Conservative scheme for a non-hydrostatic climate model

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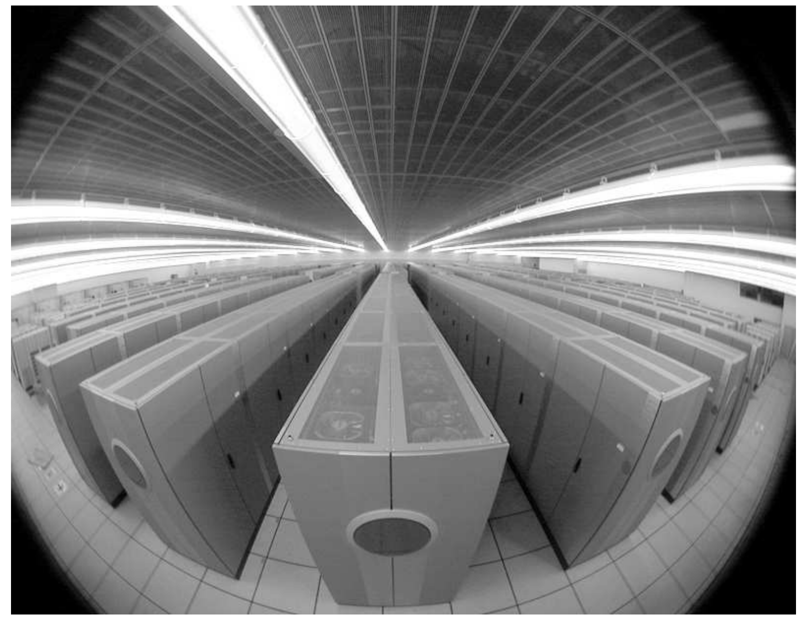
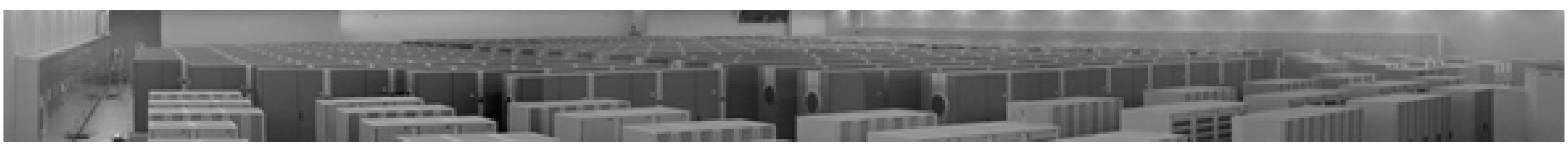
Outline

- Purposes
- Characteristics of the non-hydrostatic model
- Dry formulation and results
- Moist formulation
- Effects of precipitation
- Squall line experiments
- Tracer advection: Consistency with continuity
- Summary and tasks

ES (Earth Simulator)

- Parallel Vector Supercomputer：640node×8PE
- Peak performance: 40TFLOPS , Main memory:10TB

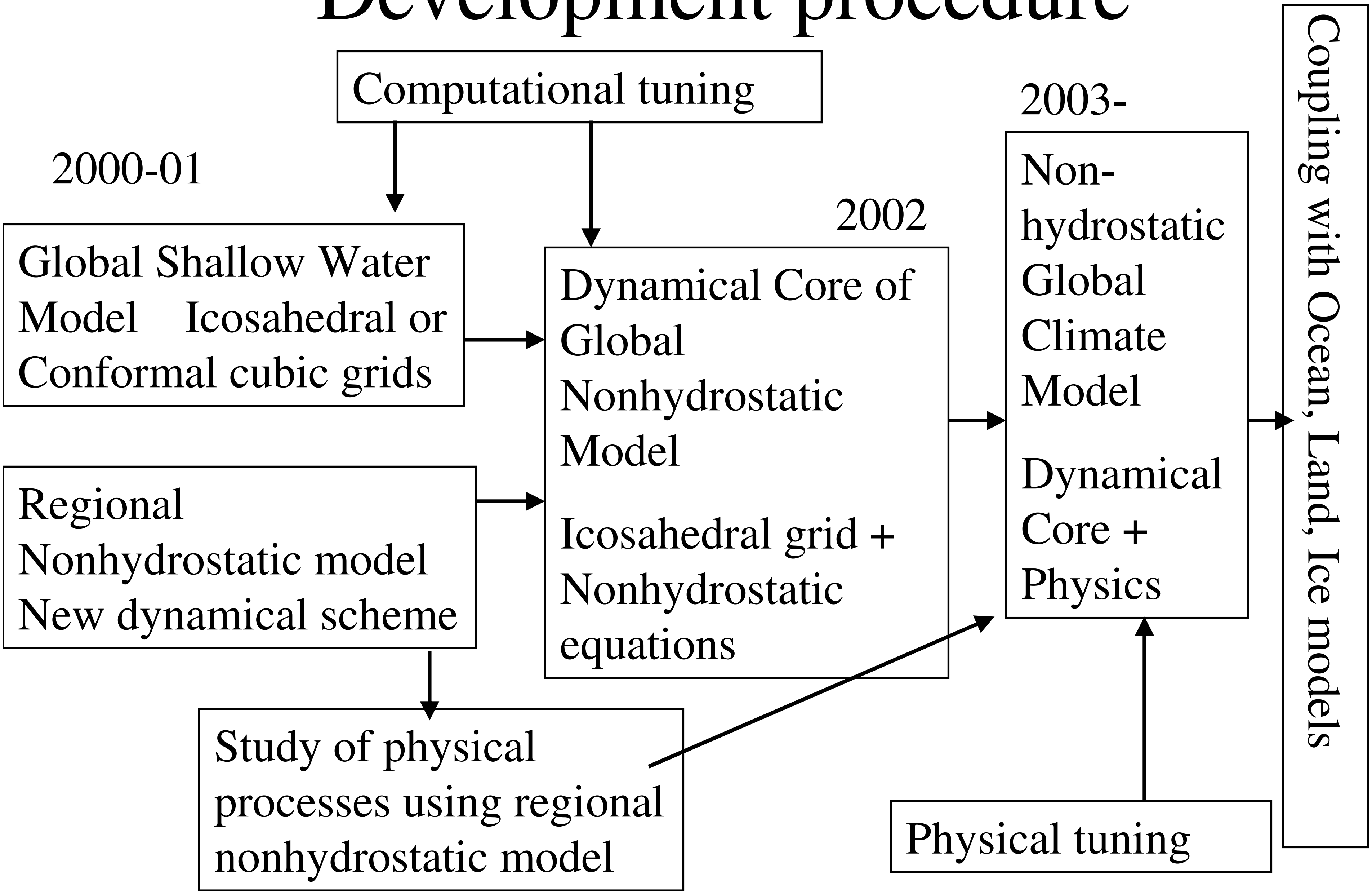




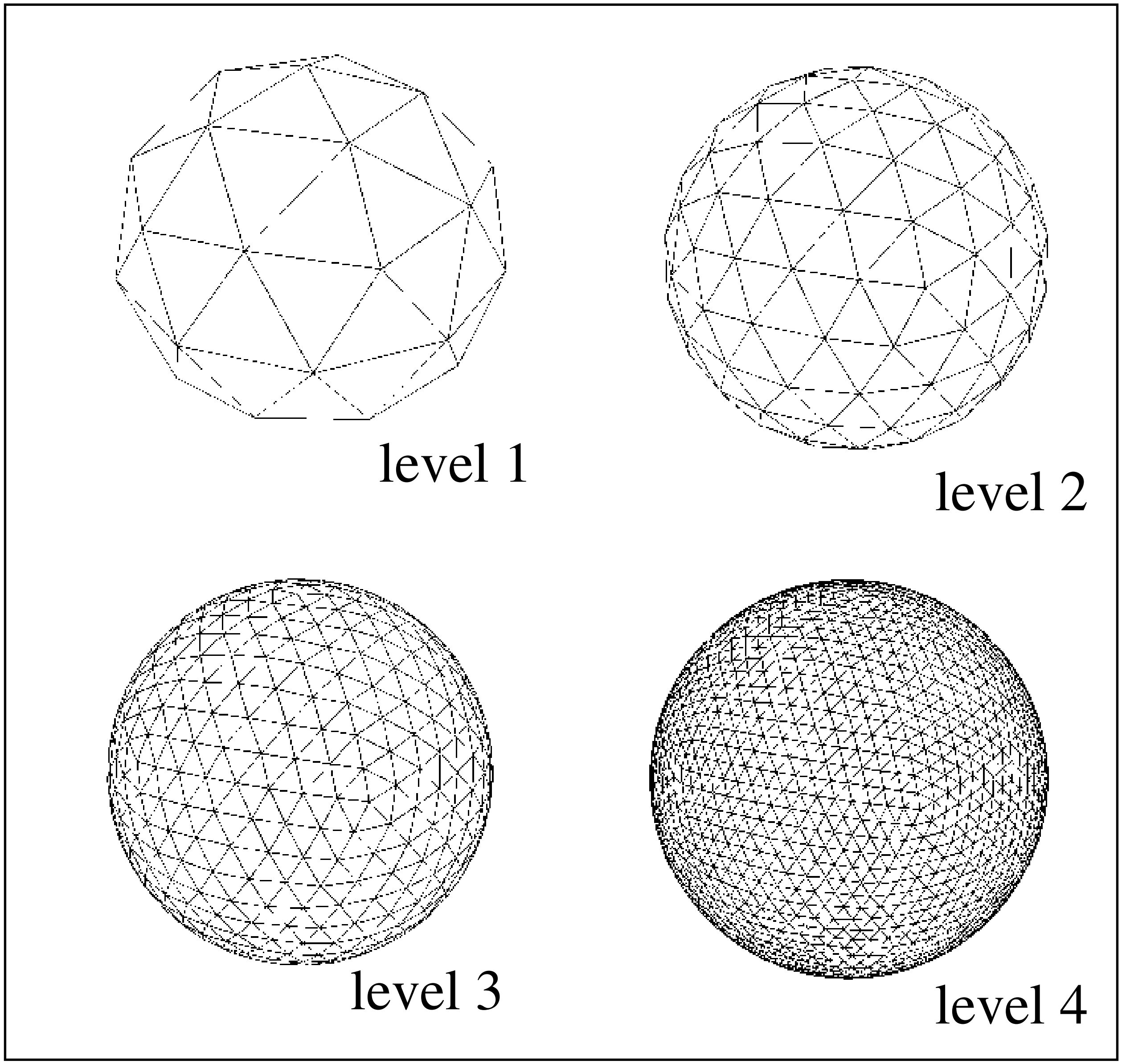
Purposes

- Development of a high resolution atmospheric general circulation model on the “Earth Simulator”: with horizontal resolution ~5km and vertical resolution ~100m: one order higher resolution than the current GCMs.
- Fully-compressible non-hydrostatic equations: less approximations to the Euler equations.
- Climate simulations for long time integrations ~ several ten years: require conservations of physical quantities (mass, energy, etc).
- A grid model with quasi-uniform grid intervals on the sphere using the icosahedral grid. Coding with the MPI parallelization using the two-dimensional domain decomposition.

Development procedure



Icosahedral grid



Number of grid points of n-level:

$$10\left(2^{n}\right)^{2}+2$$

Characteristic of the present nonhydrostatic model (1)

- Fully compressible non-hydrostatic equations:

> Horizontally explicit and vertically implicit time integration with time splitting

> No limitation on the time step for (quasi) vertical propagation of sound waves and gravity waves

> The Helmholtz equation is formulated for vertical velocity not for pressure: a switch for a hydrostatic/non-hydrostatic option can be introduced.

> The flux division method (Klemp et al 2000).

- The finite volume method using flux form equations of density, momentum and energy for conservations of the domain integrals.

> Total energy is conserved by integrating the total energy at small time integration steps.

> Domain integral entropy and potential temperature are also conserved if the vertical discretization of advection of energy is used (Taylor 1984).

Characteristic of the present nonhydrostatic model (2)

- Tracer advecion:
> Third order upwind, or UTOPIA
> Consistency with Continuity
- Exact treatment of moist thermodynamics (Ooyama 1990, 2001).
- An accurate tranport scheme for rain.
- A subset of the three-dimensional global non-hydrostatic model
> A test bed of new dynamical schemes. Development of the conservative scheme.
> Physics: cloud schemes (warm/ice), radiation, turbulence
> Study of the interaction between clound and radiation: Radiative-convective equilibrium experiments.
> Model hierarchy: can be used as 1D-vertical, 2D-horizontal-vertical, and 3D-regional models.

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Dry formulation

- Conservative flux form equations for density *R*, momentum ***V***, and internal energy *E*:

$$
\begin{aligned}
\frac{\partial}{\partial t}R + \nabla \cdot \boldsymbol{V} &= 0, \\
\frac{\partial}{\partial t}U + \frac{\partial}{\partial x}P &= -\nabla \cdot (\boldsymbol{V}u) + \frac{\partial \sigma_{xj}}{\partial x_j} \equiv G_U, \\
\frac{\partial}{\partial t}V + \frac{\partial}{\partial y}P &= -\nabla \cdot (\boldsymbol{V}v) + \frac{\partial \sigma_{yj}}{\partial x_j} \equiv G_V, \\
\frac{\partial}{\partial t}W + \frac{\partial}{\partial z}P + Rg &= -\nabla \cdot (\boldsymbol{V}w) + \frac{\partial \sigma_{zj}}{\partial x_j} \equiv G_W, \\
\frac{\partial}{\partial t}E + \nabla \cdot (\boldsymbol{V}h) - (\boldsymbol{v} \cdot \nabla P + Rwg) + Wg &= Q.
\end{aligned}
$$

where

$$
\begin{aligned}
\boldsymbol{V} &= (U, V, W) = (\rho u, \rho v, \rho w) \\
P &= p', \\
R &= \rho', \\
E &= \rho e^{in}: \quad \text{internal energy},
\end{aligned}
$$

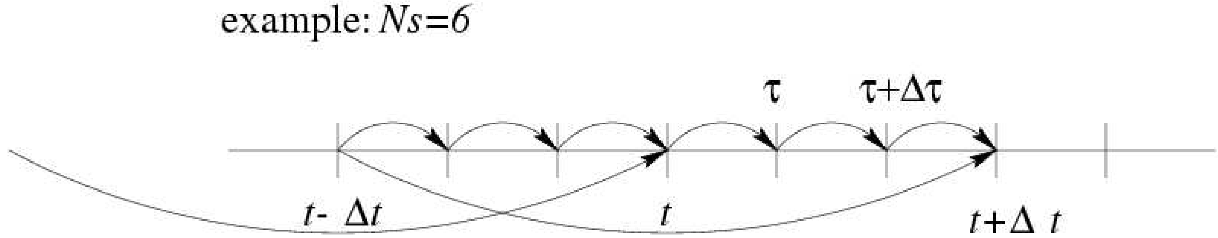
and

$$E = \rho C_v T = \frac{C_v}{R_d} p.$$

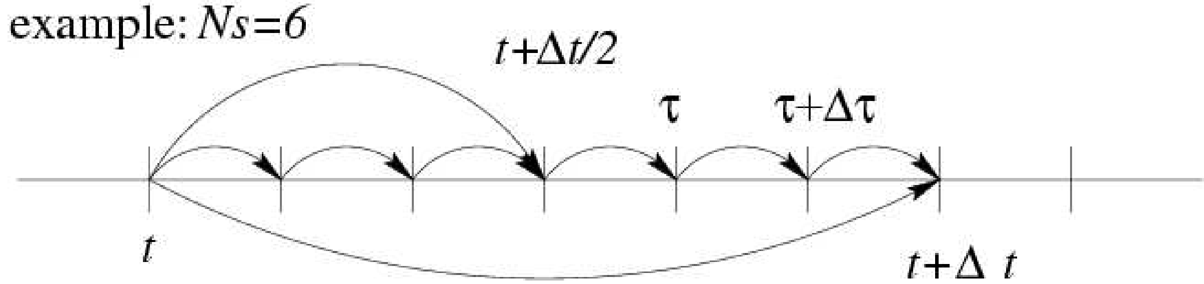
Time integration scheme: time splitting

- Large time step: t, small time step τ

Leap-frog



or RK2



The flux division method

(Klemp et al.2000)

$$A^* = A - A^t, \quad \delta_t A = \frac{A^{t+\Delta t} - A^t}{\Delta t} = \frac{A^{n+1} - A^n}{\Delta t},$$

$$\begin{aligned}
\delta_\tau R^* + \frac{\partial}{\partial x}U^* + \frac{\partial}{\partial y}V^* + \frac{\partial}{\partial z}W^* &= -\left(\frac{\partial}{\partial x}U^t + \frac{\partial}{\partial y}V^t + \frac{\partial}{\partial z}W^t\right), \\
\delta_\tau U^* + \frac{\partial}{\partial x}P^* &= -\frac{\partial}{\partial x}P^t + G_U^t, \\
\delta_\tau V^* + \frac{\partial}{\partial y}P^* &= -\frac{\partial}{\partial y}P^t + G_V^t, \\
\delta_\tau W^* + \frac{\partial}{\partial z}P^* + R^* g &= -\frac{\partial}{\partial z}P^t - R^t g + G_W^t,
\end{aligned}$$

$$\begin{aligned}
\delta_\tau E^* + \frac{\partial}{\partial x}(U^* h^t) + \frac{\partial}{\partial y}(V^* h^t) + \frac{\partial}{\partial z}(W^* h^t) &- \frac{W^*}{\rho^t}\left(\frac{\partial P^t}{\partial z} + R^t g\right) + W^* g \\
= -\frac{\partial}{\partial x}(U^t h^t) - \frac{\partial}{\partial y}(V^t h^t) - \frac{\partial}{\partial z}(W^t h^t) &+ u^t \frac{\partial}{\partial x}P^t + v^t \frac{\partial}{\partial y}P^t + w^t\left(\frac{\partial P^t}{\partial z} + R^t g\right) \\
-W^t g + Q^t &
\end{aligned}$$

Small time integration

- Explicit for *U* and *V*
- Implicit for *R*, *W*, *E*

> *W* by solving 1D-Helmholtz eq.

> Integrate for *R* in the flux form

> Energy correction: integrate for total energy in the flux form

- Explicit for U and V

$$\delta_\tau U^* = -\frac{\partial}{\partial x} P^{*\tau} + G'^{t}_{U},$$
$$\delta_\tau V^* = -\frac{\partial}{\partial y} P^{*\tau} + G'^{t}_{V}.$$

- Implicit for R, W, E: using $P^* = \frac{R_d}{C_v} E^*$

$$\delta_\tau R^* = -\frac{\partial}{\partial z} W^{*\tau+\Delta\tau} + G'^{\tau+\Delta\tau}_{R},$$
$$\underline{\delta_\tau W^*} = -\frac{\partial}{\partial z} P^{*\tau+\Delta\tau} - R^{*\tau+\Delta\tau} g + \underline{G'^{t}_{W},}$$
$$\delta_\tau P^* = -\frac{R_d}{C_v}\frac{\partial}{\partial z}(W^{*\tau+\Delta\tau} h^t) - \frac{R_d}{C_v} W^{*\tau+\Delta\tau}\bar{g} + \frac{R_d}{C_v} G'^{\tau+\Delta\tau}_{E},$$

> 1D-Helmholtz eq. for W

$$-\frac{\partial^2}{\partial z^2}\left(\Delta\tau^2 \frac{R_d}{C_v} h^t W^{*\tau+\Delta\tau}\right) - \left[\frac{\partial}{\partial z}\left(\Delta\tau^2 \frac{R_d}{C_v}\bar{g} W^{*\tau+\Delta\tau}\right) + \Delta\tau^2 g \frac{\partial}{\partial z} W^{*\tau+\Delta\tau}\right] + \underline{\alpha W^{*\tau+\Delta\tau}}$$
$$= \underline{\alpha W^{*\tau}} + \underline{\alpha \Delta\tau G'^{t}_{W}} - \Delta\tau \frac{\partial}{\partial z}\left[P^{*\tau} + \Delta\tau \frac{R_d}{C_v} G'^{\tau+\Delta\tau}_{E}\right] - \Delta\tau g \left[R^{*\tau} - \Delta\tau G'^{\tau+\Delta\tau}_{R}\right].$$

$\alpha = 0$: Hydrostatic

> Integrate for *R* in the flux form

$$R^{\tau+\Delta\tau} = R^{\tau} - \Delta t \left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} \right)^{\tau+\Delta\tau}$$

> Energy correction: integrate for total energy in the flux form

$$\delta_\tau (E + K + G) = -\nabla \cdot \left[\boldsymbol{V}^{\tau+\Delta\tau} \left(h + \frac{\boldsymbol{v}^2}{2} \right)^t \right] - \nabla \cdot \boldsymbol{F}^t + \nabla \cdot (\boldsymbol{v} \cdot \underline{\boldsymbol{\sigma}})$$

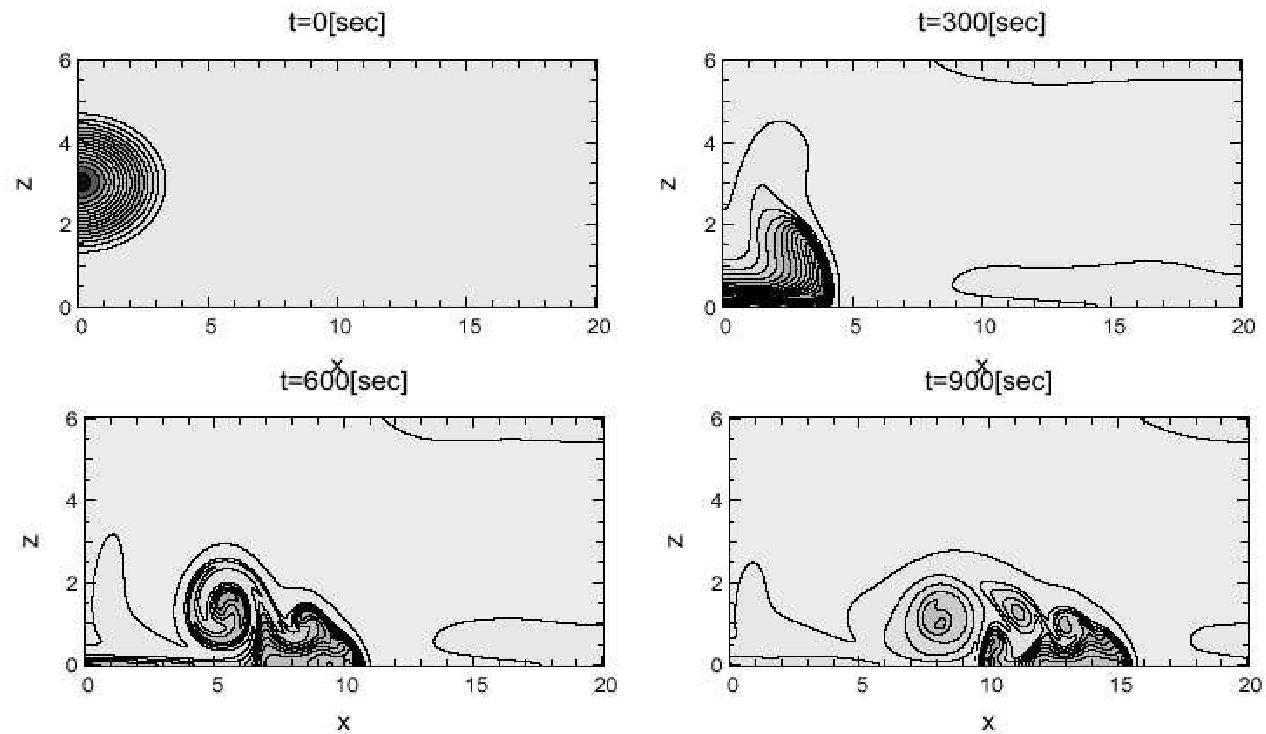
where *E*: internal energy, *K*: kinetic energy, and *G*: potential energy:

$$K = \rho \frac{u^2 + v^2 + w^2}{2}$$

$$G = \rho\Phi = \rho g z$$

Density current experiments

Straka et al.(1993)

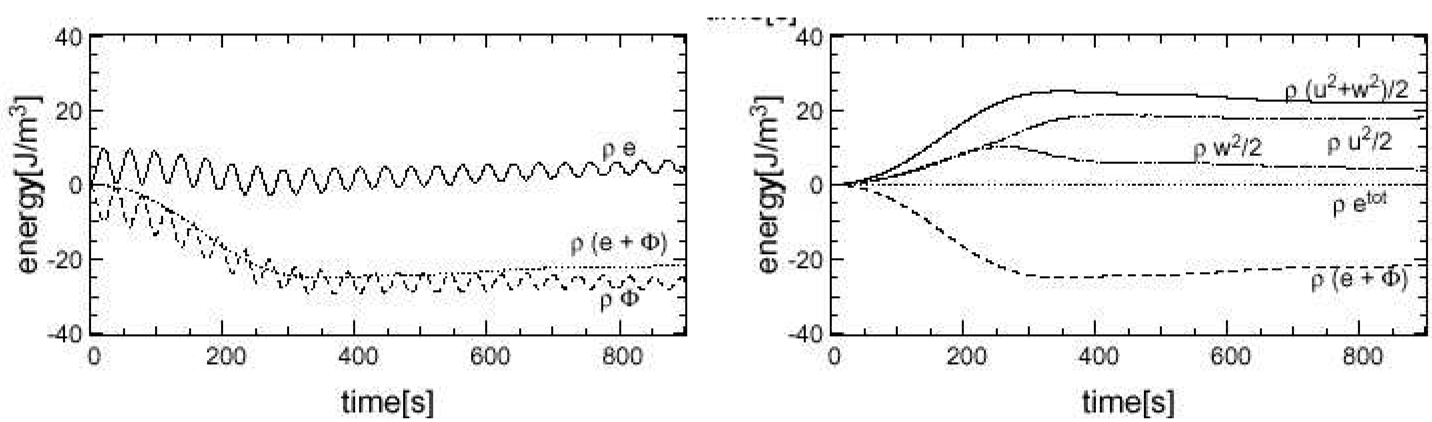


Initial cold bubble:

$\theta' = -15\text{K}$

$\Delta x = \Delta z = 50\text{m}$

$\Delta t = 0.1\text{s}$



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Moist formulation
with warm rain

- Prognostic variables:

> water vapor q_v

> cloud water q_c

> rain water q_l

> total density ρ

> momentum $\boldsymbol{V}= (U, V, W) = (\rho u, \rho v, \rho w)$

> Sensible part of internal energy E_a:

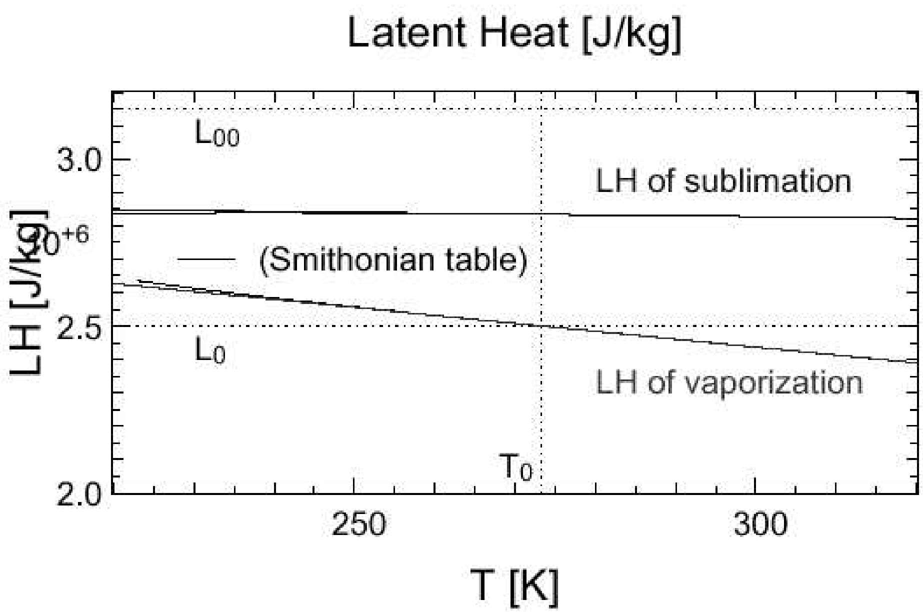
Effects of specific heats of water substance are considered:

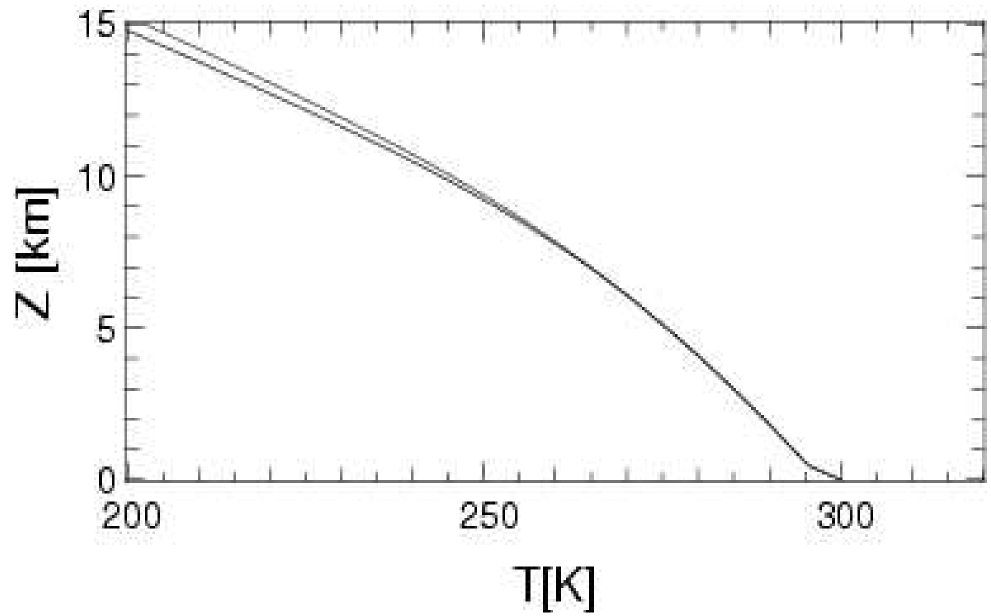
$$
\begin{aligned}
E_m &= q_d C_{vd} T + q_v (C_{vv} T + L_{00}) + q_l C_l T \\
&= (q_d C_{vd} + q_v C_{vv} + q_l C_l) T + L_{00} q_v = E_a + L_{00} q_v
\end{aligned}
$$

Effects of moisture

- Latent heat and moist adiabat:

$$L(T) \;=\; L_0 + (C_{pv} - C_l)(T - T_0) = L_{00} + (C_{pv} - C_l)T$$





moist adiabat

Governing equations (Ooyama; 1990,2000)

Water vapor:

$$\frac{\partial(\rho q_v)}{\partial t} + \nabla_H \cdot (\rho q_v \boldsymbol{v}_H) + \frac{\partial(\rho q_v w)}{\partial z} = -C + E + D_v,$$

Cloud water:

$$\frac{\partial(\rho q_c)}{\partial t} + \nabla_H \cdot (\rho q_c \boldsymbol{v}_H) + \frac{\partial(\rho q_c w)}{\partial z} = C - (S_{auto} + S_{accr}) + D_c,$$

Rain water:

$$\frac{\partial(\rho q_r)}{\partial t} + \nabla_H \cdot (\rho q_r \boldsymbol{v}_H) + \frac{\partial}{\partial z}[\rho q_r (w + W_r)] = (S_{auto} + S_{accr}) - E + D_r,$$

Density:

$$\frac{\partial \rho}{\partial t} + \nabla_H \cdot (\rho \boldsymbol{v}_H) + \frac{\partial}{\partial z}(\rho w + \rho q_r W_r) = 0,$$

Horizontal components of momentum:

$$\frac{\partial(\rho \boldsymbol{v}_H)}{\partial t} + \nabla_H \cdot (\rho \boldsymbol{v}_H \boldsymbol{v}_H) + \frac{\partial}{\partial z}(\rho \boldsymbol{v}_H w + \rho q_r \boldsymbol{v}_H W_r) = -\nabla_H p + \boldsymbol{F}_H,$$

Vertical component of momentum:

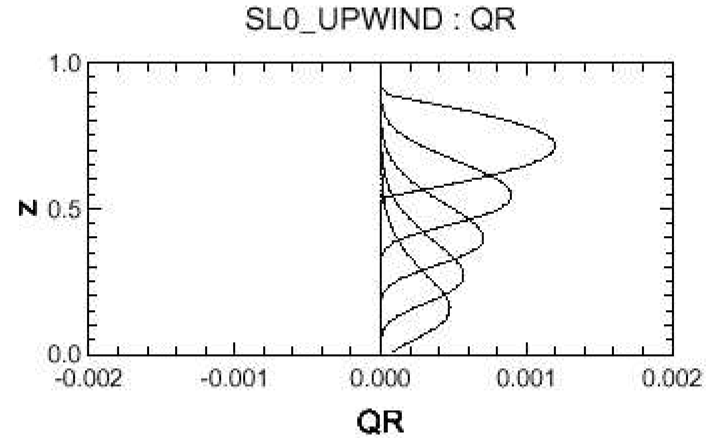
$$\frac{\partial(\rho w)}{\partial t} + \nabla_H \cdot (\rho w \boldsymbol{v}_H) + \frac{\partial}{\partial z}(\rho w w + \rho q_r w W_r) = -\frac{\partial p}{\partial z} - \rho g + F_z,$$

Internal energy:

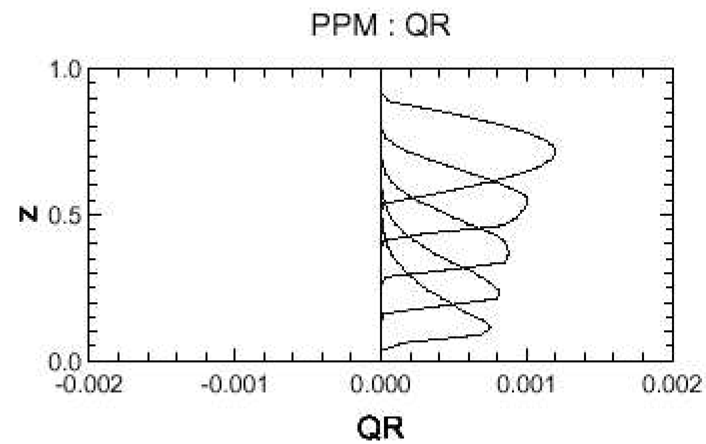
$$\frac{\partial}{\partial t}(\rho e_a) + \nabla_H \cdot (\rho h_a \boldsymbol{v}_H) + \frac{\partial}{\partial z}(\rho h_a w + \rho q_r e_r W_r)$$
$$= -\boldsymbol{v} \cdot \nabla p - \rho q_r W_r g + L_{00}(C - E) + Q_H,$$

Effects of precipitation

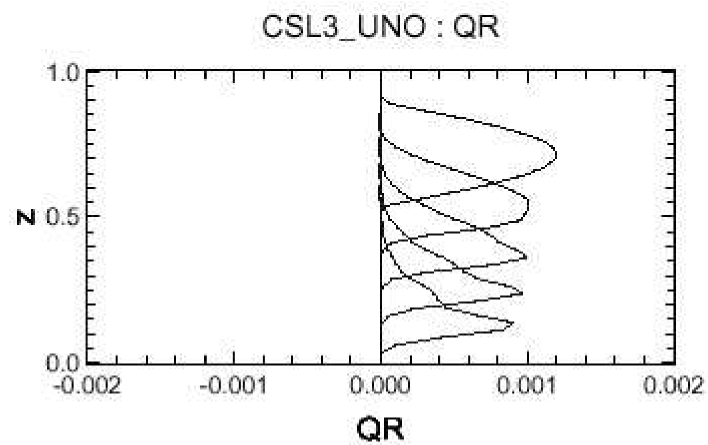
- Upwind



- PPM

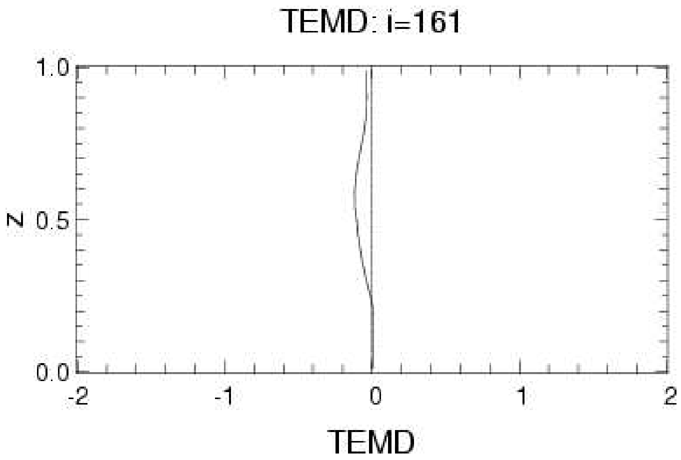


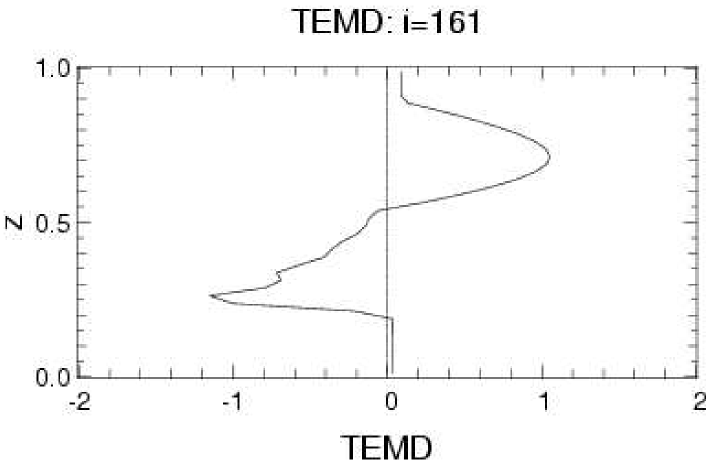
- Conservative Semi-Lagrangian (CSL3_UNO; Xiao and Yabe 2001)



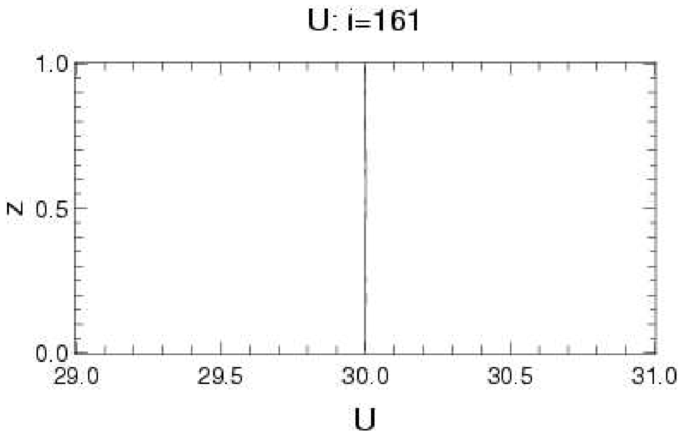
Changes of T and U due to precipitation

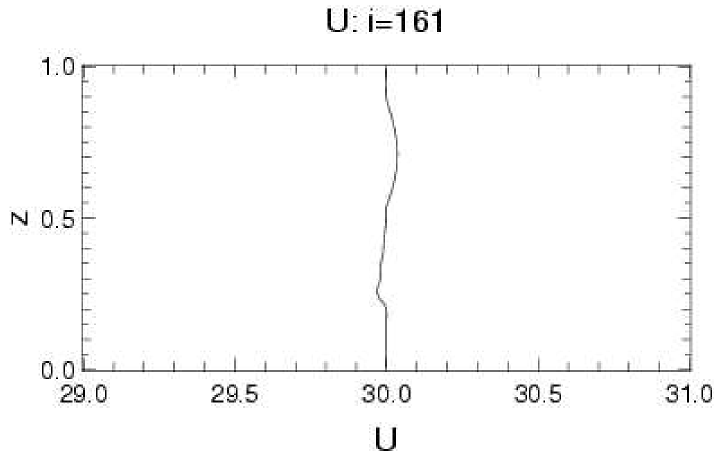
- Isentropic state with/without energy transport





- Uniform flow u=30m/s with/without momentum transport





Spurious changes are introducted if density change due to rain is neglected:

$$
\begin{aligned}
\Delta T &= T' - T = \frac{E}{\rho' C_{vd}} - \frac{E}{\rho(q_d C_{vd} + q_R C_l)} = \left(\frac{\rho(q_d C_{vd} + q_R C_l)}{\rho' C_{vd}} - 1 \right) T \\
&= \frac{q_R C_l}{(1 - q_r) C_{vd}} T \approx \frac{10^{-3} 4218}{712} 300 = 1.8[\mathrm{K}], \\
\Delta u &= u' - u = \frac{U}{\rho'} - \frac{U}{\rho} = \left(\frac{\rho}{\rho'} - 1 \right) u = \frac{q_r}{1 - q_r} u \approx 10^{-3} \times 30 = 0.03[\mathrm{m/s}].
\end{aligned}
$$

Energy transport must be considered in the flux form equations. Momentum transport can be negligible.

Squall line experiments
Redelsperger et al.(2000)

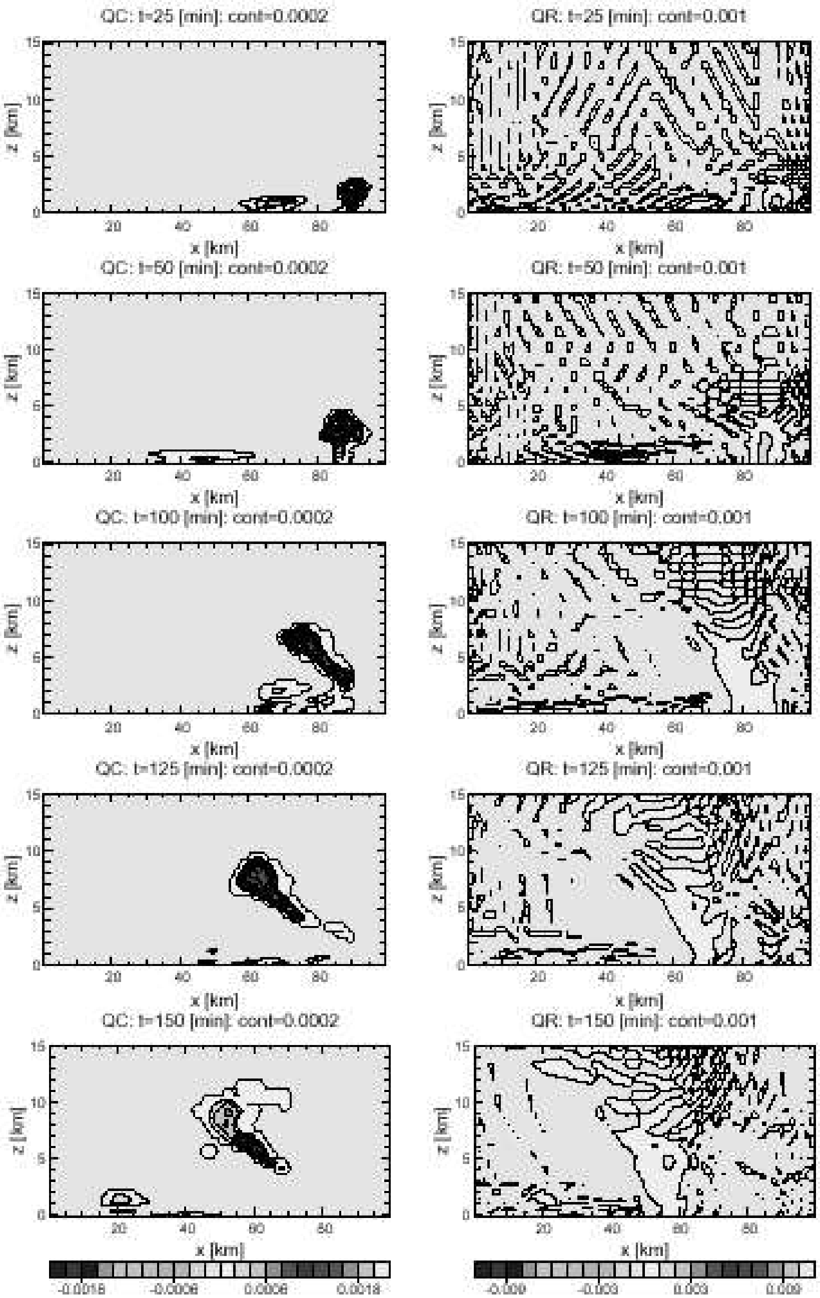
- GCSS-WG1, Case1: 2D with warm rain
- Domain length 100km and 1000km
- $\Delta x = 1.25$km, 44layers; Δt=1.5s, Ns=1
- Experiments and options

> exact moist thermodynamics vs simple

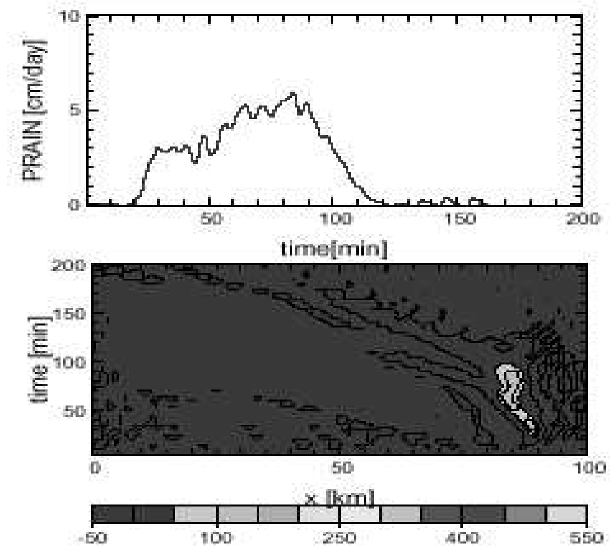
> with/without energy transport due to rain

> rain transport: CSL3, Upwind

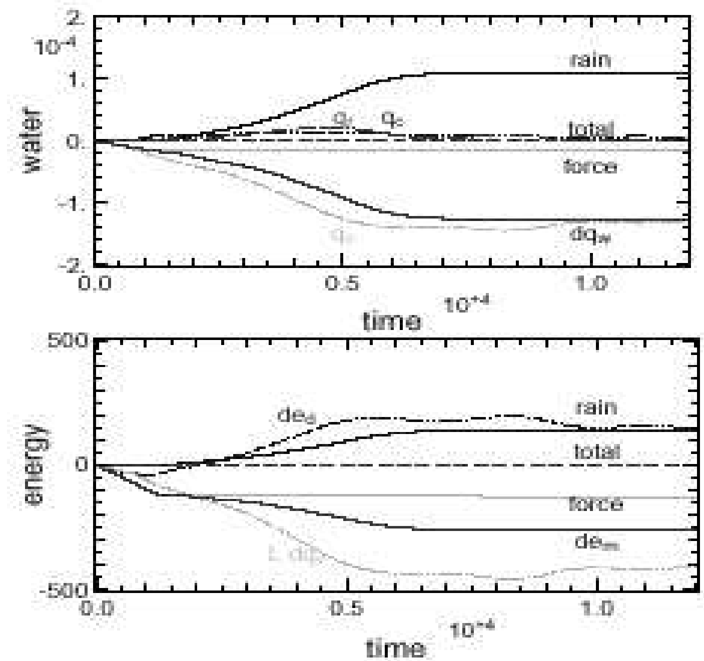
Cloud water and rain



Precipitation

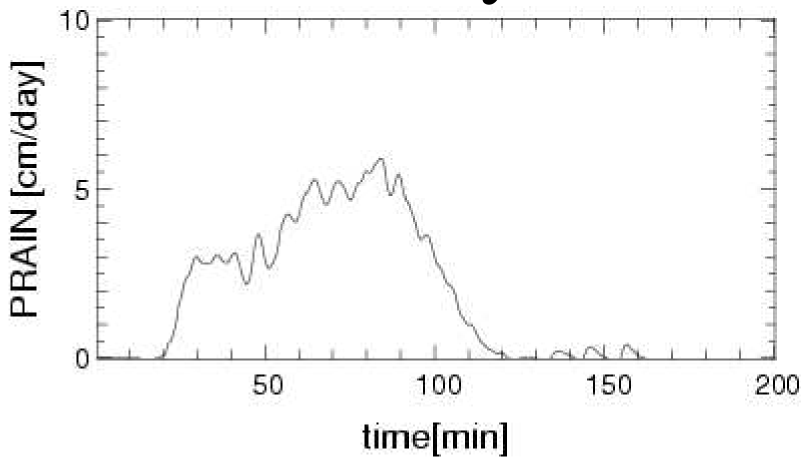


Water & Energy budgets

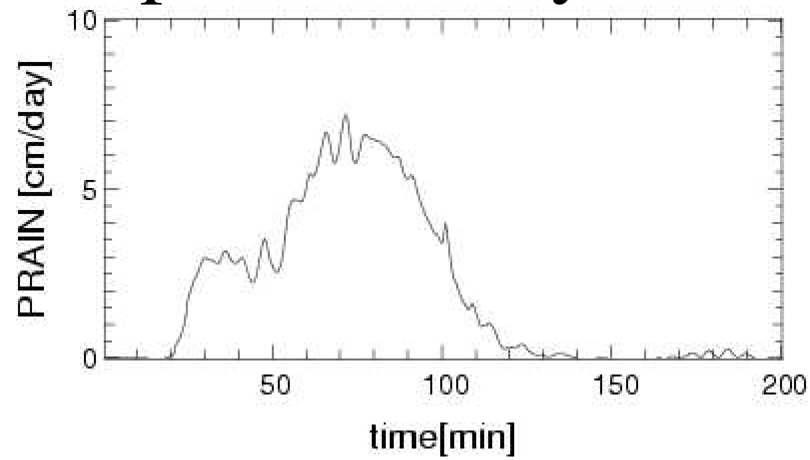


Comparison of total water

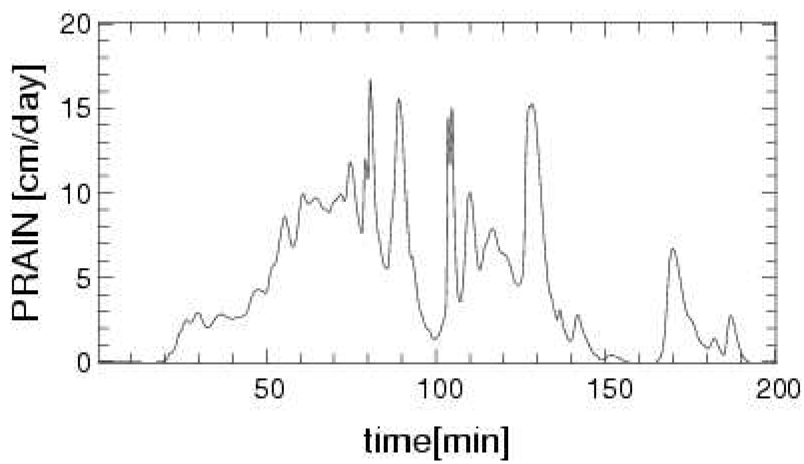
Exact thermodynamics



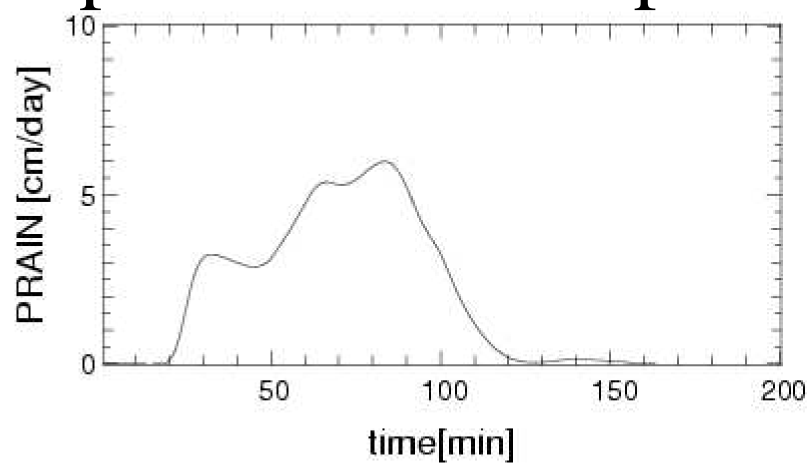
Simple thermodynamics



No rain energy transport

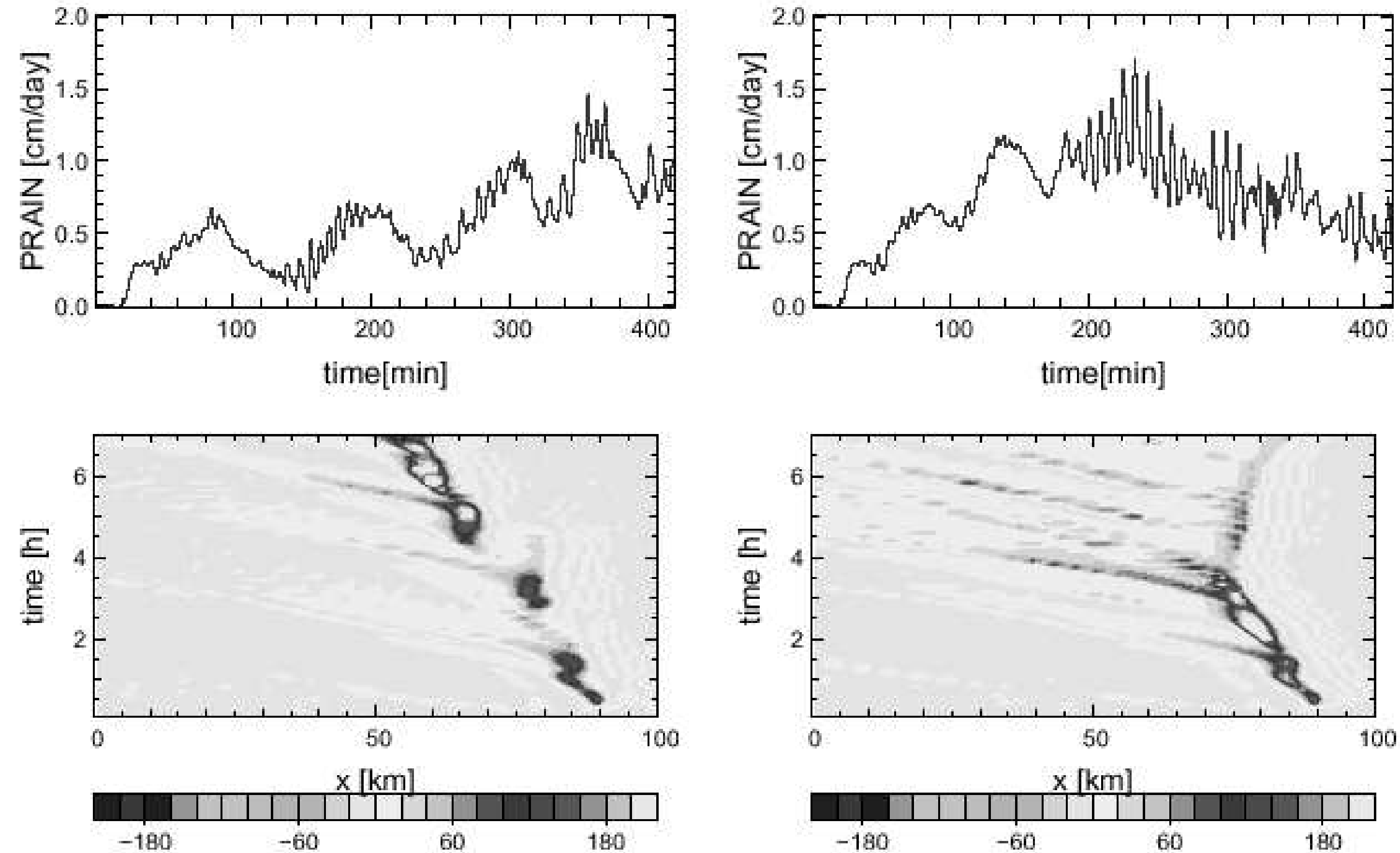


Upwind rain transport



Large domain experiments: 1000km

Exact vs simple thermodynamics



Tracer advection

Consistency with continuity(CWC)

- Small time step integration for density

$$\frac{\rho^{\tau+\Delta\tau} - \rho^{\tau}}{\Delta\tau} = -\nabla \cdot \boldsymbol{V}^{\tau+\Delta\tau}$$

- Summing up for *Ns* small time steps:

$$\frac{\rho^{t+\Delta t} - \rho^{t}}{\Delta t} = -\nabla \cdot \overline{\boldsymbol{V}}^{t}$$ where $$\overline{\boldsymbol{V}}^{t} = \frac{1}{N_s}\sum_{n=1}^{N_s} \boldsymbol{V}^{t+n\Delta\tau}$$

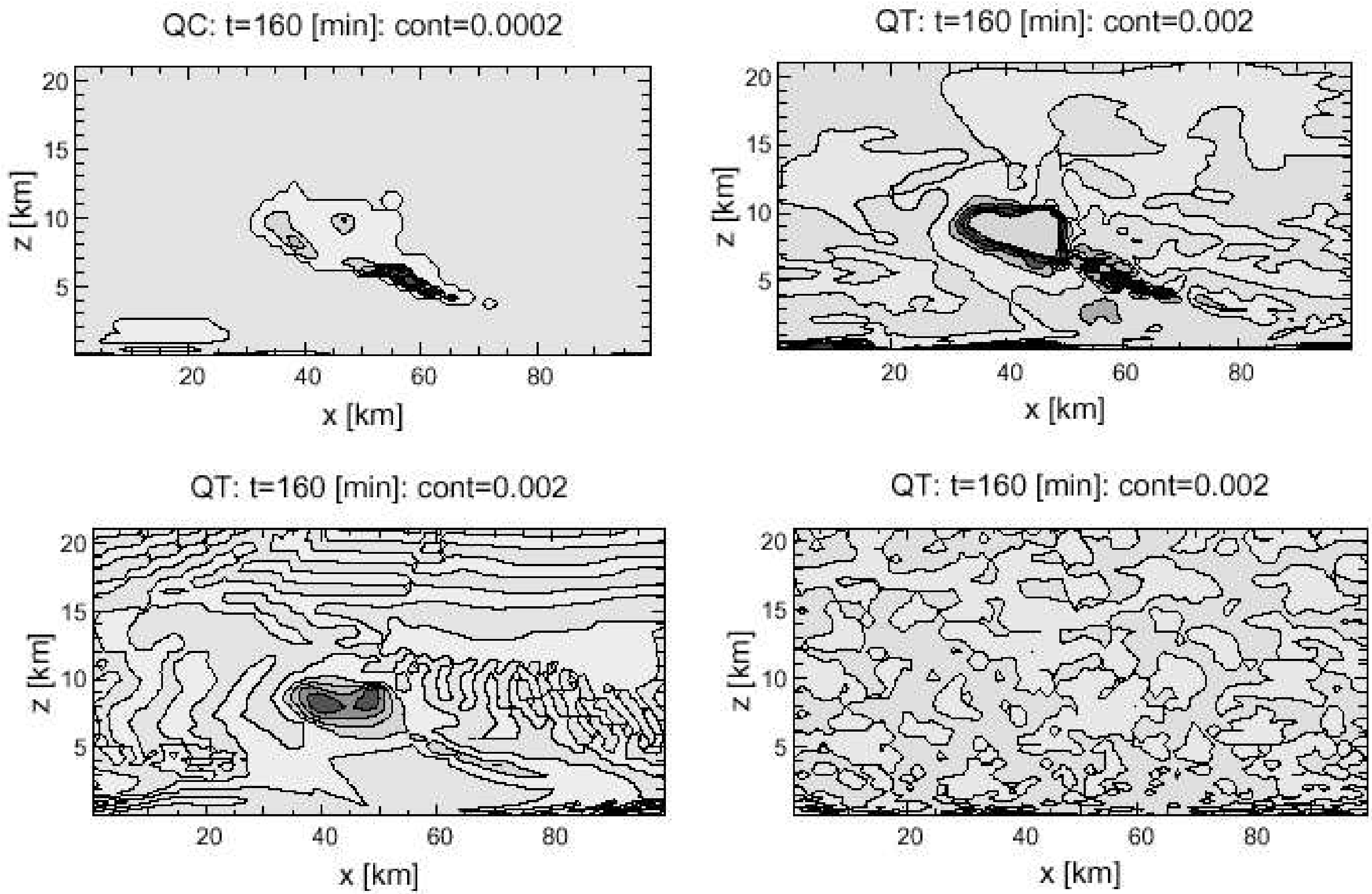
- Thus, tracer advection scheme with CWC is

$$\frac{Q^{t+\Delta t} - Q^{t}}{\Delta t} = -\nabla \cdot (\overline{\boldsymbol{V}}^{t} q^{t})$$ where $$Q = \rho q$$

$$q^{t+\Delta t} = \frac{Q^{t+\Delta t}}{\rho^{t+\Delta t}}$$

Given uniform tracer distribution q=1, initially

Cloud water Tracer: no CWC



Tracer: with CWC CWC with rain transport

Summary

- A new numerical scheme for a fully compressible non-hydrostatic model is developed with the flux form equations of mass, momentum and total energy with the time-splitting HE-VI method.
- Stable calculation with the time splitting integration using the flux splitting method of Klemp et al(2000).
- Effects of moisture

> Exact treatment of moist thermodynamics: Less precipitation if exact thermodynamics are used.

> Mass, momentum, and energy transports due to rain are appropriately considered (Ooyama 2001): Energy transport must be considered, but effect of momentum transport is small.

> The conservative CIP method (Xiao and Yabe 2001) is used for transport due to rain: Small fluctuation of precipitation is simulated if CIP or PPM is used.

- Tracer advection scheme is consistent with the continuity equation.

Tasks

- Physics: ice phase, radiation, and turbulence. (on going)
- Define standard test cases and quantify dependency of numerical schemes.

> Squall line and GCSS WG4 other cases

> Long time and wide range integration: radiative convective equilibrium in O((100 km)^2) and O(10days) run (Tompkin and Craig, 1998).

> Various cloud systems including the tropics and the mid-latitudes.

- Advection scheme: non-negative, conservative, and shape preserving scheme extensible to icosahedral grid $\Rightarrow$ conservative semi-Lagrange or UTOPIA with limiter.
- Step mountain: adaptive and boundary layer.
- Tuning and computational performance.
- Construction of a three dimensional global non-hydrostatic climate model (by Tomita): Unification of modules. Model hierarchy.

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Squall line： Δ x=1.25km vs 125m