)$$

- Pooled PD

$$PPD^{PIT}(\gamma) = \Phi(-\gamma)$$

Backtesting under a TTC System

- Realized default frequency

$$DF_{t+1}^{TTC}(\gamma) = \Phi\left(-\frac{\gamma + \beta_Y y_t + \omega v_{t+1}}{\sqrt{1-\omega^2}}\right)$$

- Unstressed PD (best forecast)

$$UPD_t^{TTC}(\gamma) = \Phi\left(-\left(\gamma + \beta_Y y_t\right)\right)$$

- Pooled PD

$$PPD^{TTC}(\gamma) = \Phi\left(-\frac{\gamma}{\sqrt{1+\beta_Y^2}}\right)$$

Backtesting Pooled PDs

- The long-run default frequency is an unbiased estimator of a grade's true pooled PD
- Over time, variance of the LRDF declines
- Variance of LRDF for a PIT risk bucket is lower than for a comparable TTC risk bucket
  - $V[LRDF^{PIT}]$ arises from systematic risk
  - $V[LRDF^{TTC}]$ arises from systematic risk and the business cycle
- Backtesting is more effective given a PIT rating system

Conclusions

- Under Basel II rating philosophy matters
  - For economic capital
  - For validation
- Basel II assigns capital based on stable pooled PDs associated with grades, not obligors
- Pooled PDs may not reflect unbiased and efficient default forecasts
  - In PIT systems pooled PDs closely approximate unstressed PDs
  - In TTC systems pooled PDs are more stable than UPDs

Conclusions: Regulatory Capital

- Dynamics of rating transitions determine dynamics of capital
- Capital requirements for PIT systems
  - Are more cyclical
  - Satisfy Basel II's fixed solvency target throughout the business cycle
- Capital requirements for TTC systems
  - Are less cyclical
  - Exceed the Basel II solvency target during upturns, but may fail to meet the target during downturns

Conclusions: Validation

- Different rating philosophies generate different pooled PDs for the same obligor
  - PIT pooled PDs are sensitive to the business cycle
  - TTC pooled PDs are stable over the cycle
- Benchmarking PDs requires that we account for differences in rating philosophy
- The efficiency of backtesting is sensitive to rating philosophy
  - Backtesting is most efficient given a PIT system

A Modest Proposal

- Basel II's requirement that a pooled PD reflect "a long-run average of realized default rates" creates several problems
  - Unlevel playing field across PIT and TTC banks
  - TTC banks may not meet 99.9% solvency target during economic downturns
  - Difficulty in benchmarking PDs across banks
  - Inefficiency in backtesting TTC systems
- Require that a pooled PD reflect "the expected default frequency of obligors *currently* assigned to the rating grade"