Optimal Asset Location and Allocation with Taxable and Tax-Deferred Investing

By

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Introduction and Motivation

- How should investors allocate their asset holdings in both taxable and tax-deferred (retirement) accounts?
  - Tax-timing options in the taxable account versus pre-tax returns in the retirement account.

- Upon what do these optimal holdings in the taxable and tax-deferred accounts depend?

- How valuable is it for the investor to locate his asset holdings efficiently? What is the relative value of tax-deferred vs. taxable wealth? Upon what does it depend?

The Model

- Framework introduces a tax-deferred account into the model of Dammon, Spatt and Zhang [*Review of Financial Studies*, 2001].

- Investor Characteristics
  - Annual consumption and portfolio allocation decisions starting at age 20
  - Mortality calibrated to that for the U.S. population (increasing with age and no investor lives beyond age 100)
  - Stochastic labor income
  - Constant relative risk-averse preferences ($\gamma = 3$).
  - Annual subjective discount factor of $\beta = 0.96$.
  - Bequest motive – At death investor's total wealth is invested in an annuity which provides consumption support for the investor's beneficiary for H periods (H = 20).

- Investment Opportunity Set
  - Riskless bond with constant return
  - Risky stock with constant dividend yield and stochastic (binomial) capital gain return

The Model

- Taxable Account
  - Dividends and Interest taxed as received
  - Capital gains and losses taxed only at sale
  - At death, investor's tax basis is *reset* to the market value, forgiving the capital gains tax.
    - Note contrast with Canadian law

- Tax-Deferred (Retirement) Account
  - Contributions based upon labor income (before-tax funds)
  - Investment returns (interest, dividends and capital gains) not taxed
  - Withdrawals taxed as ordinary income
  - At death, retirement account wealth is taxed as ordinary income

Model Simplification and Solution Technique

- The intertemporal consumption-investment problem is solved numerically using dynamic programming. Simplifications used to limit the number of state variables.

  - Investor's tax basis for equity held in the taxable account assumed to be the average purchase price.
  - The assumption of constant relative risk averse preferences ensures that the optimal consumption and portfolio decision rules (as a fraction of wealth) are independent of wealth.
  - To avoid additional state variable(s) for labor income we assume it is proportional to wealth.

- The relevant state variables for the problem are:

  - Initial equity holding in the taxable account
  - Embedded capital gains
  - Proportion of wealth held in the retirement account (not the composition of the retirement account's assets)
  - Age

General (Non-numerical) Conclusions on Asset Location

- Where is it efficient for the investor to locate his exposures (i.e., minimize taxes) while fixing the risks incurred?

- Assumptions
  a) $\tau_g < \tau_d$ (τ_g applies to both gains and losses)
  b) Unrestricted borrowing
  c) Forced Realization – No deferral of gains tax required

- No Restrictions Required on
  a) Number of Assets
  b) Calculation of tax basis (averaging rule not required)
  c) Investor preferences
  d) Joint distribution of asset returns
  e) Presence of labor income

Asset Location for Forced Realization

- Result (A): Optimal for the investor to hold the asset with the highest yield (e.g., the bond as long as the bond coupon exceeds the dividend yield) in the tax-deferred account and allocate other assets to the taxable account.

- Intuition: If the investor holds something other than the highest yielding asset in his tax-deferred account (e.g., bonds) and moves one dollar towards the high yielding and most heavily taxed asset (e.g., bonds) in the tax-deferred account, he can shift x dollars towards stocks in the taxable account to achieve a total wealth shift that is independent of the realized return on equity (for the right choice of x). The overall effect increases the investor's wealth.

Optimal Deferral

- Result (B): If the investor has the option to realize gains optimally, a sufficient (but not necessary) condition for holding only bonds in the retirement account is that the bond coupon exceeds the yields on equity.

- Intuition: Result (A) suggests that it would be optimal to hold bonds in the tax-deferred account when the bond coupon exceeds the yield on equity in the case of forced realization. But the option to realize gains optimally further increases the attractiveness of holding equity (but not bonds) in the taxable account. Hence, both the lesser taxation of ordinary income and the tax options favor locating all the desired equity in the taxable account.

Mutual Funds

It can be optimal to invest in tax-exempt bonds, while holding equity in the tax-deferred account if

$$\tau_g > \tau_d - \frac{[r(\tau_d - \tau_m)(1+d)}{r-d}.$$

This can only occur if τ_g is quite high, i.e., the form of equity is very tax-inefficient. It would not occur for index funds and other forms of tax-efficient equity.

In fact, this begs the question of why hold active (tax-inefficient) mutual funds. It could be optimal to do so only if that form of equity outperforms passive equity by about 200 basis points or so – to cover the higher fees for active funds and the implicit tax costs of the mislocation of tax-efficient equity.

Model Parameters

Financial Markets:

- Riskless one-period bond
  - Constant nominal interest rate of r = 6%.
  - Interest taxed at the rate of τ_d = 36%.

- Single risky stock
  - Constant nominal dividend yield of d = 2%.
  - Dividends taxed at the rate of τ_d = 36%.
  - Nominal capital gain return follows a binomial process (μ = 9% and σ = 20%).
  - *Realized* capital gains and losses in the taxable account are taxed at the rate of τ_g = 20%.
  - No short sales allowed.

- Constant inflation rate of i = 3.5%.

- Pre-tax nonfinancial income-wealth ratio = 15%

- Retirement account funded with 20% of pre-tax labor income

- Retirement withdrawals based upon remaining life expectancy starting at age 65

Table I

Base-Case Parameter Values

The table provides the base-case parameter values that are used to conduct the numerical analysis in Section II.

Parameters of the Model	Notation	Base-case Value
Asset Returns:		
Riskless one-period taxable interest rate	r	6.0%
Dividend yield on equity	d	2.0%
Expected capital gain return on equity	$\bar{g}$	9.0%
Standard deviation of capital gain return	σ	20.0%
Inflation rate	i	3.5%
Tax Rates:		
Ordinary income tax rate	τ_d	36%
Capital gain tax rate	τ_g	20%
Utility and Bequest Functions:		
Utility discount factor	β	0.96
Relative risk aversion	α	3.0
Bequest parameter[1]	H	20
Labor Income and Retirement Savings:		
Labor income[2]	l	15%
Retirement contribution rate[3]	k	20%
Retirement withdrawal rate[4]	h_t	1/life expectancy
Mandatory retirement age	J	65

1. The bequest parameter is the number of years of consumption the investor wishes to provide his beneficiary following his death. Higher values of H imply a stronger bequest motive.

2. Labor income is assumed to be a constant proportion of the investor's total wealth.

3. Retirement contribution rate is stated as a proportion of the investor's pre-tax labor income. Retirement contributions are mandatory prior to retirement.

4. Retirement withdrawal rate is stated as a proportion of the investor's tax-deferred wealth. Investors are not allowed to withdraw funds from their tax-deferred accounts prior to retirement.

No Borrowing and Asset Location

- Without the ability to borrow the optimal asset location involves the following:

  - Investors do not simultaneously hold taxable bonds and tax-deferred equity.
  - A mix of bonds and stock may arise in the retirement account only if the investor holds only equity in the taxable account.
  - A mix of bonds and stock may arise in the taxable account only if the investor holds all bonds in the tax-deferred account.

  - If the investor is unable to borrow, an increase in the proportion of the investor's wealth in the tax-deferred account can result in reduced overall equity exposure when the marginal holdings of equity can only be located in the tax-deferred account as the tax-deferred account is a less attractive venue to locate equity. When the taxable account is fully invested in equity, the investor does not purchase additional equity in his tax-deferred account immediately because the marginal value of equity is substantially less in the tax-deferred account than in the taxable account.

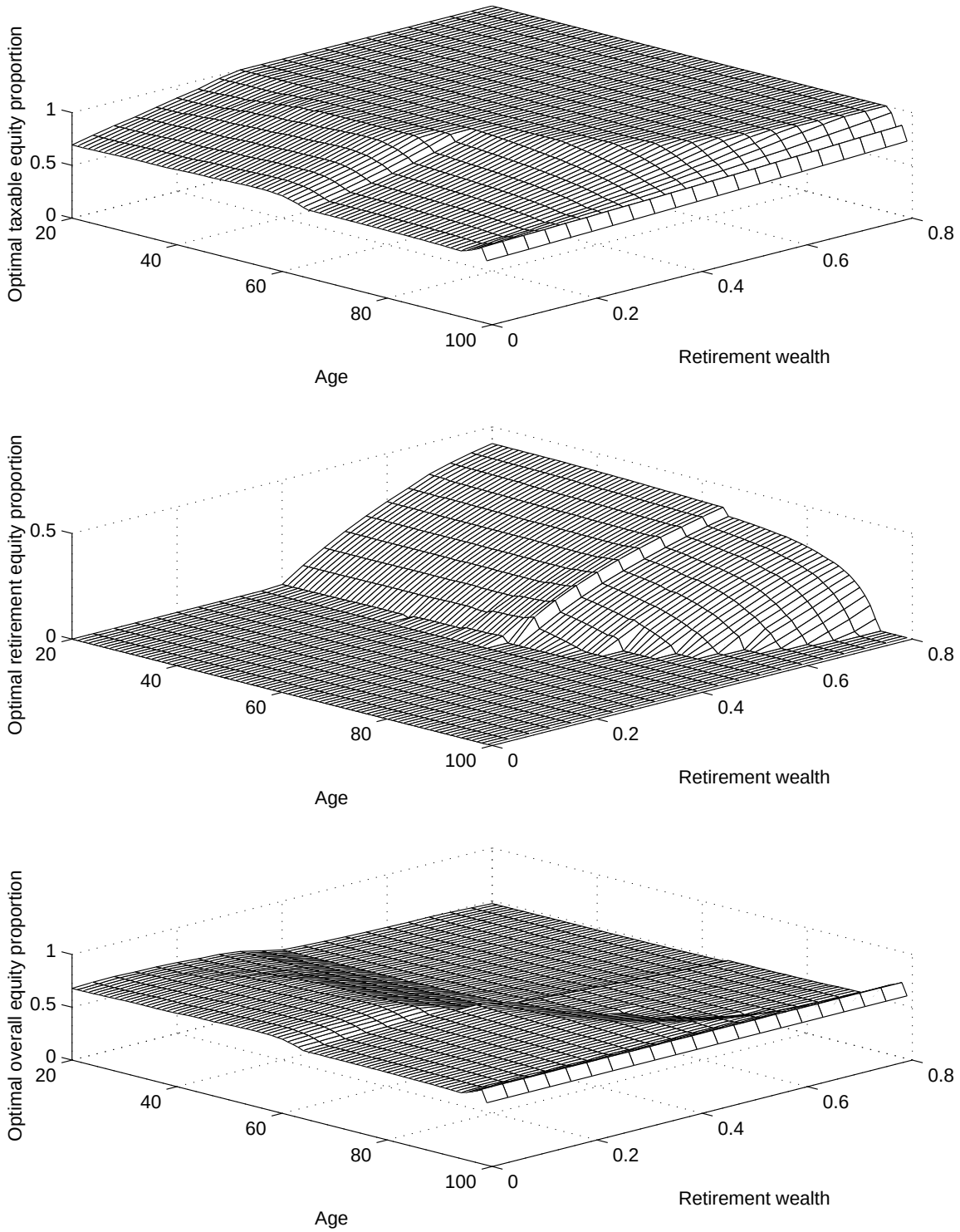

Optimal taxable equity proportion
1
0.5
0
20
40
60
80
100
Age
0
0.2
0.4
0.6
0.8
Retirement wealth
Optimal retirement equity proportion
0.5
0
20
40
60
80
100
Age
0
0.2
0.4
0.6
0.8
Retirement wealth
Optimal overall equity proportion
1
0.5
0
20
40
60
80
100
Age
0
0.2
0.4
0.6
0.8
Retirement wealth

Figure 1:

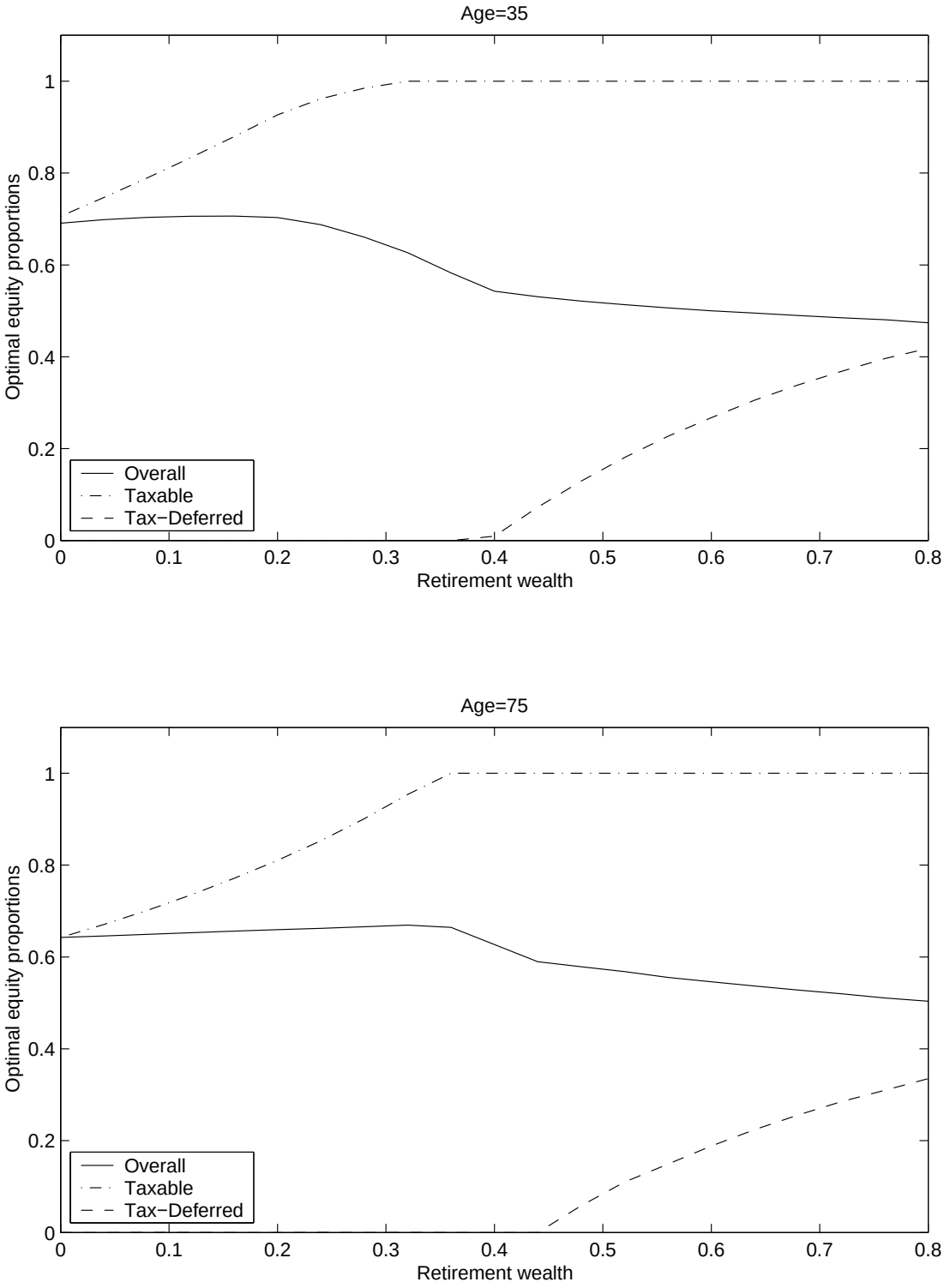

Age=35
Optimal equity proportions
Retirement wealth
Overall
Taxable
Tax−Deferred
0
0.2
0.4
0.6
0.8
1
0.1
0.3
0.5
0.7
Age=75
Optimal equity proportions
Retirement wealth
Overall
Taxable
Tax−Deferred

Figure 2:

Table II
Monte Carlo Simulation Analysis

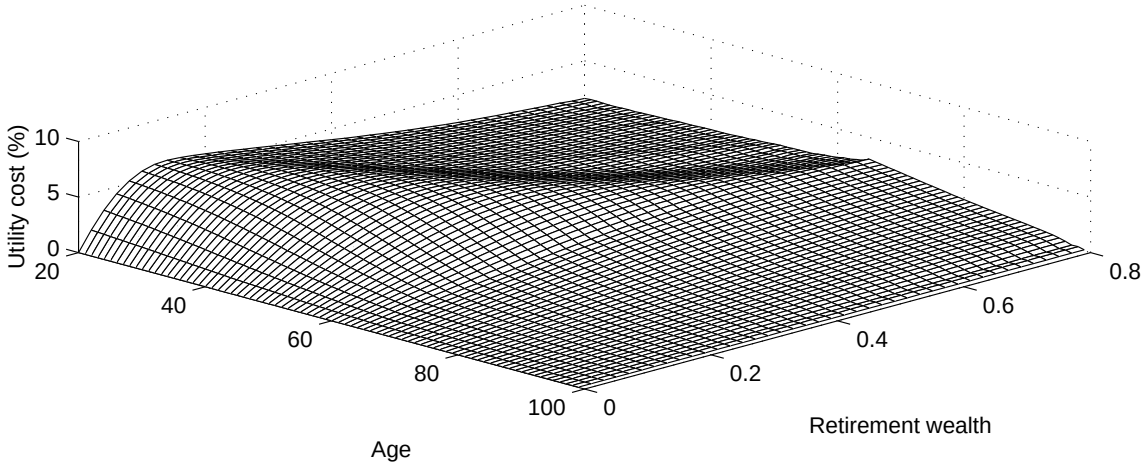
The table summarizes the results of the Monte Carlo simulation analysis conducted in Section II.C. The numbers reported in the table are averages at each age taken over 5,000 simulation trials, starting at age 20 and ending at age 100. The simulations utilize the base-case parameter values outlined in Table I. The table reports the average consumption-wealth ratio, the fraction of total wealth in the retirement account, the overall equity proportion, the equity proportion in the taxable account, the basis-price ratio of the equity in the taxable account, the frequency of reaching 100 percent equity in the taxable account, the frequency of positive equity holdings in the retirement account, and the proportion of retirement wealth allocated to equity conditional on positive equity holdings in the retirement account.

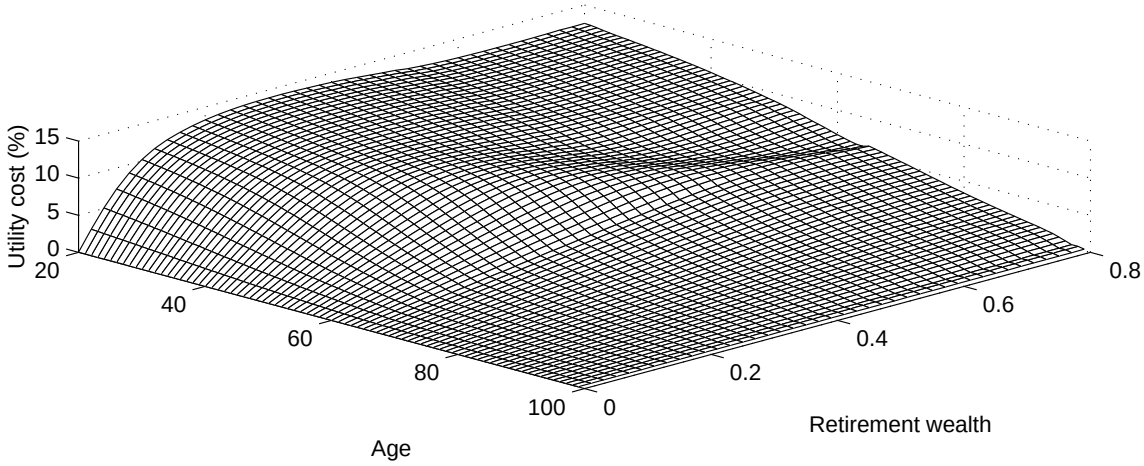
Age	Consumption-wealth ratio	Fraction of total wealth held in the retirement account	Overall equity proportion	Equity proportion in the taxable account	Basis-price ratio	Frequency of 100% equity in the taxable account	Frequency of positive equity in the retirement account	Conditional equity proportion in the retirement account
20	9.30%	1.91%	67.88%	69.35%	1.000	0.00%	0.00%	NA
25	9.19%	11.11%	71.72%	81.66%	0.666	0.00%	0.00%	NA
30	9.12%	19.70%	70.58%	90.08%	0.488	0.24%	0.24%	0.00%
35	8.92%	27.32%	67.13%	95.79%	0.367	3.62%	3.62%	5.19%
40	8.58%	33.93%	62.57%	98.36%	0.281	3.90%	3.90%	18.36%
45	8.10%	39.38%	58.56%	98.53%	0.218	1.78%	1.74%	29.53%
50	7.37%	43.14%	56.05%	98.12%	0.195	59.36%	55.04%	10.37%
55	6.35%	44.68%	55.37%	97.71%	0.233	64.70%	59.80%	11.90%
60	5.14%	43.50%	56.04%	96.70%	0.321	61.60%	60.64%	11.05%
65	3.86%	36.21%	53.76%	81.61%	0.379	10.90%	10.90%	20.02%
70	3.85%	28.96%	61.39%	87.27%	0.314	6.24%	6.24%	15.08%
75	3.90%	20.55%	66.27%	84.71%	0.244	1.98%	1.98%	9.19%
80	4.03%	12.19%	68.64%	78.91%	0.186	0.46%	0.18%	1.11%
85	4.24%	5.41%	70.83%	75.08%	0.141	0.00%	0.00%	NA
90	4.51%	1.47%	75.70%	76.87%	0.109	0.00%	0.00%	NA
95	4.88%	0.12%	86.32%	86.42%	0.084	0.00%	0.00%	NA
99	5.37%	0.00%	98.99%	98.99%	0.071	0.00%	0.00%	NA

Benefits of Tax-Deferred Investing

- Substantial welfare cost to mis-locating equity exposure in the wrong account or even equally allocating equity exposure to both accounts as a function of horizon and mix between taxable and tax-exempt wealth.

- Substantial shadow price value to tax-deferred as compared to taxable wealth as a function of horizon and mix between taxable and tax-exempt wealth.


Utility cost (%)
10
5
0
20
40
60
80
100
Age
0
0.2
0.4
0.6
0.8
Retirement wealth


Utility cost (%)
15
10
5
0
20
40
60
80
100
Age
0
0.2
0.4
0.6
0.8
Retirement wealth

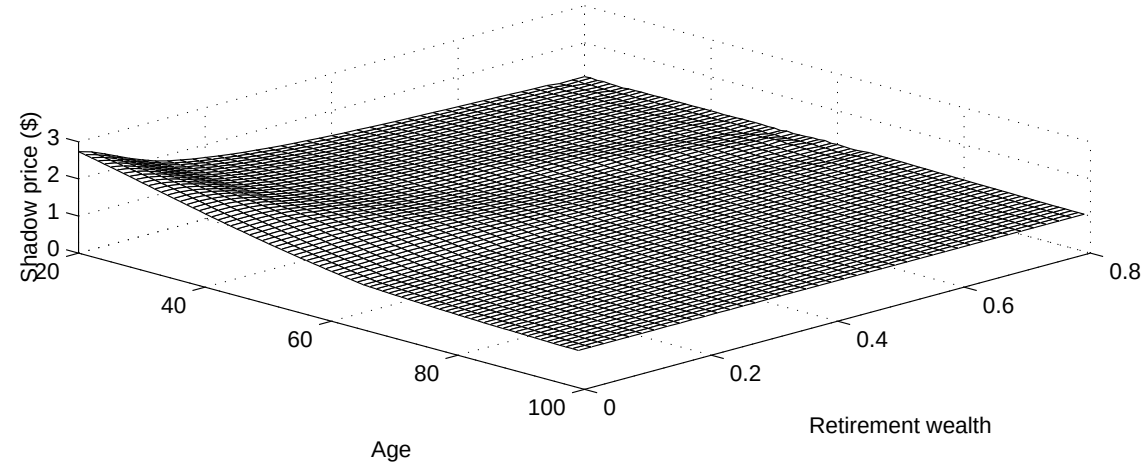

Shadow price ($)
3
2
1
0
20
40
60
80
100
Age
0
0.2
0.4
0.6
0.8
Retirement wealth

Figure 3:

Liquidity and Asset Location

- With frictionless borrowing the locational patterns are not altered since borrowing resolves the liquidity problem
- Modified Framework for Liquidity Specification without Borrowing
  - Exogenous Liquidity Shock (known 50% consumption gulp at age 30 or stochastic shock)
  - Forced Realization
  - Endogenous (partial funding) contributions and withdrawals allowed
  - 10% penalty for premature withdrawals
- For high levels of tax-deferred wealth and very large shocks the investor liquidates his entire taxable account and hedging in the taxable account would have been useful for several years before the known shock
- Some hedging also occurs by reducing or eliminating retirement contributions prior to the known shocks
- Assuming 10% probability per period of 50% shock, liquidity shocks do not influence asset location pattern, but do alter the retirement funding decision

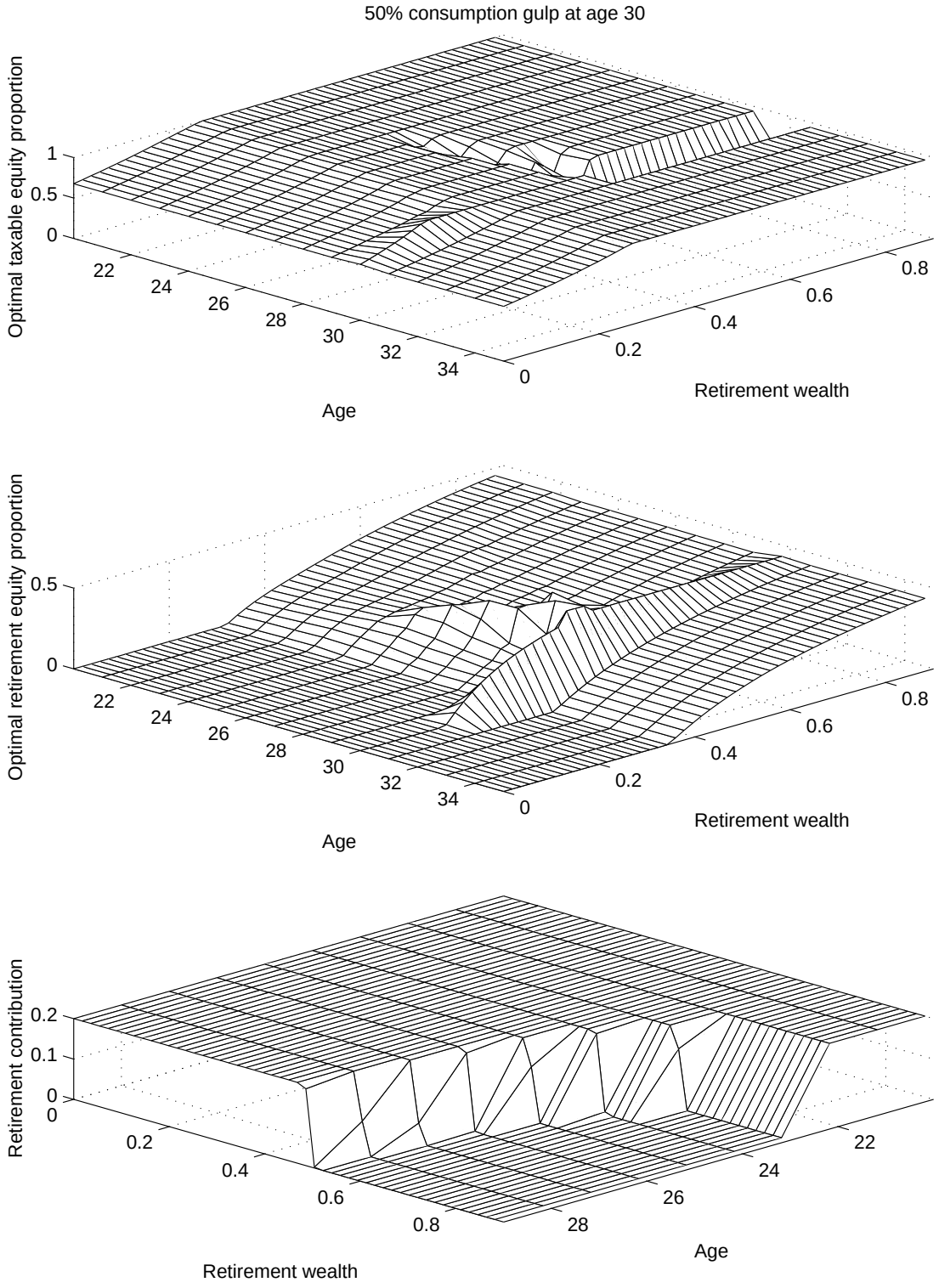

50% consumption gulp at age 30
Optimal taxable equity proportion
1
0.5
0
22
24
26
28
30
32
34
Age
0
0.2
0.4
0.6
0.8
Retirement wealth
Optimal retirement equity proportion
0.5
0
22
24
26
28
30
32
34
Age
0
0.2
0.4
0.6
0.8
Retirement wealth
Retirement contribution
0.2
0.1
0
0
0.2
0.4
0.6
0.8
Retirement wealth
22
24
26
28
Age

Figure 4:

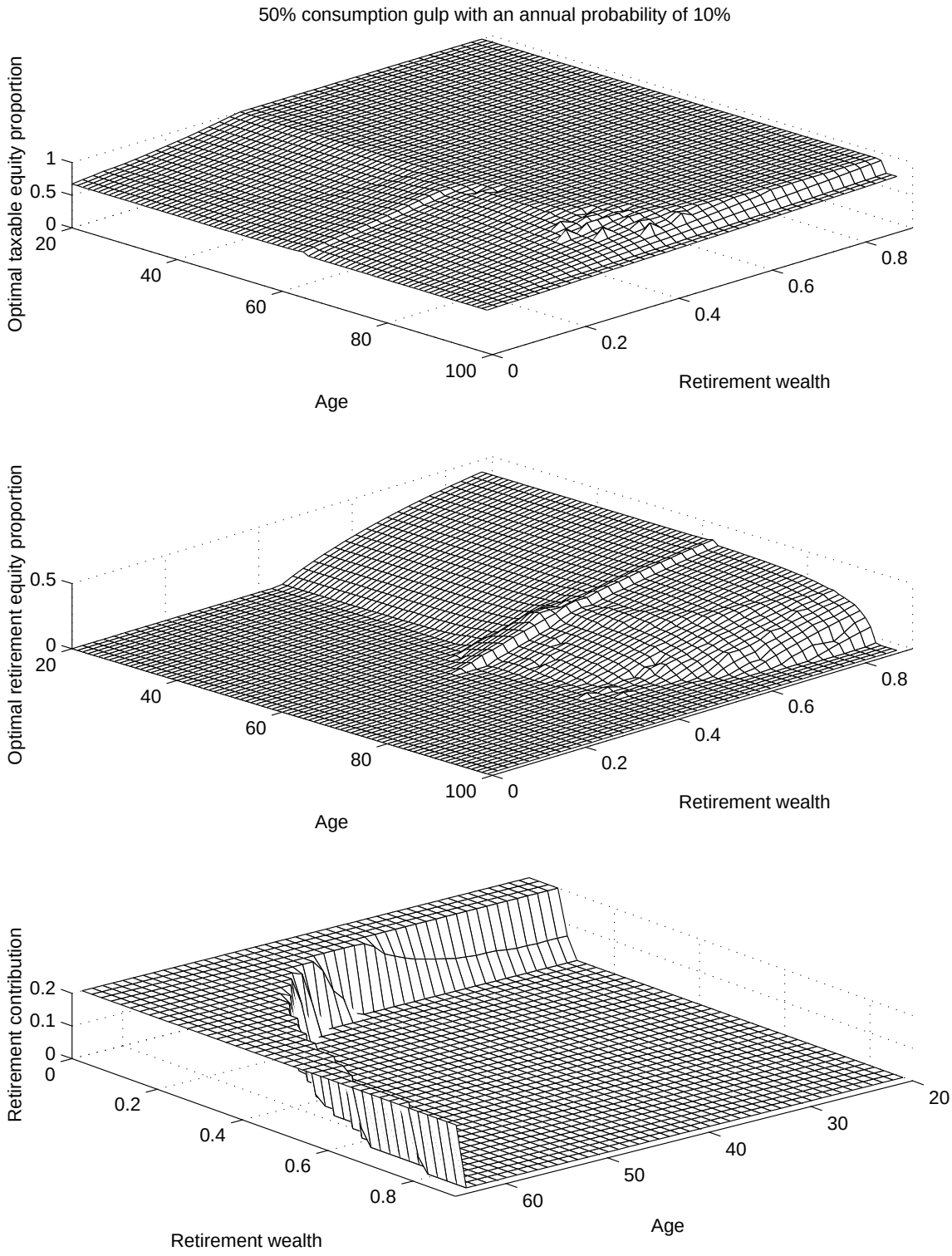

50% consumption gulp with an annual probability of 10%
Optimal taxable equity proportion
1
0.5
0
20
40
60
80
100
Age
0
0.2
0.4
0.6
0.8
Retirement wealth
Optimal retirement equity proportion
0.5
0
20
40
60
80
100
Age
0
0.2
0.4
0.6
0.8
Retirement wealth
Retirement contribution
0.2
0.1
0
0
0.2
0.4
0.6
0.8
Retirement wealth
20
30
40
50
60
Age

Figure 5:

Observed Investor Behavior

1995 Survey of Consumer Finances (Poterba and Samwick):

48.3% of holders of taxable bonds own tax-deferred equity

41.6% of holders of tax-deferred equity own taxable bonds

53.1% of holders of tax-exempt bonds own tax-deferred equity

11.3% of holders of tax-deferred equity own tax-exempt bonds

1995 TIAA-CREF Data (Ameriks and Zeldes):

	Overall equity ownership	Proportion of investors who own equity
Retirement Assets	52.4%	64.9%
Taxable Assets	36.9%	25.6%

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